Chapter 4 -Quadratic equation
Exercise – 4.1

1. Check whether the following are quadratic equations:

(i) $(x + 1)^2 = 2(x - 3)$
(ii) $x^2 - 2x = (-2)(3 - x)$
(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$
(iv) $(x - 3)(2x + 1) = x(x + 5)$
(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$
(vi) $x^2 + 3x + 1 = (x - 2)^2$
(vii) $(x + 2)^3 = 2x(x^2 - 1)$
(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Answer: (i) Given, $(x + 1)^2 = 2(x - 3)$

Hence, $(x + 1)^2 = 2x - 3$
or, $x^2 + 2x + 1 = 2x - 6$
or, $x^2 + 7 = 0$

As, the above equation is in the form of $ax^2 + bx + c = 0$, hence it is a quadratic equation.

(ii) Given, $x^2 - 2x = (-2)(3 - x)$

Hence, $x^2 - 2x = (-6) + 2x$
or, $x^2 - 4x + 6 = 0$

As, the above equation is in the form of $ax^2 + bx + c = 0$, hence it is quadratic equation.

(iii) Given, $(x - 2)(x + 1) = (x - 1)(x + 3)$

Hence, $(x - 2)(x + 1) = (x - 1)(x + 3)$
or, $x^2 - x - 2 = x^2 + 2x - 3$
or, $3x - 1 = 0$

As, the above equation is not in the form of $ax^2 + bx + c = 0$, hence, the given equation is not a quadratic equation.

(iv) Given, $(x - 3)(2x + 1) = x(x + 5)$

Hence, $2x^2 - 5x - 3 = x^2 + 5x$
or, $x^2 - 10x - 3 = 0$

As, the above equation is in the form of $ax^2 + bx + c = 0$, hence, it is quadratic equation.

(v) Given, $(2x – 1)(x – 3) = (x + 5)(x – 1)$

Hence, $2x^2 – 7x + 3 = x^2 + 4x – 5$
or, $x^2 – 11x + 8 = 0$

Since the above equation is in the form of $ax^2 + bx + c = 0$, hence, it is quadratic equation.

(vi) Given, $x^2 + 3x + 1 = (x – 2)^2$
or, $x^2 + 3x + 1 = x^2 + 4 – 4x$
or, $7x – 3 = 0$

As, the above equation is not in the form of $ax^2 + bx + c = 0$, hence it is not a quadratic equation.

(vii) Given, $(x + 2)^3 = 2x(x^2 – 1)$
or, $x^3 + 8 + x^2 + 12x = 2x^3 – 2x$
or, $x^3 + 14x – 6x^2 – 8 = 0$

Since the above equation is not in the form of $ax^2 + bx + c = 0$, hence, it is not a quadratic equation.

(viii) Given, $x^3 – 4x^2 – x + 1 = (x – 2)^3$
hence, $x^3 – 4x^2 – x + 1 = x^3 – 8 – 6x^2 + 12x$
or, $2x^2 – 13x + 9 = 0$

As, the above equation is in the form of $ax^2 + bx + c = 0$, it is a quadratic equation.

Question 2: Represent the following situations in the form of quadratic equations:

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken

Answer: (i) Let the breadth of the rectangular plot be = x m
Then, length of the plot = (2x + 1) m
Hence, area of a rectangular plot = l × b
or, 528 = (2x + 1) x
or, 528 = $2x^2 + x$
or, $2x^2 + x - 528 = 0$, is the required quadratic equation.

(ii) Let the two consecutive integers be x and (x + 1)
Then, x(x + 1) = 306
or, $x^2 + x = 306$
or, $x^2 + x - 306 = 0$, is the required quadratic equation.

(iii) Let the present age of Rohan be x years.
So, Rohan's mother present age = (x + 26) years
After 3years, Rohan's age = (x + 3) years
After 3years, Rohan's mother age = (x + 26 + 3)years = (x + 29) years
Hence, according to the question,

(x + 3) (x + 29) = 360
or, $x^2 + 29x + 3x + 87 - 360 = 0$
or, $x^2 + 32x - 273 = 0$, is the required equation.

