Chapter 8 – Applications of Trigonometry
Exercise – 8.1

Question 1: In ∆ABC right angled at B, AB = 24 cm, BC = 7 cm. Determine:
(i) sin A, cos A
(ii) sin C, cos C

Answer: By Pythagoras Theorem ,
(Hypotenuse) AC^2 = (Base) AB^2 +(Height) BC^2
or, $AC^2 = 24^2 + 7^2$
or, $AC^2 = 576 + 49$
or, $AC^2 = 625$
or, AC = 25

(i) sin A = $\frac{BC}{AC} = \frac{7}{25}$, cos A = $\frac{AB}{AC} = \frac{24}{25}$

(ii) sin C = $\frac{AB}{AC} = \frac{24}{25}$, cos C = $\frac{BC}{AC} = \frac{7}{25}$

Question 2: In Fig. 8.13, find tan P – cot R

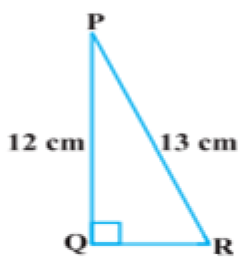

Fig. 8.13

Answer: In the ∆PQR, by Pythagoras theorem,
$PR^2 = PQ^2 + QR^2$
or, $13^2 = 12^2 + QR^2$
or, $QR^2 = 169 – 144$
or, QR = 5

tan P = $\frac{QR}{PQ} = \frac{5}{12}$

cot R = $\frac{QR}{PQ} = \frac{5}{12}$

So, tan P - cot R = $\frac{5}{12} - \frac{5}{12} = 0$

Question 3: If sin A = $\frac{3}{4}$, calculate cos A and tan A.

Answer: sin A = $\frac{3}{4}$ = $\frac{BC}{AC}$ [Given]

Let BC = 3k and AC = 4k
Hence, by using Pythagoras Theorem,

$AB^2 = AC^2 – BC^2$
$= (4k)^2 – (3k)^2$
$= 16k^2 – 9k^2$
$= 7k^2$

Or, $AB^2 = \sqrt{7}k$

Cos A = $\frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$

Tan A = $\frac{BC}{AB} = \frac{3k}{\sqrt{7}\,k} = \frac{3}{\sqrt{7}}$

Question 4: Given 15 cot A = 8, find sin A and sec A.

Answer: 15 cot A = 8
or, cot A = $\frac{8}{15}$
or, $\frac{AB}{AC} = \frac{8}{15}$

Let, AB = 8k and AC = 15k

In the ∆ABC, by using Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$= (8k)^2 + (15k)^2$
$= 64k^2 + 225k^2$
$= 289k^2$
Hence, AC = 17k

So, sin A = $\frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

Sec A = $\frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

Question 5: Given sec θ = $\frac{13}{12}$, calculate all other trigonometric ratios.

Answer: sec θ = $\frac{13}{12} = \frac{AC}{AB}$

Let AC = 13k and AB = 12k

Hence, By Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
or, $169k^2 = 144k^2 + BC^2$
or, BC = 5k

Hence,sin θ= BC/AC = 5/13
Cos θ = AB/AC = 12/13
tan θ = BC/AB = 5/12
Cosec θ = AC/BC = 13/5
cot θ = AB/BC = 12/5

Question 6: If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

Answer: Since, ∠A and ∠B. are acute angles.
Then ∠C = 90^0

Cos A = cos B [Given]
$\frac{AC}{AB} = \frac{BC}{AB}$

AC = BC
Therefore, ∠A = ∠B. [Angles opposite to equal sides are equal].

Question 7: If cot θ = $\frac{7}{8}$, evaluate:
(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$
(ii) $\cot^2\theta$

Answer: (1 + cos θ)(1 – cos θ)
We have cot θ = $\frac{7}{8} = \frac{AB}{BC}$

Let AB = 7k and BC = 8k

Then in ΔABC,
$AC^2 = AB^2 + BC^2$
$= (7k)^2 + (8k)^2$
$= 49k^2 + 64k^2$
$= 113k^2$

AC = k$\sqrt{113}$

Sin θ = $\frac{BC}{AC}$ = $\frac{8k}{\sqrt{113}\,k}$ = $\frac{8}{\sqrt{113}}$

Cos θ = $\frac{AB}{AC}$ = $\frac{7k}{\sqrt{113}\,k}$ = $\frac{7}{\sqrt{113}}$

(i) $\frac{(1+\text{Sin }\theta)(1-\text{Sin }\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-sin^2\theta}{1-cos^2\theta} = \frac{cos^2\theta}{sin^2\theta} = \frac{\left(\frac{7}{\sqrt{113}}\right)^2}{\left(\frac{8}{\sqrt{113}}\right)^2} = \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$

(ii) $cot^2\theta = \frac{cos^2\theta}{sin^2\theta} = \frac{49}{64}$

Question 8: If **3 cot A = 4, check whether $\frac{1-tan^2A}{1+tan^2A}$ = cos² A – sin² A or not**.

