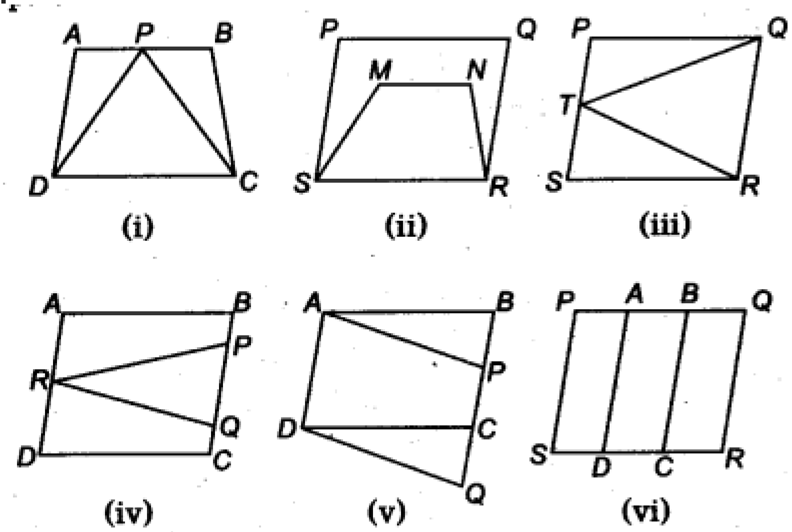
Chapter 9: Area of parallelograms and Triangles

Exercise 9.1

Question 1: Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.

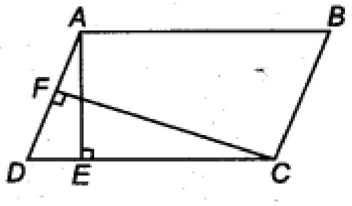


Answer: The figures (i), (iii) and (v) lie on the same base and between the same parallels.

	Common base	Two parallels
Fig (i)	DC	DC and AB
Fig (iii)	QR	QR and PS
Fig (v)	AD	AD and BQ

Exercise 9.2

Question 1: In figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.

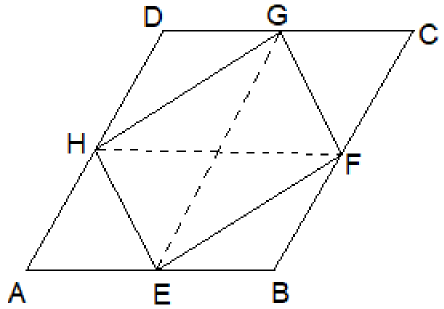


Answer: We have, AE ⊥ DC and AB = 16 cm [Given]
Since, AB = CD [Opposite sides of parallelogram]
Therefore, CD = 16 cm

Now, area of parallelogram ABCD = CD x AE
= (16 x 8) cm^2 = 128 cm^2 [AE = 8 cm]
Since, CF ⊥ AD
Therefore, Area of parallelogram ABCD = AD x CF
or, AD x CF = 128 cm
or, AD x 10 cm = 128 cm^2 [CF= 10 cm]
or, AD = $\frac{128}{10}$ cm = 12.8 cm
Thus, the required length of AD is 12.8 cm

Question 2: If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that ar (EFGH) = $\frac{1}{2}$ ar (ABCD).

Answer:



We need to join GE and HE, where GE || BC || DA and HF || AB || DC
(As, E, F, G and H are the mid-points of the sides of a parallelogram ABCD).

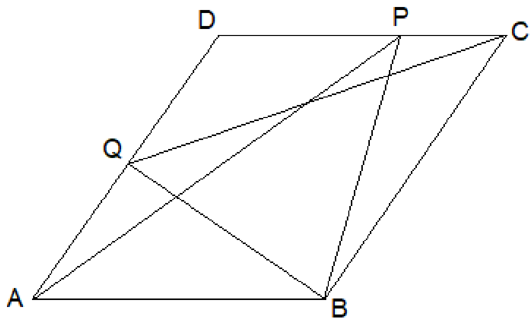
If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram.

Now, ∆EFG and parallelogram EBCG are on the same base EG and between the same parallels EG and BC.
Hence, ar(∆EFG) = $\frac{1}{2}$ ar(parallelogram EBCG) ……………………………… (1)

Similarly, ar(ΔEHG) = $\frac{1}{2}$ ar(parallelogram AEGD) …………………………………..…(2)
Adding (1) and (2), we get
ar(ΔEFG) + ar(ΔEHG) = $\frac{1}{2}$ ar(parallelogram EBCG)+ $\frac{1}{2}$ ar(parallelogram AEGD)
= $\frac{1}{2}$ ar(parallelogram ABCD)
Thus, ar(EFGH) = $\frac{1}{2}$ar(ABCD)

Question 3: P and Q are any two points lying on the sides DC and AD, respectively of a parallelogram ABCD. Show that ar (APB) = ar(BQC).

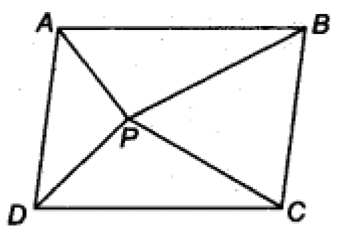
Answer:



Given, ABCD is a parallelogram.
therefore, AB || CD and BC || AD.
Now, ΔAPB and parallelogram ABCD are on the same base AB and between the same parallels AB and CD.
Hence ar(ΔAPB) = $\frac{1}{2}$ ar(parallelogram ABCD) …………………………………..….(1)

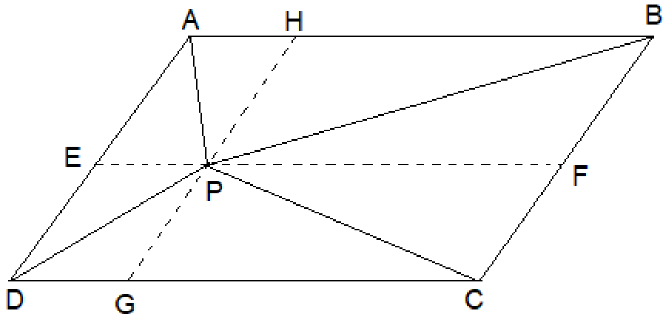
Also, ΔBQC and parallelogram ABCD are on the same base BC and between the same parallels BG and AD.
thus, ar(ΔBQC) = $\frac{1}{2}$ ar(parallelogram ABCD) …………………………………(2)
So, from (1) and (2), we have ar(ΔAPB) = ar(ΔBQC).

Question 4: In figure, P is a point in the interior of a parallelogram ABCD. Show that



(i) ar (APB) + ar (PCD) = $\frac{1}{2}$ar(ABCD)
(ii) ar (APD) + ar(PBC) = ar (APB) + ar (PCD)

Answer: We have a parallelogram ABCD, i.e., AB || CD and BC || AD. Now for the convenience of the solution let us draw EF || AB and HG || AD through P.



