**Chapter 3: Linear Equations in two variable**
**Exercise: 3.1**

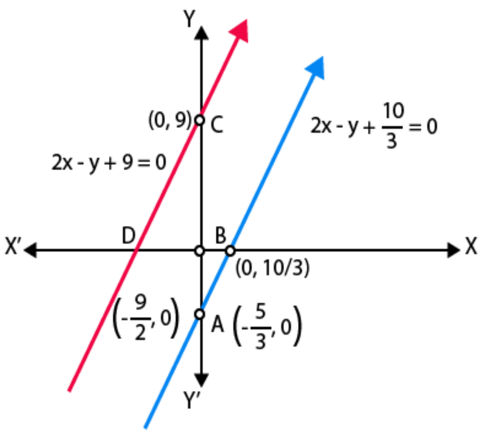
**Question 1: Graphically, the pair of equations**
**$6x - 3y + 10 = 0$**
**$2x - y + 9 = 0$**
**represents two lines which are**
**(a) intersecting at exactly one point**
**(b) intersecting exactly two points**
**(c) coincident**
**(d) parallel**

**Solution:** The given equations are $6x - 3y + 10 = 0$
or, $2x - y + \frac{10}{3} = 0$……….[dividing by 3]………………………………………..(1)
and, $2x - y + 9 = 0$ ………………………………………………………….(2)
Now, table for $2x - y + \frac{10}{3} = 0$,

| x | 0 | $-\frac{5}{3}$ |
|---|---|---|
| $y = 2x + \frac{10}{3}$ | $\frac{10}{3}$ | 0 |
| Points | A | B |

And table for $2x - y + 9 = 0$

| x | 0 | $-\frac{9}{2}$ |
|---|---|---|
| $y = 2x + 9$ | 9 | 0 |
| Points | C | D |




Hence, the pair of equations represents two parallel lines.

**Question 2: The pair of equations x + 2y + 5 = 0 and – 3x – 6y +1 = 0 has**
**(a) a unique solution** **(b) exactly two solutions**
**(c) infinitely many solutions** **(d) no solution**

**Solution :** (d)
Given, equations are x + 2y + 5 = 0 and – 3x – 6y + 1 = 0
Here, $a_1 = 1$, $b_1 = 2$, $c_1 = 5$ and
$a_2 = -3$, $b_2 = -6$, $c_2 = 1$
Therefore, $\frac{a_1}{a_2} = -\frac{1}{3}, \frac{b_1}{b_2} = -\frac{2}{6} = -\frac{1}{3}, \frac{c_1}{c_2} = \frac{5}{1}$

Hence, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Therefore, the pair of equations has no solution

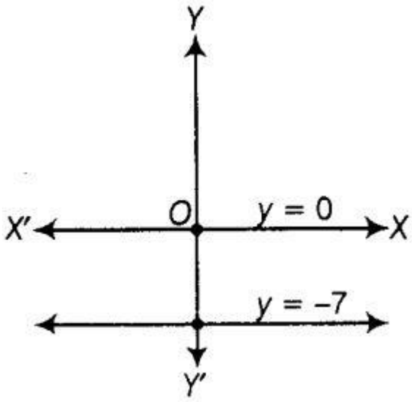
**Question 3: If a pair of linear equations is consistent, then the lines will be**
**(a) parallel** **(b) always coincident**
**(c) intersecting or coincident** **(d) always intersecting**

**Solution:** (c)
Condition for a consistent pair of linear equations,
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$…………[intersecting lines having unique solution]

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ……….[coincident or dependent]

**Question 4: The pair of equations y = 0 and y = – 7 has**
**(a) one solution** **(b) two solutions**
**(c) infinitely many solutions** **(d) no solution**

**Solution: (d)** The given pair of equations are y = 0 and y = – 7.

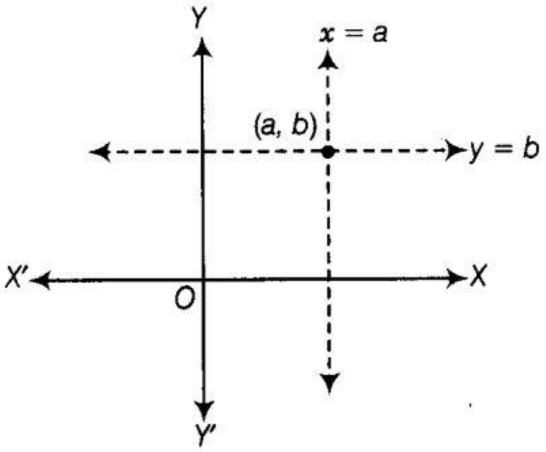



By graphically, both lines are parallel and have no solution

**Question 5: The pair of equations x = a and y = b graphically represents lines that are**

**(a) parallel** **(b) intersecting at (b, a)**
**(c) coincident** **(d) intersecting at (a, b)**

**Solution: (d)** By graphically in every condition, if a, b>>0; a, b< 0, a>0, b< 0; a<0, b>0 but a = b≠ 0.
The pair of equations x = a and y = b graphically represents lines that are intersecting at (a, b).
If a, b > 0




Similarly, in all cases, two lines intersect at (a, b).

**Question 6: For what value of k, do the equations 3x – y + 8 = 0 and 6x – ky = – 16**

**(a)** $\frac{1}{2}$ **(b)** $-\frac{1}{2}$ **(c) 2** **(d) -2**

**Solution: (c)** Condition for coincident lines is, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
Given lines, 3x – y + 8 = 0 and, 6x – ky + 16 = 0
Here, $a_1 = 3$, $b_1 = -1$, $c_1 = 8$
and, $a_2 = 6$, $b_2 = -k$, $c_2 = 16$

From eq(1), $\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$
or, $\frac{1}{k} = \frac{1}{2}$
or, k = 2

**Question 7: If the lines given by 3x+ 2ky = 2 and 2x + 5y = 1 are parallel, then the value of** k **is**

**(a)** $-\frac{5}{4}$ **(b)** $\frac{2}{5}$
**(c)** $\frac{15}{4}$ **(d)** $\frac{3}{2}$

**Solution :** (C) Condition for parallel lines is, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Given lines, $3x + 2ky – 2 = 0$ and, $2x + 5y – 1 = 0$
here, $a_1 = 3$, $b_1 = 2k$, $c_1 = -2$
and, $a_2 = 2$, $b_2 = 5$, $c_2 = -1$
Hence, $\frac{3}{2} = \frac{2k}{5}$
or, $k = \frac{15}{4}$

**Question 8: The value of c for which the pair of equations cx- y = 2 and 6x – 2y = 3**
**will have infinitely many solutions is**
**(a) 3 (b) – 3 (c)-12 (d) no value**

Solution: (d) Condition for infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$……………(1)
The given lines are $cx – y = 2$ and $6x – 2y = 3$
Here, $a_1 = c$, $b_1 = -1$, $c_1 = -2$
and, $a_2 = 6$, $b_2 = -2$, $c_2 = -3$

From eq(1), $\frac{c}{6} = \frac{-1}{-2} = \frac{-2}{-3}$
Here, $\frac{c}{6} = \frac{1}{2}$ and, $\frac{c}{6} = \frac{2}{3}$
or, $c = 3$ and, $c = 4$

Since c has different values.
Hence, for no value of c, the pair of equations will have infinitely many solutions.

**Question 9: One equation of a pair of dependent linear equations is – 5x+ 7y – 2 = 0. The second equation can be**
**(a) 10x + 14y + 4=0 (b)-10x-14y + 4 =0**
**(c) -10x + 14y + 4 = 0 (d) 10x-14y + 4=0**

Solution: (d) Condition for dependent linear equations, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{k}$……………(1)
Given, equation of line is, $-5x + 7y – 2 = 0$
Here, $a_1 = -5$, $b_1 = 7$, $c_1 = -2$
From eq(1), $-\frac{5}{a_2} = \frac{7}{b_2} = -\frac{2}{c_2} = \frac{1}{k}$ ……………[say]
or, $a_2 = -5k$, $b_2 = 7k$, $c_2 = -2k$, where k is any arbitrary constant.

Putting $k = 2$, then $a_2 = - 10$, $b_2 = 14$, $c_2 = -4$
The required eq. of the line becomes,
$a_2x + b_2y + c_2 = 0$
$-10x + 14y – 4 = 0$
$10x – 14y + 4 = 0$

**Question 10: A pair of linear equations that has a unique solution x – 2 and y = – 3 is**
**(a) x + y = 1 and 2x – 3y = – 5**

**(b) $2x+ 5y= -11$ and $4x + 10y = -22$**
**(c) $2x – y = 1$ and $3x + 2y = 0$**
**(d) $x – 4y -14 = 0$ and $5x – y -13 = 0$**

**Solution: (b)** If $x = 2$, $y = – 3$ is a unique solution of any pair of the equation, then these values must satisfy that pair of equations.
From option (b), LHS $= 2x + 5y = 2(2) + 5(- 3) = 4 – 15 = – 11 =$ RHS
and, LHS $= 4x + 10y = 4(2) + 10(- 3)= 8 – 30 = – 22 =$ RHS

**Question 11: If $x = a$ and $y = b$ is the solution of the equations $x- y = 2$ and $x + y = 4$, then the values of a and b are, respectively**
**(a) 3 and 5 (b) 5 and 3 (c) 3 and 1 (d) – 1 and – 3**

**Solution: (c)** Since, $x = a$ and $y = b$ is the solution of the equations $x – y = 2$ and $x + y = 4$, then these values will satisfy that equations
$a – b = 2$ ……………………………………………………………..(1)
and $a + b = 4$ ……………………………………………………………..(2)
On adding Eqs. (1) and (2), we get
$2a = 6$
$a = 3$ and $b = 1$

**Question 12: Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively**
**(a) 35 and 15 (b) 35 and 20 (c) 15 and 35 (d) 25 and 25**

**Solution: (d)** Let number of ₹ 1 coins = x and number of ₹ 2 coins = y
Now, by given conditions, $x + y = 50$…………………………………..(1)
Also, $x \times 1 + y \times 2 = 75$
or, $x + 2y = 75$……………………………….…(2)
On subtracting Eq. (1) from Eq. (2), we get
$(x + 2y) – (x + y) = 75 – 50$
or, $y = 25$
When $y = 25$, then $x = 25$

**Question 13: The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages (in a year) of the son and the father are, respectively**
**(a) 4 and 24 (b) 5 and 30**
**(c) 6 and 36 (d) 3 and 24**

Solution: (c) Let x be the present age of the father and y be the present age of the son.
Four years hence, it has relation by given condition,
$(x + 4) = 4(y + 4)$
or, $x – 4y = 12$ ……………………………………………………(1)
and $x = 6y$……………………………………………………………(2)

On putting the value of x from Eq. (2) in Eq. (1), we get
$6y - 4y = 12$
or, $2y = 12$
or, $y = 6$
When $y = 6$, then $x = 36$
Hence, the present age of the father is 36 year and the age of the son is 6 year.

