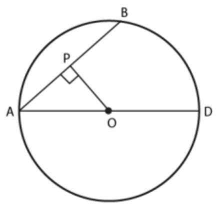
Chapter 10: Circles
Exercise 10.1 Page No: 64

1. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, the distance of AB from the centre of the circle is :

(A) 17 cm
(B) 15 cm
(C) 4 cm
(D) 8 cm

Solution:



(D) 8 cm

Given: Diameter of the circle = d = AD = 34 cm
∴ Radius of the circle = r = d/2 = AO = 17 cm
Length of chord AB = 30 cm
Since the line drawn through the center of a circle to bisect a chord is perpendicular to the chord, therefore AOP is a right angled triangle with L as the bisector of AB.
∴ AP = 1/2(AB) = 15 cm
In right angled triangle AOB, by Pythagoras theorem, we have:

$(AO)^2 = (OP)^2 + (AP)^2$
$\Rightarrow (17)^2 = (OP)^2 + (15)^2$
$\Rightarrow (OP)^2 = (17)^2 - (15)^2$
$\Rightarrow (OP)^2 = 289 - 225$
$\Rightarrow (OP)^2 = 64$
$\Rightarrow (OP) = 8$

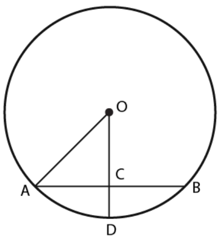
∴ The distance of AB from the center of the circle is 8 cm.

2. In Fig. 10.3, if OA = 5 cm, AB = 8 cm and OD is perpendicular to AB, then CD is equal to:

(A) 2 cm
(B) 3 cm
(C) 4 cm
(D) 5 cm

Solution:

(A) 2 cm



Given:
Radius of the circle = r = AO = 5 cm
Length of chord AB = 8 cm
Since the line drawn through the center of a circle to bisect a chord is perpendicular to the chord, therefore AOC is a right angled triangle with C as the bisector of AB.
∴ AC = ½ (AB) = 8/2 = 4 cm
In right angled triangle AOC, by Pythagoras theorem, we have:

$(AO)^2 = (OC)^2 + (AC)^2$
$\Rightarrow (5)^2 = (OC)^2 + (4)^2$
$\Rightarrow (OC)^2 = (5)^2 – (4)^2$
$\Rightarrow (OC)^2 = 25 – 16$
$\Rightarrow (OC)^2 = 9$
$\Rightarrow (OC) = 3$

∴ The distance of AC from the center of the circle is 3 cm.
Now, OD is the radius of the circle, ∴ OD = 5 cm
CD = OD – OC
CD = 5 – 3
CD = 2

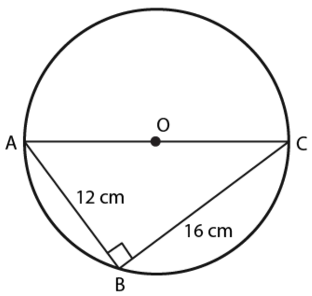
Therefore, CD = 2 cm

3. If AB = 12 cm, BC = 16 cm and AB is perpendicular to BC, then the radius of the circle passing through the points A, B and C is :

(A) 6 cm
(B) 8 cm
(C) 10 cm
(D) 12 cm

Solution:

(C) 10 cm



According to the question,
AB = 12 cm, BC = 16 cm, AB ⊥ BC.
Therefore,
AC is the diameter of the circle passing through the points A, B and C.
Now, according to the figure,
We get, ABC is a right angled triangle.

By Pythagoras theorem:

$(AC)^2 = (CB)^2 + (AB)^2$
$\Rightarrow (AC)^2 = (16)^2 + (12)^2$
$\Rightarrow (AC)^2 = 256 + 144$
$\Rightarrow (AC)^2 = 400$
Hence, (AC) = 20

Diameter of the circle = 20 cm

Thus, radius of the circle = Diameter/2 = 20/2 = 10 cm

Hence, Radius of the circle = 10 cm

4. In Fig.10.4, if ∠ABC = 20º, then ∠AOC is equal to:

(A) 20º
(B) 40º
(C) 60º
(D) 10º

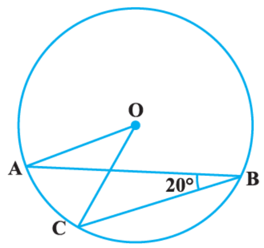


Fig. 10.4

Solution:

