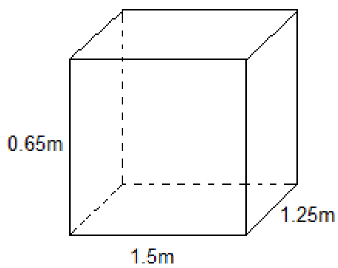
Chapter 13 – Surface areas and Volumes
Exercise- 13.1

Question 1: A plastic box 1.5 m long, 1.25 m wide and 65 cm deep is to be made. It is opened at the top. Ignoring the thickness of the plastic sheet, determine
(i) The area of the sheet required for making the box.
(ii) The cost of sheet for it, if a sheet measuring 1m^2 costs Rs.20.

Answer:



(i) Here, length (l) = 1.5 m, breadth(b) = 1.25 m and height (h) = 65 cm = $\frac{65}{100}$ m = 0.65 m
Since, the plastic box is open from the top.
Hence, its surface area
= [Lateral surface area] + [Base area]
= [2(1 + b)h] + [lb]
= [2(1.50 + 1.25)0.65] m^2 + [1.50 x 1.25] m^2
= [2 x 2.75 x 0.65] m^2 + [1.875] m^2
= 3.575 m^2+ 1.875 m^2 = 5.45 m^2
Therefore, Area of the sheet required for making the box = 5.45 m^2

(ii) Cost of 1 m^2 sheet = Rs. 20
Cost of 5.45 m^2 sheet = Rs. (20 x 5.45) = Rs. 109
Hence, cost of the required sheet = Rs. 109

Question 2: The length, breadth and height of a room are 5 m, 4 m and 3 m, respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of Rs.17.50 per m^2.

Answer: Length of the room (l) = 5 m
Breadth of the room (b) = 4 m
Height of the room (h) = 3 m
The room is like a cuboid whose four walls (lateral surface) and ceiling are to be

white washed.
Therefore, the required area for white washing
= [Lateral surface area] + [Area of the ceiling]
= [2(l + b)h] + [l x b]
= [2(5 + 4) x 3] m^2 + [5 x 4] m^2 = 54 m^2 + 20 m^2 = 74 m^2
Cost of white washing for 1m^2 area = Rs. 7.50
Therefore, cost of white washing for 74 m^2 area = Rs. (7.50 x 74) = Rs. 555
Thus, the required cost of white washing = Rs. 555

Question 3: The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs.10 per m^2 is Rs. 15000, find the height of the hall.
[Hint: Area of the four walls = Lateral surface area]

Answer: A rectangular hall means a cuboid.
Let the length and breadth of the hall be l and b respectively.
Hence, perimeter of the floor = 2(l + b)
or, 2(l + b) = 250 m
Since the area of four walls = Lateral surface area = 2(1 + b) x h, where h is the height of the hall = 250 h m^2
Cost of painting the four walls
= Rs. (10 x 250 h) = Rs. 2500h
or, 2500 h = 15000
or, h = $\frac{15000}{2500}$ = 6
Thus, the required height of the hall = 6 m

Question 4: The paint in a certain container is sufficient to paint an area equal to 9.375 m^2. How many bricks of dimensions 22.5 cm x 10 cm x 7.5 cm can be painted out of this container.

Answer: Total area that can be painted = 9.375 m^2
Here, Length of a brick (l) = 22.5 cm
Breadth of a brick (b) = 10 cm
Height of a brick (h) = 7.5 cm
Since a brick is like a cuboid, then,
Total surface area of a brick = 2[lb + bh + hl]
= 2[(225 x 1(0) + (10 x 7.5) + (7.5 x 22.5)] cm^2
= 2[(225) + (75) + (168.75)] cm^2
= 2[468.75] cm^2 = 937.5 cm^2 = $\frac{937.5}{10000}$ m^2
Let the required number of bricks be n, then,
Therefore, total surface area of n bricks = n x $\frac{937.5}{10000}$ m^2
or, n $\times$ $\frac{937.5}{10000}$ m^2 = $\frac{9375}{1000}$ m^2 [Given]

or, n = $\frac{9375}{1000} \times \frac{10000}{937.5}$
or, n = 100

Thus, the required number of bricks = 100

Question 5: A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.
(i) Which box has the greater lateral surface area and by how much?
(ii) Which box has the smaller total surface area and by how much?

Answer: For the cubical box with edge (a) = 10 cm
Lateral surface area = $4a^2 = 4 \times 10^2\ cm^2 = 400\ cm^2$
Total surface area = $6a^2 = 6 \times 10^2\ cm^2 = 600\ cm^2$
For the cuboidal box with dimensions,
Length (l) = 12.5 cm,
Breadth (b) = 10 cm,
Height (h) = 8 cm
Hence, the lateral surface area = $2[l + b] \times h = 2[12.5 + 10] \times 8\ cm^2 = 360\ cm^2$
Total surface area = $2[lb + bh + hl]$
$= 2[(12.5 \times 10) + (10 \times 8) + (8 \times 12.5)]\ cm^2$
$= 2[125 + 80 + 100]\ cm^2$
$= 2[305]\ cm^2$
$= 610\ cm^2$
(i) A cubical box has the greater lateral surface area by $(400 – 360)\ cm^2 = 40\ cm^2$.
(ii) Total surface area of a cubical box is smaller than the cuboidal box by $(610 – 600)\ cm^2 = 10\ cm^2$.

Question 6: A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.
(i) What is the area of the glass?
(ii) How much of tape is needed for all the 12 edges?

Answer: The herbarium is like a cuboid.
Here, length (l) = 30 cm,
breadth (b) = 25 cm,
height (h) = 25 cm

(i) Surface area of the herbarium glass
$= 2[lb + bh + hl]$
$= 2[(30 \times 25) + (25 \times 25) + (25 \times 30)]\ cm^2 – 2[750 + 625 + 750]\ cm^2$
$= 2[2125]\ cm^2$
$= 4250\ cm^2$
Thus, the required area of the glass = $4250\ cm^2$

(ii) Total length of 12 edges = 4l + 4b + 4h
= 4(l + b + h)
= 4(30 + 25 + 25) cm
= 4 x 80 cm = 320 cm
Thus, the required length of tape = 320 cm

Question 7: Shanti Sweets Stalll was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm x 20 cm x 5 cm and the smaller of dimensions 15 cm x 12 cm x 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs4 for 1000 cm², find the cost of cardboard required for supplying 250 boxes of each kind.

Answer: <u>For bigger box:</u>
Length (l) = 25 cm,
Breadth (b) = 20 cm,
Height (h) = 5 cm
Total surface area of a box = 2(lb + bh + hl)
= 2[(25 x 20) + (20 x 5) + (5 x 125)] cm^2
= 2 [500 + 100 + 125] cm^2
= 2[725] cm^2
= 1450 cm^2
Total surface area of 250 boxes = (250 x 1450) cm^2 = 362500 cm^2

<u>For smaller box:</u>
l = 15 cm, b = 12 cm, h = 5 cm
Total surface area of a box = 2 [lb + bh + hl]
= 2[(15 x 12) + (12 x 5) + (5 x 15)] cm^2
= 2[180 + 60 + 75] cm^2 = 2[315] cm^2 = 630 cm^2
Therefore, the total surface area of 250 boxes = (250 x 630) cm^2 = 157500 cm^2
Now, total surface area of both type of boxes = 362500 cm^2 +157500 cm^2 = 520000 cm^2 Area for overlaps = 5% of [total surface area]
= $\frac{5}{100}$ x 520000 cm^2 = 26000 cm^2
Hence, the total surface area of the cardboard required = [Total surface area of 250 boxes of each type] + [Area for overlaps]
= 520000 cm^2 + 26000 cm^2 = 546000 cm^2
Since, the cost of 1000 cm^2 cardboard = Rs. 4
Therefore, the required cost of 546000 cm^2 cardboard
= Rs.$\frac{4\times546000}{1000}$ = Rs. 2184

Question 8: Parveen wanted to make a temporary shelter, for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming

that the stitching margins are very small and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m x 3 m?