(i) ΔAPB and parallelogram AEFB are on the same base AB and between the same parallels AB and EF.

Therefore, ar(ΔAPB) = $\frac{1}{2}$ ar(parallelogram AEFB) ………………………………..…(1)

Also, ΔPCD and parallelogram CDEF are on the same base CD and between the same parallels CD and EF.

thus, ar(APCD) = $\frac{1}{2}$ ar(parallelogram CDEF) …………………………………..…(2)

Adding (1) and (2), we have

ar(ΔAPB) + ar(ΔPCD) = $\frac{1}{2}$ ar(parallelogram AEFB)+ $\frac{1}{2}$ ar(parallelogram CDEF)

or, ar(ΔAPB) + ar(ΔPCD) = $\frac{1}{2}$ ar(parallelogram ABCD) ……………………..…(3)

(ii) ΔAPD and parallelogram ΔDGH are on the same base AD and between the same parallels AD and GH.

therefore, ar(ΔAPD) = $\frac{1}{2}$ ar(parallelogram ADGH) ……………………………………..…(4)

Similarly,

ar(ΔPBC) = $\frac{1}{2}$ ar(parallelogram BCGH) ……………………………………..…(5)

Adding (4) and (5), we have

ar(ΔAPD) + ar(ΔPBC) = = $\frac{1}{2}$ ar(parallelogram ADGH)+ $\frac{1}{2}$ ar(parallelogram BCGH)

or, ar(ΔAPD) + ar(ΔPBC) = $\frac{1}{2}$ ar(parallelogram ABCD) ……………………….(6)

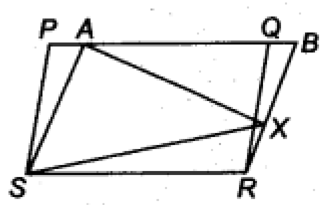
From (3) and (6), we have

ar(ΔAPD) + ar(ΔPBC) = ar(ΔAPB) + ar(ΔPCD)

Question 5: In figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

(i) ar (PQRS) = ar (ABRS)

(ii) ar (AXS) = $\frac{1}{2}$ar(PQRS)

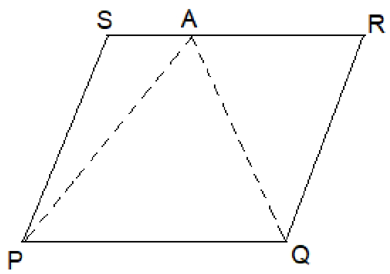


Answer: (i) Parallelogram PQRS and parallelogram ABRS are on the same base RS and between the same parallels RS and PB.
Thus, ar(PQRS) = ar(ABRS)

(ii) Triangle AXS and parallelogram ABRS are on the same base AS and between the same parallels AS and BR.
ar(AXS) = $\frac{1}{2}$ar(ABRS) ……………………………………………..(1)
But ar(PQRS) = ar(ABRS) …………………………………………..(2) [Proved in (i) part]
From (1) and (2), we have
ar(AXS) = $\frac{1}{2}$ar(PQRS)

Question 6: A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it.

Answer:



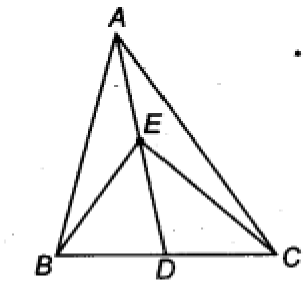
The farmer is having the field in the form of parallelogram PQRS and a point A is situated on RS. Now join AP and AQ. The given field is divided into three parts i.e., in ∆APS, ∆PAQ and ∆QAR.

Since, ∆PAQ and parallelogram PQRS are on the same base PQ and between the same parallels PQ and RS.
ar(∆PAQ) = $\frac{1}{2}$ar(parallelogram PQRS) …………………………………………………..(1)
or, ar(parallelogram PQRS) – ar(∆PAQ) = ar(parallelogram PQRS)

$-\frac{1}{2}$ar(parallelogram PQRS)
or, [ar(ΔAPS) + ar(ΔQAR)] = $\frac{1}{2}$ar(parallelogram PQRS) ……………………………..(2)
From (1) and (2), we have
ar(ΔPAQ) = ar[(ΔAPS) + (ΔQAR)]
Thus, the farmer can sow wheat in (ΔPAQ) and pulses in [(ΔAPS) + (ΔQAR)] or wheat in [(ΔAPS) + (ΔQAR)] and pulses in (ΔPAQ).

Exercise 9.3

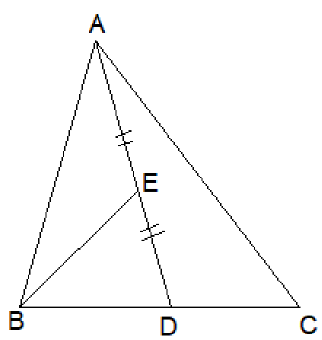
Question 1: In figure, E is any point on median AD of a ΔABC. Show that ar (ABE) = ar (ACE).



Answer: We have a ΔABC such that AD is a median.
therefore, ar(ΔABD) = ar(ΔACD) …………………(1) [A median divides the triangle into two triangles of equal areas]
Similarly, in ΔBEC, we have
ar(ΔBED) = ar(ΔDEC) ………………………………..(2)
Subtracting (2) from (1), we have
ar(ΔABD) – ar(ΔBED) = ar(ΔACD) – ar(ΔDEC)
or, ar(ΔABE) = ar(ΔACE).

Question 2: In a triangle ABC, E is the mid-point of median AD. Show that ax (BED) = $\frac{1}{2}$ar(ABC).

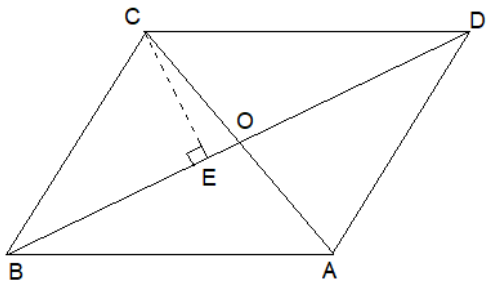
Answer:



We have a ΔABC and its median AD.
Let us join B and E.
Since, a median divides the triangle into two triangles of equal area.
ar (ΔABD) = $\frac{1}{2}$ar(ΔABC) ……………………………………………….…(1)
Now, in ΔABD, BE is a median. [E is the mid-point of AD]
∴ ar(ΔBED) = $\frac{1}{2}$ar(ΔABC) …………………………………………………..(2)
From (1) and (2), we have
ar(ΔBED) = $\frac{1}{2}$ar(ΔABC)]
or, ar(ΔBED) = $\frac{1}{4}$ar(ΔABC)

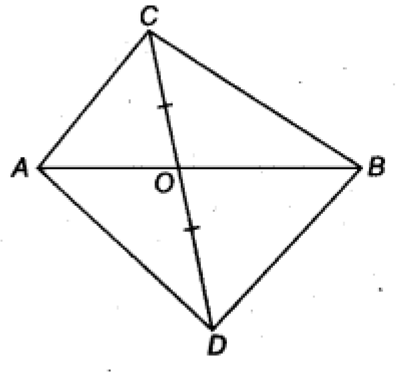
Question 3: How that the diagonals of a parallelogram divide it into four triangles of equal area.