Answer: Let, in △ABC ∠B=90°

Hence, cot A = $\frac{AB}{BC}$ = $\frac{4}{3}$

Let AB = 4k an BC =3k

Therefore, according to the Pythagorean theorem,

$AC^2 = AB^2 + BC^2$
or, $AC^2 = (4k)^2 + (3k)^2$
or, $AC^2 = 16k^2 + 9k^2$
or, $AC^2 = 25k^2$
or, AC = 5k

Now,
tan(A) = BC/AB = $\frac{3}{4}$

sin (A) = BC/AC = $\frac{3}{5}$

cos (A) = AB/AC = $\frac{4}{5}$

LHS = $\frac{1-tan^2A}{1+tan^2A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{7}{25}$

RHS = cos² A – sin² A = $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$

Hence, LHS = RHS

Question 9: In triangle ABC, right-angled at B, if tan A = $\frac{1}{\sqrt{3}}$ find the value of:
(i) sin A cos C + cos A sin C
(ii) cos A cos C – sin A sin C

Answer: Let ABC is a right angled triangle at B.
tan A = $\frac{BC}{AB}$ = $\frac{1}{\sqrt{3}}$

Let AB = $\sqrt{3}$k and BC = k

Hence, by Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$
or, $AC^2 = (\sqrt{3}\ k)^2 + (k)^2$
or, $AC^2 = 3k^2 + k^2$
or, $AC^2 = 4k^2$
or, AC = 2k

Now, sin A = $\frac{k}{2k}$ = $\frac{1}{2}$

Cos A = $\frac{\sqrt{3}k}{2k}$ = $\frac{\sqrt{3}}{2}$

Sin C = $\frac{\sqrt{3}k}{2k}$ = $\frac{\sqrt{3}}{2}$

Cos C = $\frac{k}{2k}$ = $\frac{1}{2}$

(i) sin A cos C + cos A sin C = $\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$ = $\frac{1}{4} + \frac{3}{4}$ = 1

(ii) cos A cos C – sin A sin C = $\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$ = 0

Question 10: In ΔPQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Answer: In ΔPQR
$PR^2 = PQ^2 + QR^2$
or, $PQ^2 = PR^2 – QR^2$
or, 5^2 = (PR + QR)(PR – QR)
or, 25 = 25(PR – QR)
or, PR – QR = 1…………………………………….(1)
and, PR + QR = 25………………………………..(2)

Adding eq. (1) and (2),
2PR = 26
Or, PR = 13

From eq. (1) PR – QR = 1
or, QR = 12

Sin P = $\frac{12}{13}$

Cos P = $\frac{5}{13}$

Tan P = $\frac{12}{5}$

Question 11: State whether the following statements are true or false. Justify your answer.
(i) The value of tan A is always less than 1.
(ii) sec A = $\frac{12}{5}$ for some value of angle A.
(iii) cos A is the abbreviation used for the cosecant of angle A.
(iv) cot A is the product of cot and A.
(v) sin θ = $\frac{4}{3}$ for some angle.

Answer:

(i) False, because tan 60° = $\sqrt{3} > 1$.

(ii) True, because sec A ≥ 1

(iii) False, because cos A abbreviation is used for cosine A.

(iv) False, because the term cot A is single not a product.

(v) False, because $-1 \le \sin\theta \le 1$.

Exercise 8.2

Question 1: Evaluate the following:
(i) sin 60^0 cos 30^0 + sin 30^0 cos 60^0
(ii) 2 $\tan^2 45^0 + \cos^2 30^0 - \sin^2 60^0$
(iii) $\frac{\cos 45°}{\sec 30° + cosec\ 30°}$

(iv) $\frac{\sin 30° + \tan 45° - cosec\ 60°}{\sec 30° + \cos 60° + \cot 45°}$

Answer: (i) $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4}$

= 1

(ii) $2 \tan^2 45^0 + \cos^2 30^0 - \sin^2 60^0$

$= 2\,(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

= 2 + 0 = 2

(iii) $\frac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2}$

$= \frac{\sqrt{3}}{\sqrt{2}(2+2\sqrt{3})}$

$= \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$

$= \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} \times \frac{2\sqrt{2}-2\sqrt{6}}{2\sqrt{2}-2\sqrt{6}}$