Answer: Here, length (l) = 4 m, breadth (b) = 3m and height (h) = 2.5 m
The structure is like a cuboid.
Therefore, the surface area of the cuboid, excluding the base
=[Lateral surface area] + [Area of ceiling]
= [2(l + b)h] + [lb]
= [2(4 + 3) x 2.5] m^2 + [4 x 3] m^2
= 35 m^2 + 12 m^2 = 47 m^2
Thus, 47 m^2 tarpaulin would be required.

Exercise 13.2

Question 1: The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder.

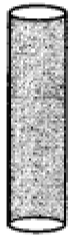
Answer: Let r be the radius of the cylinder.
Here, height (h) = 14 cm and curved surface area = 88 cm^2
Curved surface area of a cylinder = 2πrh
or, 2πrh = 88
or, 2 x $\frac{22}{7}$ x r x 14 = 88
or, r = $\frac{88\times7}{2\times22\times14}$ = 1 cm
Hence, the diameter = 2 x r = (2 x 1) cm = 2 cm

Question 2: It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?

Answer: Here, height (h) = 1 m
Diameter of the base = 140 cm = 1.40 m
Radius (r) = $\frac{1.40}{2}$m = 0.70 m
Total surface area of the cylinder = 2πr (h + r)
= 2 x $\frac{22}{7}$ x 0.70(1 + 0.70)m^2
= 2 x 22 x 0.10 x 1.70 m^2
= 2 x 22 x $\frac{10}{100}$ x $\frac{170}{100}$$m^2$
= $\frac{148}{100}$$m^2$ = 7.48 m^2
Hence, the required sheet = 7.48 m^2

Question 3: A metal pipe is 77 cm long. The inner ft diameter of a cross section is 4 cm, the outer diameter being 4.4 cm (see figure). Find its

(i) inner curved surface area.
(ii) outer curved surface area.
(iii) total surface area.



Answer: Length of the metal pipe = 77 cm
It is in the form of a cylinder.
∴ Height of the cylinder (h) = 77 cm
Inner diameter = 4 cm
Inner radius (r) = $\frac{4}{2}$ cm = 2 cm
Outer diameter = 4.4 cm
or, Outer radius (R) = $\frac{4.4}{2}$ cm = 2.2 cm

(i) Inner curved surface area = 2πrh
= 2 x $\frac{22}{7}$ x 2 x 77 cm^2
= 2 x 22 x 2 x 11 cm^2 = 968 cm^2

(ii) Outer curved surface area = 2πRh
= $2 \times \frac{22}{7} \times 2.2 \times 77$ cm^2

= $\frac{2\times22\times22\times11}{10}$ cm^2

= $\frac{10648}{10}$ cm^2

= 1064.8 cm^2

(iii) Total surface area = [Inner curved surface area] + [Outer curved surface area] + [Area of two circular ends]
= [2πrh] + [2πRh] + 2[π(R^2 – r^2)]
= $[968\ cm^2] + [1064.8\ cm^2] + 2 \times \frac{22}{7}(2.2^2 - 2^2)cm^2$
= $968\ cm^2 + 1064.8\ cm^2 + \frac{2\times22}{7}(4.84 - 4)cm^2$
= $2032.8\ cm^2 + \frac{2\times22\times0.84}{7}cm^2$
= $2032.8\ cm^2 + 5.28\ cm^2$
= $2038.8\ cm^2$

Question 4: The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2.

Answer: The roller is in the form of a cylinder of diameter = 84 cm
Or, Radius of the roller(r) = $\frac{84}{2}$ cm = 42 cm
Length of the roller (h) = 120 cm
Curved surface area of the roller = 2πrh
= 2 x $\frac{22}{7}$ x 42 x 120 cm^2
= 2 x 22 x 6 x 120 cm^2 = 31680 cm^2
Now, area of the playground levelled in one revolution of the roller = 31680 cm^2
= $\frac{31680}{10000}$ m^2

Therefore, Area of the playground levelled in 500 revolutions = 500 x $\frac{31680}{10000}$$m^2$ = 1584m^2

Question 5: A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs.12.50 per m^2 .

Answer: Diameter of the pillar = 50 cm
Hence, radius (r) = $\frac{50}{2}$ cm = 25 cm = $\frac{1}{4}$ m and height (h) = 3.5m
Curved surface area of a pillar = 2πrh

$$= 2 \times \frac{22}{7} \times \frac{1}{4} \times 3.50 \text{ m}^2$$
$$= \frac{44 \times 350}{7 \times 4 \times 100} \text{ m}^2$$
$$= \frac{11}{2} \text{ m}^2$$

Therefore, curved surface area to be painted = 112m^2
Hence, the cost of painting of 1 m^2 pillar = Rs. 12.50
so, the cost of painting of $\frac{11}{2}$ m^2 pillar
= Rs. ($\frac{11}{2}$ x 12.50)
= Rs. 68.75.

Question 6: Curved surface area of a right circular cylinder is 4.4 m2. If the radius of the base of the cylinder is 0.7 m, find its height. Curved surface area of a right circular cylinder is 4.4 m2. If the radius of the base of the cylinder is 0.7 m, find its height.

Answer: Radius (r) = 0.7 m
Let the height of the cylinder be “h” m

Hence, the curved surface area of a cylinder = 2πrh = 2 x $\frac{22}{7}$ x $\frac{7}{10}$ x h m^2
It is given that the curved surface area is 4.4 m^2.
or, 2 × $\frac{22}{7}$ × $\frac{7}{10}$ x h = 4.4
or, h = 1 m

Thus the required height is 1 m.

Question 7: The inner diameter of a circular well is 3.5m. It is 10m deep. Find
(i) its inner curved surface area,
(ii) the cost of plastering this curved surface at the rate of Rs. 40 per m^2.
(Assume π = 22/7)

Answer: Given that the inner radius of circular well, r = 3.5/2m = 1.75m
Depth of circular well, say h = 10m

(i) Inner curved surface area = 2πrh
= (2×$\frac{22}{7}$)×1.75×10)
= 110

Therefore, the inner curved surface area of the circular well is 110 m^2.

(ii)Cost of plastering 1 m^2 area = Rs.40
Cost of plastering 110 m^2 area = Rs (110×40) = Rs.4400

Therefore, the cost of plastering the curved surface of the well is Rs. 4400.

Question 8: In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.

Answer: The height of cylindrical pipe = The length of cylindrical pipe = 28m
Radius of circular end of pipe = $\frac{diameter}{2}$ = $\frac{5}{2}$ cm = 2.5cm = 0.025m

Now, the Curved Surface Area of cylindrical pipe = 2πrh,
= 2×($\frac{22}{7}$)×0.025×28 m^2
= 4.4m^2

The area of the radiating surface of the system is 4.4m^2.

Question 9: Find
(i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.
(ii) how much steel was actually used, if 112 of the steel actually used was wasted in making the tank.

Answer: Given that the storage tank is in the form of a cylinder.