Answer: Let us have a parallelogram ABCD such that its diagonals intersect at O.
Therefore, as per properties diagonals of a parallelogram bisect each other.
therefore, AO = OC and BO = OD
Let us draw CE ⊥ BD.
Now, ar(ΔBOC) = $\frac{1}{2}$BO x CE and
ar(ΔDOC) = $\frac{1}{2}$OD x CE



Since, BO = OD
Therefore, ar(ΔBOC) = ar(ΔDOC) ……………………………………..(1)
Similarly, ar(ΔAOD) = ar(ΔDOC) ……………………………….….(2)
and ar(ΔAOB) = ar(ΔBOC) ………………………………………………..(3)
From (1), (2) and (3), we have
ar(ΔAOB) = ar(ΔBOC) = ar(ΔCOD) = ar(ΔDOA)
Thus, the diagonals of a parallelogram divide it into four triangles of equal area.

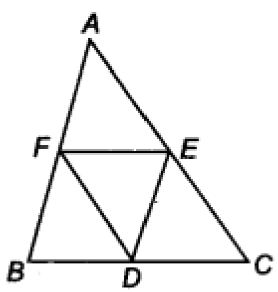
Question 4: In figure, ABC and ABD are two triangles on the same base AB. If line segment CD is bisected by AB at O, show that ar(ABC) = ar(ABD)



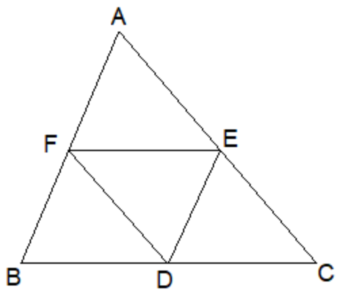
Answer: We have ΔABC and ΔABD are on the same base AB.
as, CD is bisected at O. [Given]
hence, CO = OD

Now, in ΔACD, given, AO is a median
therefore, ar(ΔOAC) = ar(ΔOAD) …………………………..(1)
Again, in ΔBCD, given, BO is a median
therefore, ar(ΔOBC) = ar(ΔODB) ………………………………(2)
Adding (1) and (2), we have
ar(ΔOAQ + ar(ΔOBQ) = ar(ΔOAD) + ar(ΔODB)
or, ar(ΔABC) = ar(ΔABD)

Question 5: D,E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Show that
(i) BDEF is a parallelogram.
(ii) ar(DEF) = $\frac{1}{4}$ar(ABC)
(iii) ar(BDEF) = $\frac{1}{4}$ar(ABC)



Answer:



We have ΔABC such
that D,E and Fare the mid-points of BC, CA and AB respectively.
(i) In ΔABC, E and F are the mid-points of AC and B D C AB respectively.
hence, EF || BC [Mid-point theorem]
or, EF || BD
Also, EF = $\frac{1}{2}$(BC)
or, EF = BD [D is the mid – point of BC]
Since BDEF is a quadrilateral whose one pair of opposite sides is parallel and of equal lengths.
Hence, BDEF is a parallelogram.

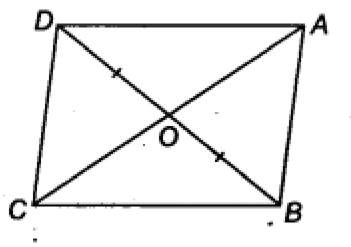
(ii) We have earlier proved that BDEF is a parallelogram.
Similarly, DCEF is a parallelogram and DEAF is also a parallelogram.
Now, parallelogram BDEF and parallelogram DCEF are on the same base EF and between the same parallels BC and EF.
Hence, ar(parallelogram BDEF) = ar(parallelogram DCEF)
or, $\frac{1}{2}$ar(parallelogram BDEF) = $\frac{1}{2}$ar(parallelogram DCEF)
or, ar(ΔBDF) = ar(ΔCDE) ……………………….(1) [Diagonal of a parallelogram divides it into two triangles of equal area]
Similarly, ar(ΔCDE) = ar(ΔDEF) …………………………(2)
and ar(ΔAEF) = ar(ΔDEF) ……………………………..(3)

From (1), (2) and (3), we have
ar(ΔAEF) = ar(ΔFBD) = ar(ΔDEF) = ar(ΔCDE)
Thus, ar(ΔABC) = ar(ΔAEF) + ar(ΔFBD) + ar(ΔDEF) + ar(ΔCDE) = 4 ar(ΔDEF)
or, ar(ΔDEF) = $\frac{1}{4}$ar(ΔABC)

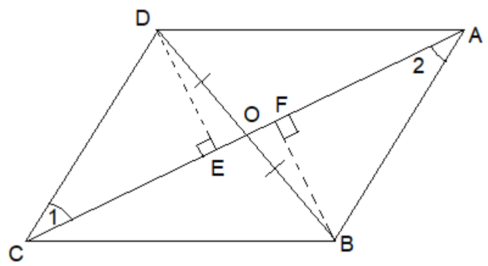
(iii) We have, ar (parallelogram BDEF) = ar(ΔBDF) + ar(ΔDEF)
= ar(ΔDEF) + ar(ΔDEF) [As, ar(ΔDEF) = ar(ΔBDF)]
2ar(ΔDEF) = 2[$\frac{1}{4}$ar(ΔABC)] = $\frac{1}{2}$ar(ΔABC)
Thus, ar (parallelogram BDEF) = $\frac{1}{2}$ar(ΔABC)

Question 6: In figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that
(i) ar(DOC) = ar(AOB)

(ii) ar (DCB) = ar (ACB)
(iii) DA || CB or ABCD is a parallelogram



Answer:



We have a quadrilateral ABCD whose diagonals AC and BD intersect at O.
We also have that OB = OD, AB = CD Let us draw DE ⊥ AC and BF ⊥ AC

(i) In ΔDEO and ΔBFO, we have
DO = BO [Given]
∠DEO = ∠BFO [Each 90°]
∠DOE = ∠BOF [Vertically opposite angles]

