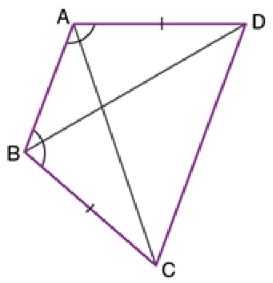
<u>CHapter 7 Triangles</u>

Q.1: ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA. Prove that

(i) ΔABD ≅ ΔBAC

(ii) BD = AC

(iii) ∠ABD = ∠BAC.



Solution:

As given in the question,

∠DAB = ∠CBA and AD = BC.

(i) ΔABD and ΔBAC are similar by SAS congruency as

AB = BA (common arm)

∠DAB = ∠CBA and AD = BC (given)

So, triangles ABD and BAC are similar

i.e. ΔABD ≅ ΔBAC. (Hence proved).

(ii) As it is already proved,

ΔABD ≅ ΔBAC

So,

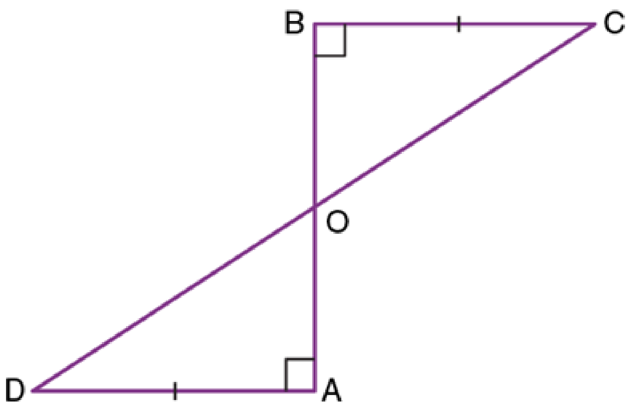
BD = AC (by CPCT)

(iii) Since ΔABD ≅ ΔBAC

So, the angles,

∠ABD = ∠BAC (by CPCT).

Q.2: AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB.



Solution:

Given, AD and BC are two equal perpendiculars to AB.

To prove: CD is the bisector of AB

Proof:

Triangles ΔAOD and ΔBOC are similar by AAS congruency

Since:

(i) ∠A = ∠B (perpendicular angles)

(ii) AD = BC (given)

(iii) ∠AOD = ∠BOC (vertically opposite angles)

∴ ΔAOD ≅ ΔBOC.

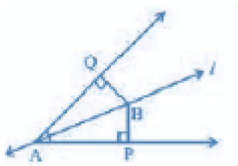
So, AO = OB (by CPCT).

Thus, CD bisects AB (Hence proved).

Q.3: Line l is the bisector of an angle ∠A and B is any point on l. BP and BQ are perpendiculars from B to the arms of ∠A. Show that:

(i) ΔAPB ≅ ΔAQB

(ii) BP = BQ or B is equidistant from the arms of ∠A.



Solution:

It is given that the line “l” is the bisector of angle ∠A and the line segments BP and BQ are perpendiculars drawn from l.

(i) ΔAPB and ΔAQB are similar by AAS congruency because;

∠P = ∠Q (both are right angles)

AB = AB (common arm)

∠BAP = ∠BAQ (As line l is the bisector of angle A)

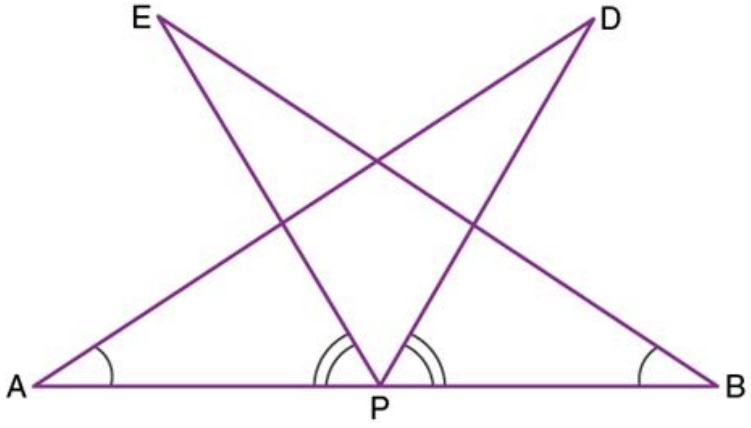
So, ΔAPB ≅ ΔAQB.

(ii) By the rule of CPCT, BP = BQ. So, we can say point B is equidistant from the arms of ∠A.

Q.4: AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB. Show that

(i) ΔDAP ≅ ΔEBP

(ii) AD = BE



Solution:

Given, P is the mid-point of line segment AB.

Also, ∠BAD = ∠ABE and ∠EPA = ∠DPB

(i) Given, ∠EPA = ∠DPB

Now, add ∠DPE on both sides,

∠EPA + ∠DPE = ∠DPB + ∠DPE

This implies that angles DPA and EPB are equal

i.e. ∠DPA = ∠EPB

Now, consider the triangles DAP and EBP.

