CBSE class 9 maths solutions

CHAPTER 2: Polynomials

Exercise 2.1

Question 1: Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer

i) $4x^2 - 3x + 7$ **ii)** $y^2 + \sqrt{2}$ **iii)** $3\sqrt{t} + t\sqrt{2}$

iv) $y + \frac{2}{y}$ **v)** $x^{10} + y^3 + t^{50}$

Answers: i) We have $4x^2 - 3x + 7 = 4x^2 - 3x^1 + 7x^0$

Hence it is a polynomial in one variable i.e. x because each exponent of x is a whole number.

ii) We have $y^2 + \sqrt{2} = y^2 + \sqrt{2}y^0$

It is a polynomial in one variable, i.e. y because each exponent of x is a whole number.

iii) We have $3\sqrt{t} + t\sqrt{2} = 3t^{\frac{1}{2}} + \sqrt{2}t$

It is not a polynomial because one of the exponents of x is $\frac{1}{2}$, which is not a whole number.

iv) We have $y + \frac{2}{y} = y + 2y^{-1}$

It is not a polynomial because one of the exponents of y is -1, which is not a whole number.

v) We have $x^{10} + y^3 + t^{50}$

Here, it is a polynomial because every exponent is a whole number, but it is not a polynomial in one variable, rather in three variables, i.e. x, y and z.

Question 2: Write the coefficients of x^2 in each of the following:

i) $2 + x^2 + x$ **ii)** $2 - x^2 + x^3$ **iii)** $\frac{\pi}{2}x^2 + x$ **iv)** $\sqrt{2}x - 1$

Answers: (i) The given polynomial is $2 + x^2 + x$.
The coefficient of x^2 is 1.

(ii) The given polynomial is $2 - x^2 + x^3$.
The coefficient of x^2 is -1.

(iii) The given polynomial is $\frac{\pi}{2}x^2 + x$.
The coefficient of x^2 is $\frac{\pi}{2}$.

(iv) The given polynomial is $\sqrt{2}\,x - 1$.
The coefficient of x^2 is 0.

Question 3: Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Answer: A binomial of degree 35 can be $2x^{35} - 5$.
A monomial of degree 100 can be $\sqrt{5}y^{100}$.

Question 4: Write the degree of each of the following polynomials:

i) $5x^3 + 4x^2 + 7x$. **ii) $4 - y^2$**

iii) $5t - \sqrt{7}$ **iv) 3**

Answer: (i) The given polynomial is $5x^3 + 4x^2 + 7x$.
The highest power of the variable x is 3.
So, the degree of the polynomial is 3.

(ii) The given polynomial is $4 - y^2$. The highest power of the variable y is 2.
So, the degree of the polynomial is 2.

(iii) The given polynomial is $5t - \sqrt{7}$. The highest power of variable t is 1. So, the degree of the polynomial is 1.

(iv) Since, $3 = 3x^\circ$ [$\because x^\circ = 1$]
So, the degree of the polynomial is 0.

Question 5: Classify the following as linear, quadratic and cubic polynomials:
i) $x^2 + x$ **ii) $x - x^3$** **iii) $y + y^2 + 4$** **iv) $1 + x$**
v) $3t$ **vi) r^2** **vii) $7x^3$**

Answer: (i) The degree of $x^2 + x$ is 2. So, it is a quadratic polynomial.
(ii) The degree of $x - x^3$ is 3. So, it is a cubic polynomial.
(iii) The degree of $y + y^2 + 4$ is 2. So, it is a quadratic polynomial.
(iv) The degree of $1 + x$ is 1. So, it is a linear polynomial.
(v) The degree of $3t$ is 1. So, it is a linear polynomial.

(vi) The degree of r^2 is 2. So, it is a quadratic polynomial.
(vii) The degree of $7x^3$ is 3. So, it is a cubic polynomial.

Exercise 2.2

Question 1: Find the value of the polynomial $5x - 4x^2 + 3$ at
i) x = 0 **ii) x = -1** **iii) x = 2**

Answer: The given polynomial is $p(x) = 5x – 4x^2 + 3$

(i) $p(0) = 5(0) – 4(0)^2 + 3 = 0 – 0 + 3 = 3$
Thus, the value of $5x – 4x^2 + 3$ at x = 0 is 3.

(ii) $p(-1) = 5(-1) – 4(-1)^2 + 3$
$= – 5x – 4x^2 + 3 = -9 + 3 = -6$
Thus, the value of $5x – 4x^2 + 3$ at x = -1 is -6.

(iii) $p(2) = 5(2) – 4(2)^2 + 3 = 10 – 4(4) + 3$
$= 10 – 16 + 3 = -3$
Thus, the value of $5x – 4x^2 + 3$ at x = 2 is – 3.