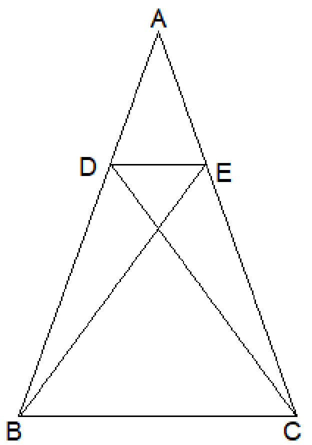
therefore, ΔDEO ≅ ΔBFO [By A AS congruency]
or, DE = BF [By C.P.C.T.]
and ar(ΔDEO) = ar(ΔBFO) …………………………………..…(1)
Now, in ΔDEC and ΔBFA, we have
∠DEC = ∠BFA [Each 90°]
DC = BA [Given]
DE = BF [Proved above]
Therefore, ΔDEC ≅ ΔBFA [By RHS congruency]
or, ar(ΔDEC) = ar(ΔBFA) ………………………………………(2)
and ∠1 = ∠2 ……………………………………………………….(3) [By C.P.C.T.]
Adding (1) and (2), we have
ar(ΔDEO) + ar(ΔDEC) = ar(ΔBFO) + ar(ΔBFA)
or, ar(ΔDOC) = ar(ΔAOB)

(ii) Since, ar(ΔDOC) = ar(ΔAOB) [Proved above]
Adding ar(ΔBOC) on both sides, we have
ar(ΔDOC) + ar(ΔBOC) = ar(ΔAOB) + ar(ΔBOC)
or, ar(ΔDCB) = ar(ΔACB)

(iii) Since, ∆DCS and ∆ACB are both on the same base CB and having equal areas.
Hence, they lie between the same parallels BC and AD i.e., CB || DA
Also ∠1 = ∠2, [By (3)] which are alternate interior angles.
So, AB || CD
Hence, ABCD is a parallelogram.

Question 7: D and E are points on sides AB and AC respectively of ∆ ABC such that ar (DBC) = ar (EBC). Prove that DE || BC.

Answer:

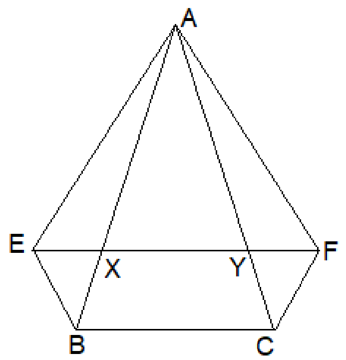


We have ∆ABC and points D and E are such that ar(DBC) = ar{EBC)
Since ∆DBC and ∆EBC are on the same base BC and having same area.

Thus, they must lie between the same parallels DE and BC.
Hence, DE || BC

Question 8: XY is a line parallel to side BC of a ∆ ABC. If BE ||AC and CF || AB meet XY at E and F respectively, show that ar (ABE) =ar (ACF)

Answer:

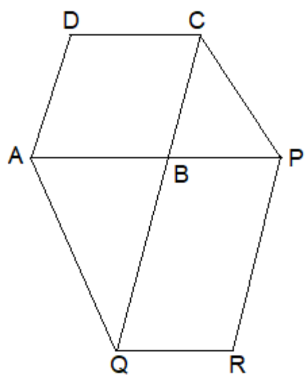


We have a ΔABC such that XY || BC,
BE || AC and CF || AB.
Since, XY ||BC and BE || CY
Hence, BCYE is a parallelogram.

Now, the parallelogram BCYE and ΔABE are on the same base 8E and between the same parallels BE and AC.

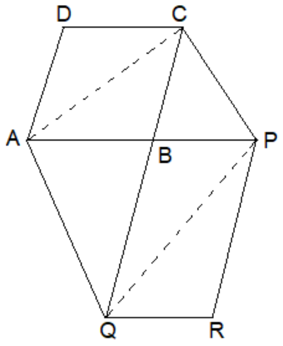
Therefore ar(ΔABE) = $\frac{1}{2}$ar(parallelogram BCYE) …………………………..(1)
Again, CF || AB [Given]
XY || BC [Given]
CF || BX and XF || BC
Therefore BCFX is a parallelogram.
Now, ΔACF and parallelogram BCFX are on the same base CF and between the same parallels AB and CF.
Therefore ar(ΔACF) = $\frac{1}{2}$ar(parallelogram BCFX) ………………………………..(2)
Also, parallelogram BCFX and parallelogram BCYE are on the same base BC and between the same parallels BC and EF.
Therefore ar(parallelogram BCFX) = ar(parallelogram BCYE) ……………………(3)
From (1), (2) and (3), we get
ar(ΔBE) = ar(ΔACF)

Question 9:The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then A parallelogram PBQR is completed (see figure).
Show that ax (ABCD) = ar(PBQR).
[Hint Join AC and PQ. Now compare ar (ACQ) and ar (APQ).]



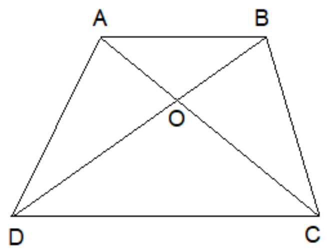
Answer: Let us join AC and PQ.
ABCD is a parallelogram [Given]
and AC is its diagonal, we know that diagonal of a parallelogram divides it into two triangles of equal areas.
Therefore ar(ΔABC) = $\frac{1}{2}$ar (parallelogram ABCD) …………………………..…(1)
Also, PBQR is a parallelogram [Given]
and QP is its diagonal.
Therefore(ΔBPQ) = $\frac{1}{2}$ar(parallelogram PBQR) ……………………………………(2)

Since, ∆ACQ and AAPQ are on the same base AQ and between A the same parallels AQ and CP.

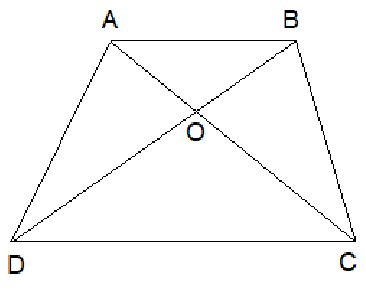


Therefore ar(∆ACQ) = ar(∆APQ)
= ar(∆ACQ) – ar(∆ABQ)
= ar(∆APQ) – ar(∆ABQ)
[Subtracting ar(∆ABQ) from both sides]
or, ar(∆ABC) = ar(∆BPQ) ……………………………………..(3)
From (1), (2) and (3), we get
$\frac{1}{2}$ar(parallelogram ABCD) = $\frac{1}{2}$ar(parallelogram PBQR)
or, ar(parallelogram ABCD) = ar(parallelogram PBQR)

Question 10: Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC)

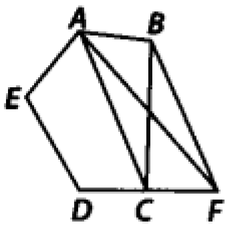


Answer: We have a trapezium ABCD having AB || CD and its diagonals AC and BD intersect each other at O.