∠DPA = ∠EPB

AP = BP (Since P is the mid-point of the line segment AB)

∠BAD = ∠ABE (given)

So, by ASA congruency criterion,

ΔDAP ≅ ΔEBP.

(ii) By the rule of CPCT,

AD = BE

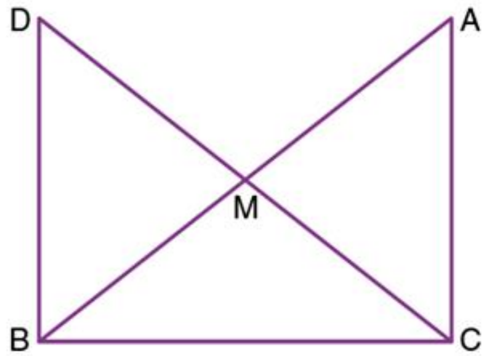
Q.5: In right triangle ABC, right-angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see the figure). Show that:

(i) ΔAMC ≅ ΔBMD

(ii) ∠DBC is a right angle.

(iii) ΔDBC ≅ ΔACB

(iv) CM = 1/2 AB



Solution:

It is given that M is the mid-point of the line segment AB, ∠C = 90°, and DM = CM

(i) Consider the triangles ΔAMC and ΔBMD:

AM = BM (Since M is the mid-point)

CM = DM (Given)

∠CMA = ∠DMB (Vertically opposite angles)

So, by SAS congruency criterion, ΔAMC ≅ ΔBMD.

(ii) ∠ACM = ∠BDM (by CPCT)

∴ AC ∥ BD as alternate interior angles are equal.

Now, ∠ACB + ∠DBC = 180° (Since they are co-interiors angles)

⇒ 90° + ∠B = 180°

∴ ∠DBC = 90°

(iii) In ΔDBC and ΔACB,

BC = CB (Common side)

∠ACB = ∠DBC (Both are right angles)

DB = AC (by CPCT)

So, ΔDBC ≅ ΔACB by SAS congruency.

(iv) DC = AB (Since ΔDBC ≅ ΔACB)

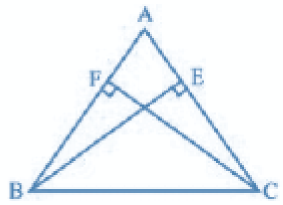
⇒ DM = CM = AM = BM (Since M the is mid-point)

So, DM + CM = BM + AM

Hence, CM + CM = AB

⇒ CM = (½) AB

Q.6: ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively. Show that these altitudes are equal.



Solution:

Given:

(i) BE and CF are altitudes.

(ii) AC = AB

To prove:

BE = CF

Proof:

Triangles ΔAEB and ΔAFC are similar by AAS congruency, since;

∠A = ∠A (common arm)

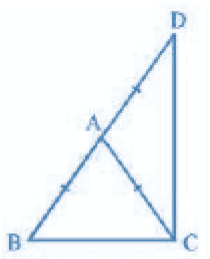
∠AEB = ∠AFC (both are right angles)

AB = AC (Given)

∴ ΔAEB ≅ ΔAFC

and BE = CF (by CPCT).

Q.7: ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB. Show that ∠BCD is a right angle.



Solution:

Given, AB = AC and AD = AB

To prove: ∠BCD is a right angle.

Proof:

Consider ΔABC,

AB = AC (Given)

Also, ∠ACB = ∠ABC (Angles opposite to equal sides)

Now, consider ΔACD,

AD = AC

Also, ∠ADC = ∠ACD (Angles opposite to equal sides)

Now,

In ΔABC,

∠CAB + ∠ACB + ∠ABC = 180°

So, ∠CAB + 2∠ACB = 180°

⇒ ∠CAB = 180° – 2∠ACB — (i)

Similarly in ΔADC,

∠CAD = 180° – 2∠ACD — (ii)

Also,

∠CAB + ∠CAD = 180° (BD is a straight line.)

Adding (i) and (ii) we get,

∠CAB + ∠CAD = 180° – 2∠ACB + 180° – 2∠ACD

⇒ 180° = 360° – 2∠ACB – 2∠ACD

⇒ 2(∠ACB + ∠ACD) = 180°

⇒ ∠BCD = 90°

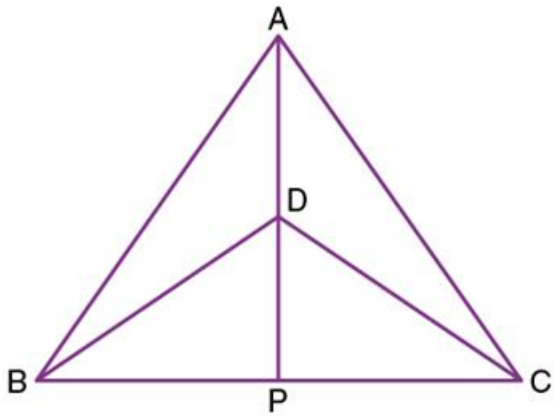
Q.8: ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see the figure). If AD is extended to intersect BC at P, show that

(i) ΔABD ≅ ΔACD

(ii) ΔABP ≅ ΔACP

(iii) AP bisects ∠A as well as ∠D.