(B) 40º

According to the question, ∠ABC = 20°

We know that, “The angle subtended by an arc at the center of a circle is twice the angle subtended by it at remaining part of the circle”

According to the theorem, we have,
∠AOC = 2 × ∠ABC
= 2 × 20°
= 40°

Therefore, ∠AOC = 40°

5. In Fig.10.5, if AOB is a diameter of the circle and AC = BC, then ∠CAB is equal to:

(A) 30º
(B) 60º
(C) 90º
(D) 45º

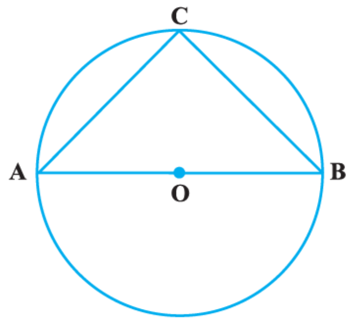


Fig. 10.5

Solution:

(D) 45º

According to the question, we have,

Diameter of the circle = AOB
AC = BC
Since, angles opposite to equal sides are equal
$\angle ABC = \angle BAC$
Let, $\angle ABC = \angle BAC = x$

Also, diameter subtends a right angle to the circle, $\angle ACB = 90°$

We also know that, by angle sum property of a triangle, sum of all angles of a triangle = 180°.

$\angle CAB + \angle ABC + \angle ACB = 180°$
$\Rightarrow x + x + 90° = 180°$
$\Rightarrow 2x = 90°$
$\Rightarrow x = 45°$

$\angle CAB = \angle ABC = 45°$

Exercise 10.2 Page No: 64

Write True or False and justify your answer in each of the following:

1. Two chords AB and CD of a circle are each at distances 4 cm from the centre. Then AB = CD.

Solution:

True
Given that AB and AC are chords that are at a distance of 4 cm from center of a circle.

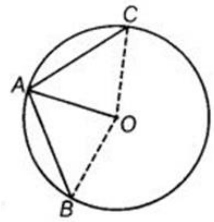
Since, chords that are equidistant from the center of a circle are equal in length,
We have, AB = CD.

2. Two chords AB and AC of a circle with centre O are on the opposite sides of OA. Then ∠OAB = ∠OAC .

Solution:

False

Let AB and AC be the chord of the circle with center O on the opposite side of OA.



Consider the triangles AOC and AOB:
AO = AO (Common side in both triangles)
OB = OC (Both OB and OC are radius of circle)

But we can't show that either the third side of both triangles are equal or any angle is equal. Therefore ΔAOB is not congruent to ΔAOC.
∴ ∠OAB ≠ ∠OAC.

3. Two congruent circles with centres O and O' intersect at two points A and B. Then ∠AOB = ∠AO'B.

Solution:

True
Equal chords of congruent circles subtend equal angles at the respective centre.
Hence, the given statement is true.

4. Through three collinear points a circle can be drawn.

Solution:

False

A circle through two points cannot pass through a point which is collinear to these two points.

5. A circle of radius 3 cm can be drawn through two points A, B such that AB = 6 cm.

Solution:

True

According to the question,
Radius of circle = 3 cm
Diameter of circle = 2 × r
= 2 × 3 cm
= 6 cm

Now, from the question we have,

AB = 6 cm
So, the given statement is true because AB will be the diameter

Exercise 10.3 Page No: 64

1. If arcs AXB and CYD of a circle are congruent, find the ratio of AB and CD.

Solution:

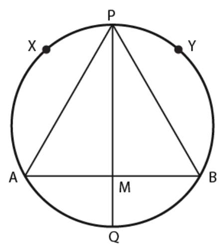
According to the question,

We have, AXB ≅ CYD.

We know that, if two arcs of a circle are congruent, then their corresponding arcs are also equal.
So, we have chord AB = chord CD.
Hence, we get,
AB/CD = 1
AB/CD = 1/1
AB : CD = 1:1

2. If the perpendicular bisector of a chord AB of a circle PXAQBY intersects the circle at P and Q, prove that arc PXA ∠ Arc PYB.