Therefore, the diameter of the tank = 4.2 m
or, Radius (r) = $\frac{4.2}{2}$ = 2.1 m
Height (h) = 4.5 m
Now,
(i) Lateral (or curved) surface area of the tank = 2πrh
= 2 x $\frac{22}{7}$ x 2.1 x 4.5 m^2
= 2 x 22 x 0.3 x 4.5 m^2 59.4 m^2

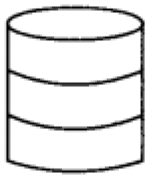
(ii) Total surface area of the tank = 2πr(r + h)
= 2 x $\frac{22}{7}$ x 2.1(2.1 + 4.5)m^2
= 44 x 0.3 x 6.6 m^2 = 87.13 m^2
Let actual area of the steel used be x m^2
therefore, area of steel that was wasted = $\frac{1}{12}$ x x m
= $\frac{x}{12}$ m^2
Area of steel used = x – $\frac{x}{12}$ m^2
= $\frac{12x-x}{12}$ m^2
= $\frac{11x}{12}$ m^2

Now, 87.12 = $\frac{11x}{12}$

Or, x = $\frac{8712}{100} \times \frac{12}{11} = \frac{104544}{1100}$ = 95.04

Thus, the required area of the steel that was actually used is 95.04 m^2.

Question 10: In figure, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.



Answer: The lampshade is in the form of a cylinder, where radius (r) = $\frac{20}{2}$ cm = 10 cm and height = 30 cm.
A margin of 2.5 cm is to be added to the top and bottom of the frame.
Hence, the total height of the cylinder, (h)
= 30 cm + 2.5 cm + 2.5 cm = 35 cm

Now, curved surface area = $2\pi rh$
$= 2 \times \frac{22}{7} \times 10 \times 35\ cm^2$
$= 2200\ cm^2$
Thus, the required area of the cloth = $2200\ cm^2$

Question 11: The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition?

Answer: Here, the penholders are in the form of cylinder.
Radius of a penholder (r) = 3 cm
Height of a penholder (h) = 10.5 cm
Since, a penholder must be open from the top.
Now, surface area of a penholder = [Lateral surface area] + [Base area]
$= [2\pi rh] + [\pi r^2]$
$=2\times(\frac{22}{7})\times3\times10.5+(\frac{22}{7})\times3^2= \frac{1585}{7}$

Hence, the area of cardboard sheet used by one competitor is $\frac{1585}{7}\ cm^2$.
So, the area of cardboard sheet used by 35 competitors = $35\times\frac{1585}{7} = 7920\ cm^2$
Therefore, $7920\ cm^2$ cardboard sheet will be needed for the competition.

Exercise 13.3

Question 1: Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

Answer: Here, diameter of the base = 10.5 cm
hence, the radius (r) = $\frac{10.5}{2}$ cm and slant height (l) =10 cm
Curved surface area of the cone = πrl
$= \frac{22}{7} \times \frac{10.5}{2} \times 10cm^2$
$= 11 \times 15 \times 1\ cm^2 = 165cm^2$

Question 2: Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Answer: Here, diameter = 24 m
Hence, Radius (r) = $\frac{24}{2}$ m = 12 m and slant height (l) = 21 m
Therefore, the total surface area of a cone = $\pi r(r +1)$
Total Surface area of the cone = $(\frac{22}{7}) \times 12 \times (21+12)\ m^2 = 1244.57m^2$ (approx.)

Question 3: Curved surface area of a cone is 308 cm² and its slant height is 14 cm. Find
(i) radius of the base and
(ii) total surface area of the cone.

Answer: Here, curved surface area = πrl = 308 cm^2
Slant height (l) = 14 cm

(i) Let the radius of the base be 'r' cm
Therefore,πrl = 308
or, $\frac{22}{7}$ x r x 14 = 308
r = $\frac{308 \times 7}{22 \times 14}$ = 7cm
Thus, radius of the cone is 7 cm

(ii) It is given that base area = $\pi r^2 = \frac{22}{7} \times 7^2$ cm^2 = 22 x 7 cm^2 = 154 cm^2 and curved surface area = 308 cm^2
Hence, Total surface area of the cone
= [Curved surface area] + [Base area] = 308 cm^2 + 154 cm^2
= 462 cm^2

Question 4: A conical tent is 10 m high and the radius of its base is 24 m. Find
(i) slant height of the tent.
(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs.70.

Answer: Here, height of the tent (h) = 10 m
Radius of the base (r) = 24 m

(i) The slant height, l = $\sqrt{(r^2 + h^2}$ = $\sqrt{(24^2 + 10^2}$ = $\sqrt{(576 + 100}$ m = $\sqrt{676}$ m = 26m
Thus, the required slant height of the tent is 26 m.

(ii) Curved surface area of the cone = πrl
Therefore, the area of the canvas required = $\frac{22}{7} \times 24 \times 26$ m^2 = $\frac{13728}{7}$ m^2

Hence, the cost of $\frac{13728}{7}$ m^2 canvas
= Rs. 70 × $\frac{13728}{7}$ = Rs. 137280

Question 5: What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. (Use π = 3.14)

Answer: Here, base radius (r) = 6 m
Height(h) = 8m
∴ Slant height (l) = $\sqrt{(r^2 + h^2}$ = $\sqrt{(6^2 + 8^2}$ m = $\sqrt{(36 + 64}$m = $\sqrt{100}$m = 10 m
Now, curved surface area = πrl
= 3.14 x 6 x 10m^2
= $\frac{314}{100}$ x 6 x 10 m^2 = 1884 m^2
Thus, area of the tarpaulin required to make the tent = 188.4 m^2
Let the length of the tarpaulin be L m
Length x Breadth = 188.4
or, L x 3 = 188.4
or, L = $\frac{18804}{3}$ = 62.8
Extra length of tarpaulin required for margins = 20cm = $\frac{20}{100}$m = 0.2m
Thus, total length of tarpaulin required = 62.8 m + 0.2 m = 63 m

Question 6: The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of ₹ 210 per 100 m^2 .

Answer: Here, base radius (r) = $\frac{14}{2}$ m = 7 m
Slant height (l) = 25 m
Hence, the curved surface area = πrl = $\frac{22}{7}$ x 7 x 25 m^2 = 550 m^2
Cost of white-washing for 100 m^2 area = Rs. 210
Hence, the cost of white-washing for 550 m^2 area
= Rs. $\frac{210}{100}$ x 550 = Rs. 1155

Question 7: A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Answer: Radius of the base (r) = 7 cm and height (h) = 24 cm
Slant height (l) = $\sqrt{(h^2 + r^2}$ = $\sqrt{(24^2 + 7^2)}$ cm = $\sqrt{576 + 49}$ = 625 cm = 25 cm
Hence, the lateral surface area = πrl = 227 x 7 x 25 cm^2 = 550 cm^2
Lateral surface area of 10 caps = 10 x 550 cm^2 = 5500 cm^2
Thus, the required area of the sheet = 5500 cm^2

Question 8: A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs. 12 per m^2, what will be the cost of painting all these cones? (Use π = 3.14 and take $\sqrt{104}$ = 1.02)

Answer: Diameter of the base = 40cm

Radius of cone, r = $\frac{diameter}{2}$ = $\frac{40}{2}$ cm = 20cm = 0.2 m

Height of cone, h = 1m

Slant height of cone is l, and $l^2 = (r^2+h^2)$

Using the given values, $l^2 = (0.2^2+1^2) = (1.04)$

Or, l = 1.02

Hence, the slant height of the cone is 1.02 m

Now,

Curved surface area of each cone = πrl

= (3.14×0.2×1.02)

= 0.64056

Curved surface area of 50 such cones = (50×0.64056) = 32.028

Curved surface area of 50 such cones = 32.028 m^2

Again,

It is given that the cost of painting 1 m^2 area = Rs 12

hence, the cost of painting 32.028 m^2 area = Rs (32.028×12)

= Rs.384.336

= Rs.384.34 (approx..)