Question 2: Find *p*(0), *p*(1) and *p*(2) for each of the following polynomials:
i) $p(y) = y^2 - y + 1$ **ii) $p(t) = 2 + t + 2t^2 - t^3$**
iii) $p(x) = x^3$ **iv) $p(x) = (x - 1)(x + 1)$**

Answer: (i) Given that $p(y) = y^2 – y + 1$.
therefore,
$p(0) = (0)^2 – 0 + 1 = 0 – 0 + 1 = 1$
$p(1) = (1)^2 – 1 + 1 = 1 – 1 + 1 = 1$
$p(2) = (2)^2 – 2 + 1 = 4 – 2 + 1 = 3$

(ii) Given that $p(t) = 2 + t + 2t^2 – t^3$
therefore,
$p(0) = 2 + 0 + 2(0)^2 – (0)^3 = 2 + 0 + 0 – 0=2$
$p(1) = 2 + 1 + 2(1)^2 – (1)^3 = 2 + 1 + 2 – 1 = 4$
$p(2) = 2 + 2 + 2(2)^2 – (2)^3 = 2 + 2 + 8 – 8 = 4$

(iii) Given that $p(x) = x^3$
therefore,
$p(0) = (0)^3 = 0$, $p(1) = (1)^3 = 1$
$p(2) = (2)^3 = 8$

(iv) Given that p(x) = (x – 1)(x + 1)
therefore,
p(0) = (0 – 1)(0 + 1) = (-1)(1) = -1
p(1) = (1 – 1)(1 +1) = (0)(2) = 0
p(2) = (2 – 1)(2 + 1) = (1)(3) = 3

Question 3: Verify whether the following are zeroes of the polynomial, indicated against them.

i) $p(x) = 3x + 1,\ x = -\frac{1}{3}$

ii) $p(x) = 5x - \pi, x = \frac{4}{5}$

iii) $p(x) = x^2 - 1, x = 1, -1$

iv) $p(x) = (x + 1)(x - 2),\ x = -1, 2$

v) $p(x) = x^2,\ x = 0$

vi) $p(x) = lx + m,\ x = -\frac{m}{l}$

vii) $p(x) = 3x^2 - 1,\ x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

viii) $p(x) = 2x + 1,\ x = \frac{1}{2}$

Answer: (i) We have , p(x) = 3x + 1
Therefore, $p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$
Since, $p\left(-\frac{1}{3}\right) = 0$, so, $x = \left(-\frac{1}{3}\right)$ is a zero of 3x+1.

(ii) We have, $p(x) = 5x – \pi$
Therefore, $p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = (4 - \pi)$
Since, $p\left(\frac{4}{5}\right) \neq 0$, so, $x = \left(\frac{4}{5}\right)$ is not a zero of $5x – \pi$

(iii) We have, $p(x) = x^2 – 1$
therefore, $p(1) = (1)^2 – 1 = 1 – 1=0$
Since, p(1) = 0, so x = 1 is a zero of $x^2 -1$.
Also, $p(-1) = (-1)^2 -1 = 1 – 1 = 0$
Since p(-1) = 0, so, x = -1, is also a zero of $x^2 – 1$.

(iv) We have, p(x) = (x + 1)(x – 2)
therefore, p(-1) = (-1 +1) (-1 – 2) = (0)(- 3) = 0
Since, p(-1) = 0, so, x = -1 is a zero of (x + 1)(x – 2).
Also, p(2) = (2 + 1)(2 – 2) = (3)(0) = 0
Since, p(2) = 0, so, x = 2 is also a zero of (x + 1)(x – 2).

(v) We have, $p(x) = x^2$
therefore, $p(0) = (0)^2 = 0$
Since, p(0) = 0, so, x = 0 is a zero of x^2.

(vi) We have p(x) = lx + m
therefore, $p\left(-\frac{l}{m}\right) = l\left(-\frac{m}{l}\right) + m = 0$
since, $p\left(-\frac{l}{m}\right) = 0$, so, $x = \left(-\frac{m}{l}\right)$ is a zero of lx + m.

(vii) We have $p(x) = 3x^2 - 1$

therefore , $p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1 = 1 - 1 = 0$

Since , $p\left(-\frac{1}{\sqrt{3}}\right) = 0$, so $x = -\frac{1}{\sqrt{3}}$ is a zero of $3x^2 - 1$.

Also, $p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{4}{3}\right) - 1 = 4 - 1 = 3$

Since , $p\left(\frac{2}{\sqrt{3}}\right) \neq 0,$ so $x = \frac{2}{\sqrt{3}}$ is not a zero of $3x^2$ - 1.

(viii) We have, p(x) = 2x + 1
∴ $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2$
Since, $p\left(\frac{1}{2}\right) \neq 0$, so, x = $\left(\frac{1}{2}\right)$ is not a zero of 2x + 1.

Question 4: Find the zero of the polynomial in each of the following cases:
i) $p(x) = x + 5$ ii) $p(x) = x - 5$ iii) $p(x) = 2x + 5$
iv) $p(x) = 3x - 2$ v) $p(x) = 3x$ vi) $p(x) = ax, a \neq 0$
vii) $p(x) = cx + d, c \neq 0, c, d$ are real numbers

Answer: (i) We have, p(x) = x + 5. Since, p(x) = 0
or, x + 5 = 0
or, x = -5.
Thus, zero of x + 5 is -5.