Solution:



According to the question, we have,
PQ is the perpendicular bisect of AB,
So, we get, AM = BM ……………………………………..eq.(1)

In △APM and △BPM, From eq.(1),

AM = BM
∠AMP = ∠BMP = 90^{o}
PM = PM [Common side]

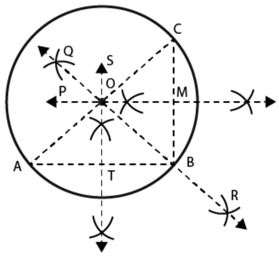
Therefore, △APM ≅ △BPM [By SAS congruence rule]

AP = BP [CPCT]
Hence, arc PXA ≅ Arc PYB
Therefore, if two chords of a circle are equal, then their corresponding arcs are congruent.

3. A, B and C are three points on a circle. Prove that the perpendicular bisectors of AB, BC and CA are concurrent.

Solution:

According to the question,
Three non-collinear points A, B and C are on a circle.



To prove: Perpendicular bisectors of AB, BC and CA are concurrent.
Construction: Join AB, BC and CA.

Draw: ST, perpendicular bisector of AB,
PM, perpendicular bisector of BC
And, QR perpendicular bisector of CA
As point A, B and C are not collinear, ST, PM and QR are not parallel and will intersect.

Proof: O lies on ST, the ⊥ bisector of AB
OA = OB ……………………………………………………………………………………(1)

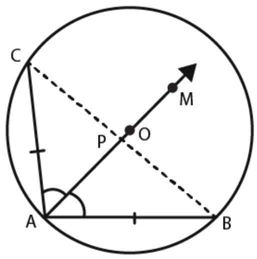
Similarly, O lies on PM, the ⊥ bisector of BC
OB = OC …..…………………………………………………………………………………(2)

And, O lies on QR, the ⊥ bisector of CA
OC = OA ……………………………………………………………………………….(3)

From (1), (2) and (3),
OA = OB = OC
Let OA = OB = OC = r
Draw circle, with centre O and radius r, passing through A, B and C.
Hence, O is the only point equidistance from A, B and C.
Therefore, the perpendicular bisectors of AB, BC and CA are concurrent.

4. AB and AC are two equal chords of a circle. Prove that the bisector of the angle BAC passes through the centre of the circle.

Solution:



According to the question,
We have, AB and AC are two chords which are equal with centre O.
AM is the bisector of ∠BAC.
To prove: AM passes through O.
Construction: Join BC. Let AM intersect BC at P.

Proof: In DBAP and DCAP
AB = AC [Given]
∠BAP = ∠CAP [Given]
And AP = BP [Common side]
△BAP ≅ △CAP [By SAS]

Hence, ∠BPA = ∠CPA [CPCT]

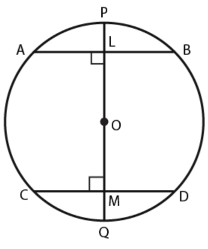
We know that, CP = PB

But, since ∠BPA and ∠CPA are linear pair angles,
We have,
$\angle BPA + \angle CPA = 180^{o}$
$\angle BPA = \angle CPA = 90^{o}$
Then, AP is perpendicular bisector of the chord BC, which will pass through the centre O on being produced.
Therefore, AM passes through O.

5. If a line segment joining mid-points of two chords of a circle passes through the centre of the circle, prove that the two chords are parallel.

Solution:

Consider AB and CD to be the chords of the circle with center O.



Let L be the midpoint of AB.
Let M be the midpoint of CD.
Let PQ be the line passing through these midpoints and the center of the circle.
Then, PQ is the diameter of the circle.

We know that, line joining center to the midpoint of a chord is always perpendicular to the chord.
Since M is the midpoint of CD, we have,
OM ⊥ CD
⇒ OMD = 90°
Similarly, L is the midpoint of AB,
OL ⊥ AB
⇒ OLA = 90°

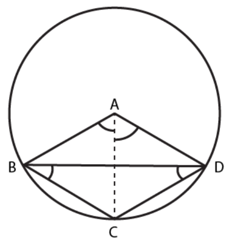
But, we know,
∠OLA and ∠OMD are alternate angles.
So, AB ∥ CD.
Hence, proved.