Therefore, the cost of painting all these cones is Rs. 384.34.

Exercise 13.4

Question 1: The surface area of a sphere (SA) = $4\pi r^2$

(i) Radius of sphere, r = 10.5 cm (given)

Surface area = $4\times(\frac{22}{7})\times10.5^2 = 1386$

Surface area of sphere is 1386 cm^2

(ii) Radius of sphere, r = 5.6cm

Using formula, SA = $4\times(\frac{22}{7})\times5.6^2 = 394.24$

Surface area of sphere is 394.24 cm^2

(iii) Radius of sphere, r = 14cm

SA = $4\pi r^2$

$= 4\times(\frac{22}{7})\times(14)^2$
$= 2464$
Surface area of sphere is 2464 cm^2

Question 2: Find the surface area of a sphere of diameter
(i) 14 cm
(ii) 21 cm
(iii) 3.5 m

Answer: (i) The radius of sphere, $r = \frac{d}{2} = \frac{14}{2}$ cm = 7 cm
Surface area of sphere = $4\pi r^2 = 4\times(\frac{22}{7})\times 7^2 = 616$
Surface area of a sphere is 616 cm^2

(ii) Radius (r) of sphere = $\frac{21}{2}$ = 10.5 cm
Surface area of sphere = $4\pi r^2 = 4\times(\frac{22}{7})\times 10.5^2 = 1386$
Surface area of a sphere is 1386 cm^2
Therefore, the surface area of a sphere having diameter 21cm is 1386 cm^2

(iii) Radius(r) of sphere = $\frac{3.5}{2}$ = 1.75 cm
Surface area of sphere = $4\pi r^2 = 4\times(22/7)\times 1.75^2 = 38.5$
Surface area of a sphere is 38.5 cm^2

Question 3: Find the total surface area of a hemisphere of radius 10 cm. (Use π = 3.14)

Answer: Given, the radius (r) = 10 cm
Total surface area of hemisphere = $3\pi r^2$ = 3 x 3.14 x 10 x 10 cm^2 = 942 cm^2

Question 4: The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Answer: Let r_1 and r_2 be the radii of spherical balloon and spherical balloon when air is pumped into it respectively. So,
r_1 = 7cm
r_2 = 14 cm

Now, The required ratio

$$= \frac{initial\ \ surface\ area}{surface\ area\ after\ pumping\ air\ into\ balloon}$$

$= \frac{4r_1^2}{4r_2^2}$

$= \left(\frac{r_1}{r_2}\right)^2$

$= \left(\frac{7}{14}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Therefore, the ratio between the surface areas is 1:4.

Question 5: A hemispherical bowl made of brass has inner diameter 10.5cm. Find the cost of tin-plating it on the inside at the rate of Rs 16 per 100 cm^2. (Assume π = 22/7)

Answer: Inner radius of hemispherical bowl, say r = $\frac{diameter}{2} = \frac{10.5}{2}$ cm = 5.25 cm

The surface area of hemispherical bowl = $2\pi r^2 = 2\times(\frac{22}{7})\times(5.25)^2 = 173.25$

Surface area of hemispherical bowl is 173.25 cm^2

Cost of tin-plating 100 cm^2 area = Rs 16

Cost of tin-plating 1 cm^2 area = Rs $\frac{16}{100}$

Cost of tin-plating 173.25 cm^2 area = Rs. $\frac{16 \times 173.25}{100}$= Rs 27.72

Question 6: Find the radius of a sphere whose surface area is 154 cm^2. (Assume π = 22/7)

Answer: Let the radius of the sphere be r.

And it is given that the surface area of sphere = 154 cm^2

Now,

$4\pi r^2 = 154$

$r^2 = \frac{154\times7}{4\times22} = \frac{49}{4}$

$r = \frac{7}{2}$ cm = 3.5 cm

Therefore, the radius of the sphere is 3.5 cm.

Question 7: he diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Answer: If diameter of earth is said d, then it is given that the diameter of moon will be $\frac{d}{4}$

Radius of earth = $\frac{d}{4}$

Radius of moon = $\frac{1}{2} \times \frac{d}{4} = \frac{d}{8}$

Surface area of moon = $4\pi \left(\frac{d}{8}\right)^2$

Surface area of earth = $4\pi \left(\left(\frac{d}{2}\right)^2$

Hence, the ratio of their surface areas = $\frac{4\pi \left(\frac{d}{8}\right)^2}{4\pi \left(\left(\frac{d}{2}\right)^2} = \frac{4}{64} = \frac{1}{16}$

Therefore, the ratio between their surface areas is 1:16.

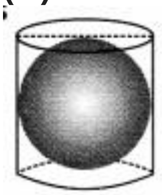
Question 8: A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5cm. Find the outer curved surface of the bowl. (Assume π =22/7)

Answer: Given that the inner radius of hemispherical bowl = 5cm
Thickness of the bowl = 0.25 cm
Outer radius of hemispherical bowl = (5+0.25) cm = 5.25 cm

Hence, the outer Curved Surface Area of hemispherical bowl = $2\pi r^2$, where r is radius of hemisphere = $2\times(\frac{22}{7})\times(5.25)^2$ = 173.25

Therefore, the outer curved surface area of the bowl is 173.25 cm^2.

Question 9: A right circular cylinder just encloses a sphere of radius r (see figure). Find
(i) surface area of the sphere,
(ii) curved surface area of the cylinder,
(iii) ratio of the areas obtained in (i) and (ii).



Answer: (i) Surface area of sphere = $4\pi r^2$, where r is the radius of the sphere.

(ii) Height of cylinder, h = r+r =2r
Let the radius of cylinder = r

Curved Surface Area of cylinder formula = $2\pi rh = 2\pi r(2r) = 4\pi r^2$ [Using the value of "h" in place of "r"]

(iii) Ratio between areas = $\frac{Surafce\ area\ of\ sphere}{curved\ surface\ area\ of\ cylinder} = \frac{4r^2}{4r^2} = \frac{1}{1}$
Ratio of the areas obtained in (i) and (ii) is 1:1.

Exercise 13.5

Question 1: A matchbox measures 4 cm×2.5cm×1.5cm. What will be the volume of a packet containing 12 such boxes?

Answer: Given dimensions of the matchbox, which is a cuboid are l × b × h = 4 cm×2.5 cm×1.5 cm
Therefore, the volume of matchbox = l × b × h = (4×2.5×1.5) = 15 cm^3
Hence, the volume of 12 such matchboxes = (15×12) cm^3 = 180 cm^3

Question 2. A cuboidal water tank is 6m long, 5m wide and 4.5m deep. How many litres of water can it hold? (1 m^3= 1000 l)

Answer: Given dimensions of the cuboidal water tank are: l = 6 m and b = 5 m and h = 4.5 m

Volume of tank, V = l × b × h = (6×5×4.5)cm^3 = 135 cm^3

Again, it is given that, amount of water that 1m^3 volume can hold = 1000 l
Amount of water, 135 m^3volume hold = (135×1000) litres = 135000 litres

Question 3: A cuboidal vessel is 10m long and 8m wide. How high must it be made to hold 380 cubic metres of a liquid?