## Exercise 3.2

**Question 1: Do the following pair of linear equations have no solution? Justify your answer.**
**(i) $2x + 4y = 3$ and $12y + 6x = 6$**
**(ii) $x = 2y$ and $y = 2x$**
**(iii) $3x + y - 3 = 0$ and $2x + - y = 2$**

Solution: Condition for no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
(i) Yes, given pair of equations,
$2x + 4y = 3$, and $12y + 6x = 6$
Here, $a_1 = 2$, $b_1 = 4$, $c_1 = -3$ and, $a_2 = 6$, $b_2 = 12$, $c_2 = -6$

Therefore, $\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{4}{12} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-3}{-6} = \frac{1}{2}$
Hence, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Thus, the given pair of linear equations has no solution.

(ii) No, given pair of equations, $x = 2y$ and $y = 2x$ or,
$x - 2y = 0$ and $2x - y = 0$
Here, $a_1 = 1$, $b_1 = -2$, $c_1 = 0$ and, $a_2 = 2$, $b_2 = -1$, $c_2 = 0$

Therefore, $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{1}$
Hence, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
Thus, the given pair of linear equations has a unique solution.

(iii) No, given pair of equations,
$3x + y - 3 = 0$ and $2x + \frac{2}{3}y - 2 = 0$

Here, $a_1 = 3$, $b_1 = 1$, $c_1 = -3$ and, $a_2 = 2$, $b_2 = \frac{2}{3}$, $c_2 = -2$

$$\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{1}{\frac{2}{3}} = \frac{3}{2}, \frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$$

Or, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{3}{2}$

Hence, the given pair of linear equations is coincident and having infinitely many solutions.

**Question 2: Do the following equations represent a pair of coincident lines? Justify your answer.**
**(i) $3x + \frac{1}{7}y = 3$ and $7x + 3y = 7$**
**(ii) $-2x - 3y = 1$ and $6y + 4x = -2$**
**(iii) $\frac{x}{2} + y + \frac{2}{5} = 0$ and $4x + 8y + \frac{5}{16} = 0$**

Solution: Condition for coincident lines, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
(i) No, given pair of linear equations,
$3x + \frac{y}{7} - 3 = 0$ and $7x + 3y - 7 = 0$, where,
$a_1 = 3, b_1 = \frac{1}{7}, c_1 = -3$
$a_2 = 7, b_2 = 3, c_2 = -7$

Now, $\frac{a_1}{a_2} = \frac{3}{7}, \frac{b_1}{b_2} = \frac{1}{21} = \frac{3}{2}, \frac{c_1}{c_2} = \frac{3}{7}$
Hence, the given pair of linear equations has a unique solution.

(ii) Yes, given pair of linear equations,
$-2x - 3y - 1 = 0$ and $6y + 4x + 2 = 0$,
where, $a_1 = -2, b_1 = -3, c_1 = -1$ and $a_2 = 4, b_2 = 6, c_2 = 2$

Now, $\frac{a_1}{a_2} = -\frac{2}{4} = -\frac{1}{2}, \frac{b_1}{b_2} = -\frac{3}{6} = -\frac{1}{2}, \frac{c_1}{c_2} = -\frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = -\frac{1}{2}$
Hence, the given pair of linear equations is coincident.

(iii) No, here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Hence, the given pair of linear equations has no solution.

**Question 3: Are the following pair of linear equations consistent? Justify your answer,**
**(i) $-3x - 4y = 12$ and $4y + 3x = 12$**
**(ii) $\frac{3}{5}x - y = \frac{1}{2}$ and $\frac{1}{5}x - 3y = \frac{1}{6}$**

**(iii) 2ax + by = a and 4ax + 2by – 2a = 0; a, b $\neq$ 0**
**(iv) x + 3y = 11 and 2(2x + 6y) = 22**

Solution: Conditions for pair of linear equations are consistent,
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ ……….[unique solution]
and, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ….[infinitely many solutions]

(i) No, the given pair of linear equations – 3x – 4y = 12 and 3x + 4y = 12
Here, $a_1 = -3$, $b_1 = -4$, $c_1 = -12$ and $a_2 = 3$, $b_2 = 4$, $c_2 = -12$

Now, $\frac{a_1}{a_2} = -\frac{3}{3} = -1, \frac{b_1}{b_2} = -\frac{4}{4} = -1, \frac{c_1}{c_2} = \frac{-12}{-12} = 1$
Hence, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, the pair of linear equations has no solutions i.e., inconsistent

(ii) Yes, the given pair of linear equations, $\frac{3}{5}x - y = \frac{1}{2}$ and $\frac{1}{5}x - 3y = \frac{1}{6}$
Here, $a_1 = \frac{3}{5}$, $b_1 = -1$, $c_1 = -\frac{1}{2}$ and $a_2 = \frac{1}{5}$, $b_2 = -3$, $c_2 = -\frac{1}{6}$

Now, $\frac{a_1}{a_2} = \frac{3}{1} = 3,\ \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3},\ \frac{c_1}{c_2} = \frac{3}{1}$ ….$\left[\frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right]$
Hence, the given pair of linear equations has a unique solution, i.e., consistent

(iii) Yes, the given pair of linear equations has infinitely many solutions i.e., consistent or dependent.
as, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

(iv) No, the pair of linear equation have no solution i.e., inconsistent, as $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

**Question 4: For the pair of equations λx + 3y + 7 = 0 and 2x + 6y -14 = 0. To have infinitely many solutions, the value of λ should be 1. Is the statement true? Give reasons.**

Solution: No, the given pair of linear equations
λx + 3y+7 = 0 and 2x + 6y – 14 = 0
Here, $a_1 = \lambda$, $b_1 = 3$ $c_1 = 7$; $a_2 = 2$, $b_2 = 6$, $c_2 = -14$

If , $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then system has infinitely many solutions.

or, $\frac{\lambda}{2} = \frac{3}{6} = -\frac{7}{14}$

or, $\frac{\lambda}{2} = \frac{3}{6}$

or, λ = 1
and, $\frac{\lambda}{2} = -\frac{7}{14}$, or, λ = -1

Hence, λ = -1 does not have a unique value.
So, for no value of λ, the given pair of linear equations has infinitely many solutions.

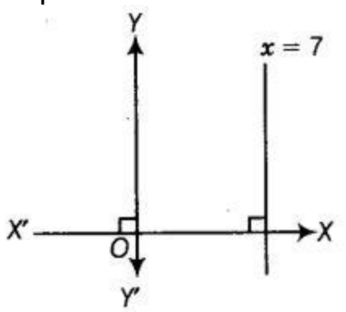
**Question 5: For all real values of c, the pair of equations x – 2y = 8 and 5x – lOy = c have a unique solution. Justify whether it is true or false.**

**Solution:** False, the given pair of linear equations
x - 2y – 8 = 0
5x - 10y = c
Here, $a_1 = 1$, $b_1 = -2$, $c_1 = -8$ and, $a_2 = 5$, $b_2 = -10$, $c_2 = -c$
Now, $\frac{a_1}{a_2} = \frac{1}{5}$, $\frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5}$, $\frac{c_1}{c_2} = \frac{-8}{c}$

But if c = 40 (real value), then the ratio $\frac{c_1}{c_2}$ becomes $\frac{1}{5}$ and then the system of linear equations have infinitely many solutions.
Hence, ate = 40, the system of linear equations does not have a unique solution.

**Question 6: The line represented by x = 7 is parallel to the X-axis, justify whether the statement is true or not.**

**Solution:**
Not true, by graphically, we observe that x = 7 line is parallel to y-axis and perpendicular to X-axis.



## Exercise 3.3

**Question 1: For which value(s) of λ, do the pair of linear equations λx + y =λ$^2$ and x + λy = 1 have**
**(i) no solution?** **(ii) infinitely many solutions?**
**(iii) a unique solution?**

Solution: The given pair of linear equations is λx + y = λ$^2$ and x + λy = 1
$a_1$ = λ, $b_1$= 1, $c_1$ = – λ$^2$
$a_2$ =1, $b_2$ =λ $c_2$= -1

(i) For no solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, we should take λ = -1 because at λ= 1 the system of linear equations has infinitely many solutions

(ii) For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, when λ ≠ 0, λ = 1

(iii) For unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

**Question 2: For which, value (s) of k will the pair of equations kx + 3y = k – 3, 12 x + ky = k has no solution?**
**Solution:**
Given pair of linear equations is
kx + 3y = k – 3 and 12x + ky = k
On comparing with ax + by + c = 0, we get
$a_1$ = k, $b_1$ = 3 and $c_1$ = -(k – 3)
$a_2$ = 12, $b_2$ = k and $c_2$ = -k

For no solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
$or, \frac{k}{12} = \frac{3}{k} \neq \frac{-(k-3)}{-k}$

Taking the first two parts, we get,
$\frac{k}{12} = \frac{3}{k}$
or, $k^2$ = 36
or, k = ±6

Taking the last two parts, we get,
$\frac{3}{k} \neq \frac{k-3}{k}$
or, 3k ≠ k(k – 3)
or, k(3 – k + 3) ≠ 0
or, k(6 – k) ≠ 0
or, k ≠ 0 and k ≠ 6

Hence, the required value of k for which the given pair of linear equations has no solution is -6.

**Question 3: For which values of a and b will the following pair of linear equations has infinitely many solutions? x + 2y = 1**
**(a – b)x + (a + b)y = a + b – 2**

**Solution:** Given pair of linear equations are
x + 2y = 1…………………………………………………(1)
(a – b)x + (a + b)y = a + b – 2 ……………………………………(2)
on comparing with ax + by + c = 0,we get,
$a_1 = 1$, $b_1 = 2$ and $c_1 = -1$ …………[From eq 1]
$a_2 = (a – b)$, $b_2 = (a + b)$ ……………[From eq 2]
$c_2 = -(a + b – 2)$
For infinitely many solutions of the pairs of linear equations,
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

or, $\frac{1}{a-b} = \frac{2}{a+b} = \frac{-1}{-(a+b-2)}$

Taking first two parts, $\frac{1}{a-b} = \frac{2}{a+b}$
or, a + b = 2(a – b)
or, a + b = 2a – 2b
or, 3b = a…………………………………………………..(3)

Taking last two parts, $\frac{2}{a+b} = \frac{1}{a+b-2}$
or, 2(a + b – 2) = a + b
or, 2a + 2b – 4 = a + b
or, a + b = 4 …………………………………………(4)

Put the value of eq(3) in eq (4),
3b + b = 4
4b = 4
b = 1

Put the value of b in eq (3), we get,
a = 3(1) = 3

So, the values(a, b) = (3, 1) satisfies all the parts. Hence, the required value of a and b is 3 and 1 respectively for which it has infinitely many solutions.