(ii) We have, p(x) = x – 5.
Since, p(x) = 0
or, x – 5 = 0
or, x = 5
Thus, zero of x – 5 is 5.

(iii) We have, p(x) = 2x + 5. Since, p(x) = 0
or, 2x + 5 =0
or, 2x = -5
or, x = −5/2
Thus, zero of 2x + 5 is −5/2

(iv) We have, p(x) = 3x – 2. Since, p(x) = 0
or, 3x – 2 = 0
or, 3x = 2
or, x = 2/3
Thus, zero of 3x – 2 is 2/3

(v) We have, p(x) = 3x. Since, p(x) = 0
or, 3x = 0
or, x = 0
Thus, zero of 3x is 0.

(vi) We have, p(x) = ax, a ≠ 0.
Since, p(x) = 0

or, ax = 0
or, x=0
Thus, zero of ax is 0.

(vii) We have, p(x) = cx + d. Since, p(x) = 0
or, cx + d = 0
or, cx = -d
or, x=−d/c
Thus, zero of cx + d is −d/c

Exercise 2.3

Question 1: Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by:
i) $x + 1$ ii) $x - \frac{1}{2}$ iii) x iv) $x + \pi$ v) $5 + 2x$

Answer: Let p(x) = $x^3 + 3x^2 + 3x + 1$

(i) The zero of x + 1 is -1.
∴ p(-1) = (-1)3 + 3(-1)2 + 3(-1) +1
= -1 + 3- 3 + 1 = 0
Thus, the required remainder = 0

(ii) The zero of $x - \frac{1}{2}$ is $\frac{1}{2}$.
Therefore , $p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$
$= \frac{1}{8} + \frac{1}{4} + \frac{1}{2} + 1 = \frac{1+6+12+8}{8} = \frac{27}{8}$
So, the remainder is $\frac{27}{8}$.

(iv) (iii) The zero of x is 0.
∴ p(0) = $(0)^3 + 3(0)^2 + 3(0) + 1$
= 0 + 0 + 0 + 1 = 1
Thus, the required remainder = 1.

(iv) The zero of x + π is $-\pi$
p($-\pi$) = $(-\pi)^3 + 3(-\pi)^2 2 + 3(-\pi) + 1$
= $-\pi^3 + 3\pi^2 + (-3\pi) + 1$
= $-\pi^3 + 3\pi^2 - 3\pi + 1$
Thus, the required remainder is $-\pi^3 + 3\pi^2 - 3\pi + 1$.

(v) The zero of 5 + 2x is $-\frac{5}{2}$

Therefore, $p\left(-\frac{5}{2}\right) = \left(-\frac{5}{2}\right)^3 + 3\left(-\frac{5}{2}\right)^2 + 3\left(-\frac{5}{2}\right) + 1$

$= -\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + 1 = -\frac{27}{8}$

So, the required remainder is $-\frac{27}{8}$.

Question 2: Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Answer: We have, p(x) = $x^3 - ax^2 + 6x - a$ and zero of x – a is a.
therefore, p(a) = $(a)^3 - a(a)^2 + 6(a) - a = a^3 - a^3 + 6a - a = 5a$
Thus, the required remainder is 5a.

Question 3: Check whether 7 + 3x is a factor of $3x^3 + 7x$.

We have, p(x) = $3x^3+7x$. and zero of 7 + 3x is $-\frac{7}{3}$
Therefore, $p\left(-\frac{7}{3}\right) = 3\left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right)$
$= 3\left(-\frac{343}{27}\right) + \left(-\frac{49}{3}\right) = -\frac{343}{9} - \frac{49}{3} = -\frac{490}{9}$
Since $-\frac{490}{9} \neq 0$, the remainder is not zero.
So , p(x) = $3x^3+7x$ is not divisible by 7+3x.
So, 7+3x is not a factor of $3x^3+7x$.

Exercise 2.4

Question 1: Determine which of the following polynomials has (x + 1) a factor:
i) $x^3 + x^2 + x + 1$
ii) $x^4 + x^3 + x^2 + x + 1$
iii) $x^4 + 3x^3 + 3x^2 + x + 1$
iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Answer: The zero of x + 1 is -1.

(i) Let p (x) = $x^3 + x^2 + x + 1$
therefore, p (-1) = $(-1)^3 + (-1)^2 + (-1) + 1$.
= -1 + 1 – 1 + 1
or, p (- 1) = 0
So, (x+ 1) is a factor of $x^3 + x^2 + x + 1$.

(ii) Let p (x) = $x^4 + x^3 + x^2 + x + 1$
therefore, p(-1) = $(-1)^4 + (-1)^3 + (-1)^2 + (-1)+1$ = 1 – 1 + 1 – 1 + 1
or, p(-1) ≠ 1
So, (x + 1) is not a factor of $x^4 + x^3 + x^2 + x+ 1$.