(iv) Let the speed of the train be = x km/hr
Total distance to be covered = 480km
Time = $\frac{distance}{speed} = \frac{480}{x}$

Decreased speed of the train = (x – 8)km/hr
Hence, time = $\frac{480}{x-8}$

Now, according to the question,
$\frac{480}{x-8} - \frac{480}{x} = 3$
or, $480\left[\frac{1}{x-8} - \frac{1}{x}\right] = 3$

or, $480\left[\frac{x-x+8}{x(x-8)}\right] = 3$

or, 480 × 8 = 3x(x – 8)
or, $3840 = 3x^2 - 24x$
or, $3x^2 - 24x - 3840 = 0$
or, $x^2 - 8x - 1280 = 0$, is the required equation.

Exercise 4.2

Question 1: Find the roots of the following quadratic equations by factorisation:
(i) x^2 -3x – 10 = 0
(ii) $2x^2 + x - 6 = 0$
(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
(iv) $2x^2 - x + \frac{1}{8} = 0$
(v) $100x^2 - 20X + 1 = 0$

Answer: (i) Given:
$x^2 - 3x - 10 = 0$
or, $x^2 - 5x + 2x - 10$
or, x(x – 5) + 2(x – 5)
or, (x – 5)(x + 2)

Hence, either x – 5 = 0 or x + 2 = 0,
therefore, x = 5 and x = (-2)

(ii) Given:
$2x^2 + x - 6 = 0$
or, $2x^2 + 4x - 3x - 6 = 0$
or, 2x(x + 2) – 3(x + 2) = 0
or, (x + 2)(2x – 3) = 0

Hence, either (x + 2) = 0 or (2x – 3) = 0
therefore, x = (- 2) and x = $\frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
or, $\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$
or, $x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$
or, $(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

Hence, either $(\sqrt{2}x + 5) = 0$ or $(x + \sqrt{2}) = 0$
Therefore, x = $-\frac{5}{\sqrt{2}}$ and $-\sqrt{2}$

(iv) $2x^2 - x + \frac{1}{8} = 0$
or, $\frac{1}{8}(16x^2 - 8x + 1) = 0$
or, $\frac{1}{8}$ [4x(4x – 1) – 1 (4x – 1)] = 0
or, $\frac{1}{8}(4x - 1)^2 = 0$
or, $(4x - 1)^2 = 0$

Hence, either (4x – 1 = 0) or (4x – 1 = 0)
Therefore, x = $\frac{1}{4}$ and $\frac{1}{4}$

(v) $100x^2 - 20x + 1 = 0$
or, $100x^2 - 10x - 10x + 1 = 0$
or, $10x(10x - 1) - 1(10x - 1) = 0$
or, $(10x - 1)^2 = 0$

Hence, either $(10x - 1) = 0$ or $(10x - 1) = 0$
Therefore, $x = \frac{1}{10}$ and $\frac{1}{10}$

Question 2: Represent the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ` 750. We would like to find out the number of toys produced on that day.

Answer: (i) Let the number of marbles John had be x
Therefore, number of marble Jivanti have = 45 – x
After losing 5 marbles each,
Number of marbles John have = x – 5
Number of marble Jivanti have = 45 – x – 5 = 40 – x
Given that the product of their marbles is 124.

Hence, according to the problem ,
$(x - 50)(40 - x) = 124$
or, $x^2 - 45x + 324 = 0$
or, $x^2 - 36x - 9x + 324 = 0$
or, $x(x - 36) - 9(x - 36) = 0$
or, $(x - 36)(x - 9) = 0$

Hence, $x - 36 = 0$ or $x = 36$
$x - 9 = 0$ or $x = 9$

Therefore, if John's marbles = 36,
Then, Jivanti's marbles = 45 – 36 = 9
And if John's marbles = 9,
Then, Jivanti's marbles = 45 – 9 = 36

(ii) Let the number of toys produced in a day be x.
Therefore, the cost of production of each toy = Rs(55 – x)

Given, the total cost of production of the toys = Rs 750

Hence, $x(55 - x) = 750$
or, $x^2 - 55x + 750 = 0$
or, $x^2 - 25x - 30x + 750 = 0$

or, $x(x – 25) – 30(x – 25) = 0$
or, $(x – 25)(x – 30) = 0$

Thus, either $(x -25) = 0$ or $(x – 30) = 0$
or, $x = 25$ or $x = 30$

Hence, the number of toys produced in a day, will be either 25 or 30.