As we know that, triangles on the same base and between the same parallels have equal areas.
∆ABD and ∆ABC are on the same base AB and between the same parallels AB and DC
Hence, ar(∆ABD) = ar(∆ABC)
Subtracting ar(∆AOB) from both sides, we get
ar(∆ABD) – ar(∆AOB) = ar(∆ABC) – ar(∆AOB)
or, ar(∆AOD) = ar(∆BOC)

Question 11: In figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that
(i) ar (ACB) = ar (ACF)
(ii) ar (AEDF) = ar (ABCDE)



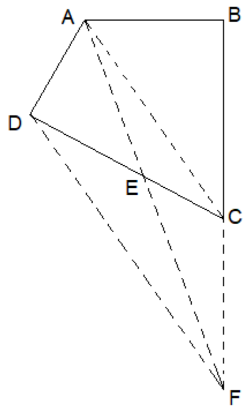
Answer: We have a pentagon ABCDE in which BF || AC and DC is extended to to F.

(i) Since, the triangles on the same base and between the same parallels are equal in area.
∆ABC and ∆ACF are on the same base i.e., AC and between the same parallels AC and BF.
Hence, ar(∆ACB) = ar(∆ACF)

(ii) Since, ar(∆ACB) = ar(∆ACF) [As proved above]
Adding ar(quad. AEDC) to both sides, we get
or, ar(∆ACB) + ar(quad. AEDC) = ar(∆ACF) + ar(quad. AEDC)
Hence, ar(ABCDE) = ar(AEDF)

Question 12: A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Answer:



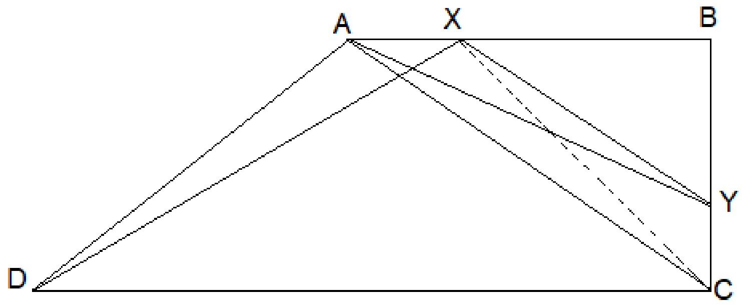
We have a plot in the form of a quadrilateral ABCD.
Let us draw DF || AC and join AF and CF.

Now, ∆DAF and ∆DCF are on the same base DF and between the same parallels AC and DF.
Hence, ar(ADAF) = ar(ADCF)

Then after subtracting ar(∆DEF) from both sides, we get
ar(∆DAF) – ar(∆DEF) = ar(∆DCF) – ar(∆DEF)
or, ar(∆ADE) = ar(∆CEF)
The portion of ∆ADE can be taken over by the Gram Panchayat by adding the land (∆CEF) to his Itwaari land so as to form a triangular plot,
i.e. ∆ABF.
To prove that ar(∆ABF) = ar(quad. ABCD), we have
ar(ACEF) = ar(AADE) [As, proved above]
Adding ar(quad. ABCE) to both sides, we get
ar(∆CEF) + ar(quad. ABCE) = ar(∆ADE) + ar (quad. ABCE)
or, ar(∆ABF) = ar (quad. ABCD)

Question 13: ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar(ADX) = ar(ACY). [Hint Join CX]

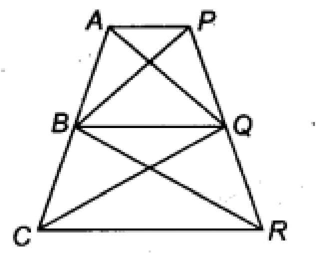
Answer:



We have a trapezium ABCD such that AB || DC.
XY || AC meets AB at X and BC at Y. Let us join CX.

ΔADX and ΔACX are on the same base AX and between the same parallels AX and DC.
Therefore, ar(ΔADX) = ar(ΔACX) ……………………………………………..(1)
As, ΔACX and ΔACY are on the same base AC and between the same parallels AC and XY.
Hence, ar(ΔACX) = ar(ΔACY) ………………………………………………….(2)
From (1) and (2), we have
ar(ΔADX) = ar(ΔACY)

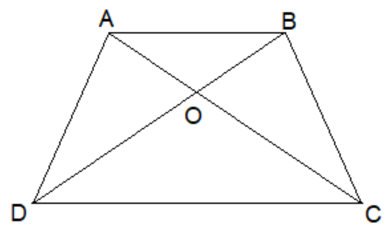
Question 14: In figure, AP || BQ || CR. Prove that ar(AQC) = ax(PBR).



Answer: We have, AP || BQ || CR
Since, ΔBCQ and ΔBQR are on the same base BQ and between the same parallels BQ and CR.
Therefore, ar(ΔBCQ) = ar(ΔBQR) ………………………………………………….(1)
Since, ΔABQ and ΔPBQ are on the same base BQ and between the same parallels AP and BQ.
Therefore, ar(ΔABQ) = ar(ΔPBQ) …………………………………………………..(2)
Adding (1) and (2), we have
ar(ΔBCQ) + ar(ΔABQ) = ar(ΔBQR) + ar(ΔPBQ)
or, ar(ΔAQC) = ar(ΔPBR)

Question 15: Diagonals AC and BD of a quadrilateral ABCD intersect at 0 in such a way that ax(AOD) = ar(BOC). Prove that ABCD is a trapezium.

Answer:

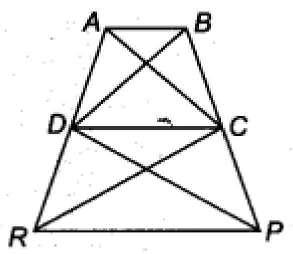


We have a quadrilateral ABCD and its diagonals AC and BD intersect at O and it is given that ar(ΔAOD) = ar(ΔBOC) [Given]

Adding ar(ΔAOB) to both sides, we have
ar(ΔAOD) + ar(ΔAOB) = ar(ΔBOC) + ar(ΔAOB)
Hence, ar(ΔABD) = ar(ΔABC)

Also, they are on the same base i.e., AB.
Since, the triangles are on the same base and having equal area.
Therefore, They must lie between the same parallels.
Hence, AB || DC
Now, ABCD is a quadrilateral having a pair of opposite sides parallel.
So, ABCD is a trapezium.