**Question 4: Find the values of p in (i) to (iv) and p and q in (v) for the following pair of equations**
**(i) 3x – y – 5 = 0 and 6x – 2y – p = 0, if the lines represented by these equations are parallel.**
**(ii) – x + py = 1 and px – y = 1 if the pair of equations has no solution.**
**(iii) – 3x + 5y = 7 and 2px – 3y** = 1,
**if the lines represented by these equations are intersecting at a unique point.**
**(iv) 2x + 3y – 5 = 0 and px – 6y – 8 = 0,**
**if the pair of equations has a unique solution.**
**(v) 2x + 3y = 7 and 2px + py = 28 – qy,**
**if the pair of equations has infinitely many solutions.**

Solution:

**(i)** Given pair of linear equations is

$$3x - y - 5 = 0 \quad \ldots(i)$$

and $$6x - 2y - p = 0 \quad \ldots(ii)$$

On comparing with $ax + by + c = 0$, we get

$$a_1 = 3, b_1 = -1$$

and $$c_1 = -5 \quad \text{[from Eq. (i)]}$$

$$a_2 = 6, b_2 = -2$$

and $$c_2 = -p \quad \text{[from Eq. (ii)]}$$

Since, the lines represented by these equations are parallel, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{-p}$$

Taking last two parts, we get $\frac{-1}{-2} \neq \frac{-5}{-p}$

$\Rightarrow$ $$\frac{1}{2} \neq \frac{5}{p}$$

$\Rightarrow$ $$p \neq 10$$

Hence, the given pair of linear equations are parallel for all real values of $p$ except 10 *i.e.*, $p \in R - \{10\}$.

**(ii)** Given pair of linear equations is

$$-x + py - 1 = 0 \quad \ldots(i)$$

and $$px - y - 1 = 0 \quad \ldots(ii)$$

On comparing with $ax + by + c = 0$, we get

$$a_1 = -1, b_1 = p \text{ and } c_1 = -1 \quad \text{[from Eq. (i)]}$$

$$a_2 = p, b_2 = -1 \text{ and } c_2 = -1 \quad \text{[from Eq. (ii)]}$$

Since, the pair of linear equations has no solution *i.e.*, both lines are parallel to each other.

$\therefore$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{-1}{p} = \frac{p}{-1} \neq \frac{-1}{-1}$$

Taking last two parts, we get

$$\frac{p}{-1} \neq \frac{-1}{-1}$$

$\Rightarrow$ $$p \neq -1$$

Taking first two parts, we get

$$\frac{-1}{p} = \frac{p}{-1}$$

$\Rightarrow$ $$p^2 = 1$$

$\Rightarrow$ $$p = \pm 1$$

but $$p \neq -1$$

$\therefore$ $$p = 1$$

Hence, the given pair of linear equations has no solution for $p = 1$.

**(iii)** Given, pair of linear equations is

$$-3x + 5y - 7 = 0 \qquad \text{...(i)}$$

and $$2px - 3y - 1 = 0 \qquad \text{...(ii)}$$

On comparing with $ax + by + c = 0$, we get

$$a_1 = -3, b_1 = 5$$

and $$c_1 = -7 \qquad \text{[from Eq. (i)]}$$

$$a_2 = 2p, b_2 = -3$$

and $$c_2 = -1 \qquad \text{[from Eq. (ii)]}$$

Since, the lines are intersecting at a unique point *i.e.*, it has a unique solution.

$\therefore$ $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow$ $$\frac{-3}{2p} \neq \frac{5}{-3}$$

$\Rightarrow$ $$9 \neq 10p$$

$\Rightarrow$ $$p \neq \frac{9}{10}.$$

Hence, the lines represented by these equations are intersecting at a unique point for all real values of $p$ except $\frac{9}{10}$

**(iv)** Given pair of linear equations is

$$2x + 3y - 5 = 0 \quad \dots(i)$$

and $$px - 6y - 8 = 0 \quad \dots(ii)$$

On comparing with $ax + by + c = 0$, we get

$$a_1 = 2, b_1 = 3$$

and $$c_1 = -5 \quad \text{[from Eq. (i)]}$$

$$a_2 = p, \; b_2 = -6$$

and $$c_2 = -8 \quad \text{[from Eq. (ii)]}$$

Since, the pair of linear equations has a unique solution.

$$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \quad \frac{2}{p} \neq \frac{3}{-6}$$

$$\Rightarrow \quad p \neq -4$$

Hence, the pair of linear equations has a unique solution for all values of $p$ except $-4$

*i.e.,* $$p \in R - \{-4\}.$$

**(v)** Given pair of linear equations is

$$2x + 3y = 7 \quad \dots(i)$$

and $$2px + py = 28 - qy$$

$$\Rightarrow \quad 2px + (p+q)y = 28 \quad \dots(ii)$$

On comparing with $ax + by + c = 0$, we get

$$a_1 = 2, b_1 = 3$$

and $$c_1 = -7 \quad \text{[from Eq. (i)]}$$

$$a_2 = 2p, b_2 = (p+q)$$

and $$c_2 = -28 \quad \text{[from Eq. (ii)]}$$

Since, the pair of equations has infinitely many solutions *i.e.,* both lines are coincident.

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{2p} = \frac{3}{(p+q)} = \frac{-7}{-28}$$

Taking first and third parts, we get

$$\frac{2}{2p} = \frac{-7}{-28}$$

$$\Rightarrow \quad \frac{1}{p} = \frac{1}{4}$$

$$\Rightarrow \quad p = 4$$

Again, taking last two parts, we get

$$\frac{3}{p+q} = \frac{-7}{-28} \Rightarrow \frac{3}{p+q} = \frac{1}{4}$$

$$\Rightarrow \quad p + q = 12$$

$$\Rightarrow \quad 4 + q = 12 \quad [\because p = 4]$$

$$\therefore \quad q = 8$$

Here, we see that the values of $p = 4$ and $q = 8$ satisfies all three parts.

Hence, the pair of equations has infinitely many solutions for the values of $p = 4$ and $q = 8$

**Question 5: Two straight paths are represented by the equations x-3y = 2 and –2x + 6y =5. Check whether the paths cross each other or not.**

Solution:

Given linear equations are

$$x - 3y - 2 = 0 \quad \text{...(i)}$$

and

$$-2x + 6y - 5 = 0 \quad \text{...(ii)}$$

On comparing both the equations with $ax + by + c = 0$, we get

$$a_1 = 1, b_1 = -3$$

and $c_1 = -2$ [from Eq. (i)]

$$a_2 = -2, \ b_2 = 6$$

and $c_2 = -5$ [from Eq. (ii)]

Here,

$$\frac{a_1}{a_2} = \frac{1}{-2}$$

$$\frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

*i.e.,*

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$ [parallel lines]

Hence, two straight paths represented by the given equations never cross each other, because they are parallel to each other.

**Question 6: Write a pair of linear equations that has the unique solution x = – 1 and y = 3. How many such pairs can you write?**

**Solution:**

Condition for the pair of the system to have a unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Let the equations are,

$$a_1x + b_1y + c_1 = 0$$

and $$a_2x + b_2y + c_2 = 0$$

Since, $x = -1$ and $y = 3$ is the unique solution of these two equations, then

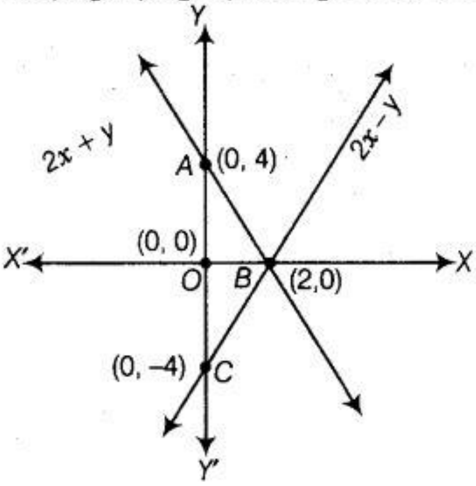
$$a_1(-1) + b_1(3) + c_1 = 0$$

$\Rightarrow$ $$-a_1 + 3b_1 + c_1 = 0 \quad \ldots(i)$$

and $$a_2(-1) + b_2(3) + c_2 = 0$$

$\Rightarrow$ $$-a_2 + 3b_2 + c_2 = 0 \quad \ldots(ii)$$

So, the different values of $a_1$, $a_2$, $b_1$, $b_2$, $c_1$ and $c_2$ satisfy the Eqs. (i) and (ii).




Hence, infinitely many pairs of linear equations are possible.

**Question 7:**

If 2x+ y = 23 and 4x- y = 19, then find the values of 5y – 2x and $\frac{y}{x} - 2.$

**Solution:**

Given equations are

$$2x + y = 23 \quad \ldots(i)$$

and $$4x - y = 19 \quad \ldots(ii)$$

On adding both equations, we get

$$6x = 42 \Rightarrow x = 7$$

Put the value of $x$ in Eq. (i), we get

$$2(7) + y = 23$$

$\Rightarrow$ $$14 + y = 23$$

$\Rightarrow$ $$y = 23 - 14$$

$\Rightarrow$ $$y = 9$$

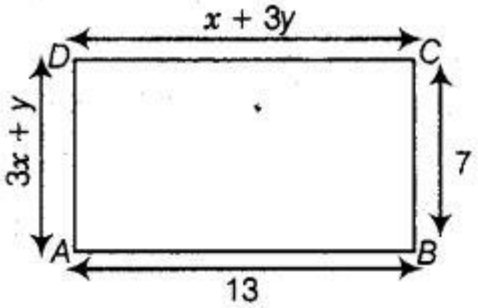
We have, $$5y - 2x = 5 \times 9 - 2 \times 7$$

$$= 45 - 14 = 31$$

and $$\frac{y}{x} - 2 = \frac{4}{x} - 2 = \frac{9}{7} - 2 = \frac{9-14}{7} = -\frac{5}{7}$$

Hence, the values of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$ are 31 and $\frac{-5}{7}$, respectively.

**Question 8:**
Find the values of x and y in the following rectangle




**Solution:**

By property of rectangle,

Lengths are equal, *i.e.*, $CD = AB$

$\Rightarrow \quad x + 3y = 13 \quad \ldots(i)$

Breadth are equal, *i.e.*, $AD = BC$

$\Rightarrow \quad 3x + y = 7 \quad \ldots(ii)$

On multiplying Eq. (ii) by 3 and then subtracting Eq. (i), we get

$$\begin{array}{r} 9x + 3y = 21 \\ x + 3y = 13 \\ - \quad - \quad\quad - \\ \hline 8x \quad\quad = 8 \\ x = 1 \end{array}$$

On putting $x = 1$ in Eq. (i), we get

$$3y = 12 \Rightarrow y = 4$$

Hence, the required values of $x$ and $y$ are 1 and 4, respectively.