(iii) Let p (x) = $x^4 + 3x^3 + 3x^2 + x + 1$.
therefore, p (-1)= $(-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$= 1 – 3 + 3 – 1 + 1 = 1

or, p (-1) ≠ 0
So, (x + 1) is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$.

(iv) Let p (x) = $x^3 – x^2 – (2 + \sqrt{2}) x + \sqrt{2}$
therefore, p (- 1) =(- 1)3- (-1)2 – (2 + √2)(-1) + √2
= -1 – 1 + 2 + √2 + √2
= 2√2
or, p (-1) ≠ 0
So, (x + 1) is not a factor of $x^3 – x^2 – (2 + \sqrt{2}) x + \sqrt{2}$.

Question 2: Use the Factor Theorem to determine whether *g*(*x*) is a factor of *p*(*x*) in each of the following cases:
i) $p(x) = 2x^3 + x^2 - 2x - 1,\ \ g(x) = x + 1$
ii) $p(x) = x^3 + 3x^2 + 3x + 1,\ \ g(x) = x + 2$
iii) $p(x) = x^3 - 4x^2 + x + 6,\ \ g(x) = x - 3$

Answer: We have, p (x)= $2x^3 + x^2 – 2x – 1$ and g (x) = x + 1
therefore, p(-1) = $2(-1)^3 + (-1)^2 – 2(-1) - 1$
= 2(-1) + 1 + 2 – 1
= -2 + 1 + 2 -1 = 0
or, p(-1) = 0, so g(x) is a factor of p(x).

(ii) We have, p(x) $x^3 + 3x^2 + 3x + 1$ and g(x) = x + 2
therefore, p(-2) = $(-2)^3 + 3(-2)^2 + 3(-2) + 1$
= -8 + 12 – 6 + 1
= -14 + 13
= -1
or, p(-2) ≠ 0, so g(x) is not a factor of p(x).

(iii) We have, = $x^3 – 4x^2 + x + 6$ and g (x) = x – 3
therefore, p(3) = $(3)^3 – 4(3)^2 + 3 + 6$
= 27 – 4(9) + 3 + 6
= 27 – 36 + 3 + 6 = 0
or, p(3) = 0, so g(x) is a factor of p(x).

Question 3: Find the value of *k*, if *x* – 1 is a factor of *p*(*x*) in each of the following cases:
i) $p(x) = x^2 + x + k$
ii) $p(x) = 2x^2 + kx + \sqrt{2}$
iii) $p(x) = kx^2 - \sqrt{2}x + 1$
iv) $p(x) = kx^2 - 3x + k$

Answer: For (x – 1) to be a factor of p(x), p(1) should be equal to 0.

(i) Here, p(x) = $x^2 + x + k$
Since, p(1) = $(1)^2 + 1 + k$

⇒ p(1) = k + 2 = 0
⇒ k = -2.

(ii) Here, p (x) = $2x^2 + kx + \sqrt{2}$
Since, p(1) = $2(1)^2 + k(1) + \sqrt{2}$
= $2 + k + \sqrt{2} = 0$
k = $-2 - \sqrt{2} = -(2 + \sqrt{2})$

(iii) Here, p (x) = $kx^2 - \sqrt{2}x + 1$
Since, p(1) = $k(1)^2 - (1) + 1$
= $k - \sqrt{2} + 1 = 0$
⇒ k = $\sqrt{2} - 1$

(iv) Here, p(x) = $kx^2 - 3x + k$
p(1) = $k(1)^2 - 3(1) + k$
= k – 3 + k
= 2k – 3 = 0
⇒ k = $\frac{3}{2}$

Question 4: Factorise:

i) $12x^2 - 7x + 1$ **ii) $2x^2 + 7x + 3$**
iii) $6x^2 + 5x - 6$ **iv) $3x^2 - x - 4$**

Answer: (i) We have, $12x^2 - 7x + 1$
= $12x^2 - 4x - 3x + 1$
= 4x (3x – 1) -1 (3x – 1)
= (3x -1) (4x -1)
Thus, $12x^2 - 7x + 3$ = (3x-1)(4x-1)

(ii) We have, $2x^2 + 7x + 3$
= $2x^2 + x + 6x + 3$
= x(2x + 1) + 3(2x + 1)
= (2x + 1)(x + 3)
Thus, 2×2 + 7x + 3 = (2x + 1)(x + 3)

(iii) We have, $6x^2 + 5x - 6$
= $6x^2 + 9x - 4x - 6$
= 3x(2x + 3) – 2(2x + 3)
= (2x + 3)(3x – 2)
Thus, $6x^2 + 5x - 6$ = (2x + 3)(3x – 2)

(iv) We have, $3x^2 - x - 4$
= $3x^2 - 4x + 3x - 4$
= x(3x – 4) + 1(3x – 4) = (3x – 4)(x + 1)
Thus, $3x^2 - x - 4$ = (3x – 4)(x + 1)

