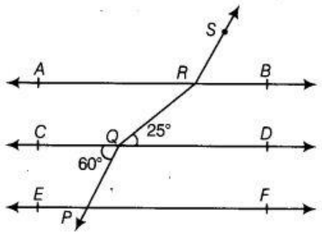
Chapter 6: Lines and Angles

Exercise 6.1

Question 1: In figure, if AB || CD || EE, PQ || RS, ∠RQD = 25° and ∠CQP = 60°, then ∠QRS is equal to



(a) 85° (b)135° (c)145° (d) 110°

Answer: (c) Given, PQ || RS
∠PQC = ∠BRS = 60° [alternate exterior angles and ∠PQC = 60° (given)] and ∠DQR = ∠QRA = 25° [alternate interior angles]
[∠DQR = 25°, given]
∠QRS = ∠QRA + ∠ARS
= ∠QRA + (180° – ∠BRS) [linear pair axiom]
= 25° + 180° – 60°= 205° – 60°= 145°

Question 2: If one angle of a triangle is equal to the sum of the other two angles, then the triangle is
(a) an isosceles triangle (b) an obtuse triangle
(c) an equilateral triangle (d) a right triangle

Answer: (d) Let the angles of a AABC be ∠A, ∠B and ∠C.
Given, ∠A = ∠B+∠C …(i)
InMBC, ∠A+ ∠B+ ∠C-180° [sum of all angles of atriangle is 180°]…(ii)
From Eqs. (i) and (ii),
∠A+∠A = 180°
or, 2 ∠A = 180°
or, 180° ÷ 2
∠A = 90°
Hence, the triangle is a right triangle.

Question 3: An exterior angle of a triangle is 105° and its two interior opposite angles are equal. Each of these equal angles is
(a) 37 ½° (b) 52 ½° (c)72 ½° (d) 75°

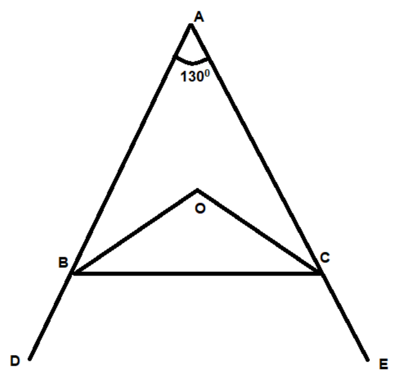
Answer: Let one of interior angle be x°.
Sum of two opposite interior angles = Exterior angle
Hence, x° + x° = 105°
2x° = 105°
x° = 105° ÷ 2
x°=52 ½°
Hence, each angle of a triangle is 52 ½°.

Question 4: If the angles of a triangle are in the ratio 5:3:7, then the triangle is
(a) an acute angled triangle
(b) an obtuse angled triangle
(c) a right angled triangle
(d) an isosceles triangle

Answer: (a) Given, the ratio of angles of a triangle is 5 : 3 : 7.
Let angles of a triangle be ∠A,∠B and ∠C.
Then, ∠A = 5x, ∠B = 3x and ∠C = 7x
In ΔABC, ∠A + ∠B + ∠C = 180° [since, sum of all angles of a triangle is 180°]
5x + 3x + 7x = 180°
or, 15x = 180°
x = 180° ÷ 15= 12°
∠A = 5x = 5 x 12° = 60°
∠B = 3x= 3 x 12°= 36°
and ∠C =7x = 7 x 12° = 84°
Since, all angles are less than 90°, hence the triangle is an acute angled triangle.

Question 5: If one of the angles of a triangle is 130°, then the angle between the bisectors of the other two angles can be
(a) 50° (b) 65° (c) 145° (d) 155

Answer:



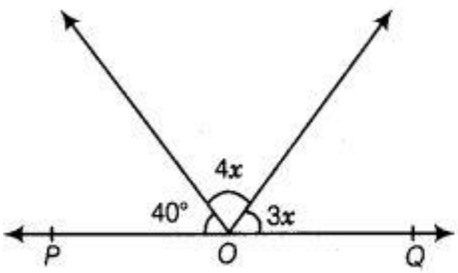
In ΔABC, ∠A + ∠B + ∠C = 180^0
or, $\frac{1}{2}$∠A + $\frac{1}{2}$∠B + $\frac{1}{2}$∠C = $\frac{180°}{2}$ = 90^0
or, $\frac{1}{2}$∠B + $\frac{1}{2}$∠C = 90^0 - $\frac{1}{2}$∠A ………..[in ΔOBC, ∠OBC + ∠BCO + ∠COB = 180^0]
or, 180^0 - ∠BOC = 90^0 - $\frac{1}{2}$∠A