**Question 9:**

Solve the following pairs of equations

(i) $x + y = 3.3,\quad \dfrac{0.6}{3x-2y} = -1,\ 3x - 2y \neq 0$

(ii) $\dfrac{x}{3} + \dfrac{y}{4} = 4,\quad \dfrac{5x}{6} - \dfrac{y}{8} = 4$

(iii) $4x + \dfrac{6}{y} = 15,\quad 6x - \dfrac{8}{y} = 14,\ y \neq 0$

(iv) $\dfrac{1}{2x} - \dfrac{1}{y} = -1,\quad \dfrac{1}{x} + \dfrac{1}{2y} = 8,\ x,\ y \neq 0$

(v) $43x + 67y = -24,\quad 67x + 43y = 24$

(vi) $\dfrac{x}{a} + \dfrac{y}{b} = a + b,\quad \dfrac{x}{a^2} + \dfrac{y}{b^2} = 2,\ a,\ b \neq 0$

(vii) $\dfrac{2xy}{x+y} = \dfrac{3}{2},\quad \dfrac{xy}{2x-y} = \dfrac{-3}{10},\ x + y \neq 0,\ 2x - y \neq 0$

**Solution:**

**(i)** Given pair of linear equations are is

$$x + y = 3.3 \quad \text{...(i)}$$

and

$$\frac{0.6}{3x-2y} = -1$$

$$\Rightarrow \quad 0.6 = -3x + 2y$$

$$\Rightarrow \quad 3x - 2y = -0.6 \quad \text{...(ii)}$$

Now, multiplying Eq. (i) by 2 and then adding with Eq. (ii), we get

$$\Rightarrow \quad 2x + 2y = 6.6$$

$$\Rightarrow \quad 3x - 2y = -0.6$$

$$5x = 6 \Rightarrow x = \frac{6}{5} = 1.2$$

Now, put the value of $x$ in Eq. (i), we get

$$1.2 + y = 3.3$$

$$\Rightarrow \quad y = 3.3 - 1.2$$

$$\Rightarrow \quad y = 2.1$$

Hence, the required values of $x$ and $y$ are 1.2 and 2.1, respectively.

**(ii)** Given, pair of linear equations is

$$\frac{x}{3} + \frac{y}{4} = 4$$

On multiplying both sides by LCM (3, 4) = 12, we get

$$4x + 3y = 48 \qquad \text{...(i)}$$

and

$$\frac{5x}{6} - \frac{y}{8} = 4$$

On multiplying both sides by LCM (6, 8) = 24, we get

$$20x - 3y = 96 \qquad \text{...(ii)}$$

Now, adding Eqs. (i) and (ii), we get

$$24x = 144$$

$$\Rightarrow \quad x = 6$$

Now, put the value of $x$ in Eq. (i), we get

$$4 \times 6 + 3y = 48$$

$$\Rightarrow \quad 3y = 48 - 24$$

$$\Rightarrow \quad 3y = 24 \Rightarrow y = 8$$

Hence, the required values of $x$ and $y$ are 6 and 8, respectively.

**(iii)** Given pair of linear equations are

$$4x + \frac{6}{y} = 15 \qquad \text{...(i)}$$

and

$$6x - \frac{8}{y} = 14, \; y \neq 0 \qquad \text{...(ii)}$$

Let $u = \frac{1}{y}$, then above equation becomes

$$4x + 6u = 15 \qquad \text{...(iii)}$$

and

$$6x - 8u = 14 \qquad \text{...(iv)}$$

On multiplying Eq. (iii) by 8 and Eq. (iv) by 6 and then adding both of them, we get

$$32x + 48u = 120$$

$$36x - 48u = 84 \;\Rightarrow\; 68x = 204$$

$$\Rightarrow \qquad x = 3$$

Now, put the value of $x$ in Eq. (iii), we get

$$4 \times 3 + 6u = 15$$

$$\Rightarrow \qquad 6u = 15 - 12 \;\Rightarrow\; 6u = 3$$

$$\Rightarrow \qquad u = \frac{1}{2} \Rightarrow \frac{1}{y} = \frac{1}{2} \qquad \left[\because u = \frac{1}{y}\right]$$

$$\Rightarrow \qquad y = 2$$

Hence, the required values of $x$ and $y$ are 3 and 2, respectively.

**(iv)** Given pair of linear equations is

$$\frac{1}{2x} - \frac{1}{y} = -1 \qquad \text{...(i)}$$

and $$\frac{1}{x} + \frac{1}{2y} = 8, x, y \neq 0 \qquad \text{...(ii)}$$

Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$, then the above equations becomes

$$\frac{u}{2} - v = -1$$

$\Rightarrow$ $$u - 2v = -2 \qquad \text{....(iii)}$$

and $$u + \frac{v}{2} = 8$$

$\Rightarrow$ $$2u + v = 16 \qquad \text{...(iv)}$$

On, multiplying Eq. (iv) by 2 and then adding with Eq. (iii), we get

$$\begin{array}{r} 4u + 2v = 32 \\ \underline{u - 2v = -2} \\ 5u = 30 \end{array}$$

$\Rightarrow$ $$u = 6$$

Now, put the value of $u$ in Eq. (iv), we get

$$2 \times 6 + v = 16$$

$\Rightarrow$ $$v = 16 - 12 = 4$$

$\Rightarrow$ $$v = 4$$

$\therefore$ $$x = \frac{1}{u} = \frac{1}{6} \text{ and } y = \frac{1}{v} = \frac{1}{4}$$

Hence, the required values of $x$ and $y$ are $\frac{1}{6}$ and $\frac{1}{4}$, respectively.

**(v)** Given pair of linear equations is

$$43x + 67y = -24 \qquad \text{...(i)}$$

and $$67x + 43y = 24 \qquad \text{...(ii)}$$

On multiplying Eq. (i) by 43 and Eq. (ii) by 67 and then subtracting both of them, we get

$$\begin{array}{r} (67)^2 x + 43 \times 67 y = 24 \times 67 \\ \underline{\underset{-}{(43)^2} x + \underset{-}{43} \times 67 y = -\underset{+}{24} \times 43} \\ \{(67)^2 - (43)^2\} x = 24 (67 + 43) \end{array}$$

$\Rightarrow$ $$(67 + 43)(67 - 43) x = 24 \times 110 \qquad [\because (a^2 - b^2) = (a - b)(a + b)]$$

$\Rightarrow$ $$110 \times 24 x = 24 \times 110$$

$\Rightarrow$ $$x = 1$$

Now, put the value of $x$ in Eq. (i), we get

$$43 \times 1 + 67y = -24$$

$\Rightarrow$ $$67y = -24 - 43$$

$\Rightarrow$ $$67y = -67$$

$\Rightarrow$ $$y = -1$$

Hence, the required values of $x$ and $y$ are 1 and $-1$, respectively.

**(vi)** Given pair of linear equations is

$$\frac{x}{a} + \frac{y}{b} = a + b \quad \text{...(i)}$$

and

$$\frac{x}{a^2} + \frac{y}{b^2} = 2, a, b \neq 0 \quad \text{...(ii)}$$

On multiplying Eq. (i) by $\frac{1}{a}$ and then subtracting from Eq. (ii), we get

$$\begin{array}{r} \frac{x}{a^2} + \frac{y}{b^2} = 2 \\ \underset{-}{\frac{x}{a^2}} + \underset{-}{\frac{y}{ab}} = \underset{-}{1} + \frac{b}{a} \\ \hline y\left(\frac{1}{b^2} - \frac{1}{ab}\right) = 2 - 1 - \frac{b}{a} \end{array}$$

$$\Rightarrow \quad y\left(\frac{a-b}{ab^2}\right) = 1 - \frac{b}{a} = \left(\frac{a-b}{a}\right)$$

$$\Rightarrow \quad y = \frac{ab^2}{a} \Rightarrow y = b^2$$

Now, put the value of $y$ in Eq. (ii), we get

$$\frac{x}{a^2} + \frac{b^2}{b^2} = 2$$

$$\Rightarrow \quad \frac{x}{a^2} = 2 - 1 = 1$$

$$\Rightarrow \quad x = a^2$$

Hence, the required values of $x$ and $y$ are $a^2$ and $b^2$, respectively.

**(vii)** Given pair of equations is

$$\frac{2xy}{x+y} = \frac{3}{2}, \text{ where } x + y \neq 0$$

$$\Rightarrow \quad \frac{x+y}{2xy} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{x}{xy} + \frac{y}{xy} = \frac{4}{3}$$

$$\Rightarrow \quad \frac{1}{y} + \frac{1}{x} = \frac{4}{3} \quad \text{...(i)}$$

and

$$\frac{xy}{2x-y} = \frac{-3}{10}, \text{ where } 2x - y \neq 0$$

$$\Rightarrow \quad \frac{2x-y}{xy} = \frac{-10}{3}$$

$$\Rightarrow \quad \frac{2x}{xy} - \frac{y}{xy} = \frac{-10}{3}$$

$$\Rightarrow \quad \frac{2}{y} - \frac{1}{x} = \frac{-10}{3} \quad \text{...(ii)}$$

Now, put $\frac{1}{x} = u$ and $\frac{1}{y} = v$, then the pair of equations becomes

$$v + u = \frac{4}{3} \quad \text{...(iii)}$$

and

$$2v - u = \frac{-10}{3} \quad \text{...(iv)}$$

On adding both equations, we get

$$3v = \frac{4}{3} - \frac{10}{3} = \frac{-6}{3}$$

$$\Rightarrow \quad 3v = -2$$

$$\Rightarrow \quad v = \frac{-2}{3}$$

Now, put the value of $v$ in Eq. (iii), we get

$$\frac{-2}{3} + u = \frac{4}{3}$$

$$\Rightarrow \quad u = \frac{4}{3} + \frac{2}{3} = \frac{6}{3} = 2$$

$$\therefore \quad x = \frac{1}{u} = \frac{1}{2}$$

and

$$y = \frac{1}{v} = \frac{1}{(-2/3)} = \frac{-3}{2}$$

Hence, the required values of $x$ and $y$ are $\frac{1}{2}$ and $\frac{-3}{2}$, respectively.

**Question 10:**

Find the solution of the pair of equations $\frac{x}{10} + \frac{y}{5} - 1 = 0$

$\frac{x}{8} + \frac{y}{6}$= 15 and find A, if y = λx + 5.