Question 3: Find two numbers whose sum is 27 and product is 182.

Answer: Let the two numbers be x and (27 – x) respectively.
Now according to the question ,

$x(27 – x) = 182$
or, $27x – x^2 = 182$
or, $x^2 – 27x + 182 = 0$
or, $x^2 – 14x – 13x + 182 = 0$
or, $x(x – 14) – 13(x – 14) = 0$
or, $(x – 14)(x – 13) = 0$
Hence, either $x = 14$ or 13

The numbers be $x = 14$; $(27 – x) = 27 – 14 = 13$

Therefore, the numbers are 14 and 13.

Question 4: Find two consecutive positive integers, sum of whose squares is 365.

Answer: Let the two consecutive be x and (x + 1)

Hence, $x^2 + (x + 1)^2 = 365$
or, $x^2 + x^2 + 1 + 2x = 365$
or, $2x^2 + 2x = 365 – 1$
or, $x^2 + x – 364 = 0$
or, $x^2 + 14x – 13x – 182 = 0$
or, $x(x + 14) – 13(x + 14) = 0$
or, $(x + 14)(x – 13) = 0$
hence, $x = (-14)$ or 13

Since, the integers are positive, so x can be 13, only.
Therefore, $x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers are 13 and 14.

Question 5: The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Answer: Let the base of the right-angled triangle be x cm.
The altitude of right triangle = (x – 7) cm [Given]

By using Pythagoras theorem, we get,

$x^2 + (x - 7)^2 = 132$
or, $x^2 + x^2 + 49 - 14x = 169$
or, $2x^2 - 14x - 120 = 0$
or, $x^2 - 7x - 60 = 0$
or, $x^2 - 12x + 5x - 60 = 0$
or, $x(x - 12) + 5(x - 12) = 0$
or, $(x - 12)(x + 5) = 0$

Hence, either $x - 12 = 0$ or $x + 5 = 0$; $x = 12$ or $x = -5$

As, sides cannot be negative, x can only be 12.

Therefore, the base of the given triangle is 12 cm and the altitude of this triangle will be
(12 – 7) cm = 5 cm.

Question 6: A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs.90, find the number of articles produced and the cost of each article.

Answer: Let the number of pottery articles produced in a day = x
Cost of production of each article = Rs. $\frac{90}{x}$
According to the problem,
$2x + 3 = \frac{90}{x}$
or, $2x^2 + 3x - 90 = 0$
or, $2x^2 + 15x - 12x - 90 = 0$
or, $x(2x + 15) - 6(2x + 15) = 0$
or, $(2x + 15)(x - 6) = 0$
Hence, $x = -\frac{15}{2}$ or 6

As the number of articles produced can not be a negative integer, therefore, x can only be 6.
Hence, the number of articles produced = 6
And the cost of each article = 2 × 6 + 3 = Rs 15.

Exercise 4.3

Question 1: Find the roots of the following quadratic equations, if they exist, by the method of completing the square:
(i) $2x^2 - 7x + 3 = 0$
(ii) $2x^2 + x - 4 = 0$
(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$
(iv) $2x^2 + x + 4 = 0$

Answer: (i) Given $2x^2 - 7x + 3 = 0$

or, $2\left(x^2 - \frac{7}{2}x + \frac{3}{2}\right) = 0$
or, $x^2 - \frac{7}{2}x + \frac{3}{2} = 0$
or, $(x)^2 - \left(\frac{7}{2}x\right) + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$
or, $\left(x - \frac{7}{4}\right)^2 - \left(\frac{49}{16} - \frac{3}{2}\right) = 0$
or, $\left(x - \frac{7}{4}\right)^2 - \left(\frac{49-24}{16}\right) = 0$
or, $\left(x - \frac{7}{4}\right)^2 - \left(\frac{25}{16}\right) = 0$
or, $\left(x - \frac{7}{4}\right)^2 - \left(\frac{5}{4}\right)^2 = 0$
or, $\left(x - \frac{7}{4} + \frac{5}{4}\right)\left(x - \frac{7}{4} - \frac{5}{4}\right) = 0$
or, $\left(x - \frac{12}{4}\right)\left(x - \frac{2}{4}\right) = 0$
or, $(x - 3)\left(x - \frac{1}{2}\right) = 0$
or, $x = 3 \;\; or \;\; x = \frac{1}{2}$

Hence, the required roots are 3 and $\frac{1}{2}$.