Answer: Given, the length of cuboidal vessel, l = 10 m
width of cuboidal vessel, b = 8m
Hence, the volume of cuboidal vessel, V = 380 m^3

Now, let the height of the given vessel be h.
Volume of a cuboid, V = l×b×h = 380
or, 10×8×h= 380
or, h = 4.75

Therefore, the height of the vessels is 4.75 m.

Question 4. Find the cost of digging a cuboidal pit 8m long, 6m broad and 3m deep at the rate of Rs 30 per m^3.

Answer: The given pit has its length(l) = 8m, width (b) = 6m and depth (h) = 3 m.
Volume of the cuboidal pit = l × b × h = (8 × 6 × 3) = 144
Hence, the required Volume is 144 m^3

Now,
The cost of digging per m^3 volume = Rs 30
So, the cost of digging 144 m^3 volume = Rs (144×30) = Rs 4320

Question 5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Answer: The length (l) of the tank = 2.5m and depth (h) =10 m.
Let the breadth be b.
So, the volume of a tank = l × b × h = (2.5× b×10) m^3= 25b m^3
Capacity of the tank= 25b m^3, which is equal to 25000b litres

Also, it is given that the capacity of a cuboidal tank is 50000 litres of water.
Therefore, 25000 b = 50000
or, b = 2

Therefore, the breadth of the tank is 2 m.

Question 6. A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m×15 m×6 m. For how many days will the water of this tank last?

Answer: The length of the tank = l = 20 m, breadth of the tank = b = 15 m and height of the tank = h = 6 m
The total population of a village = 4000
Hence the consumption of the water per head per day = 150 litres

Water consumed by the people in 1 day = (4000×150) litres = 600000 litres …………………..…(1)
The capacity of tank, C = l × b × h = (20×15×6) m^3= 1800 m^3
or, C = 1800000 litres

Let water in this tank last for d days.
Water consumed by all people in d days = Capacity of tank (using equation (1))
or, 600000 d =1800000
d = 3

Therefore, the water of this tank will last for 3 days.

Question 7. A godown measures 40 m×25m×15 m. Find the maximum number of wooden crates each measuring 1.5m×1.25 m×0.5 m that can be stored in the godown.

Answer: It is given that the length of the godown = 40 m, breadth = 25 m and height = 15 m
Whereas, the length of the wooden crate = 1.5 m, breadth = 1.25 m and height = 0.5 m

Since godown and wooden crate, both are in cuboidal shape.
Hence, V = l × b × h.

Now, Volume of the godown = (40×25×15) m^3 = 15000 m^3
And volume of the wooden crate = (1.5×1.25×0.5) m^3 = 0.9375 m^3

Let us consider that, n wooden crates can be stored in the godown,
then, volume of n wooden crates = Volume of godown
or, 0.9375×n =15000
or, n= 15000/0.9375 = 16000

Hence, the number of wooden crates that can be stored in the godown is 16,000.

Question 8: A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

Answer: Given, the side of the cube = 12cm
Volume of the cube = $(\text{Side})^3 = (12)^3 \text{cm}^3 = 1728 \text{cm}^3$

Surface area of the cube with side 12 cm = $6a^2$ = $6(12)^2$ cm^2 ………………………………..…(1)

Given that the cube is cut into eight small cubes of equal volume, say side of each cube is p.
Hence, the volume of a small cube = p^3

Surface area = $6p^2$ ……………………………………………………………………..…(2)

Volume of each small cube = $\frac{1728}{8}$ cm^3 = 216 cm^3
or, $(p)^3$ = 216 cm^3
or, p = 6 cm

Now, Surface areas of the cubes ratios = $\frac{surface\ area\ of\ bigger\ cube}{surafce\ area\ of\ smaller\ cubes}$
From equation (1) and (2), we get
Surface areas of the cubes ratios = $\frac{6a^2}{6p^2} = \frac{a^2}{p^2} = \frac{12^2}{6^2}$ = 4

Therefore, the required ratio is 4 : 1.

Question 9. A river 3m deep and 40m wide is flowing at the rate of 2km per hour. How much water will fall into the sea in a minute?

Answer: Given the depth of river, h = 3 m, width of river, b = 40 m
Rate of the water flow = 2km per hour = $\frac{2000}{60}$ min = $\frac{100}{3}$ metre/min

Now, the volume of water flowed in 1 min = $\frac{100}{3} \times 40 \times 3 = 4000m^3$
Therefore, 4000 m^3 water will fall into the sea in a minute.

Exercise 13.6

Question 1: The circumference of the base of cylindrical vessel is 132cm and its height is 25cm. How many litres of water can it hold? (1000 cm^3= 1L) (Assume π = 22/7)

Answer: Given, the circumference of the base of cylindrical vessel = 132 cm and height of vessel, h = 25 cm Let r be the radius of the cylindrical vessel.

To find the radius of vessel
We know that, circumference of base = 2πr
So, it is given that, 2πr = 132
or, = (132/(2 π)) $\frac{132}{2\pi}$
or, = $66\times\frac{7}{22} = 21$

Hence, the radius is 21 cm

To find the volume of vessel
We know that, the volume of cylindrical vessel = $\pi r^2 h$
= $(\frac{22}{7})\times21^2\times25$
= 34650

Therefore, volume is 34650 cm^3

Since, we know that, 1000 cm^3 = 1L
So, the vessel can hold = $\frac{34650}{1000}$ L= 34.65L of waters

Question 2: The inner diameter of a cylindrical wooden pipe is 24cm and its outer diameter is 28 cm. The length of the pipe is 35cm.Find the mass of the pipe, if 1cm^3 of wood has a mass of 0.6g. (Assume π = 22/7)

Answer: Given the inner radius of cylindrical pipe, $r_1 = \frac{diameter_1}{2} = \frac{24}{2}$ cm = 12cm

And the outer radius of cylindrical pipe, say $r_2 = \frac{diameter_2}{2} = \frac{28}{2}$ cm = 14 cm

Height of pipe, h = Length of pipe = 35cm

Now, the Volume of pipe = $\pi(r_2^2 - r_1^2)h$ cm^3
Volume of pipe = 110×52 cm^3 = 5720 cm^3

Since, it is given that, Mass of 1 cm^3 wood = 0.6 g
Mass of 5720 cm^3 wood = (5720×0.6) g = 3432 g or 3.432 kg

Question 3: A soft drink is available in two packs
(i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm.
(ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

Answer: (i) For rectangular pack,
Length (l) = 5 cm,
Breadth (b) = 4 cm
Height (h) = 15 cm
Hence, volume = l x b x h = 5 x 4 x 15 cm^3 = 300 cm^3
Therefore, capacity of the rectangular pack = 300 cm^3

(ii) For cylindrical pack,
Base diameter = 7 cm
Therefore, the radius of the base (r) = $\frac{7}{2}$ cm
Height (h) = 10 cm
Volume = $\pi r^2 h = \frac{22}{7} \times (\frac{7}{2})^2 \times 10$cm
= $\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10$cm
= 11 x 7 x 5cm^3 = 385cm^3
Therefore, capacity of the cylindrical pack = 385 cm^3
So, the cylindrical pack has greater capacity by (385 – 300) cm^3 = 85 cm^3

Question 4: If the lateral surface of a cylinder is 94.2cm^2 and its height is 5cm, then find
(i) radius of its base
(ii) its volume. [Use π = 3.14]

Answer: Curved Surface Area of cylinder = 94.2 cm^2
Height of cylinder, h = 5cm

(i) Let radius of cylinder be r.
Using Curved Surface Are of cylinder, we get
$2\pi rh = 94.2$
$2 \times 3.14 \times r \times 5 = 94.2$
$r = 3$

Hence, the radius is 3 cm

(ii) Volume of cylinder
Volume of cylinder = $\pi r^2 h$
Now, $\pi r^2 h = (3.14 \times (3)^2 \times 5)$ (using value of r from part (i)) = 141.3

Hence, the volume is 141.3 cm^3

Question 5: It costs Rs 2200 to paint the inner curved surface of a cylindrical vessel 10m deep. If the cost of painting is at the rate of Rs 20 per m^2, find
(i) inner curved surface area of the vessel
(ii) radius of the base
(iii) capacity of the vessel (Assume π = 22/7)

Answer: i) Rs 20 is the cost of painting 1 m^2 area.
Rs 1 is the cost to paint 1/20 m^2 area
So, Rs 2200 is the cost of painting = $(\frac{1}{20}\times 2200)$ m^2 = 110 m^2 area

The inner surface area of the vessel is 110m^2.