Question 5: Factorise:

i) $x^3 - 2x^2 - x + 2$ **ii)** $x^3 - 3x^2 - 9x - 5$

iii) $x^3 + 13x^2 + 32x + 20$ **iv)** $2y^3 + y^2 - 2y - 1$

Answer: (i) We have, $x^3 - 2x^2 - x + 2$
Rearranging the terms, we have $x^3 - x - 2x^2 + 2$
$= x(x^2 - 1) - 2(x^2 - 1) = (x^2 - 1)(x - 2)$
$= [(x)^2 - (1)^2](x - 2)$
$= (x - 1)(x + 1)(x - 2)$ [since, $(a^2 - b^2) = (a + b)(a - b)$]
Thus, $x^3 - 2x^2 - x + 2 = (x - 1)(x + 1)(x - 2)$

(ii) We have, $x^3 - 3x^2 - 9x - 5$
$= x^3 + x^2 - 4x^2 - 4x - 5x - 5$,
$= x^2 (x + 1) - 4x(x + 1) - 5(x + 1)$
$= (x + 1)(x^2 - 4x - 5)$
$= (x + 1)(x^2 - 5x + x - 5)$
$= (x + 1)[x(x - 5) + 1(x - 5)]$
$= (x + 1)(x - 5)(x + 1)$
Thus, $x^3 - 3x^2 - 9x - 5 = (x + 1)(x - 5)(x + 1)$

(iii) We have, $x^3 + 13x^2 + 32x + 20$
$= x^3 + x^2 + 12x^2 + 12x + 20x + 20$
$= x^2(x + 1) + 12x(x + 1) + 20(x + 1)$
$= (x + 1)(x^2 + 12x + 20)$
$= (x + 1)(x^2 + 2x + 10x + 20)$
$= (x + 1)[x(x + 2) + 10(x + 2)]$
$= (x + 1)(x + 2)(x + 10)$
Thus, $x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 2)(x + 10)$

(iv) We have, $2y^3 + y^2 - 2y - 1$
$= 2y^3 - 2y^2 + 3y^2 - 3y + y - 1$
$= 2y^2(y - 1) + 3y(y - 1) + 1(y - 1)$
$= (y - 1)(2y^2 + 3y + 1)$
$= (y - 1)(2y^2 + 2y + y + 1)$
$= (y - 1)[2y(y + 1) + 1(y + 1)]$
$= (y - 1)(y + 1)(2y + 1)$
Thus, $2y^3 + y^2 - 2y - 1 = (y - 1)(y + 1)(2y + 1)$

Exercise 2.5

Question 1: Use suitable identities to find the following products:

i) $(x + 4)(x + 10)$ **ii)** $(x + 8)(x - 10)$ **iii)** $(3x + 4)(3x - 5)$

iv) $(y^2 + \frac{3}{2})(y^2 - \frac{3}{2})$ **v)** $(3 - 2x)(3 + 2x)$

Answer: (i) We have, (x+ 4) (x + 10)
Using identity, (x+ a) (x+ b) = x^2 + (a + b) x+ ab.
So, (x + 4) (x + 10) = x^2+(4 + 10) x + (4 x 10)
= x^2 + 14x+40

(ii) We have, (x+ 8) (x -10)
Using identity, (x + a) (x + b) = x^2 + (a + b) x + ab
So, (x + 8) (x – 10) = x^2 + [8 + (-10)] x + (8) (- 10)
= x^2 – 2x – 80

(iii) We have, (3x + 4) (3x – 5)
Using identity, (x + a) (x + b) = x^2 + (a + b) x + ab
So, (3x + 4) (3x – 5) = $(3x)^2$ + (4 – 5) x + (4) (- 5)
= $9x^2$ – 3x – 20

(iv) We have, $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$
Using the identity, (a+b)(a-b) = $a^2 - b^2$
So, $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$
= $(y^2)^2$ - $\left(\frac{3}{2}\right)^2$
= y^4 - $\frac{9}{4}$

(v) We have, (3 – 2x)(3 + 2x)
Using the identity, (a+b)(a-b) = $a^2 - b^2$
So, (3 – 2x)(3 + 2x)
= $3^2 - (2x)^2$
= 9 – $4x^2$

Question 2: Evaluate the following products without multiplying directly:
i) 103×107 ii) 95×96 iii) 104×96

Answer: (i)We have, 103 x 107 = (100 + 3) (100 + 7)
= $(100)^2$ + (3 + 7) (100)+ (3 x 7) [Using (x + a)(x + b) = x^2 + (a + b)x + ab]
= 10000 + (10) x 100 + 21
= 10000 + 1000 + 21=11021

(ii) We have, 95 x 96 = (100 – 5) (100 – 4)
= $(100)^2$ + [(- 5) + (- 4)] 100 + (- 5 x – 4) [Using (x + a)(x + b) = x^2 + (a + b)x + ab]
= 10000 + (-9) + 20 = 9120
= 10000 + (-900) + 20 = 9120