**Solution:**

Given pair of equations is

$$\frac{x}{10} + \frac{y}{5} - 1 = 0 \quad \text{...(i)}$$

and $$\frac{x}{8} + \frac{y}{6} = 15 \quad \text{...(ii)}$$

Now, multiplying both sides of Eq. (i) by LCM (10, 5) = 10 , we get

$$x + 2y - 10 = 0$$

$$\Rightarrow \quad x + 2y = 10 \quad \text{...(iii)}$$

Again, multiplying both sides of Eq. (iv) by LCM (8,6) = 24, we get

$$3x + 4y = 360 \quad \text{...(iv)}$$

On, multiplying Eq. (iii) by 2 and then subtracting from Eq. (iv), we get

$$\begin{array}{r} 3x + 4y = 360 \\ \underline{\overset{}{\underset{-}{2x}} + \underset{-}{4y} = \underset{-}{20}} \\ x = 340 \end{array}$$

Put the value of $x$ in Eq. (iii), we get

$$340 + 2y = 10$$

$$\Rightarrow \quad 2y = 10 - 340 = -330$$

$$\Rightarrow \quad y = -165$$

Given that, the linear relation between $x$, $y$ and $\lambda$ is

$$y = \lambda x + 5$$

Now, put the values of $x$ and $y$ in above relation, we get

$$-165 = \lambda (340) + 5$$

$$\Rightarrow \quad 340\lambda = -170$$

$$\Rightarrow \quad \lambda = -\frac{1}{2}$$

Hence, the solution of the pair of equations is $x = 340, y = -165$ and the required value of $\lambda$ is $-\frac{1}{2}$.

**Question 11:**

**By the graphical method, find whether the following pair of equations is consistent or not. If consistent, solve them.**

**(i) 3x + y + 4 = 0,6x- 2y + 4 = 0**

**(ii) x — 2y — 6, 3x — 6y = 0**

**(iii) x + y = 3, 3x + 3y =9**

**Solution:**

**(i)** Given pair of equations is

$$3x + y + 4 = 0 \quad \text{...(i)}$$

and $$6x - 2y + 4 = 0 \quad \text{...(ii)}$$

On comparing with $ax + by + c = 0$, we get

$$a_1 = 3, b_1 = 1$$

and $$c_1 = 4 \quad \text{[from Eq. (i)]}$$

$$a_2 = 6, b_2 = -2$$

and $$c_2 = 4 \quad \text{[from Eq. (ii)]}$$

Here, $$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{1}{-2}$$

and $$\frac{c_1}{c_2} = \frac{4}{4} = \frac{1}{1}$$

$\because$ $$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, the given pair of linear equations are intersecting at one point, therefore these lines have unique solution.

Hence, given pair of linear equations is consistent.

We have, $$3x + y + 4 = 0$$

$\Rightarrow$ $$y = -4 - 3x$$

When $x = 0$, then $y = -4$

When $x = -1$, then $y = -1$

When $x = -2$, then $y = 2$

| $x$ | 0 | $-1$ | $-2$ |
|---|---|---|---|
| $y$ | $-4$ | $-1$ | 2 |
| **Points** | $B$ | $C$ | $A$ |

and $$6x - 2y + 4 = 0$$

$\Rightarrow$ $$2y = 6x + 4$$

$\Rightarrow$ $$y = 3x + 2$$

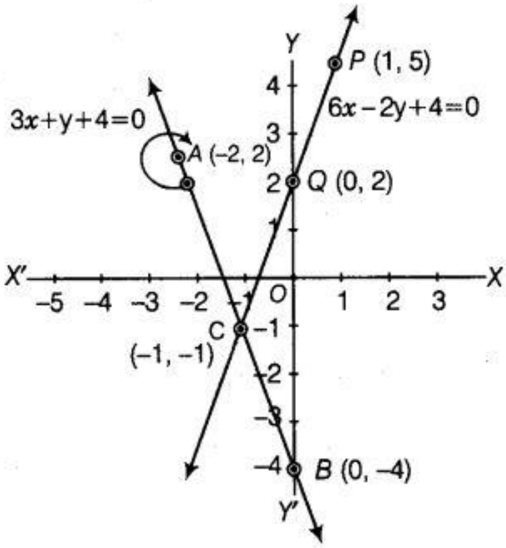
When $x = 0$, then $y = 2$

When $x = -1$, then $y = -1$

When $x = 1$, then $y = 5$

| $x$ | $-1$ | 0 | 1 |
|---|---|---|---|
| $y$ | $-1$ | 2 | 5 |
| **Points** | $C$ | $Q$ | $P$ |

Plotting the points $B\ (0, -4)$ and $A\ (-2, 2)$, we get the straight tine $AB$. Plotting the points $Q\ (0, 2)$ and $P\ (1, 5)$, we get the straight line $PQ$. The lines $AB$ and $PQ$ intersect at $C\ (-1, -1)$.




**(ii)** Given pair of equations is $\qquad x - 2y = 6 \qquad$ ...(i)

and $\qquad 3x - 6y = 0 \qquad$ ...(ii)

On comparing with $ax + by + c = 0$, we get

$$a_1 = 1,\ b_1 = -2 \text{ and } c_1 = -6 \qquad \text{[from Eq. (i)]}$$

$$a_2 = 3,\ b_2 = -6 \text{ and } c_2 = 0 \qquad \text{[from Eq. (ii)]}$$

Here, $$\frac{a_1}{a_2} = \frac{1}{3},\ \frac{b_1}{b_2}\ \frac{-2}{-6} = \frac{1}{3} \text{ and } \frac{c_1}{c_2} = \frac{-6}{0}$$

$\therefore$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the lines represented by the given equations are parallel. Therefore, it has no solution. So, the given pair of lines is inconsistent.

**(iii)** Given pair of equations is $x + y = 3$ ...(i)

and $3x + 3y = 9$ ...(ii)

On comparing with $ax + by + c = 0$, we get

$$a_1 = 1, b_1 = 1 \text{ and } c_1 = -3 \quad \text{[from Eq. (i)]}$$
$$a_2 = 3, b_2 = 3 \text{ and } c_2 = -9 \quad \text{[from Eq. (ii)]}$$

Here, $$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{1}{3} \text{ and } \frac{c_1}{c_2} = \frac{-3}{-9} = \frac{1}{3}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, the given pair of lines is coincident. Therefore, these lines have infinitely many solutions. Hence, the given pair of linear equations is consistent.

Now, $x + y = 3 \Rightarrow y = 3 - x$

If $x = 0$, then $y = 3$, If $x = 3$, then $y = 0$

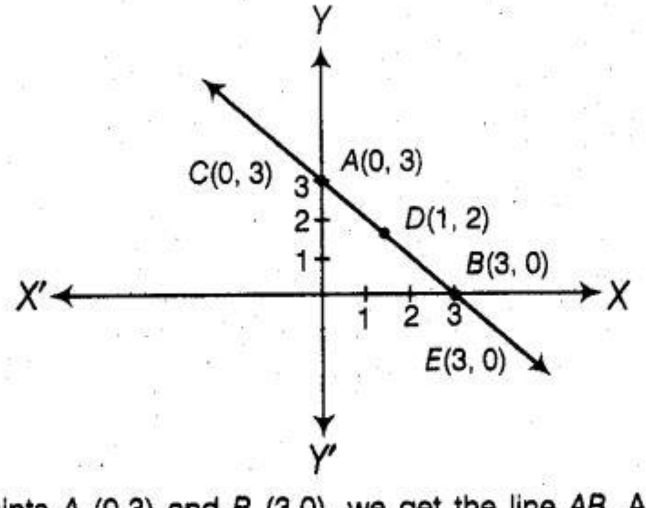
| $x$ | 0 | 3 |
|---|---|---|
| $y$ | 3 | 0 |
| **Points** | $A$ | $B$ |

and $3x + 3y = 9 \Rightarrow 3y = 9 - 3x$

$$\Rightarrow y = \frac{9 - 3x}{3}$$

If $x = 0$, then $y = 3$; if $x = 1$, then $y = 2$ and if $x = 3$, then $y = 0$

| $x$ | 0 | 1 | 3 |
|---|---|---|---|
| $y$ | 3 | 2 | 0 |
| **Points** | $C$ | $D$ | $E$ |




Plotting the points $A$ (0,3) and $B$ (3,0), we get the line $AB$. Again, plotting the points $C$ (0,3) $D$ (1, 2) and $E$ (3,0), we get the line $CDE$.

We observe that the lines represented by Eqs. (i) and (ii) are coincident.

**Question 12:**

**Draw the graph of the pair of equations 2x + y= 4 and 2x – y = 4. Write the**

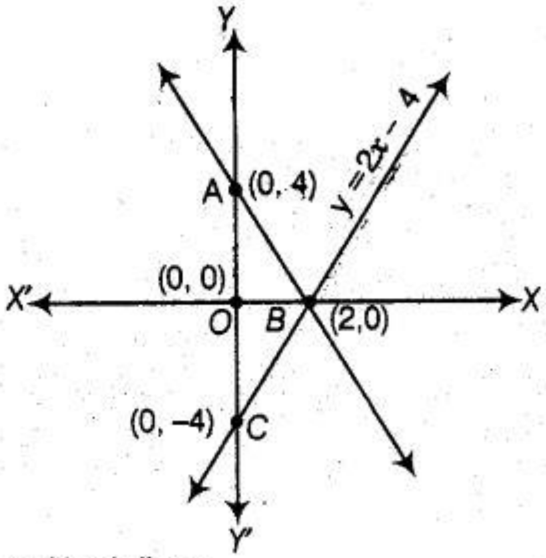
**vertices of the triangle formed by these lines and the y-axis, find the area of this triangle? '**
**Solution:**

The given pair of linear equations
Table for line $2x + y = 4$,

| $x$ | 0 | 2 |
|---|---|---|
| $y = 4 - 2x$ | 4 | 0 |
| **Points** | $A$ | $B$ |

and table for line $2x - y = 4$,

| $x$ | 0 | 2 |
|---|---|---|
| $y = 2x - 4$ | $-4$ | 0 |
| **Points** | $C$ | $B$ |




Graphical representation of both lines.
Here, both lines and $Y$-axis form a $\Delta ABC$.

Hence, the vertices of a $\Delta ABC$ are $A(0, 4)$ $B(2, 0)$ and $C(0, -4)$.

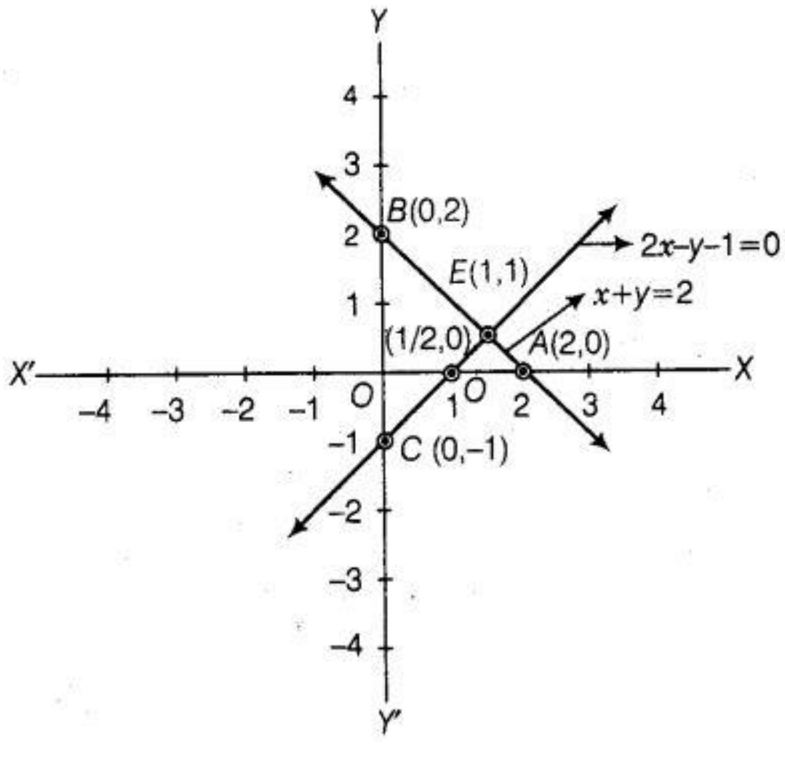
$\therefore$ Required area of $\Delta ABC = 2 \times$ Area of $\Delta AOB$

$$= 2 \times \frac{1}{2} \times 4 \times 2 = 8 \text{ sq units}$$

Hence, the required area of the triangle is 8 sq units.