(ii) Radius of the base of the vessel be r.
Height (h) = 10 m and
Surface area formula = $2\pi rh$

Using result of part (i)
$2\pi rh$ = 110 m^2
or, $2 \times \frac{22}{7} \times r \times 10 = 110$
r = 1.75
Hence, the radius is 1.75 m.

(iii) Volume of vessel formula = $\pi r^2 h$
Given r = 1.75 and h = 10
Volume of the vessel = $(\frac{22}{7})\times(1.75)^2\times 10 = 96.25$
Therefore, the capacity of the vessel is 96.25 m^3 or 96250 litres.

Question 6: The capacity of a closed cylindrical vessel of height 1m is15.4 liters. How many square meters of metal sheet would be needed to make it? (Assume π = 22/7)

Answer: Height of cylindrical vessel, h = 1 m
Capacity of cylindrical vessel = 15.4 litres = 0.0154 m^3

Let r be the radius of the circular end.

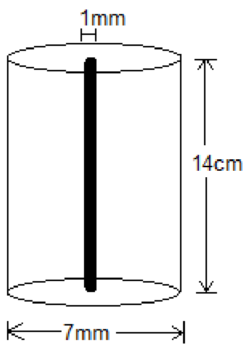
Now, the capacity of cylindrical vessel = $(\frac{22}{7}) \times r^2 \times 1 = 0.0154$
or, r = 0.07 m

Again, total surface area of vessel = $2\pi r (r + h)$
= 2×22/7×0.07(0.07+1)
= 0.44×1.07
= 0.4708

Total surface area of vessel is 0.4708 m^2

Question 7: A lead pencil consists of a cylinder of wood with solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite. (Assume π = 22/7)

Answer:



Radius of pencil, $r_1 = \frac{7}{2}$ mm = $\frac{0.7}{2}$ cm = 0.35 cm
Radius of graphite, $r_2 = \frac{1}{2}$ mm = $\frac{0.1}{2}$ cm = 0.05 cm

Height of pencil, h = 14 cm
Hence, volume of wood in pencil = $(r_1^2 - r_2^2)h$ cubic units

We have
= $[(\frac{22}{7})\times(0.35^2-0.05^2)\times 14]$
= 44×0.12
= 5.28

Hence, the volume of wood in pencil = 5.28 cm^3

Again, Volume of graphite = r_2^2h cubic units
We have,
= $(\frac{22}{7})\times 0.05^2\times 14$

$= 44 \times 0.0025$
$= 0.11$

So, the volume of graphite is 0.11 cm^3.

Question 8: A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7cm. If the bowl is filled with soup to a height of 4cm, how much soup the hospital has to prepare daily to serve 250 patients? (Assume π = 22/7)

Answer: The diameter of cylindrical bowl = 7 cm
Radius of cylindrical bowl, $r = \frac{7}{2}$ cm = 3.5 cm
Given that, bowl is filled with soup to a height of 4cm, so h = 4 cm

Hence, the volume of soup in one bowl= $\pi r^2 h = (\frac{22}{7}) \times 3.5^2 \times 4 = 154$
Volume of soup in one bowl is 154 cm^3

Therefore, the volume of soup given to 250 patients = (250×154) cm^3= 38500 cm^3 = 38.5litres

Exercise 13.7

Question 1: Find the volume of the right circular cone with
(i) radius 6cm, height 7 cm (ii) radius 3.5 cm, height 12 cm (Assume π = 22/7)

Answer: Volume of cone = $(\frac{1}{3}) \pi r^2 h$ cube units
Where r be radius and h be the height of the cone

(i) Radius of cone, r = 6 cm and height of cone, h = 7cm
Let, V be the volume of the cone, we have
$V = \frac{1}{3} \times \frac{22}{7} \times 36 \times 7$
$= (12 \times 22)$
$= 264$

The volume of the cone is 264 cm^3.

(ii) Radius of cone, r = 3.5cm and height of cone, h = 12cm
Hence, the volume of cone = $\frac{1}{3} \times \frac{22}{7} \times 3.5^2 \times 7 = 154$

Question 2: Find the capacity in litres of a conical vessel with (i) radius 7cm, slant height 25 cm (ii) height 12 cm, slant height 12 cm (Assume π = 22/7)

Answer: (i) Radius of the cone, r =7 cm, slant height, l = 25 cm

Height of the cone, h = $\sqrt{l^2 - r^2}$ = $\sqrt{25^2 - 7^2}$ = $\sqrt{625 - 49}$ = $\sqrt{576}$
or, h = 24 cm

Volume of cone, V = $\frac{1}{3}\pi r^2 h$ (formula)
V = $\frac{1}{3} \times \frac{22}{7} \times 7^2 \times 24$ cm^3
= (154×8) cm^3
= 1232

Therefore, capacity of the conical vessel = (1232/1000) litres (as, 1L = 1000 cm^3) = 1.232 Litres.

(ii) Height of cone, h = 12 cm and slant height of cone, l = 13 cm
Radius of the cone, r = $\sqrt{l^2 - h^2}$ = $\sqrt{13^2 - 12^2}$ = $\sqrt{169 - 144}$ = $\sqrt{25}$
or, r = 5 cm

Now, Volume of the cone, V = (1/3)$\pi r^2 h$

Volume of the cone = $\frac{1}{3} \times \frac{22}{7} \times 52 \times 12$ cm^3 = $\frac{2200}{7}$ cm^3
Now, the capacity of the conical vessel= $\frac{2200}{7000}$ litres (1L = 1000 cm^3) = $\frac{11}{35}$ litres

Question 3: The height of a cone is 15cm. If its volume is 1570cm^3, find the diameter of its base. (Use π = 3.14)

Answer: Given, the height of the cone, h = 15 cm
Volume of cone =1570 cm^3

Let the radius of the cone be r.
We know that, volume of a cone, V = $\frac{1}{3}\pi r^2 h$

So, $\frac{1}{3}\pi r^2 h$ = 1570
or, $\frac{1}{3} \times 3.14 \times r^2 \times 15 = 1570$
or, r^2 = 100
or, r = 10

Hence, the radius is 10cm.

Question 4: If the volume of a right circular cone of height 9cm is 48πcm^3, find the diameter of its base.

Answer: Height of cone, h = 9cm and volume of cone =48π cm^3
Let the radius of the cone be r.