$= \frac{2\sqrt{6}-2\sqrt{18}}{-16}$

$= \frac{2\sqrt{6}-6\sqrt{2}}{-16}$

$= \frac{2(\sqrt{6}-3\sqrt{2})}{-16}$

$= \frac{3\sqrt{2}-\sqrt{6}}{8}$

(iv) $\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1}$

$= \dfrac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}}$

$= \dfrac{\sqrt{3}+2\sqrt{3}-4}{4+\sqrt{3}+2\sqrt{3}}$

$= \dfrac{3\sqrt{3}-4}{4+3\sqrt{3}} \times \dfrac{4-3\sqrt{3}}{4-3\sqrt{3}}$

$= \dfrac{24\sqrt{3}-43}{-11} = \dfrac{43-24\sqrt{3}}{11}$

Question 2: Choose the correct option and justify your choice :

(i) $\dfrac{2\tan 30°}{1+\tan^2 30°} =$

(A) sin 60° (B) cos 60° (C) tan 60° (D) sin 30°

(ii) $1-\tan^2 45°/1+\tan^2 45° =$

(A) tan 90° (B) 1 (C) sin 45° (D) 0

(iii) sin 2A = 2 sin A is true when A =

(A) 0° (B) 30° (C) 45° (D) 60°

(iv) $2\tan 30°/1-\tan^2 30° =$

(A) cos 60° (B) sin 60° (C) tan 60° (D) sin 30°

Answer:

(i) $$\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\frac{1}{1}+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3+1}{3}} = \frac{2}{\sqrt{3}}\times\frac{3}{4} = \frac{3}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3\sqrt{3}}{2\times 3} = \frac{\sqrt{3}}{2} = \sin 60^\circ$$ Correct option is (A)

(ii) $$\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} = \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = 0$$ **Correct option is (D)**

(iii) **$\sin 2A = 2\sin A$, for $A = 0^\circ$**

LHS = $\sin 2A = \sin 2\times 0 = \sin 0^\circ = 0$

RHS = $2\sin A = 2\sin 0^\circ = 2\times 0 = 0$ **Correct option is (A)**

(iv) $$\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{\frac{1}{1}-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}} = \frac{2}{\sqrt{3}}\times\frac{3}{2} = \frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3\sqrt{3}}{3} = \sqrt{3} = \tan 60^\circ$$ **Correct option is (C)**

Question 3: If tan (A + B) = √3 and tan (A – B) = $\frac{1}{\sqrt{3}}$; 0° < A + B ≤ 90°; A > B, find A and B.

Answer: tan (A + B) = √3 = tan 60^0
or, A + B = 60^0 ……………………………………………….(1)

tan (A – B) = $\frac{1}{\sqrt{3}}$ = tan 30^0
or, A – B = 30^0 …………………………………………………(2)

Adding eq. (1) and (2),
2A = 90^0
or, A = 45^0

From (1) we get,
45^0 + B = 60^0
or, B = 15^0

Thus, A = 45^0 and B = 15^0

Question 4: State whether the following statements are true or false. Justify your answer.
(i) sin (A + B) = sin A + sin B.
(ii) The value of sin θ increases as θ increases.
(iii) The value of cos θ increases as θ increases.
(iv) sin θ = cos θ for all values of θ.
(v) cot A is not defined for A = 0°.

Answer:
(i) Let, $A = 60^0$ and $B = 30^0$
Then, LHS = $\sin(60^0 + 30^0)$
RHS = $\sin A + \sin B = \sin 60^0 + \sin 30^0 = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$

Hence, LHS is not equal to RHS.
Thus, the given statement is false.

(ii) $\sin 0^0 = 0$
$\sin 30^0 = \frac{1}{2}$
$\sin 45^0 = \frac{1}{\sqrt{2}}$
$\sin 60^0 = \frac{\sqrt{3}}{2}$
$\sin 90^0 = 1$

Value of $\sin\theta$ increases as θ increases.
Hence, the given statement is true.

(iii) $\cos 0^0 = 1$
$\cos 30^0 = \frac{\sqrt{3}}{2} = 0.87$
$\cos 45^0 = \frac{1}{\sqrt{2}}$
$\cos 60^0 = \frac{1}{2}$
$\cos 90^0 = 0$

Value of $\cos\theta$ decreases as θ increases.
Hence, the given statement is false

(iv) $\sin 30^0 = \frac{1}{2}$, $\cos 30^0 = \frac{\sqrt{3}}{2}$

Hence, $\sin 30^0 \neq \cos 30^0$

The given statement is false.