(ii) Given : $2x^2 + x = 4$

Dividing both sides of the equation by 2, we get
$x^2 + \frac{x}{2} = 2$

or, $x^2 + \frac{x}{2} - 2 = 0$
or, $x^2 + \frac{x}{2} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 = 0$
or, $\left(x + \frac{1}{4}\right)^2 - \frac{1}{16} - 2 = 0$
or, $\left(x + \frac{1}{4}\right)^2 - \frac{33}{16} = 0$
or, $\left(x + \frac{1}{4}\right)^2 - \left(\frac{\sqrt{33}}{4}\right)^2 = 0$
or, $\left(x + \frac{1}{4} + \frac{\sqrt{33}}{4}\right)\left(x + \frac{1}{4} - \frac{\sqrt{33}}{4}\right) = 0$

Hence, $x = \frac{-1-\sqrt{33}}{4}$ or, $x = \frac{-1+\sqrt{33}}{4}$

(iii) Given: $4x^2 + 4\sqrt{3}x + 3 = 0$
Converting the equation into $a^2 + 2ab + b^2$ form, we get,
or, $(2x)^2 + 2 \times 2x \times \sqrt{3} + (\sqrt{3})^2 = 0$
or, $(2x + \sqrt{3})^2 = 0$
or, $(2x + \sqrt{3}) = 0$ and $(2x + \sqrt{3}) = 0$

Therefore, either $x = \frac{-\sqrt{3}}{2}$ or $\frac{-\sqrt{3}}{2}$

(iv) Given: $2x^2 + x + 4 = 0$
or, $x^2 + \frac{x}{2} + 2 = 0$

or, $x^2 + \frac{x}{2} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 + 2 = 0$

or, $\left(x + \frac{1}{4}\right)^2 - \left(\frac{1}{16} - 2\right) = 0$

or, $\left(x + \frac{1}{4}\right)^2 - \left(\frac{-31}{16}\right) = 0$

or, $\left(x + \frac{1}{4}\right)^2 - \left(\frac{\sqrt{-31}}{4}\right)^2 = 0$

Since, $\frac{\sqrt{-31}}{4}$ is an imaginary value.

Hence, real roots does not exist.

Question 2: Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

Answer: (i) $2x^2 - 7x + 3 = 0$
On comparing the given equation with $ax^2 + bx + c = 0$, we get, a = 2, b = -7 and c = 3

By using quadratic formula, we get, $x = \frac{-b \pm \sqrt{D}}{2a}$

$x = \frac{7 \pm \sqrt{49} - 24}{4}$

or, $x = \frac{7 \pm \sqrt{25}}{4}$

or, $x = \frac{7 \pm 5}{4}$

or, $x = \frac{7+5}{4}$ or $x = \frac{7-5}{4}$

or, $x = \frac{12}{4}$ or $\frac{2}{4}$

Hence, $x = 3$ or $\frac{1}{2}$

(ii) $2x^2 + x - 4 = 0$
On comparing the given equation with $ax^2 + bx + c = 0$, we get, a = 2, b = 1 and c = -4

By using quadratic formula, we get, $x = \frac{-b \pm \sqrt{D}}{2a}$

$x = \frac{-1 \pm \sqrt{33}}{4}$

or, $x = \frac{-1+\sqrt{33}}{4}$ or, $\frac{-1-\sqrt{33}}{4}$

Question 3: Find the roots of the following equations:

(i) x - $\frac{1}{x}$ = 3, x$\neq$ 0
(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$, x $\neq$ (- 4), 7

Answer: (i) x–$\frac{1}{x}$ = 3
or, x^2 – 3x -1 = 0

And on comparing the given equation with $ax^2 + bx + c = 0$, we get a = 1, b = -3 and c = -1

By using quadratic formula, we get, x = $\frac{-b \pm\sqrt{D}}{2a}$, [D = b^2 – 4ac]
Hence, x = $\frac{3 \pm\sqrt{(9+4)}}{2}$
or, x = $\frac{3 \pm\sqrt{13}}{2}$

Therefore, x = $\frac{3+\sqrt{13}}{2}$ or, $\frac{3-\sqrt{13}}{2}$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

or, $\frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$

or, $\frac{-11}{(x+4)(x-7)} = \frac{11}{30}$

or, (x + 4)(x - 7) = (-30)
or, x^2 – 3x – 28 = 30
or, x^2 – 3x + 2 = 0

We can solve this equation by factorization method now,
or, x^2 – 2x – x + 2 = 0
or, x(x – 2) – 1(x – 2) = 0
or, (x – 2)(x – 1) = 0
either, x = 1 or 2

Question 4: The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is 1/3. Find his present age.

