<u>Chapter 9: Appplication of Trigonometry</u>
<u>Exercise – 9.1</u>

Question 1: A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°. (see fig. 9.11)

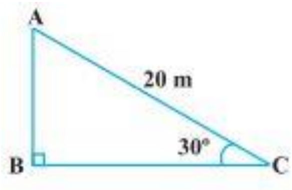


Fig. 9.11

Answer: Length of the rope is 20 m and $\angle ACB$ = 30°.
AC = 20 m and ∠C = 30° [Given]

<u>To Find</u>: Height of the pole

Let AB be the vertical pole
In the right ΔABC, using sine formula
sin 30° = $\frac{AB}{AC}$

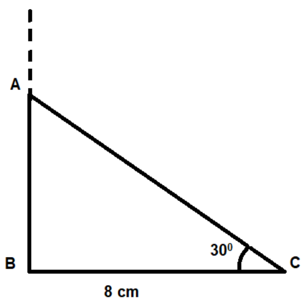
Or, $\frac{1}{2} = \frac{AB}{20}$
Or, AB = 10

Therefore, the height of the pole is 10 m.

Question 2: A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the tree's foot to the point where the top touches the ground is eight m. Find the height of the tree.

Solution: Let AC be the broken part of the tree. ∠C = 30° and BC = 8 m

To Find: Height of the tree, which is AB



So, the total height of the tree is the sum of AB and AC, i.e. AB + AC

In the right ΔABC,
cos 30° = $\frac{AC}{BC}$

We know that, cos 30° = $\frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} = \frac{8}{AC}$

Or, AC = $\frac{16}{\sqrt{3}}$ ……………………………………………………….………(1)

Also, tan 30° = AB/BC

$\frac{1}{\sqrt{3}} = \frac{AB}{8}$

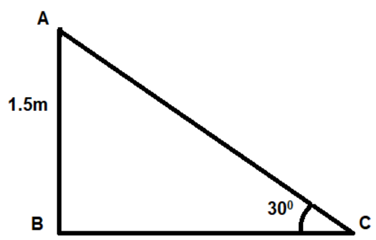
AB = $\frac{8}{\sqrt{3}}$……………………………………………………………………..(2)

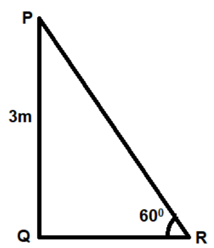
Therefore, total height of the tree = AB + AC
$= \frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}} = \frac{24}{\sqrt{3}} = \frac{8}{\sqrt{3}}$ m.

Question 3: A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a decline whose top is at the height of 1.5 m and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at the height of 3m, and inclined at an angle of 60° to the ground. What should be the length of the decline in each case?

Answer: As per the contractor's plan,

Ages	Height of Slide	Inclined Angle
Below 5years	1.5m	30 degrees
Above 5years	3m	60 degrees





In right ΔABC,

sin 30° = $\frac{AB}{AC}$

Or, $\frac{1}{2} = \frac{1.5}{AC}$
or, AC = 3

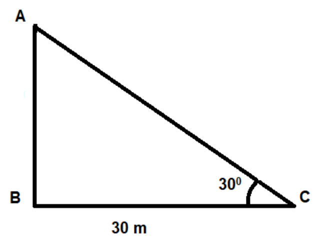
Also, in the right ΔPQR,

sin 60° = $\frac{PQ}{PR}$
Or, $\frac{\sqrt{3}}{2} = \frac{3}{PR}$
Or, PR = $2\sqrt{3}$

Hence, the length of the slide for below 5 = 3 m and length of the slide for elders children = $2\sqrt{3}$ m

Question 4: The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the tower's foot, is 30°. Find the height of the tower.

Answer: Let AB be the height of the tower, C be the point of elevation which is 30 m away from the building's foot.



In right ABC tan 30° = $\frac{AB}{BC}$

Or, $\frac{1}{\sqrt{3}} = \frac{AB}{30}$

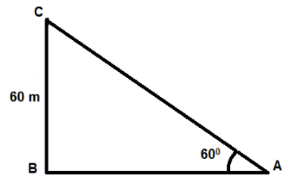
Or, AB = $10\sqrt{3}$

Thus, the height of the tower is $10\sqrt{3}$ m.