(v) $\cot 0^0 = \frac{\cos 0°}{\sin 0°} = \frac{1}{0}$ (not defined)

The given statement is true.

Exercise 8.3

Question 1: (i) $\frac{\sin 18°}{\cos 72°}$ (ii) $\frac{\tan 26°}{\cot 64°}$ (iii) $\cos 48^0 – \sin 42^0$ (iv) $\text{cosec } 31^0 – \sec 59^0$

Answer: (i) $\frac{\sin 18°}{\cos 72°} = \frac{\sin(90°-72°)}{\cos 72°} = \frac{\cos 72°}{\cos 72°} = 1$

(ii) $\frac{\tan 26°}{\cot 64°} = \frac{\tan(90°-64°)}{\cot 64°} = \frac{\cot 64°}{\cot 64°} = 1$

(iii) $\cos 48^0 – \sin 42^0 = \cos (90^0 – 42^0) – \sin 42^0$

$= \sin 42^0 – \sin 42^0$

$= 0$

(iv) $\text{cosec } 31^0 – \sec 59^0 = \text{cosec } 31^0 – \sec (90^0 - 31^0)$

$= \text{cosec } 31^0 – \text{cosec } 31^0$

$= 0$

Question 2: Show that:

(i) tan 48° tan 23° tan 42° tan 67° = 1

(ii) cos 38° cos 52° – sin 38° sin 52° = 0

Answer: (i) tan 48° tan 23° tan 42° tan 67°

$= \tan 48° \tan 23° \tan (90^0 – 48^0) \tan (90^0 – 23^0)$

$= \tan 48° \tan 23° \cot 48^0 \cot 23^0$

$= \tan 48° \tan 23° \frac{1}{\tan 48°} \frac{1}{\tan 23°}$

$= 1$

(ii) LHS = cos 38° cos 52° – sin 38° sin 52°

$= \cos 38° \cos (90^0 – 38^0) – \sin 38° \sin (90^0 – 38^0)$

= cos 38° sin 38° - sin 38° cos 38°
= 0 = RHS

Question 3: If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.

Answer: We have, tan 2A = cot (A – 18°)
or, cot (90^0 – 2A) = cot (A – 18^0)   [As, cot ($90^0 - \theta$) = tan θ
or, 90^0 – 2A = A – 18^0
or, 2A + A = 90^0 + 18^0
or, 3A = 108^0
or, A = $\frac{108°}{3}$ = 36^0

Question 4: If tan A = cot B, prove that A + B = 90°.

Answer: tan A =cot B
or, tan A = tan (90^0 – B)   [tan ($90^0 - \theta$) = cot θ]
or, A = 90^0 – B
or, A + B = 90^0 [proved]

Question 5: If **sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A.**

Answer: sec 4A = cosec (A – 20^0)
or, cosec (90 – 4A) = cosec (A – 20^0)
or, 90^0 – 4A = A – 20^0
or, 90^0 + 20^0 = 5A
or, 110^0 = 5A
or, $\frac{110}{5}$ = A
or, A = 22^0

Question 6: If A**, B and C are interior angles of a triangle ABC, then show that: sin $\left(\frac{B+C}{2}\right)$ = cos $\frac{A}{2}$**

Answer: In triangle ABC, A + B + C = 180^0
or, B+C = 180^0 – A
or, $\frac{B+C}{2}$ = $\frac{180°-A}{2}$

or, $\frac{B+C}{2}$ = 90^0 - $\frac{A}{2}$
or, sin $\left(\frac{B+C}{2}\right)$ = sin $\left(90° - \frac{A}{2}\right)$
or, sin $\left(\frac{B+C}{2}\right)$ = cos $\frac{A}{2}$ [Proved]

Question 7: Express (sin 67° + cos 75°) in terms of trigonometric ratios of angles between 0° and 45°.

Answer:
sin 67^0 + cos 75^0 = sin (90^0 – 23^0) + cos (90^0 – 15^0) = cos 23^0 + sin 15^0

Exercise 8.4

Question 1: Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.Solution:
(i) $\frac{\boldsymbol{sin\ 18°}}{\boldsymbol{cos\ 72°}}$
(ii) $\frac{\boldsymbol{tan\ 26°}}{\boldsymbol{cot\ 64°}}$

Answer: (i) $cosec^2$ A – cot^2 A = 1
or, $\frac{1}{sin^2 A}$ = 1 + cot^2 A

or, sin^2 A = $\frac{1}{1 + \cot^2 A}$

or, sin A = $\sqrt{\frac{1}{1+cot^2 A}}$

or, sin A = $\frac{1}{\sqrt{1+ cot^2 A}}$

(ii) sec^2 A = 1 + tan^2 A
or, sec^2 A = 1 + $\frac{1}{cot^2 A}$ = $\frac{cot^2 A+1}{cot^2 A}$

or, sec A = $\sqrt{\frac{cot^2 A+1}{cot^2 A}}$ = $\frac{\sqrt{cot^2 A+1}}{\cot A}$

Question 2: Write all the other trigonometric ratios of ∠A in terms of sec A.