(iii) We have 104 x 96 = (100 + 4) (100 – 4)
= $(100)^2$-4^2 [Using (a + b)(a -b) = a^2– b^2]
= 10000 – 16 = 9984

Question 3: Factorise the following using appropriate identities:
i) $9x^2 + 6xy + y^2$ ii) $4y^2 - 4y + 1$ iii) $x^2 - \frac{y^2}{100}$

Answer: (i) We have, $9x^2 + 6xy + y^2$
$= (3x)^2 + 2(3x)(y) + (y)^2$
$= (3x + y)^2$ [Using $a^2 + 2ab + b^2 = (a + b)^2$]
$= (3x + y)(3x + y)$

(ii) We have, $4y^2 – 4y + 1^2$
$= (2y)^2 + 2(2y)(1) + (1)^2$
$= (2y -1)^2$ [Using $a^2 – 2ab + b^2 = (a- b)^2$]
$= (2y – 1)(2y – 1)$

(iii) We have, $x^2 - \frac{y^2}{100} = x^2 - \left(\frac{y}{2}\right)^2 = \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$ [Using $a^2 – b^2 = (a + b)(a – b)$]

Question 4: Expand each of the following, using suitable identities:

i) $(x + 2y + 4z)^2$ **ii)** $(2x - y + z)^2$ **iii)** $(-2x + 3y + 2z)^2$

iv) $(3a - 7b - c)^2$ **v)** $(-2x + 5y - 3z)^2$ **vi)** $[\frac{1}{4}a - \frac{1}{2}b + 1]^2$

Answer: We know that
$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(i) $(x + 2y + 4z)^2$
$= x^2 + (2y)^2 + (4z)^2 + 2 (x) (2y) + 2 (2y) (4z) + 2(4z) (x)$
$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8 zx$

(ii) $(2x – y + z)^2$
$= (2x)^2 + (- y)^2 + z^2 + 2 (2x) (- y)+ 2 (- y) (z) + 2 (z) (2x)$
$= 4x^2 + y^2 + z^2 – 4xy – 2yz + 4zx$

(iii) $(- 2x + 3y + 2z)^2$
$= (- 2x)^2 + (3y)^2 + (2z)^2 + 2 (- 2x) (3y)+ 2 (3y) (2z) + 2 (2z) (- 2x)$
$= 4x^2 + 9y^2 + 4z^2 – 12xy + 12yz – 8zx$

(iv) $(3a -7b- c)^2$
$= (3a)^2 + (- 7b)^2 + (- c)^2 + 2 (3a) (- 7b) + 2 (- 7b) (- c) + 2 (- c) (3a)$
$= 9a^2 + 49b^2 + c^2 – 42ab + 14bc – 6ac$

(v) $(- 2x + 5y- 3z)^2$
$= (- 2x)^2 + (5y)^2 + (- 3z)^2 + 2 (- 2x) (5y) + 2 (5y) (- 3z) + 2 (- 3z) (- 2x)$
$= 4x^2 + 25y^2 + 9z^2 – 20xy – 30yz + 12zx$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$
$= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right) + 2\left(-\frac{1}{2}b\right)(1) + 2(1)\left(\frac{1}{4}a\right)$
$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

Question 5: Factorise:

i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Answer: i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$
$=(2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$
$=(2x + 3y - 4z)^2 = (2x + 3y - 4z)(2x + 3y - 4z)$

2). $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$
$=\left(-\sqrt{2}x\right)^2 + y^2 + \left(2\sqrt{2}z\right)^2 + 2\left(-\sqrt{2}x\right)(y) + 2(y)\left(2\sqrt{2}z\right) + 2\left(-\sqrt{2}x\right)\left(2\sqrt{2}z\right)$
$= \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)^2 = \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)\left(-\sqrt{2}x + y + 2\sqrt{2}z\right)$

Question 6: Write the following cubes in expanded form :

i) $(2x+1)^3$ ii) $(2a-3b)^3$ iii) $[\frac{3}{2}x+1]^3$ iv) $[x - \frac{2}{3}y]^3$

Answer: i) $(2x+1)^3$
$= (2x)^3 + (1)^3 + 3(2x)(1)(2x+1)$ [as $(x+y)^3 = (x)^3 + (y)^3 + 3(x)(y)(x+y)$]
$= 8x^3 + 1 + 6x(2x+1)$
$= 8x^3 + 12x^2 + 6x + 1$

ii) $(2a-3b)^3$
$= (2a)^3 - (3b)^3 - 3(2a)(3b)(2a-3b)$ [as $(x-y)^3 = (x)^3 - (y)^3 - 3(x)(y)(x-y)$]
$= 8a^3 - 27b^3 - 18ab(2a-3b)$
$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

