Chapter 12: Heron's formula

Exercise: 12.1(MCQ)

Question 1. An isosceles right triangle has an area of 8 cm^2. The length of its hypotenuse is

(A) $\sqrt{32}$ cm
(B) $\sqrt{16}$ cm
(C) $\sqrt{48}$ cm
(D) $\sqrt{24}$ cm

Solution: (A) $\sqrt{32}$ cm

Let the height of triangle = h
As the triangle is isosceles,
Let base = height = h

According to the question,
Area of triangle = $8cm^2$
$\Rightarrow$ ½ × Base × Height = 8
$\Rightarrow$ ½ × h × h = 8
$\Rightarrow h^2 = 16$
$\Rightarrow$ h = 4cm

Base = Height = 4cm

Since the triangle is right-angled,
$Hypotenuse^2 = Base^2 + Height^2$
$\Rightarrow Hypotenuse^2 = 4^2 + 4^2$
$\Rightarrow Hypotenuse^2 = 32$
$\Rightarrow$ Hypotenuse = $\sqrt{32}$

Question 2. The perimeter of an equilateral triangle is 60 m. The area is

(A) $10\sqrt{3}\ m^2$
(B) $15\sqrt{3}\ m^2$
(C) $20\sqrt{3}\ m^2$
(D) $100\sqrt{3}\ m^2$

Solution: (D) $100\sqrt{3}\ m^2$

Area of an equilateral triangle of side a = $\sqrt{3}/4\ a^2$

According to the question,
The perimeter of triangle = 60m
$\Rightarrow$ a + a + a = 60
$\Rightarrow$ 3a = 60
$\Rightarrow$ a = 20m

Area of the triangle = $(\sqrt{3}/4)\ a^2$
$= (\sqrt{3}/4)\ (20)^2$
$= (\sqrt{3}/4)\ (400)$
$= 100\sqrt{3}$

Question 3. The sides of a triangle are 56 cm, 60 cm and 52 cm long. Then the area of the triangle is

(A) 1322 cm^2
(B) 1311 cm^2
(C) 1344 cm^2
(D) 1392 cm^2

Solution: (C) 1344 cm^2

According to the question,
Sides of a triangle,
a = 56, b = 60, c = 52
s = $\frac{a+b+c}{2}$
s = (56 + 60 + 52)/2 = 168/2 = 84.

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{84(84-56)(84-60)(84-52)}$
= $\sqrt{\sqrt{84 \times 28 \times 24 \times 32}}$
= 1344cm^2

4. The area of an equilateral triangle with side 2√3 cm is

(A) 5.196 cm^2
(B) 0.866 cm^2
(C) 3.496 cm^2
(D) 1.732 cm^2

Solution: (A) 5.196 cm^2

Area of an equilateral triangle of side a = √3/4 a^2
According to the question,
a = 2√3
Area of triangle = (√3/4) a^2
= (√3/4) (2√3)^2
= (√3/4)(12)
= 3√3
= 5.196

Exercise 12.2

Write True or False and justify your answer:

Question 1. The area of a triangle with a base of 4 cm aa and height of 6 cm is 24 cm^2.

Solution: False

Area of triangle = ½ × Base × Altitude
= ½ × 4 × 6
= 12cm^2

Hence, the statement "the area of a triangle with base 4 cm and height 6 cm is 24 cm^2" is False.

2. The area of △ ABC is 8 cm^2 in which AB = AC = 4 cm and ∠A = 90º.

Solution: True

Area of triangle = ½ × Base × Altitude
= ½ × 4 × 4
= 8cm^2

Hence, the statement is "area of △ABC is 8 cm^2 in which AB = AC = 4 cm and ∠A = 90º" is True.

3. The area of the isosceles triangle is 5/4 √11cm^2 if the perimeter is 11 cm and the base is 5 cm.

Solution: True

According to the question, Perimeter = 11cm and base, a = 5

As the triangle is isosceles, b = c
Perimeter = 11
⇒ a + b + c = 11
⇒ 5 + b + b = 11
⇒ 5 + 2b = 11
⇒ 2b = 6
⇒ b = 3

So, we have, a = 5, b = 3, c = 3

s = $\frac{a+b+c}{2}$
s = (5 + 3 + 3)/2 = 11/2

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{\frac{11}{2}(\frac{11}{2}-5)(\frac{11}{2}-3)(\frac{11}{2}-3)}$
= $\sqrt{\frac{11}{2}\left(\frac{1}{2}\right)\left(\frac{5}{2}\right)\left(\frac{5}{2}\right)}$

⇒ Area of triangle = (5√11)/4cm^2

Hence, the statement “The area of the isosceles triangle is 5/4 √11cm^2 if the perimeter is 11 cm and the base is 5 cm” is True.