Answer: $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A = \frac{1}{1} = \frac{1}{sec^2 A} = \frac{sec^2 A - 1}{sec^2 A}$

or, $\sin A = \frac{\sqrt{sec^2 A - 1}}{\sec A}$

$\text{Cos } A = \frac{1}{\sec A}$

$\text{Tan } A = \frac{\sin A}{\cos A} = \frac{\frac{\sqrt{sec^2 A - 1}}{\sec A}}{\frac{1}{\sec A}} = \sqrt{sec^2 A - 1}$

$\text{Cosec } A = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{sec^2 A - 1}}{\sec A}} = \frac{\sec A}{\sqrt{sec^2 A - 1}}$

$\text{Cot } A = \frac{1}{\tan A} = \frac{1}{\sqrt{sec^2 A - 1}}$

Question 3: Show that:

(i) $\frac{sin^2 63° + sin^2 27°}{cos^2 17° + cos^2 73°}$

(ii) sin 25° cos 65° + cos 25° sin 65°

Answer: (i) $\frac{sin^2 63° + sin^2 27°}{cos^2 17° + cos^2 73°} = \frac{sin^2 63° + sin^2 (90° - 63°)}{cos^2 (90° - 73°) + cos^2 73°} = \frac{sin^2 63° + cos^2 63°}{sin^2 73° + cos^2 73°} = 1$

(ii) sin 25° cos 65° + cos 25° sin 65°

$= \sin 25^0 \cos(90^0 - 25^0) + \cos 25^0 \sin(90^0 - 25^0)$

$= \sin^2 25^0 + \cos^2 25^0$

$= 1$

Question 4: **Choose the correct option. Justify your choice.**

(i) 9 sec² A – 9 tan² A = ……

(A) 1 (B) 9 (C) 8 (D) 0

(ii) (1 + tan θ + sec θ) (1 + cot θ – cosec θ)

(A) 0 (B) 1 (C) 2 (D) – 1

(iii) (sec A + tan A) (1 – sin A) =

(A) sec A (B) sin A (C) cosec A (D) cos A

(iv) $\frac{1 + tan^2 A}{1 + cot^2 A}$ **=**

(A) $\sec^2 A$ **(B) -1 (C)** $\cot^2 A$ **(D)** $\tan^2 A$

Answer: (i) $9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A) = 9 \times 1 = 9$
correct option is (B).

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$

$= \left(\frac{1}{1} + \frac{sin\theta}{cos\theta} + \frac{1}{cos\theta}\right)\left(\frac{1}{1} + \frac{cos\theta}{sin\theta} - \frac{1}{sin\theta}\right)$

$= \left(\frac{cos\theta + sin\theta + 1}{cos\theta}\right)\left(\frac{sin\theta + cos\theta - 1}{sin\theta}\right)$

$= \frac{(cos\theta + \sin\theta)^2 - 1^2}{cos\theta sin\theta}$

$= \frac{1 + 2cos\theta sin\theta - 1}{cos\theta sin\theta}$

$= \frac{2cos\theta sin\theta}{cos\theta sin\theta} = 2$

Correct option is (C).

(iii) $(\sec A + \tan A)(1 - \sin A)$

$= \left(\frac{1}{cos\,A} + \frac{\sin A}{\cos A}\right)\left(\frac{1 - \sin A}{1}\right)$

$= \left(\frac{1 + \sin A}{\cos A}\right)\left(\frac{1 - \sin A}{1}\right)$

$= \frac{1^2 - (sinA)^2}{\cos A}$

$= \frac{1 - sin^2 A}{\cos A}$

$= \frac{cos^2 A}{\cos A} = \cos A$

Correct option is (D)

(iv) $\frac{1 + tan^2 A}{1 + cot^2\,A} = = \frac{sec^2 A}{cosec^2 A} = \frac{\frac{1}{cos^2\,A}}{\frac{1}{sin^2 A}} = \frac{cos^2 A}{cos^2 A} = \tan$