If $x = 0$, then $y = -1$; if $x = \frac{1}{2}$, then $y = 0$ and if $x = 1$, then $y = 1$

| $x$ | 0 | 1/2 | 1 |
|---|---|---|---|
| $y$ | $-1$ | 0 | 1 |
| **Points** | $C$ | $D$ | $E$ |




**Question 13:**
**Write an equation of a line passing through the point representing the solution of the pair of Linear equations x + y = 2 and 2x – y = 1, How many such lines can we find?**
**Solution:**
Plotting the points A (2,0) and B (0, 2), we get the straight line AB. Plotting the points C (0, – 1)andD (1/2, 0), we get the straight line CD. The lines AB and CD intersect at E(1,1), Hence, infinite lines can pass through the intersection point of linear equations x + y = 2 and 2x – y = 1 i.e., E(1,1) like as y = x, 2x + y = 3, x + 2y = 3. so on.

**Question 14: If (x + 1) is a factor of $2x^3 + ax^2 + 2bx + l$, then find the value of a and b given that 2a – 3b = 4.**
**Solution:**

Given that, (x + 1) is a factor of $f(x) = 2x^s + ax^2 + 2bx + 1$, then f(-1) = 0.
[if (x + a) is a factor of $f(x) = ax^2 + bx + c$, then f(-) = 0]

$$\Rightarrow \quad 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$$
$$\Rightarrow \quad -2 + a - 2b + 1 = 0$$
$$\Rightarrow \quad a - 2b - 1 = 0 \qquad \text{...(i)}$$
Also, $$2a - 3b = 4$$
$$\Rightarrow \quad 3b = 2a - 4$$
$$\Rightarrow \quad b = \left(\frac{2a-4}{3}\right)$$

Now, put the value of $b$ in Eq. (i), we get
$$a - 2\left(\frac{2a-4}{3}\right) - 1 = 0$$
$$\Rightarrow \quad 3a - 2(2a - 4) - 3 = 0$$
$$\Rightarrow \quad 3a - 4a + 8 - 3 = 0$$
$$\Rightarrow \quad -a + 5 = 0$$
$$\Rightarrow \quad a = 5$$
Now, put the value of $a$ in Eq. (i), we get
$$5 - 2b - 1 = 0$$
$$\Rightarrow \quad 2b = 4$$
$$\Rightarrow \quad b = 2$$
Hence, the required values of a and b are 5 and 2, respectively.

**Question 15: If the angles of a triangle are x, y and 40° and the difference between the two angles x and y is 30°. Then, find the value of x and y,**
**Solution:**
Given that, x, y and 40° are the angles of a triangle.
x + y + 40° = 180°
[since the sum of all the angles of a triangle is 180°]
$x + y = 140^0$……………………………………………..(i)
or, $x – y = 30^0$ ………………………………………………..(ii)
Also, on adding eq(i) and (ii), we get
$2x = 170^0$
$x = 85^0$
On putting $x = 85^0$ in eq(i), we get
$85^0 + y = 140^0$
$y = 55^0$

Hence, the required values of x and y are 85° and 55°, respectively.

**Question 16: Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. How old are they now?**
**Solution:**
Let Salim and his daughter's age be x and y yr respectively.
Now, by the first condition
Two years ago, Salim was thrice as old as his daughter.
i.e., x – 2 = 3(y – 2)
or, x – 2 = 3y – 6

or, $x - 3y = -4$……………………………………………………..(i)
and by the second condition, 6years later, Salim will be 4yers older than twice her age.

$x + 6 = 2(y + 6) + 4$
or, $x + 6 = 2y + 12 + 4$
or, $x - 2y = 16 - 6$
or, $x - 2y = 10$……………………………………………………..(ii)
On subtracting eq(i) from eq(ii), we get

$$\begin{array}{r} x - 2y = 10 \\ x - 3y = -4 \\ (-)\ \ (+)\quad\ (+) \\ \hline y = 14 \end{array}$$

Put the value of y in eq(ii), we get
$x - 2 \times 14 = 10$
or, $x = 10 + 28$
or, $x = 38$

Hence, Salim and his daughter's ages are 38 yr and 14 yr, respectively.

**Question 17: The age of the father is twice the sum of the ages of his two children. After 20 yr, his age will be equal to the sum of the ages of his children. Find the age of the father.**
**Solution:**
Let the present age (in a year) of the father and his two children be x, y and z yr, respectively.
Now by given condition, $x = 2(y + z)$ ……………………………..………………..(i)
and after 20 yr, $(x + 20) = (y + 20) + (z + 20)$
$\Rightarrow y+z +40 = x +20$
$\Rightarrow y + z = x - 20$
On putting the value of (y + z) in Eq. (i) and get the present age of the father
$x =2(x - 20)$ x
$= 2x - 40 = 40$
Hence, the father's age is 40 yr.

**Question 18: Two numbers are in the ratio 5: 6. If 8 is subtracted from each of the numbers, the ratio becomes 4: 5, then find the numbers.**
**Solution:**
Let the two numbers be x and y.
Then, by the first Condition, the ratio of these two numbers = 5:6
$x : y = 5 : 6$
or, $\frac{x}{y} = \frac{5}{6}$
or, $y = \frac{6x}{5}$ ………………………………………(i)
and by the second condition, then, 8 is subtracted from each of the numbers, then the ratio becomes 4: 5
$\frac{x-8}{y-8} = \frac{4}{5}$
or, $5x - 40 = 4y - 32$
or, $5x - 4y = 8$ ……………………………………(2)

Now, put the value of y in eq(2) we get,

$5x - 4\left(\frac{6x}{5}\right) = 8$

or, $25x - 24x = 40$

or, $x = 40$

Put the value of x in eq(i), we get,

$y = \frac{6}{5} \times 40$

$= 6 \times 8 = 48$

Hence, the required numbers are 40 and 48.

**Question 19:**

**There are some students in the two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B but, if 20 students are sent from B to A, the number of students in A becomes double the number of students in B, then find the number of students in both halls.**

Solution:

Let the number of students in halls A and 8 are x and y, respectively.

Now, by given condition, $x-10=y+10$

$\Rightarrow x - y = 20$ … (i)

and $(x + 20) = 2(y-20)$

$\Rightarrow x-2y=-60$ …(ii)

On subtracting Eq. (ii) from Eq. (i), we get

(x-y)-(x- 2 y) = 20+60 „ x-y-x + 2y~ 80 => y= 80

On putting y = 80 in Eq. (i), we get

x – 80 = 20 => x = 100 and y = 80

Hence, 100 students are in hall A and 80 students are in hall 8.

**Question 20:**

**A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days and an additional charge for each day thereafter. Latika paid ₹ 22 for a book kept for six days, while Anand paid ₹ 16 for the book kept for four days. Find the fixed charges and the charge for each extra day.**

**Solution:**

Let Latika takes a fixed charge for the first two days is ₹ x and the additional charge for each day thereafter is ₹ y.

Now by the first condition.

Latika paid ₹ 22 for a book kept for six days i.e.,

$x + 4y = 22$ …(i)

and by the second condition,

Anand paid ₹ 16 for a book kept for four days i.e.,

$x+2y=16$ …(ii)

Now, subtracting Eq. (ii) from Eq. (i), we get

$2y=6 \Rightarrow y = 3$

On putting the value of y in Eq. (ii), we get

x + 2 x 3 =16

x = 16-6 = 10
Hence, the fixed charge = ₹ 10
and the charge for each extra day = ₹ 3

**Question 21: In a competitive examination, 1 mark is awarded for each correct answer while 1/2 mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?**
**Solution:**
Let x be the number of correct answers to the questions in a competitive examination, then (120 – x) be the number of wrong answers to the questions.
Then, by given condition, $x(1) - (120 - x)\frac{1}{2} = 90$

or, $x - 60 + \frac{x}{2} = 90$
or, $\frac{3x}{2} = 150$
or, $x = \frac{150 \times 2}{3} = 100$

Hence, Jayanti answered correctly 100 questions.

**Question 22: The angles of a cyclic quadrilateral ABCD are ∠A – (6x +10)°, ∠B = (5x)°, ∠C = (x+ y)° and ∠D = (3y – 10)°.Find x and y and hence the values of the four angles.**
**Solution:**
We know that, by property of cyclic quadrilateral,
Sum of opposite angles = 180°
$\angle A + \angle C = (6x + 10)^\circ + (x + y)^\circ = 180^\circ$……………………………………(1)

or, $7x + y = 170$
and, $\angle B + \angle D = (5x)^0 + (3y - 10)^0 = 180^0$…………………………………………………(2)
or, $5x + 3y = 190^0$
On multiplying eq(1) by 3 and then subtracting, we get
$3(7x + y) - (5x + 3y) = 510^0 - 190^0$
or, $21x + 3y - 5x - 3y = 320^0$
or, $16x = 320^0$
or, $x = 20^0$

On putting $x = 20^0$ in eq(2) we get,
$7(20) + y = 170^0$
or, $y = 170^0 - 140^0$
or, $y = 30^0$

Thus, $\angle A = (6x + 10)^0 = 6(20^0) + 10^0 = 120^0 + 10^0 = 130^0$
$\angle B = (5x)^0 = 5(20^0) = 100^0$
$\angle C = (x + y)^0 = 20^0 + 30^0 = 50^0$
$\angle D = (3y - 10)^0 = 3(30^0) - 10^0 = 90^0 - 10^0 = 80^0$
Hence, the required values of x and y are 20° and 30° respectively and the values of the four angles;.e., ZA, ZB, ZC and ZD are 130°, 100°, 50° and 80°, respectively.