Question 5: A kite is flying at the height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination

of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Answer: Let BC = 60 m and A is the point where a string of the kite is tied.



From the above figure,

sin 60° = $\frac{BC}{AC}$

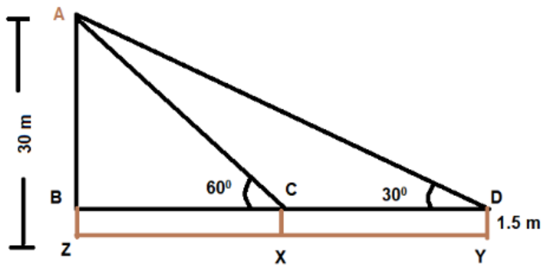
or, $\frac{\sqrt{3}}{2} = \frac{60}{AC}$

Or, AC = 40√3 m

Thus, the length of the string from the ground is 40√3 m.

Question 6: A 1.5 m tall boy is standing at some distance from a 30 m tall building. The elevation angle from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Answer:



From the figure, XY = CD and height of the building = AZ = 30 m.
AB = AZ – BZ = (30 – 1.5)m = 28.5m

In right ΔABD, tan 30° = $\frac{AB}{BD}$
Or, $\frac{1}{\sqrt{3}}$ = $\frac{28.5}{BD}$
Or, BD = 28.5√3 m

Again, in right ΔABC,

tan 60° = $\frac{AB}{BC}$
or, √3 = $\frac{28.5}{BC}$
or,BC = $\frac{28.5}{\sqrt{3}}$ = $\frac{28.5\sqrt{3}}{3}$

Therefore, the length of BC is $\frac{28.5\sqrt{3}}{3}$ m.

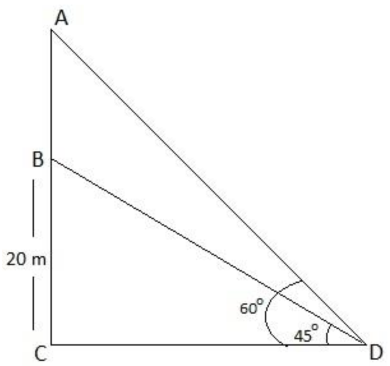
XY = CD = BD – BC
= (28.5√3 - $\frac{28.5\sqrt{3}}{3}$)
= 28.5√3(1 - $\frac{1}{3}$)
= 28.5√3 × $\frac{2}{3}$ = $\frac{57}{\sqrt{3}}$ m.

Thus, the distance boy walked towards the building is $\frac{57}{\sqrt{3}}$ m.

Question 7: From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Answer: Let BC = 20 m
D is the point where the elevation is taken.

Height of transmission tower = AB = AC – BC



From the figure, in the right ΔBCD,
tan 45° = $\frac{BC}{CD}$

Or, 1 = $\frac{20}{CD}$
or, CD = 20

Again, in the right ΔACD,
or, tan 60° = $\frac{AC}{CD}$
Or. √3 = $\frac{AC}{20}$
or, AC = 20√3

Now, AB = AC – BC = (20√3 - 20) = 20(√3-1)

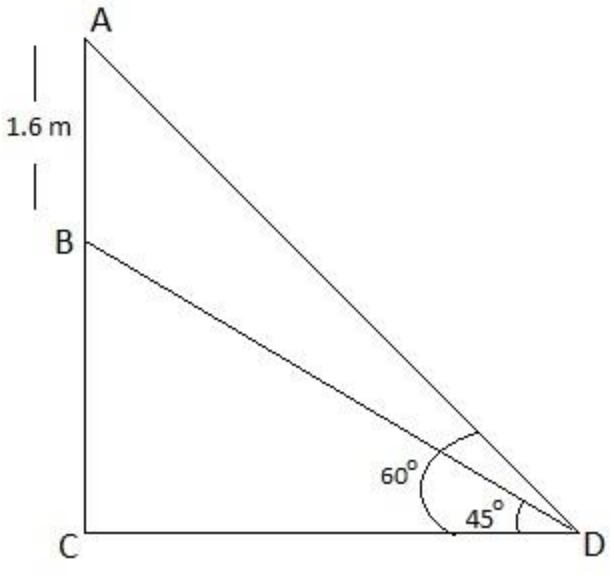
Height of the tower = 20(√3 – 1) m.