Question 16: In figure ax(DRC) = ar(DPC) and ai(BDP) = ar(ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.



Answer: We have, ar(ΔDRC) = ar(ΔDPC) [Given]
And they are on the same base i.e., DC.
Therefore, ΔDRC and ΔDPC lies between the same parallels.
So, DC || RP i.e. a pair of opposite sides of quadrilateral DCPR is parallel.
Hence, quadrilateral DCPR is a trapezium.

Again, we have

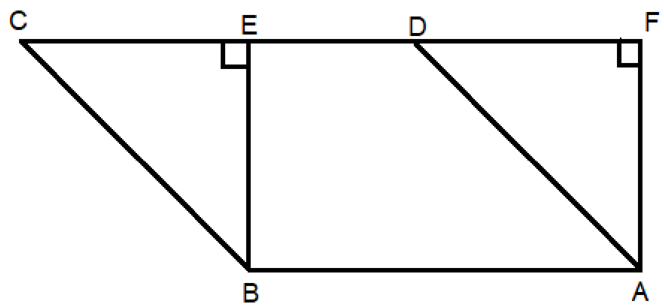
ar(ΔBDP) = ar(ΔARC) [Given] …………………………………………………..(1)
Also, ar(ΔDPC) = ar(ΔDRC) [Given] ……………………………………………….(2)
Subtracting (2) from (1), we get
ar(ΔBDP) – ar(ΔDPC) = ar(ΔARQ – ar(ΔDRQ)
or, ar(ΔBDC) = ar(ΔADC)
They are on the same base DC.

Hence, ΔBDC and ΔADC must lie between the same parallels.
So, AB || DC i.e. a pair of opposite sides of quadrilateral ABCD is parallel.
therefore, Quadrilateral ABCD is a trapezium.

Exercise 9.4

Question 1: Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Answer:

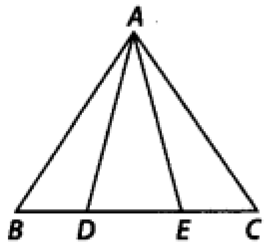


We have a parallelogram ABCD and rectangle ABEF such that
ar(parallelogram ABCD) = ar(rectangle ABEF)

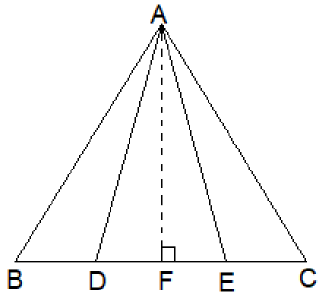
AB = CD [Opposite sides of parallelogram]
and AB = EF [Opposite sides of a rectangle]
or, CD = EF
or, AB + CD = AB + EF ……………………………………………………. (1)
BE < BC and AF < AD [In a right triangle, hypotenuse is the longest side]
or, (BC + AD) > (BE + AF) ……………………………………………………….(2)

From (1) and (2), we have
(AB + CD) + (BC+AD) > (AB + EF) + BE + AF)
or, (AB + BC + CD + DA) > (AB + BE + EF + FA)
or, Perimeter of parallelogram ABCD > Perimeter of rectangle ABEF.

Question 2: In figure, D and E are two points on BC such that BD = DE = EC. Show that ar(ABD) = ar(ADE) = ar(AEC).



Answer:



Let us draw AF for the convenience of the solution, which is perpendicular to BC in such a way that AF is the height of ∆ABD, ∆ADE and ∆AEC.

As we know that,

Area of the triangle = $\frac{1}{2}$ × base × height

Therefore, ar(∆ABD) = $\frac{1}{2}$ × BD × AF

Similarly, ar(∆ADE) = $\frac{1}{2}$ × DE × AF

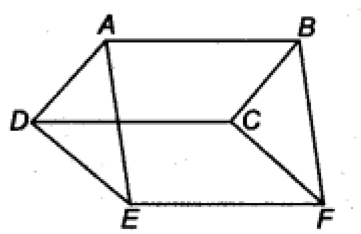
or, ar(∆AEC) = $\frac{1}{2}$ × EC × AF

Since, BD = DE = EC

Therefore, $\frac{1}{2}$ × BD × AF = $\frac{1}{2}$ × DE × AF = $\frac{1}{2}$ × EC × AF

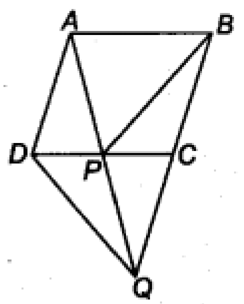
or, ar(∆ABD) = ar(∆ADE) = ar(∆AEC)

Question 3: In figure, ABCD, DCFE and ABFE are parallelograms. Show that ar(ADE) = ax(BCF).



Answer: Since, it is given that ABCD is a parallelogram.
Therefore, its opposite sides are parallel and equal.
i.e., AD = BC ………………………………………………………………….(1)
Now, ∆ADE and ∆BCF are on equal bases AD = BC [from (1)] and between the same parallels AB and EF.
So, ar(∆ADE) = ar(∆BCF).

Question 4: In figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that ar(BPC) = ax(DPQ).[Hint Join AC.]

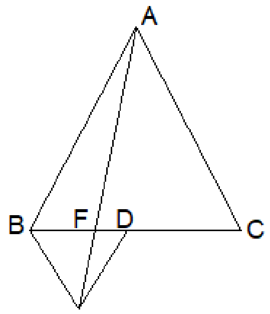


Answer: Given a parallelogram ABCD and AD = CQ. We join AC.
We know that triangles on same base and between the same parallels have equal area.
Since, ∆QAC and ∆QDC are on the same base QC and between the same parallels AD and BQ.
Therefore, ar(∆QAC) = ar(∆QDC)

Subtracting ar(∆QPC) from both sides, we have
ar(∆QAQ – ar(∆QPC) = ar(∆QDC) – ar(∆QPC)
Hence, ar(∆PAQ = ar(∆QDP) ………………………………………………….(1)
Since, ∆PAC and ∆PBC are on the same base PC and between the same parallels AB and CD.
Therefore, ar(∆PAC) = ar(∆PBC)
…………………………………………………………..(2)
From (1) and (2), we get
ar(∆PBC) = ar(∆QDP)

Question 5: In figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, Show that

i) ar(ΔBDE) = $\frac{1}{4}$ ar(ΔABC)
ii) ar(ΔBDE) = $\frac{1}{2}$ ar(ΔBAE)
iii) ar(ΔABC) = 2ar(ΔBEC)
iv) ar(ΔBFE) = ar(ΔAFD)
v) ar(ΔBFE) = 2ar(ΔFED)
vi) ar(ΔFED) = $\frac{1}{8}$ ar(ΔAFC)



