Chapter 2: Polynomials

1. Give an example of a monomial and a binomial having degrees as 82 and 99, respectively.

Solution: An example of a monomial having a degree of 82 = x^{82}
An example of a binomial having a degree of 99 = $x^{99} + x$

2. Compute the value of $9x^2 + 4y^2$ if xy = 6 and 3x + 2y = 12.

Solution: Consider the equation 3x + 2y = 12
Now, square both sides:
$(3x + 2y)^2 = 12^2$
or, $9x^2 + 12xy + 4y^2 = 144$
or, $9x^2 + 4y^2 = 144 – 12xy$

From the questions, XY = 6

So, $9x^2 + 4y^2 = 144 – 72$
Thus, the value of $9x^2 + 4y^2 = 72$

3. Find the value of the polynomial $5x – 4x^2 + 3$ at x = 2 and x = –1

Solution: Let the polynomial be $f(x) = 5x – 4x^2 + 3$

Now, for x = 2,
$f(2) = 5(2) – 4(2)^2 + 3$
or, f(2) = 10 – 16 + 3 = –3
Or, the value of the polynomial $5x – 4x^2 + 3$ at x = 2 is -3.

Similarly, for x = –1,
$f(–1) = 5(–1) – 4(–1)^2 + 3$
or, f(–1) = –5 –4 + 3 = -6
The value of the polynomial $5x – 4x^2 + 3$ at x = -1 is -6.

4. Calculate the perimeter of a rectangle whose area is $25x^2 – 35x + 12$.

Solution: Given, the area of rectangle = $25x^2 – 35x + 12$
We know, area of rectangle = length × breadth
So, by factoring $25x^2 – 35x + 12$, the length and breadth can be obtained.
$25x^2 – 35x + 12 = 25x^2 – 15x – 20x + 12$
Or, $25x^2 – 35x + 12 = 5x(5x – 3) – 4(5x – 3)$
or, $25x^2 – 35x + 12 = (5x – 3)(5x – 4)$
So, the length and breadth are (5x – 3)(5x – 4).

Now, perimeter = 2(length + breadth)
So, perimeter of the rectangle = 2[(5x – 3)+(5x – 4)]
= 2(5x – 3 + 5x – 4) = 2(10x – 7) = 20x – 14
So, the perimeter = 20x – 14

5. Find the value of $x^3 + y^3 + z^3 – 3xyz$ if $x^2 + y^2 + z^2 = 83$ and x + y + z = 15

Solution: Consider the equation x + y + z = 15
From algebraic identities, we know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

So, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + xz)$

From the question, $x^2 + y^2 + z^2 = 83$ and $x + y + z = 15$
So, $15^2 = 83 + 2(xy + yz + xz)$
or, $225 – 83 = 2(xy + yz + xz)$
Or, $xy + yz + xz = 142/2 = 71$

Using algebraic identity $a^3 + b^3 + c^3 – 3abc = (a + b + c)(a^2 + b^2 + c^2 – ab – bc – ca)$,
$x^3 + y^3 + z^3 – 3xyz = (x + y + z)(x^2 + y^2 + z^2 – (xy + yz + xz))$

Now, $x + y + z = 15$, $x^2 + y^2 + z^2 = 83$ and $xy + yz + xz = 71$
So, $x^3 + y^3 + z^3 – 3xyz = 15(83 – 71)$
or, $x^3 + y^3 + z^3 – 3xyz = 15 \times 12$
Or, $x^3 + y^3 + z^3 – 3xyz = 180$

6. Without actually Calculating the cubes, find the value of $(-12)^3 + (7)^3 + (5)^3$
Solution: Using $a^3+ b^3 + c^3 = 3abc$,
if $a + b + c = 0$
$(-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5) = 1260$, as $-12 + 7 + 5 = -12 + 12 = 0$

7. Is the following expressions are polynomials in one variable? State your reasons for both yes and no conditions.
(i) $4x^2 – 3x + 7$
(ii) $3\sqrt{t} + t\sqrt{2}$

Solution: (i) $4x^2 – 3x + 7$, we can observe that in the given polynomial, we have x as the only variable and the powers of x in each term are a whole number. Hence, we can conclude that $4x^2 – 3x + 7$ is a polynomial with one variable.

(ii) $3\sqrt{t} + t$, we can observe that in the given polynomial, we have "t" as the only variable and the powers of "t" is not whole numbers. Therefore, we conclude that $3\sqrt{t} + t\sqrt{2}$ is not a polynomial in one variable