Question 8: A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60°, and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Answer: Let AB be the height of the statue.

D is the point on the ground from where the elevation is taken.

To Find: Height of pedestal = BC = AC-AB



From the figure, in right triangle BCD,

tan 45° = $\frac{BC}{CD}$
or, 1 = $\frac{BC}{CD}$
or, BC = CD ……………………………………………………………..(1)

Again, in the right ΔACD,

tan 60° = $\frac{AC}{AD}$
or, √3 = $\frac{AB+BC}{CD}$
or,√3CD = 1.6 + BC
or, √3BC = 1.6 + BC ………………………………..[using equation (1)]
or, √3BC – BC = 1.6
or, BC(√3-1) = 1.6
or, BC = $\frac{1.6}{\sqrt{3}-1}\ m$

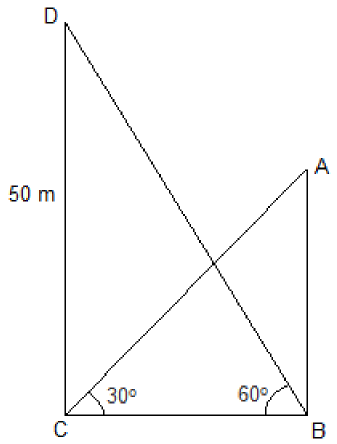
BC = 0.8(√3+1)

Thus, the height of the pedestal is 0.8(√3+1) m.

Question 9. The angle of elevation of the top of a building from the foot of the tower is 30°, and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Answer:

Let CD be the height of the tower. AB be the height of the building. BC be the distance between the foot of the building and the tower. The elevation is 30 degree and 60 degrees from the tower and the building respectively.



In right ΔBCD,

tan 60° = $\frac{CD}{BC}$

or, √3 = $\frac{50}{BC}$

or, BC = $\frac{50}{\sqrt{3}}$ …(1)

Again,

In right ΔABC,

tan 30° = $\frac{AB}{BC}$

or, $\frac{1}{\sqrt{3}} = \frac{AB}{BC}$

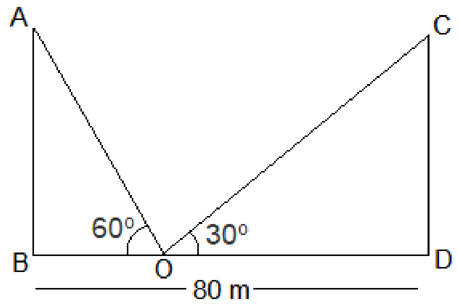
Use result obtained in equation (1)

AB = $\frac{50}{3}$

Thus, the height of the building is $\frac{50}{3}$ m.

Question 10. Two poles of equal heights are standing opposite each other on either side of the road, 80 m wide. From a point between them on the road, the elevation angles of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the bars.

Answer: Let AB and CD be the poles of equal height.



As per above figure, AB = CD,

OB + OD = 80 m

Now,

In right ΔCDO,

tan 30° = $\frac{CD}{OD}$

or, $\frac{1}{\sqrt{3}} = \frac{CD}{OD}$

Or, CD = OD/√3 ………………………………….… (1)

Again, in right ΔABO,

tan 60° = $\frac{AB}{OB}$

or, √3 = $\frac{AB}{(80-OD)}$

or, AB = $\frac{\sqrt{3}}{(80-OD)}$

AB = CD (Given)

√3(80-OD) = $\frac{OD}{\sqrt{3}}$ (Using equation (1))

or, 3(80-OD) = OD

or, 240 – 3 OD = OD

or, 4 OD = 240

OD = 60

Putting the value of OD in equation (1)