Answer: Let the present age of Rehman be x years.
3 years ago Rehman's age was = (x – 3)years
5 years from now Rehman's age will be = (x + 5)years

According to the question,
$\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{3}$

or, $\frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$

or, $\frac{2x+2}{x^2+2x-15} = \frac{1}{3}$

or, $6x + 6 = x^2 + 2x – 15$

or, $x^2 – 4x – 21 = 0$
or, $x^2 – 7x + 3x – 21 = 0$
or, $x(x – 7) + 3(x – 7) = 0$
or, $(x – 7)(x + 3) = 0$
Hence, x = 7 or (- 3)

As, age cannot be negative, hence, present age of Rehman is 7years.

Question 5: In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Answer: Let the marks of Shefali in Maths be x.
Then, the marks in English will be 30 – x.

As per the given problem,
$(x + 2)(30 – x – 3) = 210$
or,$(x + 2)(27 – x) = 210$
or, $-x^2 + 25x + 54 = 210$
or, $x^2 – 25x + 156 = 0$
or, $x^2 – 12x – 13x + 156 = 0$
or, $x(x – 12) -13(x – 12) = 0$
or, $(x – 12)(x – 13) = 0$
or, x = 12, 13

Therefore, if the marks in Maths are 12, then marks in English will be 30 – 12 = 18
And if the marks in Maths is 13, then marks in English will be 30 – 13 = 17**.**

Question 6: The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Answer: Let the shorter side of the rectangle be *x* m then, larger side of the rectangle = (*x* + 30)m
Diagonal of the rectangle = $\sqrt{x^2 + (x + 30)^2}$ = x + 60
$x^2 + (x + 30)^2 = (x + 60)^2$
or, $x^2 + x^2 + 900 + 60x = x^2 + 3600 + 120x$
or, $x^2 – 60x – 2700 = 0$
or, $x^2 – 90x + 30x – 2700 = 0$
or, $x(x – 90) + 30(x -90) = 0$
or, $(x – 90)(x + 30) = 0$
or, x = 90, -30

Hence, the side of the field cannot be negative.
Therefore, the length of the shorter side will be 90 m and the length of the larger side will be
(90 + 30) m = 120 m.

Question 7: The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Answer:

Let the smaller number be x
Then the larger be $\frac{x^2}{8}$

So , we have, $\left(\frac{x^2}{8}\right)^2 - x^2 = 180$
= $\frac{x^4}{64} - x^2 - 180 = 0$
= $x^4 - 64x^2 - 11520 = 0$
= $y^2 - 64y - 11520 = 0$
Here a =1, b = -64 and c = -11520

Therefore, D = b^2 - 4ac
= $(-64)^2 - 4 \times 1 \times (-11520)$
= $4096 + 46080 = 50176$

Therefore , $y = \frac{-b \pm \sqrt{D}}{2a}$
= $\frac{-(-64) \pm \sqrt{50176}}{2 \times 1} = \frac{64 \pm 224}{2}$

or, y = -80 or 144

y can't be negative as y = x^2 can't be negative.

So, y = 144

or, x = 144 = $(12)^2$

or, x = ± 12

Therefore the smaller number = ± 12
Therefore, greater number when smaller number is $+12 = \frac{1}{8} \times 144 = 18$
Therefore, greater number when smaller number is $-12 = \frac{1}{8} \times 144 = 18$

Hence the numbers are either 12 and 18, or -12 and 18.