Answer: Let us join EC and AD. Draw EP ⊥ BC.
Let AB = BC = CA = a, then
BD = $\frac{a}{2}$ = DE = BE

$ar(\Delta \text{ABC}) = \frac{\sqrt{3}}{4}a^2$ and

$$ar(\Delta \text{BDE}) = \frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2 = \frac{\sqrt{3}}{16}a^2$$

$or, ar(\Delta \text{BDE}) = \frac{1}{4}ar(\Delta \text{ABC})$

ii) Since, ∆ABC and ∆BED are equilateral triangles.
or, ∠ACB = ∠DBE = 60°
or, BE || AC
∆BAE and ∆BEC are on the same base BE and between the same parallels BE and AC.
ar(∆BAE) = ar(∆BEC)
or, ar(∆BAE) = 2 ar(∆BDE) [DE is median of ∆EBC. Hence, ar(∆BEC) = || ar(∆BDE)]
or, ar(ABDE) = $\frac{1}{2}$ar(∆BAE)

(iii) ar(∆ABC) = 4 ar(∆BDE) [Proved in (i) part]
ar(∆BEC) = 2 ar(∆BDE) [DE is median of ∆BEC]
or, ar(∆ABC) = 2 ar(∆BEC)

(iv) Since, ∆ABC and ∆BDE are equilateral triangles.
or, ∠ABC = ∠BDE = 60°
or, AB || DE

ΔBED and ΔAED are on the same base ED and between the same parallels AB and DE.
therefore, ar(ΔBED) = ar(ΔAED)
Subtracting ar(AEFD) from both sides, we get
or, ar(ΔBED) – ar(ΔEFD) = ar(ΔAED) – ar(ΔEFD)
or, ar(ΔBEE) = ar(ΔAFD)

v) $AD^2 = AB^2 - BD^2$

$or, AD^2 = a^2 - \frac{a^2}{4} = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4}$

$or, AD = \frac{\sqrt{3}a}{2}$

In right angled ΔPED,

$EP^2 = DE^2 - DP^2$

$or, EP^2 = \left(\frac{a}{2}\right)^2 - \left(\frac{a}{4}\right)^2 = \frac{a^2}{4} - \frac{a^2}{16} = \frac{3a^2}{16}$

$or, EP = \frac{\sqrt{3}a}{4}$

Therefore, $\Delta\text{AFD} = \frac{1}{2} \times FD \times AD$

$= \frac{1}{2} \times FD \times \frac{\sqrt{3}a}{2}$ …………………………………..(1)

and $ar(\Delta\text{EFD}) = \frac{1}{2} \times FD \times EP$

$= \frac{1}{2} \times FD \times \frac{\sqrt{3}a}{4}$ ……………………………………(2)

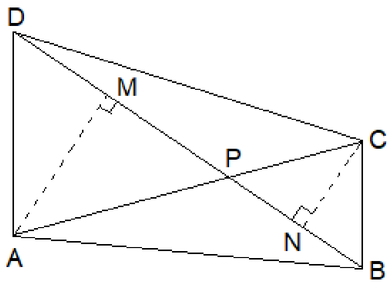
From (1) and (2), we get
ar(ΔAFD) = 2 ar(ΔEFD)
ar(ΔAFD) = ar(ΔBEF) ……………………..[From (iv) part]
or, ar(ΔBFE) = 2 ar(ΔEFD)

(vi) ar(ΔAFC) = ar(ΔAFD) + ar(ΔADC)
= ar(ΔBFE) + $\frac{1}{2}$ ar(ΔABC) ……………..[From (iv) part]
= ar(ΔBFE) + $\frac{1}{2}$ x 4 x ar(ΔBDE)……………………… [From (i) part]
= ar(ΔBFE) + 2ar(ΔBDE)
= 2ar(ΔFED) + 2[ar(ΔBFE) + ar(ΔFED)]

= 2ar(ΔFED) + 2[2ar(ΔFED) + ar(ΔFED)………………………….. [From (v) part]
= 2ar(ΔFED) + 2[3ar(ΔFED)]
= 2ar(ΔFED) + 6ar(ΔFED)
= 8ar(ΔFED)
Therefore, ar(ΔFED) = $\frac{1}{8}$ ar(ΔAFC)

Question 6: Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that ar(APB) x ar(CPD) = ar(APD) x ar(BPC).
[Hint From A and C, draw perpendiculars to BD.]

Answer: We have a quadrilateral ABCD such that its diagonals AC and BD intersect at P.
Let us draw AM ⊥ BD and CN ⊥ BD.



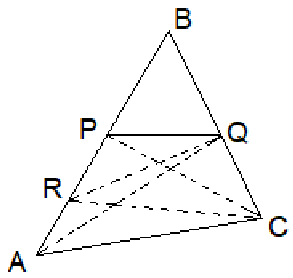
Now , $ar(\Delta APB) = \frac{1}{2} \times BP \times AM$
and , $ar(\Delta CDP) = \frac{1}{2} \times DP \times CN$
Therefore $ar(\Delta APB) \times ar(\Delta CPD) = \left(\frac{1}{2} \times BP \times AM\right) \times \left(\frac{1}{2} \times DP \times CN\right)$
$= \frac{1}{4} \times BP \times DP \times AM \times CN$ ……………………………….(1)
Similarly, $ar(\Delta APD) \times ar(\Delta BPC)$
$= \left(\frac{1}{2} \times DP \times AM\right) \times \left(\frac{1}{2} \times BP \times CN\right)$
$= \frac{1}{4} \times BP \times DP \times AM \times CN$ …………………………….(2)

From (1) and (2), we get $ar(\Delta APB) \times ar(\Delta CPD) = ar(\Delta APD) \times ar(\Delta BPC)$

Question 7: P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that
i) ar(ΔPQR) = $\frac{1}{2}$ ar(ΔARC)
ii) ar(ΔPBQ) = ar(ΔARC)
iii) ar(ΔRQC) = $\frac{3}{8}$ ar(ΔABC)

Answer: We have a ΔABC such that P is the mid-point of AB and Q is the mid-point of BC. Also, R is the mid-point of AP. Let us join AQ, RQ, PC and PC.