As, Volume of cone, V = $\frac{1}{3}$ πr^{2}h
So, $\frac{1}{3}$ π r^2(9) = 48 π
or, r^2 = 16
or, r = 4

Hence, the radius of cone is 4 cm. So, diameter = 2×radius = 2 × 4 cm = 8cm

Question 5: A conical pit of top diameter 3.5m is 12m deep. What is its capacity in kilolitres? (Assume π = 22/7)

Answer: Given the diameter of conical pit = 3.5 m, radius of conical pit, r = diameter/2 = (3.5/2)m = 1.75m and height of pit, h = Depth of pit = 12m

Volume of cone, V = $\frac{1}{3}$ πr^{2}h
V = $\frac{1}{3}$ × $\frac{22}{7}$ ×(1.75)^2×12 m^3 = 38.5 m^3

Hence, capacity of the pit = (38.5×1) kilolitres = 38.5 kilolitres.

Question 6: The volume of a right circular cone is 9856cm^3. If the diameter of the base is 28cm, find (i) height of the cone
(ii) slant height of the cone
(iii) curved surface area of the cone (Assume π = 22/7)

Answer: Given the volume of a right circular cone = 9856 cm^3 , diameter of the base = 28 cm

(i) Radius of cone, r = $\frac{28}{2}$ cm = 14 cm and the height of the cone be h
As we know that, the volume of cone, V = $\frac{1}{3}$ πr^{2}h
or, $\frac{1}{3}$ πr^{2}h = 9856
or, $\frac{1}{3}$ × $\frac{22}{7}$ ×14×14×h = 9856
h = 48

The height of the cone is 48 cm.

(ii) Slant height of the cone, l = r = $\sqrt{r^2 + h^2}$
or, l = $\sqrt{14^2 + 48^2}$
or, l = $\sqrt{196 + 2304}$
or, l = 50

Hence, the slant height is 50cm.

(iii) Curved surface area of a cone = $\pi rl = \frac{22}{7} \times 14 \times 50$ cm^2 = 2200 cm^2

Question 7: A right triangle ABC with sides 5cm, 12cm and 13cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Answer: Given, Height (h)= 12 cm, radius (r) = 5 cm, and slant height (l) = 13 cm
Volume of cone, $V = \frac{1}{3}\pi r^2 h$
or, $V = \frac{1}{3}\times\pi\times5^2\times12$ cm^3 = 100π cm^3

Question 8: If the triangle ABC in the Question 7 is revolved about the side 5cm, then find the volume of the solids so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Answer: A right-angled ΔABC is revolved about its side 5cm, a cone will be formed of radius as 12 cm, height as 5 cm, and slant height as 13 cm.

Volume of cone = $\frac{1}{3}\pi r^2 h$; where r is the radius and h be the height of cone
$= \frac{1}{3}\times\pi\times12\times12\times5$ cm^3
= 240 π cm^3

So, required ratio = $\frac{100\pi}{240\pi} = \frac{10}{24}$ = 5:12.

Question 9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas. (Assume π = 22/7)

Answer: Radius of the given heap, $r = \frac{10.5}{2}$ m = 5.25, height of the heap, h = 3m and volume of heap = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7}\times5.25\times5.25\times3$ m^3 = 86.625 m^3
Again, area of canvas required = curved surface area of cone = πrl

Hence, the curved surface area of cone
$= \frac{22}{7} \times 5.25 \times \sqrt{5.25^2 + 3^2}$ m^2
= 99.825 m^2

<u>Exercise 13.8</u>

Question 1: Find the volume of a sphere whose radius is (i) 7 cm (ii) 0.63 m (Assume π =22/7)

Answer: (i) We know that, volume of a sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 7^3$ $cm^3 = \frac{4321}{3}$ cm^3

(ii) Given, the radius of sphere, r = 0.63 m
Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 0.63^3$ m^3 = 1.0478 m^3 (approx.)

Question 2: Find the amount of water displaced by a solid spherical ball of diameter (i) 28 cm (ii) 0.21 m (Assume π =22/7)

Answer: i) Given, the diameter = 28 cm, radius, r = $\frac{28}{2}$ cm = 14cm
Volume of the solid spherical ball = $\frac{4}{3}\pi r^3$

Volume of the ball = $\frac{4}{3} \times \frac{22}{7} \times 14^3$ cm^3 = $\frac{34496}{3}$ cm^3

(ii) Given, the diameter = 0.21 m
Radius of the ball = $\frac{0.21}{2}$ m= 0.105 m
Volume of the ball = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 0.105^3$ m^3

Question 3: The diameter of a metallic ball is 4.2cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3? (Assume π=22/7)

Answer: Given, the diameter of a metallic ball = 4.2 cm
Radius of the metallic ball, r = $\frac{4.2}{2}$ cm = 2.1 cm

Volume of the metallic ball = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 2.1$ cm^3 = 38.808 cm^3
Now, the density = $\frac{mass}{volume}$

Mass = Density × volume = (8.9×38.808) g = 345.3912 g (approx.)

Question 4: The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Answer: Let the diameter of earth be d.
Therefore, the radius of earth will be will be $\frac{d}{2}$
Diameter of moon will be $\frac{d}{4}$ and the radius of moon will be $\frac{d}{8}$

Volume of the moon = $\frac{4}{3}$ πr^3 = $\frac{4}{3}$ π $^3 \times \frac{d^3}{8^3} = \frac{4}{3} \times \pi \times \frac{d^3}{512}$

Volume of the earth = = $\frac{4}{3}$ πr^3 = $\frac{4}{3}$ π $\times \frac{d^3}{8}$

$$\frac{volume\ of\ the\ moon}{volume\ of\ the\ earth} = \frac{\frac{4}{3}\times \pi \times \frac{d^3}{512}}{\frac{4}{3}\ \pi\ \times \frac{d^3}{8}} = \frac{8}{512} = \frac{1}{64}$$

Hence, Volume of moon is of the 1/64 volume of earth.

Question 5: How many litres of milk can a hemispherical bowl of diameter 10.5cm hold? (Assume π = 22/7)

Answer: Given the diameter of hemispherical bowl = 10.5 cm
Radius of hemispherical bowl, r = $\frac{10.5}{2}$ cm = 5.25 cm

Hence, the volume of the hemispherical bowl = $\frac{2}{3}$ πr^3
Volume of the hemispherical bowl = $\frac{2}{3} \times \frac{22}{7} \times 5.25^3$ cm^3= 303.1875 cm^3

Capacity of the bowl = $\frac{303.1875}{1000}$ L = 0.303 litres(approx.)

Question 6: A hemi spherical tank is made up of an iron sheet 1cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank. (Assume π = 22/7)

Answer: Inner Radius of the tank, (r) = 1m and Outer Radius (R) = 1.01m
Volume of the iron used in the tank = $\frac{2}{3}$ π(R^3– r^3)
Volume of the iron used in the hemispherical tank = $\frac{2}{3} \times \frac{22}{7} \times(1.01^3 - 1^3)$ = 0.06348

7. Find the volume of a sphere whose surface area is 154 cm^2. (Assume π = 22/7)

Answer: Let r be the radius of a sphere. And we know that, surface area of sphere = 4πr^2
So, 4πr^2 = 154 cm^2 [Given]
r^2 = $\frac{154\times7}{4\times22}$ cm
r = $\frac{7}{2}$ cm

Now, Volume of the sphere = $\frac{4}{3}$ πr^3

Volume of the sphere, = $\frac{4}{3} \times \frac{22}{7} \times \frac{7^3}{2^3}$ $= 179\frac{2}{3}$ cm^3

Question 8: A dome of a building is in the form of a hemi sphere. From inside, it was white-washed at the cost of Rs. 4989.60. If the cost of white-washing isRs20 per square meter, find the

(i) inside surface area of the dome (ii) volume of the air inside the dome

(Assume π = 22/7)

Answer: (i) Cost of white-washing the dome from inside = Rs 4989.60
Cost of white-washing 1m^2 area = Rs 20
CSA of the inner side of dome = 498.96/2 m^2 = 249.48 m^2

(ii) Let the inner radius of the hemispherical dome be r.
CSA of inner side of dome = 249.48 m^2 (from (i))
We know that, CSA of a hemi sphere = 2πr^2

$2\pi r^2 = 249.48$

$2 \times \frac{22}{7} \times r^2 = 249.48$

$r^2 = \frac{249.48 \times 7}{2 \times 22}$

$r^2 = 39.69$

$r^2 = 6.3$

So, the radius is 6.3m.