Question 8: A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Answer: Total distance travelled = 360 km
Let uniform speed be x km/hr
Then, increased speed = (x + 5)km/hr

According to the question,
$\frac{360}{x} - \frac{360}{x+5} = 1$ (Time = $\frac{distance}{speed}$)
or, $\frac{360(x+5)-360x}{x(x+5)}$ = 1
or, 360x + 1800 – 360x = x(x + 5)

or, 1800 = x^2 + 5x
or, x^2 + 5x – 1800 = 0
or, x^2 + 45x – 40x – 1800 = 0
or, x(x + 45) – 40(x + 45) = 0
or, (x + 45)(x – 40) = 0

Either, x = (-45) or 40

As, speed cannot be negative then, x will be 40.

Therefore, speed of the train = 40km/hr

Question 9: Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Answer: Let the smaller tap takes x hours to fill the tank.
Then, larger tap will take (x – 10)hours to fill the same tank.
If the two work together, then the amount of water following in our hour = $\frac{1}{x} + \frac{1}{x-10}$

$\frac{1}{x} + \frac{1}{x-10}$ = $9\frac{3}{8}$

or, $\frac{x-10+x}{x(x-10)}$ = $\frac{8}{75}$ [Since, the amount of water flowing in 1hour = $\frac{1}{\frac{75}{8}}$]

or, 75(2x – 10) = 8x(x – 10)
or, 150x – 750 = $8x^2$ – 80x
or, $8x^2$ – 230x + 750 = 0

Here, a = 8, b = -230, and c = 750

Therefore, D = b^2 – 4ac
= $(-230)^2$ – 4 × 8 × 750
= 52900 – 24000
= 28900

Hence, x = $\frac{-b\pm\sqrt{D}}{2a}$ = $\frac{-(-230)\pm\sqrt{28900}}{2\times8}$ = $\frac{230\pm170}{16}$

Either, x = $\frac{230+170}{16}$ or $\frac{230-170}{16}$

= 25 or $\frac{15}{4}$

As, time cannot be fraction, hence, x = 25
Hence, the smaller tap takes 25hours and the larger tap takes 15hours to fill the tank.

Question 10: An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speeds of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Answer: Let the speed of passenger train = x km/hr
Then, the speed of express train be (x + 11)km/hr

According to the problem,
$\frac{132}{x} - \frac{132}{x+11} = 1$

or, $\frac{132(x+11)-132x}{x(x+11)} = 1$
or, $132x + 1452 – 132x = x^2 + 11x$
or, $x^2 + 11x – 1452 = 0$
or, $x^2 + 44x – 33x – 1452 = 0$
or, $x(x + 44) – 33(x + 44) = 0$
or, $(x – 33)(x + 44) = 0$
Hence, Either x = 33 or, x = - 44
As, speed cannot be negative, hence, x = 33
Therefore, speed of the passenger train = 33km/hr
and speed of the express train = 33 + 11 = 44 km/hr

Question 11: Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Answer:

Let the side of the smaller square be x, and the side of the larger square be y.
Perimeter of smaller square = 4x
Perimeter of larger square = 4y

So according to question,

4y – 4x = 24
or, y – x = 6 [Dividing both sides by 4]
or, y = x + 6 …………………………………………..… (i)

Now, given sum of areas of squares = 468 m^2

or, $x^2 + y^2 = 468$
or, $x^2 + (x+6)^2 = 468$…………………………………………[From (i)]
or, $x^2 + x^2 + 12x + 36 = 468$
or, $2x^2 + 12x – 432 = 0$
or, $x^2 + 6x – 216 = 0$

Therefore, D = $b^2 - 4ac$
$= (6)^2 - 4 \times 1 \times (-216) = 36 + 864$
$= 900$

Therefore, $x = \frac{-b \pm \sqrt{D}}{2a}$

= $\frac{-6 \pm \sqrt{900}}{2 \times 1} = \frac{-6 \pm 30}{2}$

So, either $x = \frac{-6+30}{2}$ or $x = \frac{-6-30}{2}$
or, x = 12 or x = -18

But length of a square cannot be negative. So,
x = 12 and y = 12 + 6 = 18

Hence the sides of the squares are 12m and 18m.

Exercise 4.4

Question 1: Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:
(i) $2x^2 - 3x + 5 = 0$
(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$
(iii) $2x^2 - 6x + 3 = 0$

Answer: (i) $2x^2 - 3x + 5 = 0$
Hence, a = 2; b = (-3); c = 5

Therefore, D = $b^2 - 4ac$
$= (-3)^2 - 4 \times 2 \times 5$
$= 9 - 40$
$= -31 < 0$
Hence, the roots are imaginary.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$
Hence, a = 3; b = $(-4\sqrt{3})$; c = 4

Therefore, D = $b^2 - 4ac$
$= (-4\sqrt{3})^2 - 4 \times 3 \times 4$
$= 48 - 48 = 0$

Hence, the roots are real and ewqual.