4. The area of the equilateral triangle is 20√3 cm^2 whose each side is 8 cm.

Solution: False

Area of an equilateral triangle of side a = √3/4 a^2
According to the question,
Area of a triangle = 20√3 cm^2
⇒ √3/4 a^2 = 20√3
⇒ a^2 = 20× 4
⇒ a^2 = 80
⇒ a = 4√5 cm

Hence, the statement “the area of the equilateral triangle is 20√3 cm^2 whose each side is 8 cm” is False.

Exercise 12.3

Question 1 Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs 7 per m^2.

Solution: According to the question,

The sides of the triangular field are 50 m, 65 m and 65 m.
Cost of laying grass in a triangular field = Rs 7 per m^2
Let a = 50, b = 65, c = 65
s = (a + b + c)/2
⇒ s = (50 + 65 + 65)/2
= 180/2
= 90.

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{90(90-50)(90-65)(90-65)}$

= $\sqrt{90 \times 40 \times 25 \times 25}$
= 1500m^2

Cost of laying grass = Area of triangle ×Cost per m^2
= 1500×7
= Rs.10500

2 The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m and 15 m. The advertisements yield an earning of Rs 2000 per m^2 a year. A company hired one of its walls for 6 months. How much rent did it pay?

Solution: According to the question,

The sides of the triangle are 13 m, 14 m and 15 m
Let a = 13, b = 14, c = 15
s = (a + b + c)/2
⇒ s = (13 + 14 + 15)/2 = 42/2 = 21.

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{21(21-13)(21-14)(21-15)}$
= 84m^2

Cost of advertisements for a year = Area of triangle × Cost per m^2
= 84×2000 = Rs. 168000

Since the board is rented for only 6 months:
Cost of advertisements for 6 months = (6/12) × 168000 = Rs. 84000

Question 3 From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.

Solution: According to the question,

The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm.
We know that, Area of an equilateral triangle of side a = √3/4 a^2

We divide the triangle into three triangles,

Area of triangle = (1/2 × a × 14) + (1/2 × a × 10) + (1/2 × a × 6)
√3/4 a^2 = ½ × a × (14 + 10 + 6)
√3/4 a^2 = ½ × a × 30
a = 60/√3 = 20√3

Area of the triangle = √3/4 a^2 =√3/4 (20√3)^2 = 300√3 cm^2

Question 4 The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to its base is 3: 2. Find the area of the triangle.

Solution: According to the question,

The perimeter of the isosceles triangle = 32 cm

It is also given that, the ratio of equal side to base = 3: 2
Let the equal side = 3x
So, base = 2x
The perimeter of the triangle = 32
⇒ 3x + 3x + 2x = 32
⇒ 8x = 32
⇒ x = 4.

Equal side = 3x = 3×4 = 12
Base = 2x = 2×4 = 8
The sides of the triangle = 12cm, 12cm and 8cm.

Let a = 12, b = 12, c = 8
s = (a + b + c)/2
⇒ s = (12 + 12 + 8)/2
= 32/2
= 16.

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{16(16-12)(16-12)(16-8)}$
= 32√2 cm^2

Question 5 Find the area of a parallelogram given in Fig. 12.2. Also, find the length of the altitude from vertex A on the side DC.

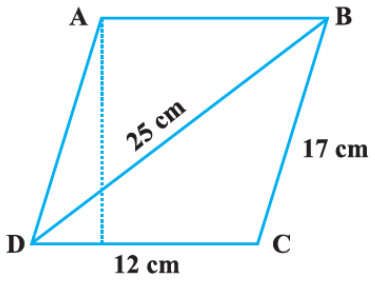


Fig. 12.2

Solution:

We know that, Area of parallelogram(ABCD) = Area(ΔBCD) + Area(ΔABD)

For Area (ΔBCD),
We have,
a = 12, b = 17, c = 25
s = (a + b + c)/2
⇒ s = (12 + 17 + 25)/2 = 54/2 = 27.

Area of (ΔBCD) = $\sqrt{s(s-a)(s-b)(s-c)}$
=$\sqrt{27(27-12)(27-17)(27-25)}$

= 90 cm^2

Since, ABCD is a parallelogram,
Area(ΔBCD) = Area(ΔABD)
Area of parallelogram(ABCD) = Area(ΔBCD) + Area(ΔABD)
= 90 + 90 = 180 cm^2

Let altitude from A be = x
Also, Area of parallelogram(ABCD) = CD × (Altitude from A)
⇒ 180 = 12 × x
⇒ x = 15 cm

Exercise 12.4

Question 1. How much paper of each shade is needed to make a kite given in Fig. 12.4, in which ABCD is a square with a diagonal of 44 cm.