CD = $\frac{OD}{\sqrt{3}}$

CD = $\frac{60}{\sqrt{3}}$

CD = $\frac{20}{\sqrt{3}}$ m

Also,

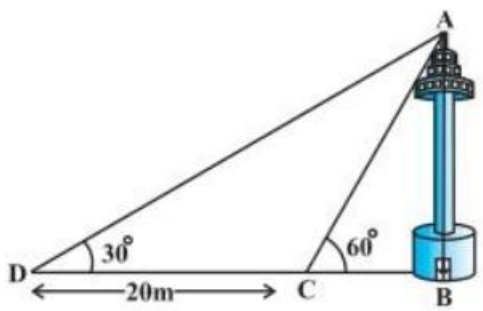
OB + OD = 80 m

or, OB = (80-60) m = 20 m

Thus, the height of the poles are $\frac{20}{\sqrt{3}}$ m, and distance from the point of elevation are 20 m and

60 m respectively.

Question 11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see Fig. 9.12). Find the height of the tower and the width of the canal.



Answer: Given, AB is the height of the tower and DC = 20 m

In right ΔABD,

tan 30° = $\frac{AB}{BD}$

or, $\frac{1}{\sqrt{3}} = \frac{AB}{20+BC}$

AB = $\frac{20+BC}{\sqrt{3}}$ ………………………………………….. (i)

Again,

In right ΔABC,

tan 60° = $\frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{BC}$

AB = √3 BC … (ii)

From equation (i) and (ii)

$\sqrt{3}$ BC = $\frac{20+BC}{\sqrt{3}}$

3 BC = 20 + BC

2 BC = 20

BC = 10

Putting the value of BC in equation (ii)

AB = 10√3

This implies, the height of the tower is 10√3 m, and the width of the canal is 10 m.

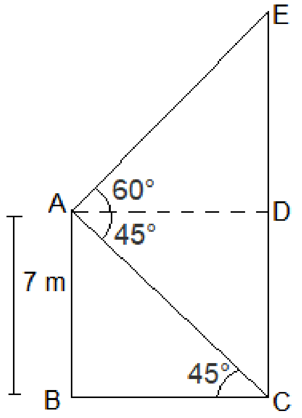
Question 12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60°, and the angle of depression of its foot is 45°. Determine the height of the tower.

Solution:

Let AB be the building of height 7 m and EC be the height of the tower.

EC = DE + CD

Also, CD = AB = 7 m. and BC = AD



In right ΔABC,

tan 45° = $\frac{AB}{BC}$

OR, 1= $\frac{7}{BC}$

or, BC = 7

Since BC = AD, So AD = 7

Again, from right triangle ADE,

tan 60° = $\frac{DE}{AD}$

or, √3 = $\frac{DE}{7}$

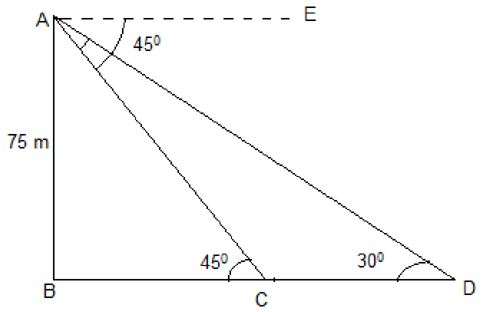
or, DE = 7√3 m

Now: EC = DE + CD = (7√3 + 7) = 7(√3+1)

Therefore, Height of the tower is 7(√3+1) m.

13. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Answer: 30° and 45° are the angles of depression from the lighthouse.



From right triangle ABC,

tan 45° = $\frac{AB}{BC}$

or, 1= $\frac{75}{BC}$

or, BC = 75 m

Now, from right triangle ABD,

tan 30° = $\frac{AB}{BD}$

or, $\frac{1}{\sqrt{3}}$ = $\frac{75}{BD}$

BD = 75√3

Now, CD = BD – BC = (75√3 – 75) = 75(√3-1)

Hence, the distance between the two ships is 75(√3-1) m.

Question 14: A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at the height of 88.2 m from the ground. The angle of elevation of the balloon from the girl's eyes at any instant is 60°. After some time, the angle of elevation reduces to 30° (see Fig. 9.13). Find the distance travelled by the balloon during the interval.