## Exercise 3.4

**Question 1: Graphically, solve the following pair of equations $2x + y = 6$ and $2x - y + 2 = 0$. Find the ratio of the areas of the two triangles formed by the lines representing these equations with the X-axis and the lines with the Y-axis.**
**Solution:**

Given equations are $2x + y = 6$ and $2x - y + 2 = 0$

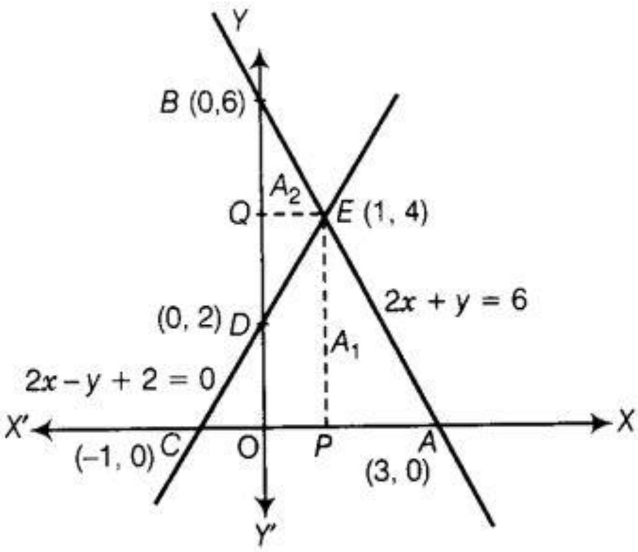
Table for equation $2x + y = 6$,

| $x$ | 0 | 3 |
|---|---|---|
| $y$ | 6 | 0 |
| **Points** | B | A |

Table for equation $2x - y + 2 = 0$,

| $x$ | 0 | −1 |
|---|---|---|
| $y$ | 2 | 0 |
| **Points** | D | C |

Let $A_1$ and $A_2$ represent the areas of $\Delta ACE$ and $\Delta BDE$, respectively.




Now, $$A_1 = \text{Area of } \Delta ACE = \frac{1}{2} \times AC \times PE$$
$$= \frac{1}{2} \times 4 \times 4 = 8$$

and $$A_2 = \text{Area of } \Delta BDE = \frac{1}{2} \times BD \times QE$$
$$= \frac{1}{2} \times 4 \times 1 = 2$$
$$\therefore A_1 : A_2 = 8:2 = 4:1$$

Hence, the pair of equations intersect graphically at point $E\,(1, 4)$, *i.e.*, $x = 1$ and $y = 4$.

**Question 2: Determine graphically, the vertices of the triangle formed by the lines y = x, 3y = x and x + y = 8**

**Solution:** Given linear equations are, y = x……………………………………(i)

3y = x …………………………………………………………….(ii)

x + y = 8 ………………………………………………………….(iii)

For equation $y = x$,

| | | | |
|---|---|---|---|
| x | 0 | 1 | 2 |
| y | 0 | 1 | 2 |
| Points | O | A | B |

For equation, $x = 3y$

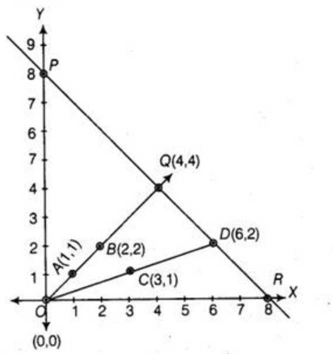
| | | | |
|---|---|---|---|
| x | 0 | 3 | 6 |
| y | 0 | 1 | 2 |
| Points | O | C | D |

For equation,

$x + y = 8$

or, $y = 8 – x$

| | | | |
|---|---|---|---|
| x | 0 | 4 | 8 |
| y | 8 | 4 | 0 |
| Points | P | Q | R |



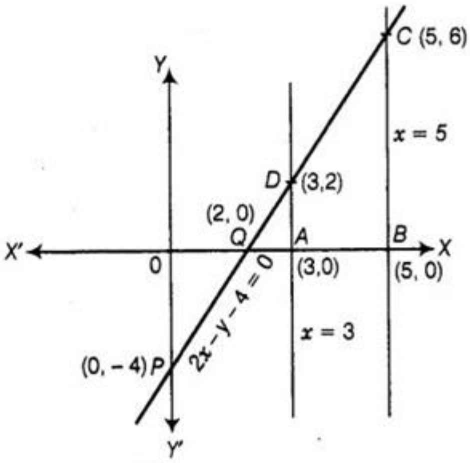

Plotting the points A(1,1) and 6(2,2), we get the straight line AB. Plotting the points C(3,1) and 0(6,2), we get the straight line CD. Plotting the points P(0, 8), Q(4, 4) and R{8, 0), we get the straight line PQR. We see that lines AB and CD intersecting the line PR on Q and D, respectively.
So, AOQD is formed by these lines. Hence, the vertices of the A00D formed by the given lines are0(0, 0), Q(4, 4)and 0(6,2).

**Question 3: Draw the graphs of the equations x = 3, x = 5 and 2x – y – 4 = 0. Also, find the area of the quadrilateral formed by the lines and the X-axis.**

**Solution:** Given equation of lines 2x – y – 4 = 0, x = 3 and x = 5 Table for line 2x – y – 4 = 0 Also we have to find the area of the quadrilateral formed by the lines and the X-axis.

| | | |
|---|---|---|
| x | 0 | 2 |
| y = 2x – 4 | -4 | 0 |
| Points | P | Q |

Draw the points P(0, -4) and Q(2, 0) and join these points and form a line PQ also draw the lines x = 3 and x = 5




$\therefore$ Area of quadrilateral $ABCD = \frac{1}{2} \times$ distance between parallel lines $(AB) \times (AD + BC)$

[since, quadrilateral $ABCD$ is a trapezium]

$= \frac{1}{2} \times 2 \times (6 + 2)$

$[\because AB = OB - OA = 5 - 3 = 2,\ AD = 2$ and $BC = 6]$

$= 8$ sq units

Hence, the required area of the quadrilateral formed by the lines and the X-axis is 8 sq units.

**Question 4:**
The cost of 4 pens and 4 pencils boxes is 1100. Three times the cost of a pen is ₹ 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.
**Solution:**
Let the cost of a pen be ₹ x and the cost of a pencil box be ₹ y.
Then, by given condition,
$4x + 4y = 100$
or, $x + y = 25$……………………………………………….(i)
and, $3x = y + 15$
or, $3x – y = 15$……………………………………………….(ii)
On adding eq(i) and (ii), we get
$4x = 40$
or, $x = 10$
By substituting $x = 10$, in eq(i) we get,
$y = 25 – 10 = 15$
Hence, the cost of a pen and a pencil box are ₹ 10 and ₹ 15, respectively.

**Question 5: Determine, algebraically, the vertices of the triangle formed by the lines**
**$3x – y = 3$**
**$2x – 3y = 2$**

**and x + 2y = 8**
**Solution:**
Given the equation of lines are
$3x - y = 3$ …………………………………………………………….(i)
$2x - 3y = 2$ …………………………………………………………...(ii)
and $x + 2y = 8$ ………………………………………………………… ….(iii)
Let lines (i), (ii) and (iii) represent the sides of a ΔABC i.e., AB, BC and CA, respectively. On solving lines (i) and (ii), we will get the intersecting point B.
On multiplying Eq. (i) by 3 in Eq. (i) and then subtracting, we get,

$$9x - 3y = 9$$
$$2x - 3y = 2$$
$$(-) \quad (+) \quad (-)$$
$$\overline{\quad 7x = 7 \quad}$$
or, $x = 1$

On putting the value of x in eq(1) we get,
$3(1) - y = 3$
or, $y = 0$
So, the coordinate of point or vertex B(1,0)
On solving lines (ii) and (iii), we will get the intersecting point C.
On multiplying eq(iii) by 2 then subtracting, we get,

$$2x + 4y = 16$$
$$2x - 3y = 2$$
$$(-) \quad (+) \quad (-)$$
$$\overline{\quad 7y = 14 \quad}$$
or, $y = 2$

On putting the value of y in eq(iii), we get,
$x + 2(2) = 8$
or, $x = 8 - 4$
or, $x = 4$
Hence, the coordinate of point or vertex C is (4,2)
On solving lines (iii) and (i), we will get the intersecting point A.
On multiplying in eq(i) by 2 and then adding eq(iii), we get

$$6x - 2y = 6$$
$$x + 2y = 8$$
$$\overline{\quad 7x = 14 \quad}$$
or, $x = 2$

On putting the value of x in eq(i) we get,
$3(2) - y = 3$
$y = 6 - 3$
$y = 3$
So, the coordinate of point or vertex A is (2, 3).
Hence, the vertices of the ΔABC formed by the given lines are A (2, 3), B (1, 0) and C(4,2).

**Question 6: Ankita travels 14 km to her home partly by rickshaw and partly by bus. It takes half an hour if she travels 2 km by rickshaw and the remaining distance by bus. On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 min longer. Find the speed of the rickshaw and the bus**.

**Solution:** Let the speed of the rickshaw and the bus are km/hr and ykm/hr, respectively.

Now, she has taken time to travel 2km by rickshaw, $t_1 = \frac{2}{x}$ h

and she has taken time to travel the remaining distance i.e., (14 – 2) = 12km by bus = $t_2 = \frac{12}{y}$ h

By first condition, $t_1 + t_2 = \frac{1}{2}$

or, $\frac{2}{x} + \frac{12}{y} = \frac{1}{2}$..........................................................(1)

Now, she has taken the time to travel 4km by rickshaw, $t_3 = \frac{4}{x}$h

and she has taken time to travel remaining distance i.e., (14 – 4) = 10km by bus = $t_2$ = $\frac{12}{y}$ h

By second condition, $t_3 + t_4 = \frac{1}{2} + \frac{9}{60} = \frac{1}{2} + \frac{3}{20}$

or, $\frac{4}{x} + \frac{10}{y} = \frac{13}{20}$ ..............................................................(2)

Let $\frac{1}{x}$ = u and $\frac{1}{y}$ = v, then eq(1) and (2) becomes,

$2u + 12v = \frac{1}{2}$ ........................................................................... (3)

$4u + 10v = \frac{13}{20}$ ...........................................................................(4)

On multiplying eq(2) and then subtracting, we get

$$\begin{array}{rcl} 4u + 24v &=& 1 \\ 4u + 10v &=& \frac{13}{20} \\ (-) \quad (-) & & (-) \\ \hline 14v &=& 1 - \frac{13}{20} = \frac{7}{20} \end{array}$$

or, $2v = \frac{1}{20}$

or, v = $\frac{1}{40}$
Now, put the value of v in eq(3) we get,

$$2u + 12\left(\frac{1}{40}\right) = \frac{1}{2}$$

or, 2u = $\frac{1}{2} - \frac{3}{10} = \frac{5-3}{10}$
or, 2u = $\frac{2}{10}$
or, u = $\frac{1}{10}$

Thus, $\frac{1}{x}$ = u
or, $\frac{1}{x} = \frac{1}{10}$
or, x = 10km/hr
and, $\frac{1}{y}$ = v
or, $\frac{1}{y} = \frac{1}{40}$
or, y = 40km/hr