(i) In ΔAPQ, R is the mid-point of AP. [Given] B



Therefore, RQ is a median of ΔAPQ.
or, ar(ΔPRQ) = $\frac{1}{2}$ar(ΔAPQ) ………………………………………………………..(1)
In ΔABQ, P is the mid-point of AB.
Hence, QP is a median of ΔABQ.
Therefore, ar(ΔAPQ) = $\frac{1}{2}$ar(ΔABQ) ……………………………………………………(2)

From (1) and (2), we get,
ar(ΔPRQ) = $\frac{1}{2} \times \frac{1}{2}$ar(ΔABQ) = $\frac{1}{4}$ar(ΔABQ) = $\frac{1}{4} \times \frac{1}{2}$ar(ΔABC) [AQ is median of ΔABC]
Therefore, ar(ΔPRQ) = $\frac{1}{8}$ar(ΔABC)
……………………………………………………………(3)

Now, ar(ΔARC) = $\frac{1}{2}$ar(ΔAPC) [CR is the median of ΔAPC]
= $\frac{1}{2} \times \frac{1}{2}$ar(ΔABC) [CP is a median of ΔABC]
Therefore, ar(ΔARC) = $\frac{1}{4}$ar(ΔABC)……………………………………………(4)

Now From (3), we get
ar(ΔPRQ) = $\frac{1}{8}$ ar(ΔABC)
= $\frac{1}{2} \times \frac{1}{4}$ar(ΔABC)
= $\frac{1}{2}$ar(ΔARC) [using (4)]

Thus, ar(ΔPRQ) = $\frac{1}{2}$ar(ΔARC)

ii) In ΔRBC, RQ is a median.
Therefore, ar(ΔRQC) = ar(ΔRBQ) = ar(ΔPRQ) + ar(ΔBPQ)

= $\frac{1}{8}$ ar(ΔABC) + ar(ΔBPQ) [From eq. (3) of part (i)]

= $\frac{1}{8}$ ar(ΔABC) + $\frac{1}{2}$ar(ΔPBC) [since, PQ is the median of ΔBPC]

= $\frac{1}{8}$ ar(ΔABC) + $\frac{1}{2} \times \frac{1}{2}\, ar$(ΔABC) [Since, CP is the median of ΔABC]

= $\frac{1}{8}$ ar(ΔABC) + $\frac{1}{4}\, ar$(ΔABC)
= $\left(\frac{1}{8} + \frac{1}{4}\right)$ ar(ΔABC)

$= \frac{3}{8}$ ar(ΔABC)

Thus, ar(ΔRQC) = $\frac{3}{8}$ ar(ΔABC)

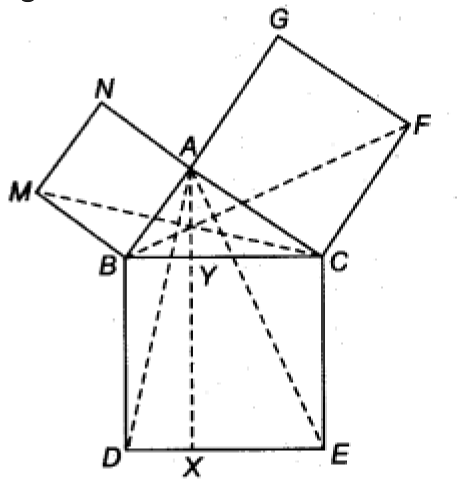
iii) QP is the median of ΔABQ

$= \frac{1}{2} \times \frac{1}{2}$ ar(ΔABC) [Since, AQ is the median of ΔABC]

$= \frac{1}{4}$ ar(ΔABC) = ar(ΔARC) [from eq. (4) of part (i)]

Thus, ar(ΔPBQ) = ar(ΔARC)

Question 8: In figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y. Show that



(i) ΔMBC = ΔABD
(ii) ar(BYXD) = 2 ar(MBC)
(iii) ar(BYXD) = ax(ABMN)
(iv) ΔFCB ≅ ΔACE
(v) ar(CYXE) = 2 ar(FCB)
(vi) ar(CYXE) = ax(ACFG)
(vii) ar(BCED) = ar(ABMN) + ar(ACFG)

Answer: We have a right-angled ΔABC such that BCED, ACFG and ABMN are squares on its sides BC, CA and AB respectively. Line segment AX ⊥ DE is also drawn such that it meets BC at Y.

(vii) $\angle CBD = \angle MBA$ [Each90°]
Hence, $\angle CBD + \angle ABC = \angle MBA + \angle ABC$ (By adding $\angle ABC$ on both sides)
or $\angle ABD = \angle MBC$
In ΔABD and ΔMBC, we have
AB = MB [Sides of a square]
Therefore, BD = BC
$\angle ABD = \angle MBC$ [Proved above]
Hence, $\Delta ABD = \Delta MBC$ [By SAS congruency]

(ii) Since parallelogram BYXD and ΔABD are on the same base BD and between the same parallels BD and AX.
Therefore, ar(ΔABD) = $\frac{1}{2}$ar(parallelogram BYXD)
But $\Delta ABD \cong \Delta MBC$…………………………… [From (i) part]
Since, congruent triangles have equal areas.
Hence, ar(ΔMBC) = $\frac{1}{2}$ar(parallelogram BYXD)
or, ar(parallelogram BYXD) = 2ar(ΔMBC)

(vii) Since, ar(parallelogram BYXD) = 2ar(ΔMBC)
……………………………….(1) [From (ii) part]
and ar(square ABMN) = 2ar(ΔMBC)
……………………………………….(2) [ABMN and Triangle MBC are on the same base MB and between the same parallels MB and NC]
From (1) and (2), we have
ar(BYXD) = ar(ABMN) .

(iv) $\angle FCA = \angle BCE$ (Each 90°)
or $\angle FCA + \angle ACB = \angle BCE + \angle ACB$ [By adding $\angle ACB$ on both sides]
or, $\angle FCB = \angle ACE$
In ΔFCB and ΔACE, we have
FC = AC [Sides of a square]
CB = CE [Sides of a square]
$\angle FCB = \angle ACE$ [Proved above]
Hence,. $\Delta FCB \cong \Delta ACE$ [By SAS congruency]

(vii) Since, parallelogram CYXE and ΔACE are on the same base CE and between the same parallels CE and AX.
Therefore, ar(parallelogram CYXE) = 2ar(ΔACE)
But $\Delta ACE \cong \Delta FCB$………………………………… [From (iv) part]
Since, congruent triangles are equal in areas.
Hence, ar (parallelogram CYXE) = 2ar(ΔFCB)

(vi) Since, ar(parallelogram CYXE) = 2ar(ΔFCB) ………………..…(3) [From (v) part]
Also (quad. ACFG) and ΔFCB are on the same base FC and between the same parallels FC and BG.
or, ar(quad. ACFG) = 2ar(ΔFCB) ………………………………(4)

From (3) and (4), we get
ar(quad. CYXE) = ar(quad. ACFG) ……………………………………..…(5)

(vii) We have ar(quad. BCED)
= ar(quad. CYXE) + ar(quad. BYXD)
= ar(quad. CYXE) + ar(quad. ABMN)……………………….[From (iii) part]
Thus, ar (quad. BCED)
= ar(quad. ABMN) + ar(quad. ACFG) ………………………………..[From (vi) part]