(iv) AP is the perpendicular bisector of BC.



Solution:

In the above question, it is given that ΔABC and ΔDBC are two isosceles triangles.

(i) ΔABD and ΔACD are similar by SSS congruency because:

AD = AD (It is the common arm)

AB = AC (Since ΔABC is isosceles)

BD = CD (Since ΔDBC is isosceles)

∴ ΔABD ≅ ΔACD.

(ii) ΔABP and ΔACP are similar as:

AP = AP (common side)

∠PAB = ∠PAC (by CPCT since ΔABD ≅ ΔACD)

AB = AC (Since ΔABC is isosceles)

So, ΔABP ≅ ΔACP by SAS congruency.

(iii) ∠PAB = ∠PAC by CPCT as ΔABD ≅ ΔACD.

AP bisects ∠A. ………… (1)

Also, ΔBPD and ΔCPD are similar by SSS congruency as

PD = PD (It is the common side)

BD = CD (Since ΔDBC is isosceles.)

BP = CP (by CPCT as ΔABP ≅ ΔACP)

So, ΔBPD ≅ ΔCPD.

Thus, ∠BDP = ∠CDP by CPCT. ……………. (2)

Now by comparing equation (1) and (2) it can be said that AP bisects ∠A as well as ∠D.

(iv) ∠BPD = ∠CPD (by CPCT as ΔBPD ≅ ΔCPD)

and BP = CP — (1)

also,

∠BPD + ∠CPD = 180° (Since BC is a straight line.)

⇒ 2∠BPD = 180°

⇒ ∠BPD = 90° —(2)

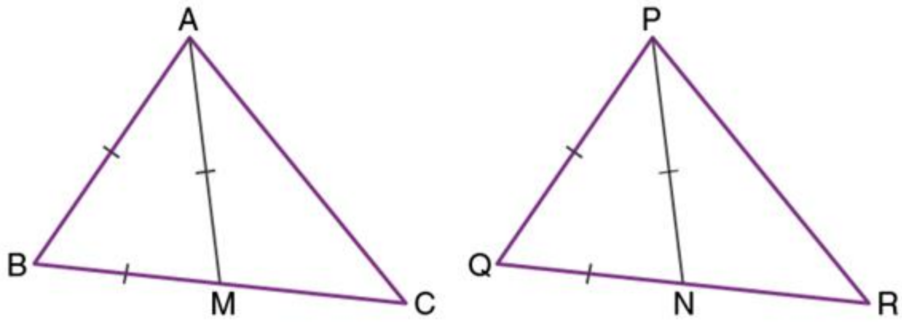
Now, from equations (1) and (2), it can be said that

AP is the perpendicular bisector of BC.

Q.9: Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see the figure). Show that:

(i) ΔABM ≅ ΔPQN

(ii) ΔABC ≅ ΔPQR



Solution:

Given;

AB = PQ,

BC = QR and

AM = PN

(i) 1/2 BC = BM and 1/2QR = QN (Since AM and PN are medians)

Also, BC = QR

So, 1/2 BC = 1/2QR

⇒ BM = QN

In ΔABM and ΔPQN,

AM = PN and AB = PQ (Given)

BM = QN (Already proved)

∴ ΔABM ≅ ΔPQN by SSS congruency.

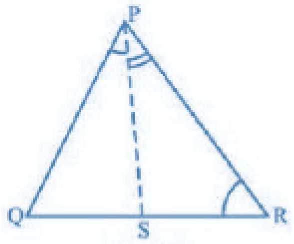
(ii) In ΔABC and ΔPQR,

AB = PQ and BC = QR (Given)

∠ABC = ∠PQR (by CPCT)

So, ΔABC ≅ ΔPQR by SAS congruency.

Q.10: In the Figure, PR > PQ and PS bisect ∠QPR. Prove that ∠PSR > ∠PSQ.



Solution:

Given, PR > PQ and PS bisects ∠QPR

To prove: ∠PSR > ∠PSQ

Proof:

∠QPS = ∠RPS — (1) (PS bisects ∠QPR)

∠PQR > ∠PRQ — (2) (Since PR > PQ as angle opposite to the larger side is always larger)

∠PSR = ∠PQR + ∠QPS — (3) (Since the exterior angle of a triangle equals the sum of opposite interior angles)

∠PSQ = ∠PRQ + ∠RPS — (4) (As the exterior angle of a triangle equals the sum of opposite interior angles)

By adding (1) and (2)

∠PQR + ∠QPS > ∠PRQ + ∠RPS

Now, from (1), (2), (3) and (4), we get

∠PSR > ∠PSQ