6. ABCD is such a quadrilateral that A is the centre of the circle passing through B, C and D. Prove that ∠CBD + ∠CDB = ½ ∠BAD

Solution:

According to the question, we have, a quadrilateral ABCD such that A is the centre of the circle passing through B, C and D.

Construction: Join CA and BD.



We know that, in a circle, angle subtended by an arc at the center is twice the angle subtended by it at any other point in the remaining part of the circle

So, the arc DC subtends ∠DAC at the center and ∠CAB at point B in the remaining part of the circle,

We get, ∠DAC = 2∠CBD ………………………………………………………(1)

Similarly,
the arc BC subtends ∠CAB at the center and ∠CDB at point D in the remaining part of the circle,

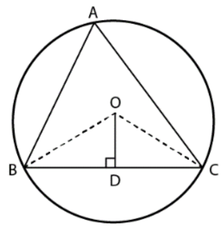
We get,
∠CAB = 2∠CDB ……………………………………………………………………(2)

From equations (1) and (2),
We have:
DAC + ∠CAB = 2∠CDB + 2∠CBD
⇒ ∠BAD = 2(∠CDB + ∠CBD)
⇒ (∠CDB + ∠CBD) = ½ (∠BAD)

7. O is the circumcentre of the triangle ABC and D is the mid-point of the base BC. Prove that ∠BOD = ∠A.

Solution:

According to the question, we have, O is the circumcenter of the triangle ABC and D is the midpoint of BC.



To prove: ∠BOD = ∠A
Construction: Join OB and OC.
In ΔOBD and ΔCD:
OD = OD (common)
DB = DC (D is the midpoint of BC)
OB = OC (radius of the circle)
By SSS congruence rule, ΔOBD ≅ ΔOCD.

∠BOD = ∠COD (By CPCT)

Let ∠BOD = ∠COD = x

We know that, angle subtended by an arc at the center of the circle is twice the angle subtended by it at any other point in the remaining part of the circle.

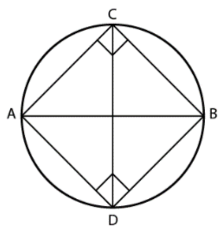
So, we have,
2∠BAC = ∠BOC
⇒ 2∠BAC = ∠BOD + ∠DOC
⇒ 2∠BAC = x + x
⇒ 2∠BAC = 2x
⇒ ∠BAC = x
⇒ ∠BAC = ∠BOD

Hence, proved.

8. On a common hypotenuse AB, two right triangles ACB and ADB are situated on opposite sides. Prove that ∠BAC = ∠BDC.

Solution:

According to the question, we have, ACB and ADB are two right triangles.



To Prove: ∠BAC = ∠BDC
We know that, ACB and ADB are right angled triangles,

Then, ∠C + ∠D = 90° + 90°
∠C + ∠D = 180°
Therefore ADBC is a cyclic quadrilateral as sum of opposite angles of a cyclic quadrilateral = 180°

We also have, ∠BAC and ∠BDC lie in the same segment BC and angles in the same segment of a circle are equal.
∴ ∠BAC = ∠BDC.

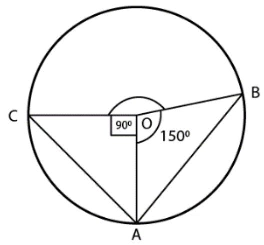
Hence Proved.

9. Two chords AB and AC of a circle subtends angles equal to 90º and 150º, respectively at the centre. Find ∠BAC, if AB and AC lie on the opposite sides of the centre.

Solution:

According to the question, we have,

In ΔAOB,
OA = OB (radius of the circle)



Since angle opposite to equal sides are equal, we get, ∠OBA = ∠OAB

We know that, according to angle sum property, sum of all angles of a triangle = 180º

Using the angle sum property in ΔAOB, we get,
∠OAB + ∠AOB +∠OBA = 180°
⇒ ∠OAB +90° + ∠OAB = 180°
⇒ 2∠OAB = 180° – 90°
⇒ 2∠OAB = 90°
⇒ ∠OAB = 45°

Now, in ΔAOC, OA = OC (radius of the circle)
Since, angle opposite to equal sides are equal
∴ ∠OCA = ∠OAC