Volume of air inside the dome = Volume of hemispherical dome
As we know, volume of the hemisphere = 2/3 πr^3

= $\frac{2}{3} \times \frac{22}{7} \times 6.3 \times 6.3 \times 6.3$

= $523.908 = 523.9\ (approx.)$

Hence, Volume of air inside dome is 523.9 m^3.

Question 9: Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S'. Find the

(i) radius r' of the new sphere,

(ii) ratio of Sand S'.

Answer: As we know, Volume of a cube = $\frac{4}{3}\pi r^3$
Hence, the volume of 27 solid spheres = $27 \times \frac{4}{3}\pi r^3 = 36\pi r^3$

i) Radius of new solid iron sphere = r'
Volume of this sphere = $\frac{4}{3}\pi(r')^3$

$\frac{4}{3}\pi(r')^3 = 36\pi r^3$

$(r')^3 = 27r^3$

$(r') = 3r$

Therefore the radius of the new sphere will be 3r.

ii) Surface area of a sphere with radius r (S)= $4\pi r^2$

Surface area of sphere with radius r' (S') = $4\pi(r')^2$

Therefore, $\frac{S}{S\prime} = \frac{4\pi r^2}{4\pi(r\prime)^2}$

or, $\frac{S}{S\prime} = \frac{r^2}{(3r)^2} = \frac{1}{9}$

The ratio of S and S' is 1:9.

Question 10: A capsule of medicine is in the shape of a sphere of diameter 3.5mm. How much medicine (in mm^3) is needed to fill this capsule? (Assume π = 22/7)

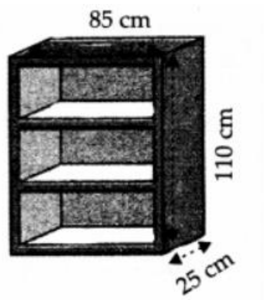
Answer:

Diameter of capsule = 3.5 mm
Radius of capsule, say r = diameter/ 2 = (3.5/2) mm = 1.75mm
Volume of spherical capsule = $\frac{4}{3}\pi r^3$
Volume of spherical capsule = $\frac{4}{3} \times \frac{22}{7} \times (1.75)^3$ = 22.458

Hence the volume of the spherical capsule is 22.46 mm^3.

Exercise 13.9

Question 1: A wooden bookshelf has external dimensions as follows: Height = 110cm, Depth = 25cm, Breadth = 85cm. The thickness of the plank is 5cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.



Answer: Given dimensions of book shelf,

Length, l = 85cm
Breadth, b = 25 cm
Height, h = 110 cm

External surface area of shelf excluding the front face of the shelf

= lh+2(lb+bh)

= [85×110+2(85×25+25×110)] = (9350+9750) = 19100

External surface area of shelf is 19100 cm^2
Area of front face = [85×110-75×100+2(75×5)] = 1850+750

So, the area is 2600 cm^2

Now, Area to be polished = (19100+2600) cm^2 = 21700 cm^2.
Given cost of polishing 1 cm^2 area = Rs 0.20

Cost of polishing 21700 cm^2 area Rs. (21700×0.20) = Rs 4340

Dimensions of row of the book shelf

Length(l) = 75 cm
Breadth (b) = 20 cm
Height(h) = 30 cm

Area to be painted in one row= 2(l+h)b+lh = [2(75+30)× 20+75×30] = (4200+2250) = 6450

So, the area is 6450 cm^2.

Area to be painted in 3 rows = (3×6450)cm^2 = 19350 cm^2.

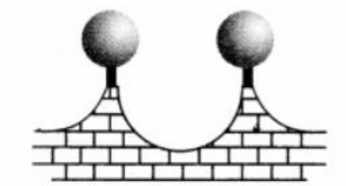
Cost of painting 1 cm^2 area = Rs. 0.10

Therefore, Cost of painting 19350 cm^2 area = Rs (19350 x 0.1) = Rs 1935

Total expense required for polishing and painting = Rs. (4340+1935) = Rs. 6275

Hence the cost for polishing and painting the surface of the book shelf is Rs. 6275.

Question 2: The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in figure. Eight such spheres are used forth is purpose, and are to be painted silver. Each support is a cylinder of radius 1.5cm and height 7cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.



Answer: Diameter of wooden sphere = 21 cm
Radius of wooden sphere, r = diameter/ 2 = (21/2) cm = 10.5 cm

We know, Surface area of wooden sphere = $4\pi r^2$

= $4 \times \frac{22}{7} \times (10.5)^2$ = 1386

So, surface area is 1386 cm^3

Given Radius of the circular end of cylindrical support = 1.5 cm
Height of cylindrical support = 7 cm

We know, Curved surface area = $2\pi rh$

= $2 \times \frac{22}{7} \times 1.5 \times 7 = 66$

So, CSA is 66 cm^2

Area of the circular end of cylindrical support = πr^2

= $\frac{22}{7} \times (1.5)^2$

= 7.07

Area of the circular end is 7.07 cm^2
Area to be painted silver = [8 ×(1386-7.07)] = 8×1378.93 = 11031.44
Area to be painted is 11031.44 cm^2

Therefore, Cost for painting with silver colour = Rs(11031.44×0.25) =Rs 2757.86

Area to be painted black = (8×66) cm^2 = 528 cm^2

Cost for painting with black colour =Rs (528×0.05) = Rs26.40

Therefore, the total painting cost is:

= Rs(2757.86 +26.40)

= Rs 2784.26

Question 3: The diameter of a sphere is decreased by 25%. By what percent does its curved surface area decrease?

Answer: Let the diameter of the sphere "d".
Radius of sphere, r_1=d/2

New radius of sphere , let $r_2=\frac{d}{2} \times \left(1 - \frac{25}{100}\right) = \frac{3d}{8}$

Curved surface area of sphere, $(CSA)_1=4\pi r_1{}^2 = 4\pi \left(\frac{d}{2}\right)^2 = \pi d^2 \quad ...(1)$

Curved surface area of sphere when radius is decreased $(CSA)_2=4\pi r_2{}^2$

= $4\pi \left(\frac{3d}{8}\right)^2 = \frac{9}{16}\pi d^2 \quad ...(2)$

From (1) and (2) we get the decrease in surface area = $(CSA)_1$-$(CSA)_2$

=$\pi d^2 - \frac{9}{16}\pi d^2 = \frac{7}{16}\pi d^2$

Percentage decrease in surface area of sphere = $\frac{(\mathrm{CSA})_1-(\mathrm{CSA})_2}{(\mathrm{CSA})_2} \times 100$

$= \frac{7d^2}{16d^2} \times 100 = \frac{700}{16} = 43.75\%$

Therefore the percentage decrease in the surface area of the sphere is 43.75%