Hence, the speed of the rickshaw and the bus are 10km/hr and 40km/hr, respectively

**Question 7: A person, rowing at the rate of 5 km/h in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.**
**Solution:**
Let the speed of the stream be v km/h.
Given that, a person rowing in still water = 5 km/h
The speed of a person rowing downstream = (5+ v) km/h
and the speed of a person has rowed in upstream = (5 – v) km/h
Now, the person took time to cover 40 km downstream,
$t_1 = \frac{40}{5+v}$ h ……………………..$\left[\text{Speed} = \frac{\text{distance}}{\text{time}}\right]$
and the person has taken time to cover 40km upstream,
$t_2 = \frac{40}{5-v}$ h

By condition, $t_2 = t_1 \times 3$
or, $\frac{40}{5-v} = \frac{40}{5+v} \times 3$

or, $\frac{1}{5-v} = \frac{3}{5+v}$

or, 5 + v = 15 – 3v
or, 4v = 10
or, v = 2.5km/hr

Hence, the speed of the stream is 2.5 km/h,

**Question 8: A motorboat can travel 30 km upstream and 28 km downstream in 7 h. It can travel 21 km upstream and return in 5 h. Find the speed of the boat in still water and the speed of the stream.**

**Solution:** Let the speed of the motorboat in still water and the speed of the stream are u km/h and v km/h, respectively.
Then, a motorboat speed in downstream = (u + v) km/h and a motorboat speed in upstream = (u – v) km/h.
Motorboat has taken time to travel 30 km upstream,
$t_1 = \frac{30}{u-v}\ h$
and motorboat has taken time to travel 28km downstream,
$t_2 = \frac{28}{u+v}$ h

By the first condition, a motorboat can travel 30km upstream and 28 km downstream in 7hours i.e.,
$t_1 + t_2 = 7h$
or, $\frac{30}{u-v} = \frac{28}{u+v} = 7$ ………………………………………….(1)
Now, motorboat has taken time to travel 21km upstream and return i.e., $t_3 = \frac{21}{u-v}$ [for upstream]
$t_4 = \frac{21}{u+v}$ [for downstream]

By second condition, $t_3 + t_4 = 5$hours
or, $\frac{21}{u-v} + \frac{21}{u+v} = 5$ ……………………………………………………….(2)
Let $x = \frac{1}{u+v}$ and $y = \frac{1}{u-v}$
Eq(1) and (2) becomes, $30x + 28y = 7$ ………………………………….(3)
and, $21x + 21y = 5$
or, $x + y = \frac{5}{21}$………………………………………………………………..(4)

Now, multiplying in eq(4) by 28 and then subtracting from eq(3) we get

$$
\begin{aligned}
30x + 28y &= 7 \\
28x + 28y &= \frac{140}{21} \\
(-) \quad (-) \quad &(-) \\
\hline
2x = 7 - \frac{20}{3} &= \frac{21-20}{3}
\end{aligned}
$$

or, $2x = \frac{1}{3}$
or, $x = \frac{1}{6}$

On putting the value of x in eq(4) we get,
$\frac{1}{6} + y = \frac{5}{21}$
or, $y = \frac{5}{21} - \frac{1}{6} = \frac{10-7}{42} = \frac{3}{42}$
or, $y = \frac{1}{4}$

$x = \frac{1}{u+v} = \frac{1}{6}$

or, u + v = 6 ……………………………………………………………….(5)

and, y = $\frac{1}{u-v}$ = $\frac{1}{14}$
or, u – v = 14
Now adding eq(5) and (6), we get,
    2u = 20
or, u = 10
On putting the value of u in eq(5) we get,
    10 + v = 6
or, v = -4
Hence, the speed of the motorboat in still water is 10 km/h and the speed of the stream 4 km/h.

**Question 9:**
**A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.**
**Solution:** Let the two-digit number = 10x + y

Case 1:
Multiplying the sum of the digits by 8 and then subtracting 5 = 2-digit number
or, 8(x + y) – 5 = 10x + y
or, 8x + 8y – 5 = 10x + y
or, 2x – 7y = -5 …………………………………………………..(1)

Case 2:
Multiplying the difference of the digits by 16 and then adding 3 = 2-digit number
or, 16(x – y) + 3 = 10x + y
or, 16x – 16y + 3 = 10x + y
or, 6x – 17y = -3…………………………………………………..(2)

Now, multiplying in eq(1) by 3 and then subtracting from eq(2), we get

    6x – 17y = -3
    6x – 21y = -15
  (-)    (+)        (+)

      4y = 12
or, y = 3

Now put the value of y in eq(1) we get
    2x – 7(3) = -5
or, 2x = 21 – 5 = 16
or, x = 8

Hence, the required 2-digit number = 10x + y
= 10(8) + 3
= 80 + 3 = 83

**Question 10: A railway half ticket cost half the full fare but the reservation charges are the same on a half ticket as on a full ticket. One reserved first-class ticket from the station's A to B costs ₹ 2530. Also, one reserved first-class ticket and one reserved first-class half ticket from stations A to B costs ₹ 3810. Find the full first-class fare from stations A to B and also the reservation charges for a ticket.**

**Solution:** Let the cost of full and half first-class fare be Rs. x and Rs $\frac{x}{2}$ respectively and reservation charges are ₹ y per ticket.

<u>Case 1:</u>
The cost of one reserved first-class ticket from the station's A to B = Rs. 2530
or, x + y = 2530…………………………………………………………(1)

<u>Case 2:</u>
The cost of one reserved first-class ticket and one reserved first-class half ticket from stations A and B = Rs 3810
or, x + y + $\frac{x}{2}$ + y = 3810
or, $\frac{3x}{2}$ + 2y = 3810
or, 3x + 4y = 7620……………………………………………………………(2)

Now multiplying eq(1) by 4 and then subtracting from eq(2) we get,

3x + 4y = 7620
4x + 4y = 10120
(-) (-) (-)

- x = = 2500
x = 2500

On putting the value of x in eq(1) we get,
2500 + y = 2530
y = 2530 – 2500
y = 30

Hence, full first-class fare from stations A to 6 is ₹ 2500 and the reservation for a ticket is ₹30.

**Question 11: A shopkeeper sells a saree at 8% profit and a sweater at a 10% discount, thereby, getting a sum of ₹ 1008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got ₹ 1028 then find the cost of the saree and the list price (price before discount) Of the sweater**.

**Solution:** Let the cost price of the saree and the list price of the sweater be Rs x and Rs y, respectively.

Case 1:

Sells a saree at 8% profit + Sells a sweater at 10% discount = Rs 1008
or, (100 + 8)% of x + (100 – 10)% of y = 1008
or, 108% of x + 90% of y = 1008
or, 108x + 0.9y = 1008……………………………………(1)

Case 2:

Sold the saree at 10% profit + Sold the sweater at 8% discount = Rs 1028
or, (100 + 10)% of x + (100 – 8)% of y = Rs 1028
or, 110% of x + 92% of y = 1028
or, 1.1x + 0.92y = 1028…………………………(2)
On putting the value of y from eq(1) into eq(2), we get,
$1.1x + 0.92\left[\frac{1008-1.08x}{0.9}\right] = 1028$
or, 1.1(0.9x) + 927.36 – 0.9936x = 1028(0.9)
or, 0.99x – 0.9936x = 9252 – 927.36
or, -0.0036x = -2.16
or, $x = \frac{2.16}{0.0036} = 600$

On putting the value of x in eq(1) we get,
1.08(600) + 0.9y = 1008
or, 108(6) + 0.9y = 1008
or, 0.9y = 1008 – 648
or, 0.9y = 360
or, $y = \frac{360}{0.9} = 400$

Hence, the cost price of the saree and the list price (price before discount) of the sweater are ₹ 600 and ₹ 400, respectively.

**Question 12: Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received ₹ 1860 as annual interest. However, had she interchanged the amount of investments in the two schemes, she would have received ₹ 20 more as annual interest. How much money did she invest in each scheme?**

**Solution:** Let the amount of investments in schemes A and 6 be ₹ x and ₹ y, respectively.

Case 1:

Interest at the rate of 8% per annum on scheme A+ Interest at the rate of 9% per annum on scheme 6 = Total amount received

or, $\frac{x\times8\times1}{100}+\frac{y\times9\times1}{100}$ = Rs 1860……….$\left[\mathrm{SI}\ =\ \frac{\text{principal}\ \times\text{rate}\times\text{time}}{100}\right]$
or, 8x + 9y = 186000

<u>Case 2:</u>
Interest at the rate of 9% per annum on scheme A + interest at the rate of 8% per annum on scheme B = Rs 20 more as annual interest
or, $\frac{x\times9\times1}{100}+\frac{y\times8\times1}{100}$ = Rs (20 + 1860)
or, $\frac{9x}{100}+\frac{8y}{100}$ = 1880
or, 9x + 8y = 188000……………………………………………………..(2)

On subtracting eq(1) by 9 and eq(2) by 8 and then subtracting them we get,

72x + 81y = 9(186000)
72x + 64y = 8(188000)
(-)__(-)____(-)_______

17y = 1000[(9×186) – (8×188)]
= 1000 (1674 – 1504)
= 1000(170)
y = 10000

On putting the value of y in eq(1), we get
8x + 9(10000) = 186000
or, 8x = 186000 – 90000
or, 8x = 96000
or, x = 12000

**Question 13: Vijay had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of ₹ 2 for 3 bananas and the second lot at the rate of ₹ 1 per banana and got a total of ₹ 400 If he had sold the first lot at the rate of ₹ 1 per banana and the second lot at the rate of ₹ 4 for 5 bananas, his total collection would have been ₹ 460. Find the total number of bananas he had.**
Solution: Let the number of bananas in lots A and B be x and y, respectively.
<u>Case 1:</u> Cost of the first lot at the rate of Rs. 2 for 3 bananas + cost of the second lot at the rate of Rs 1 per banana = Amount received
or, $\frac{2}{3}$x + y = 400
or, 2x + 3y = 1200……………………………………………(1)

Case 2: Cost the first lot at the rate of Rs 1 per banana + cost of the second lot at the rate of Rs 4 for 5 bananas = Amount received

or, $x + \frac{4}{5}y = 460$

or, $5x + 4y = 2300$……………………………………………………….(2)

On multiplying in eq (1) by 4 and eq(2) by 3 and then subtracting them, we get,

$$\begin{array}{r} 8x + 12y = 4800 \\ 15x + 12y = 6900 \\ \hline -7x = -2100 \\ x = 300 \end{array}$$

Now put the value of x in eq (1),

$2(300) + 3y = 1200$

or, $600 + 3y = 1200$

or, $3y = 1200 – 600$

or, $3y = 600$

or, $y = 200$

Therefore, total number of bananas = Number of bananas in lot A + Number of bananas in lot B = x + y = 300 + 200 = 500

Hence, he had 500 bananas.