iii) $\left[\frac{3}{2}x + 1\right]^3$
$= \left(\frac{3}{2}x\right)^3 + (1)^3 + 3(\frac{3}{2}x)(1)\left(\frac{3}{2}x + 1\right)$ [as $(x+y)^3 = (x)^3 + (y)^3 + 3(x)(y)(x+y)$]
$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x + 1\right)$
$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$
$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

iv) $\left(x - \frac{2}{3}y\right)^3$
$= (x)^3 - \left(\frac{2}{3}y\right)^3 - 3(x)(\frac{2}{3}y)\left(x - \frac{2}{3}y\right)$ [as $(x-y)^3 = (x)^3 - (y)^3 - 3(x)(y)(x-y)$]
$= x^3 - \frac{8}{27}y^3 - 2xy\left(x - \frac{2}{3}y\right)$
$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

Question 7: Evaluate the following using suitable identities

i) $(99)^3$ ii) $(102)^3$ iii) $(998)^3$

Answer: i) We have, $(99)^3 = (100-1)^3$
$= (100)^3 - (1)^3 - 3(100)(1)(100-1)$ [as $(x-y)^3 = (x)^3 - (y)^3 - 3(x)(y)(x-y)$]
$= 1000000 - 1 - 300(100-1)$
$= 1000000 - 1 - 30000 + 300$
$= 1000300 - 30001 = 970299$

ii) We have $(102)^3 = (100+2)^3$
$= (100)^3 + (2)^3 + 3(100)(2)(100+2)$ $[as\ (x+y)^3 = (x)^3 + (y)^3 + 3(x)(y)(x+y)]$
$= 1000000 + 8 + 600(100+2)$
$= 1000000 + 8 + 60000 + 1200 = 1061208$

iii) We have $(998)^3 = (1000-2)^3$
$= (1000)^3 - (2)^3 - 3(1000)(2)(1000-2)$
[as $(x-y)^3 = (x)^3 - (y)^3 - 3(x)(y)(x-y)$]
$= 1000000000 - 8 - 6000(1000-2)$
= 1000000000 – 8 – 6000000 +12000
= 994011992

Question 8: Factorise each of the following:

i) $8a^3 + b^3 + 12a^2b + 6ab^2$
ii) $8a^3 - b^3 - 12a^2b + 6ab^2$
iii) $27 - 125a^3 - 135a + 225a^2$
iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$
v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Answer: i) $8a^3 + b^3 + 12a^2b + 6ab^2$
$= (2a)^3 + (b)^3 + 6ab(2a+b)$
$= (2a)^3 + (b)^3 + 3(2a)(b)(2a+b)$
$= (2a+b)^3$ $[as\ (x+y)^3 = (x)^3 + (y)^3 + 3(x)(y)(x+y)]$
$= (2a+b)(2a+b)(2a+b)$

ii) $8a^3 - b^3 - 12a^2b + 6ab^2$
$= (2a)^3 - (b)^3 - 6ab(2a-b)$
$= (2a)^3 - (b)^3 - 3(2a)(b)(2a-b)$
$= (2a-b)^3$ [as $(x-y)^3 = (x)^3 - (y)^3 - 3(x)(y)(x-y)$]
$= (2a-b)(2a-b)(2a-b)$

iii) $27 - 125a^3 - 135a + 225a^2$
$= (3)^3 - (5a)^3 - 3(3)(5a)(3-5a)$
$= (3-5a)^3$ [as $(x-y)^3 = (x)^3 - (y)^3 - 3(x)(y)(x-y)$]
$= (3-5a)(3-5a)(3-5a)$

iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$
$= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a-3b)$
$= (4a-3b)^3$ [as $(x-y)^3 = (x)^3 - (y)^3 - 3(x)(y)(x-y)$]
$= (4a-3b)(4a-3b)(4a-3b)$

iv) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$
$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)(\frac{1}{6})\left(3p - \frac{1}{6}\right)$
$= \left(3p - \frac{1}{6}\right)^3$ [as $(x-y)^3 = (x)^3 - (y)^3 - 3(x)(y)(x-y)$]
$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

Question 9: Verify: i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Answer: i) We have $(x + y)^3 = (x)^3 + (y)^3 + 3(x)(y)(x + y)$
or, $(x + y)^3 - 3(x)(y)(x + y) = (x)^3 + (y)^3$
or, $(x + y)[(x + y)^2 - 3xy] = (x)^3 + (y)^3$
or, $(x + y)(x^2 + y^2 + 2xy - 3xy) = (x)^3 + (y)^3$
or, $(x + y)(x^2 + y^2 - xy) = (x)^3 + (y)^3$
Hence verified.

ii) We have $(x - y)^3 = (x)^3 - (y)^3 - 3(x)(y)(x - y)$
or, $(x - y)^3 + 3(x)(y)(x + y) = (x)^3 - (y)^3$
or, $(x - y)[(x - y)^2 + 3xy] = (x)^3 - (y)^3$
or, $(x - y)(x^2 + y^2 - 2xy + 3xy) = (x)^3 - (y)^3$
or, $(x - y)(x^2 + y^2 + xy) = (x)^3 - (y)^3$
Hence verified.