Using the angle sum property in ΔAOB, sum of all angles of the triangle is 180°, we have:
∠OAC + ∠AOC +∠OCA = 180°
⇒ ∠OAC +150° + ∠OAC = 180°
⇒ 2∠OAC = 180° – 150°

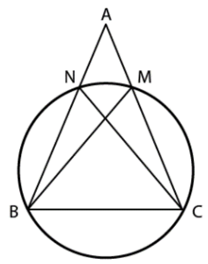
⇒ 2∠OAC = 30°
⇒ ∠OAC = 15°

Now, ∠BAC = ∠OAB + ∠OAC
= 45° + 15°
= 60°

∴ ∠BAC = 60°

10. If BM and CN are the perpendiculars drawn on the sides AC and AB of the triangle ABC, prove that the points B, C, M and N are concyclic.

Solution:



According to the question,
BM and CN are the perpendiculars drawn on the sides AC and AB of the triangle ABC.
So, we have, ∠BMC = ∠BNC = 90º

We know that, if a line segment joining two points subtends equal angles on the same side of the line containing the segment, then the four points are concyclic.

Considering the question, since BC joins the two points, B and C, subtending equal angles, ∠BMC and ∠BNC, at M and N on the same side BC containing the segment, then B, C, M and N are concyclic.

Hence, we get that, B, C, M and N are concyclic.

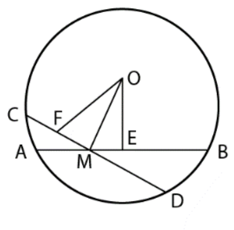
<u>Exercise 10.4 Page No: 64</u>

1. If two equal chords of a circle intersect, prove that the parts of one chord are separately equal to the parts of the other chord.

Solution:

According to the question, AB and CD are two equal chords of a circle with centre O, intersect each other at M.

To prove:
(i) MB = MC and
(ii) AM = MD



Proof: AB is a chord and OE ⊥ to AB from the centre O,

Since, perpendicular from the centre to a chord bisect the chord we get,

AE = ½ AB
Similarly, FD = ½ CD

It is given that, AB = CD
⇒ ½ AB = ½ CD

So, AE = FD ……………………………………………………………………(1)

Since equal chords are equidistance from the centre,
And AB = CD
So, OE = OF

Now, as proved, in right triangles MOE and MOF,
hyp. OE = hyp. OF [Common side]
OM = OM
ΔMOE ≅ ΔMOF
ME = MF ………………………………………………………………………(2)

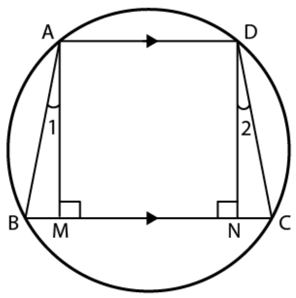
Subtracting equations (2) from (1), we get
AE – ME = FD – MF
⇒ AM = MD [Proved part (ii)]

Again, AB = CD [Given]
And AM = MD [Proved]
AB – AM = CD – MD [Equals subtracted from equal]

Hence, MB = MC [Proved part (i)]

2. If non-parallel sides of a trapezium are equal, prove that it is cyclic.

Solution:



According to the question, we have,

ABCD is a trapezium in which AD||BC non-parallel sides AB and DC of the trapezium ABCD are equal i.e., AB = DC.

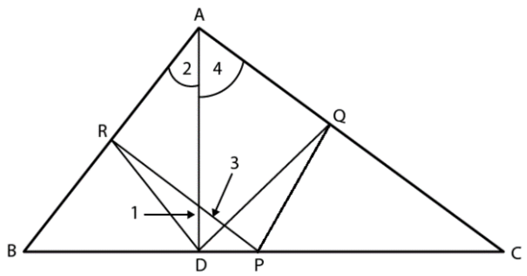
To prove: Trapezium ABCD is cyclic.
Construction: Draw AM and DN such that they are perpendicular on BC.
Proof: In right triangles AMB and DNC,
$\angle AMB = \angle DNC = 90^o$
AB = DC [Given]
Since perpendicular distance between two parallel lines are same,
AM = DN
$\Delta AMB \cong \Delta DNC$ [By RHS congruence rule]
$\angle B = \angle C$ [CPCT]
And $\angle 1 = \angle 2$
$\angle BAD = \angle 1+ 90$
$= \angle 2 + 90$
$= \angle CDA$

Now, in quadrilateral ABCD
$\angle B + \angle C + \angle CDA + \angle BAD = 360$
Or, $\angle B + \angle B + \angle CDA + \angle CDA = 360$
or, $2(\angle B + \angle CDA) = 360$
or, $\angle B + \angle CDA = 180$

We know that, if any pair of opposite angles of a quadrilateral is 180º, then the quadrilateral is cyclic. Hence, the trapezium ABCD is cyclic.