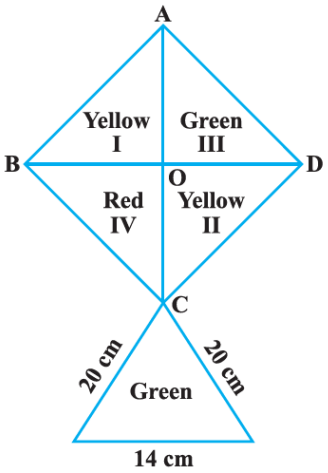


Fig. 12.4

Solution:

According to the figure,
AC = BD = 44cm
AO = 44/2 = 22cm
BO = 44/2 = 22cm

From ΔAOB,
$AB^2 = AO^2 + BO^2$
$\Rightarrow AB^2 = 22^2 + 22^2$
$\Rightarrow AB^2 = 2 \times 22^2$
$\Rightarrow AB = 22\sqrt{2}$ cm

Area of square = $(Side)^2$
$= (22\sqrt{2})^2$
$= 968\ cm^2$

Area of each triangle (I, II, III, IV) = Area of square /4
= 968 /4
$= 242\ cm^2$

To find the area of the lower triangle,
Let a = 28, b = 28, c = 14
s = (a + b + c)/2
⇒ s = (28 + 28 + 14)/2 = 70/2 = 35.

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{35(35-28)(35-28)(35-14)}$

$= 49\sqrt{15} = 189.77 cm^2$

Therefore, we get,

Area of Red = Area of IV = $242\ cm^2$

Area of Yellow = Area of I + Area of II = 242 + 242 = $484\ cm^2$

Area of Green = Area of III + Area of the lower triangle
= 242 + 189.77
$= 431.77\ cm^2$

Question 2. The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

Solution: Let the smaller side be = x cm
Then, larger side = (x + 4) cm
And, third side = (2x-6) cm

Given that, Perimeter = 50 cm
⇒ x + (x + 4) + (2x-6) = 50
⇒ 4x-2 = 50
⇒ 4x = 52
⇒ x = 13

Therefore, smaller side = 13cm
Larger side = x + 4 = 13 + 4 = 17cm
Third side = 2x-6 = 2×13 – 6 = 26-6 = 20cm

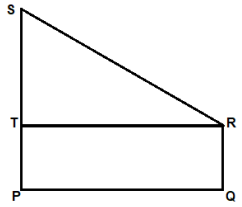
To find the area of a triangle,
Let a = 13, b = 17, c = 20
s = (a + b + c)/2
⇒ s = (13 + 17 + 20)/2 = 50/2 = 25.

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
= $\sqrt{25(25-13)(25-17)(25-20)}$

= 20√30 cm^2

Question 3. The area of a trapezium is 475 cm^2 and the height is 19 cm. Find the lengths of its two parallel sides if one side is 4 cm greater than the other.

Solution:



Let the given trapezium be PQRS, given in the figure.
According to the question,
PQ = 19cm
Let RQ = x cm

Then, PS = (x + 4)cm

Construction: Draw a perpendicular from R on PS which will also be parallel to PQ.

Now, we get,

PQRT is a rectangle,
Area of rectangle PQRT = PQ × QR
⇒ Area(PQRT) = 19×x = 19x

Now, PS = PT + TS
Since PT = QR = x cm
(x + 4) = x + TS
⇒ TS = 4cm

Area of triangle RST = ½ × RT × ST
Since RT = PQ = 19cm
⇒ Area(ΔRST) = ½ × 19 × 4 = 38cm^2

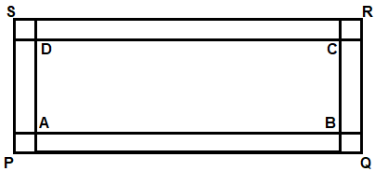
Area(PQRS) = Area(PQRT) + Area(ΔRST)
⇒ 475 = 19x + 38
⇒ 19x = 475 -38
⇒ 19x = 437
⇒ x = 23 cm

(x + 4) = 23 + 4 = 27cm

Therefore, the lengths of the parallel sides are 23cm and 27cm.

Question 4. A rectangular plot is given for constructing a house, having a measurement of 40 m long and 15 m in the front. According to the laws, a minimum of 3 m, wide space should be left in the front and back each and 2 m wide space on each of the other sides. Find the largest area where the house can be constructed.

Solution:



Let the given rectangle be rectangle PQRS,

According to the question,
PQ = 40m and QR = 15m

As 3m is left in both front and back,
AB = PQ – 3
⇒ AB = 40 – 6
⇒ AB = 34m

Also, Given that 2m has to be left on both sides,

BC = QR – 2
⇒ BC = 15 – 4
⇒ BC = 11m

Now, Area left for house construction is an area of ABCD.

Hence,
Area(ABCD) = AB × CD
= 34 × 11
= 374 m^2