Question 10: Factorise each of the following:
i) $27y^3 + 125z^3$ ii) $64m^3 - 343n^3$

Answer: i) $27y^3 + 125z^3$
$= (3y)^3 + (5z)^3$
$= (3y + 5z)[(3y)^2 + (5z)^2 - (3y)(5z)]$
$= (3y + 5z)(9y^2 + 25z^2 - 15yz)$
[as $(x + y)(x^2 + y^2 - xy) = (x)^3 + (y)^3$]

ii) $64m^3 - 343n^3$
$= (4m)^3 - (7n)^3$
$= (4m - 7n)[(4m)^2 + (7n)^2 + (4m)(7n)]$
$= (4m - 7n)[16m^2 + 49n^2 + 28mn]$
$[as\ (x - y)(x^2 + y^2 + xy) = (x)^3 - (y)^3]$

Question 11: Factorise: $27x^3 + y^3 + z^3 - 9xyz$

Answer: i) We have $27x^3 + y^3 + z^3 - 9xyz$
$= (3x)^3 + y^3 + z^3 - 3(3x)(y)(z)$
Using the identity : $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2+y^2+z^2 - xy - yz - zx)$
$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$

Question 12: Verify that: $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$
Answer: Considering the R.H.S.

$\frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$
$= \frac{1}{2}(x + y + z)[(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (x^2 + z^2 - 2xz)]$
$= \frac{1}{2}(x + y + z)[2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz]$
$= (x + y + z)[x^2 + y^2 + z^2 - xy - yz - xz]$

= $x^3 + y^3 + z^3 - 3xyz$

[as $(x + y + z)[x^2 + y^2 + z^2 - xy - yz - xz] = x^3 + y^3 + z^3 - 3xyz$

Hence proved, L.H.S. = R.H.S.

Question 13: If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$

Answer: Since $x + y + z = 0$
=> $x + y = -z$
=> $(x + y)^3 = (-z)^3$
=> $x^3 + y^3 + 3xy(x + y) = -z^3$ [taking cubes on both sides]
=> $x^3 + y^3 + 3xy(-z) = -z^3$ [as $x + y = -z$]
=> $x^3 + y^3 + z^3 - 3xyz = 0$
=> $x^3 + y^3 + z^3 = 3xyz$
Hence Proved.

Question 14: Without actually calculating the cubes, find the value of each of the following:
i) $(-12)^3 + (7)^3 + (5)^3$
i) $(28)^3 + (-15)^3 + (-13)^3$

Answer: i) Given $(-12)^3 + (7)^3 + (5)^3$

Let x=-12, y=7, z=5;
Then $x + y + z = -12 + 7 + 5 = 0$

Hence , by the identity $x^3 + y^3 + z^3 = 3xyz$
So, $(-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5) = 3(-420) = -1260$

ii) Given $(28)^3 + (-15)^3 + (-13)^3$
Let x=28, y=-15, z=-13.
Then $x + y + z = 28 + (-15) + (-13) = 28 - 28 = 0$
Hence , by the identity $x^3 + y^3 + z^3 = 3xyz$
So, $(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13) = 3(5460) = 16380$

Question 15: Give possible expressions for the length and breadth of each of the following
rectangles, in which their areas are given:
i) Area: $25a^2 - 35a + 12$ **ii) Area: $35y^2 + 13y - 12$**

Answer: i) Given area = $25a^2 - 35a + 12$ = Length x Breadth
= $25a^2 - 20a - 15a + 12$
= $5a(5a - 4) - 3(5a - 4)$
= $(5a - 3)(5a - 4)$
Hence, the possible length and breadth are (5a-3) and (5a-4).

ii) Given area = $35y^2 + 13y - 12$ = Length x Breadth

= $35y^2 + 28y - 15y - 12$
= 7y(5y + 4) – 3(5y + 4)
= (5 y + 4)(7y – 3)
Hence, the possible length and breadth are 5y+4 and 7y-3.

Question 16: What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

i) Volume: $3x^2 - 12x$

ii) Volume: $12ky^2 + 8ky - 20k$

Answer: i) We have given volume = $3x^2 - 12x$ = Length x Breadth x Height
= 3(x^2 – 4x)
=3.x.(x - 4)
Hence, the possible dimensions are 3, x and x-4.

ii) We have given volume = $12ky^2 + 8ky - 20k$
= 4[$3ky^2$ + 2ky – 5k]
= 4[k($3y^2$ + 2y – 5)]
= 4.k.($3y^2$ + 2y – 5)
= 4k($3y^2$ – 3y + 5y – 5)
= 4k[3y(y – 1) + 5(y – 1)]
= 4k[(3y + 5)(y – 1)]
= 4k (3y + 5) (y – 1)
Hence the possible dimensions of the cuboid are 4k, 3y+5, and y-1.