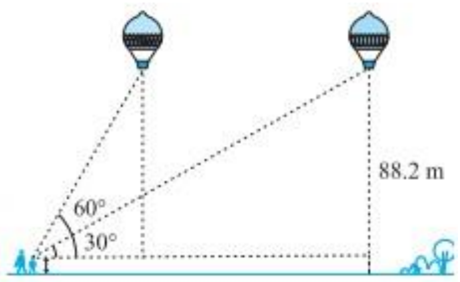
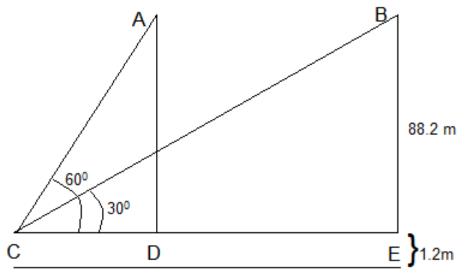


Fig. 9.13

Answer:



In the right ΔBEC,

tan 30° = $\frac{BE}{CE}$

or, $\frac{1}{\sqrt{3}}$= $\frac{87}{CE}$

or, CE = 87√3……….(1)

Now, in the right ΔADC,

tan 60° = $\frac{AD}{CD}$

or, √3= $\frac{87}{CD}$

or, CD = $\frac{87}{\sqrt{3}}$ = 29√3…………..(2)

DE = CE – CD = (87√3 – 29√3) [From (1) and (2)]

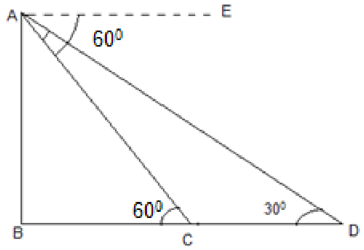
= 29√3(3 – 1)

= 58√3

Distance travelled by the balloon = 58√3 m.

Question A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the rise of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Answer:



In the right ΔABC,

tan 60° = $\frac{AB}{BC}$

or, √3 = $\frac{AB}{BC}$

or, BC = $\frac{AB}{\sqrt{3}}$

or, AB = √3 BC……………………………..(1)

Now, in right ΔABD,

tan 30° = $\frac{AB}{BD}$ = $\frac{1}{\sqrt{3}}$

or, AB = $\frac{BD}{\sqrt{3}}$ …………………………………………(2)

from eq. (1) and (2) we have

√3 BC = $\frac{BD}{\sqrt{3}}$

or, 3 BC = BD

or, 3 BC = BC + CD

or, 2BC = CD

or, or BC = $\frac{CD}{2}$

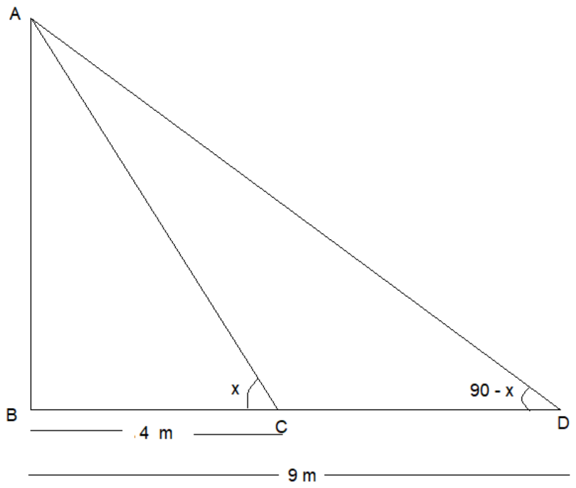
Here, distance of BC =$\frac{CD}{2}$. Thus, the time taken is also half.

Time is taken by car to travel distance CD = 6 secs.

Hence, time taken by car to travel BC = $\frac{6}{2}$ = 3 sec.

Question 16: The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the tower base and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Answer:



In the right ΔABC,
tan x = $\frac{AB}{BC}$
or, tan x = $\frac{AB}{4}$
or, AB = 4 tan x ……………………………………………… (1)

Again, from right ΔABD,
tan (90°-x) = = $\frac{AB}{BD}$
or, cot x = = $\frac{AB}{9}$
or, AB = 9 cot x … ………………………………………………(2)

Now, multiplying equation (1) and (2)

AB^2 = 9 cot x × 4 tan x
or, AB^2 = 36
or, AB = ± 6

Since height cannot be negative, therefore, the height of the tower is 6 m.