Now using the formula, we get

$x = \frac{-b \pm \sqrt{D}}{2a}$

$x = \frac{-(-4\sqrt{3}) \pm \sqrt{(-4\sqrt{3})^2 - 4 \times 3 \times 4}}{2 \times 3}$

$x = \frac{4\sqrt{3} \pm \sqrt{48-48}}{6}$

$x = \frac{4\sqrt{3}}{6} = \frac{2}{\sqrt{3}}$

Hence, the equal roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(iii) Given : $2x^2$-6x + 3 = 0 , where, a = 2, b = -6, c = 3

Hence, D = $b^2 - 4ac$
$= (-6)^2 - 4 \times 2 \times 3$
$= 36 - 24 = 12 > 0$

Now using the formula,

$x = \frac{-b \pm \sqrt{D}}{2a}$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 3 \times 2}}{2 \times 2}$

$x = \frac{6 \pm \sqrt{36-24}}{4}$

$x = \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$

Hence, $x = \frac{3 + \sqrt{3}}{2}$ or, $\frac{3 - \sqrt{3}}{2}$

Question 2: Find the values of *k* for each of the following quadratic equations, so that they have two equal roots.
(i) $2x^2 + kx + 3 = 0$
(ii) kx (x – 2) + 6 = 0

Answer: (i) $2x^2 + kx + 3 = 0$, where a = 2. B = k and c = 3

Hence, D = $b^2 - 4ac$
$= k^2 - 4 \times 2 \times 3$
$= k^2 - 24$

We know that, for real roots, D = 0
Hence, $k^2 - 24 = 0$
or, $k = \pm 2\sqrt{6}$ i.e., $2\sqrt{6}$ or $-2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$
or, $kx^2 - 2kx + 6 = 0$
Now comparing with $ax^2 - bx + c = 0$, we get, $a = k$, $b = -2k$ and $c = 6$

$D = b^2 - 4ac$
$= (-2k)^2 - 4 \times k \times 6$
$= 4k^2 - 24k$

As we know that, for real roots,
$D = 0$
or, $4k^2 - 24k = 0$
or, $k(4k - 24) = 0$
or, $k = 0$(not possible) or $4k - 24 = 0$

or, $k = 6$

Question 3: Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.

Answer: Let the breadth be x m
Then, the length of rectangular will be 2x m

According to the problem,
$l \times b = \text{area}$
or, $x \times 2x = 800$
or, $2x^2 = 800$
or, $x^2 = 400$
or, $x = 20$
Hence, $2x = 40$

Hence, the rectangular mango grove is possible whose length and breadth will be 40m and 20m respectively.

Question 4: Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Answer: Let the present age of one friend be x years
Then ,the present age of other friend be $(20 - x)$years
4 years ago, one friend was $(x - 4)$ years
4 years ago , other friend's age was $(20 - x - 4) = (16 - x)$ years

Hence, according to the problem,
$(x - 4)(16 - x) = 48$
or, $16x - x^2 - 64 + 4x = 48$

or, $x^2 - 20x + 112 = 0$
After comparing with $ax^2 + bx + c = 0$, $a = 1$, $b = (-20)$ and $c = 112$

Therefore, $D = b^2 - 4ac$
$= (-20)^2 - 4 \times 1 \times 112$
$= 400 - 448$
$= (-48) < 0$

Since, no real roots exist.
So, the given situation is not possible.

Question 5: Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.

Answer: Let the rectangular park be x.
Then the perimeter will be 2(l + b)

Therefore, $2(x + b) = 80$
or, $b = 40 - x$

According to the problem,
area of the rectangular park = $l \times b$
or, $400 = x(40 - x)$
or, $400 = 40x - x^2$
or, $x^2 - 40x + 400 = 0$
or, $x^2 - 20x - 20x + 400 = 0$
or, $(x - 20)(x - 20) = 0$
or, $x = 20$

Thus the length of the rectangular park be 20m and the breadth be (40 – 20) m = 20m