3. If P, Q and R are the mid-points of the sides BC, CA and AB of a triangle and AD is the perpendicular from A on BC, prove that P, Q, R and D are concyclic.

Solution: To prove: R, D, P and Q are concyclic.



Construction: Join RD, QD, PR and PQ. RP joins the mid-point of AB, i.e., R, and the mid-point of BC, i.e., P.

Using midpoint theorem, RP||AC
Similarly, PQ||AB.

So, we get, ARPQ is a parallelogram.
So, $\angle RAQ = \angle RPQ$ [Opposite angles of a ||gm]……………………….(1)

ABD is a right angled triangle and DR is a median,
.RA = DR and $\angle 1 = \angle 2$ ……………………………………………….….(2)

Similarly $\angle 3 = \angle 4$ …………………………………………………………..….(3)
Adding equations (2) and (3),

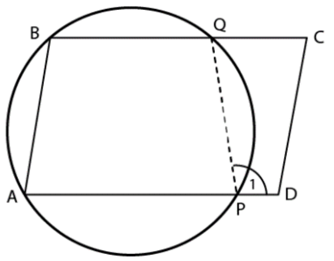
We get, ∠1+ ∠3 = ∠2 +∠4
⇒ ∠RDQ = ∠RAQ

Since ∠D and ∠P are subtended by RQ on the same side of it, we get the points R, D, P and Q concyclic. Hence, R, D, P and Q are concyclic.

4. ABCD is a parallelogram. A circle through A, B is so drawn that it intersects AD at P and BC at Q. Prove that P, Q, C and D are concyclic.

Solution:

According to the question, ABCD is a parallelogram.
A circle through A, B is so drawn that it intersects AD at P and BC at Q.
To prove: P, Q, C and D are concyclic.
Construction: Join PQ.



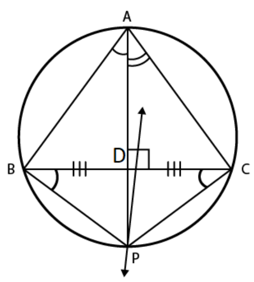
Extend side AP of the cyclic quadrilateral APQB to D.
External angle, ∠1= interior opposite angle, ∠B
Since, BA||CD and BC cuts them
∠B +∠C =180º

Since, Sum of interior angles on the same side of the transversal = 180º
Or ∠1+∠C =180º
So, PDCQ is cyclic quadrilateral.
Hence, the points P, Q, C and D are concyclic.

5. Prove that angle bisector of any angle of a triangle and perpendicular bisector of the opposite side if intersect, they will intersect on the circumcircle of the triangle.

Solution:

According to the question, triangle ABC and *l* is perpendicular bisector of BC.



To prove: Angles bisector of ∠A and perpendicular bisector of BC intersect on the circumcircle of ΔABC.

Proof: Let the angle bisector of ∠A intersect circumcircle of ΔABC at D.

Construction: Join BP and CP.
Since, angles in the same segment are equal
We have, $\angle BAP = \angle BCP$
We know that, AP is bisector of $\angle A$.

Then, $\angle BAP = \angle BCP = \frac{1}{2} \angle A$ ……………………………………………………….(1)

Similarly, $\angle PAC = \angle PBC = \frac{1}{2} \angle A$ ………………………………………………..(2)

From equations (1) and (2), We have

$\angle BCP = \angle PBC$

We know that, if the angles subtended by two Chords of a circle at the centre are equal, the chords are equal.

So, BP = CP
Here, P is on perpendicular bisector of BC.
Hence, angle bisector of $\angle A$ and perpendicular bisector of BC intersect on the circumcircle of ΔABC.