Therefore, ∠BOC = 180^0 – 90^0 + $\frac{1}{2}$∠A
= 90^0 + $\frac{1}{2}$∠A

$= 90^0 + \frac{1}{2} \times 130°$
$= 90^0 + 65^0$

Hence, the required angle is 155^0

Question 6: In the figure, POQ is a line. The value of x is

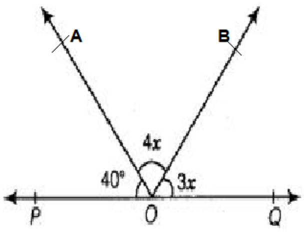


(a)20° (b)25° (c)30° (d) 35°

Thinking Process

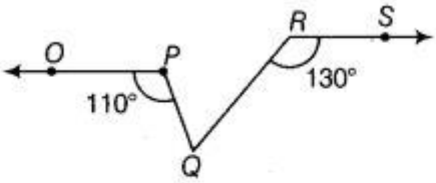
When two or more rays are initiated from a same point of a line, then the sum of all angles made between the rays and line at the same point is 180°.

Answer: Since, POQ is a line segment,



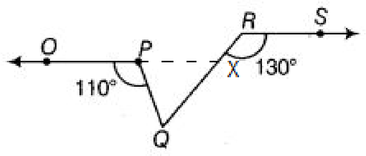
Therefore, $\angle POQ = 180^0$
or, $\angle POA + \angle AOB + \angle BOQ = 180^0$
or, $40^0 + 4x + 3x = 180^0$
or, $7x = 180^0 - 40^0$
or, $7x = 140^0$
or, $x = 20^0$

Question 7: In the figure, if OP || RS, ∠OPQ = 110° and ∠QRS = 130°, then ∠PQR is equal to



(a) 40° (b) 50° (c) 60° (d) 70°

Answer: (c)



In the given figure, producing OP to intersect RQ at X.
Since, OP || RS and RX is a transversal.
So, ∠RXP = ∠XRS [alternate angles]
or, ∠RXP = 130^0 ….[Since, Given, ∠QRS = 130^0] ………………………...(1)
Now, RQ is a line-segment.
So, ∠PXQ + ∠RXP = 180^0 [Linear pair axiom]
or, ∠PXQ = 180^0 - ∠RXP
or, ∠PXQ = $180^0 – 130^0$
or, ∠PXQ = 50^0

In triangle PQX, ∠OPQ is an exterior angle.
Thus, ∠OPQ = ∠PXQ + ∠PQX [Since, exterior angle = sum of the opposite interior angles]
or, $110^0 = 50^0$ + ∠PQX
or, ∠PQX = $110^0 - 50^0$
Therefore, ∠PQR = 60^0

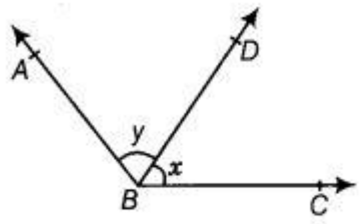
Question 8: Angles of a triangle are in the ratio 2:4:3. The smallest angle of the triangle is
(a) 60° (b) 40° (c) 80° (d) 20°
Thinking Process
Use the concept, the sum of all angles in a triangle is 180°. Further, simplify it and get the smallest angle.

Answer: (b) Given, the ratio of angles of a triangle is 2 : 4 : 3.
Let the angles of a triangle be ∠A, ∠B and ∠C.
∠A = 2x, ∠B = 4x
∠C = 3x , ∠A+∠B+ ∠C= 180°
[sum of all the angles of a triangle is 180°]
2x + 4x + 3x = 180°
9x = 180°
x=180°/9 =20°
∠A=2x=2 x 20° = 40°
∠B = 4x = 4 x 20° = 80°

∠C = 3x = 3 x 20° = 60°
Hence, the smallest angle of a triangle is 40°.

Exercise 6.2(Very short Answer Question)

Question 1: For what value of x + y in figure will ABC be a line? Justify your answer.



Answer: For ABC to be a line, the sum of the two adjacent angles must be 180° i.e.,x + y = 180°.

Question 2: Can a triangle have all angles less than 60°? Give reason for your answer.

Answer: No, a triangle cannot have all angles less than 60°, because if all angles will be less than 60°, then their sum will not be equal to 180°. Hence, it will not be a triangle.

Question 3: Can a triangle have two obtuse angles? Give reason for your answer.

Answer: No, because if the triangle have two obtuse angles i.e., more than 90° angle, then the sum of all three angles of a triangle will not be equal to 180°.

Question 4: How many triangles can be drawn having its angles as 45°, 64° and 72°? Give reason for your answer.

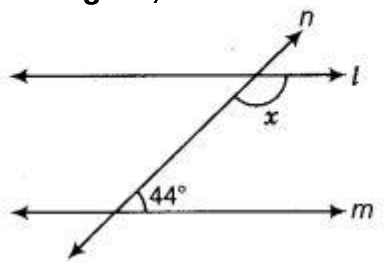
Answer: None, the sum of given angles = 45° + 64° + 72° = 181° ≠ 180°.
Hence, we see that sum of all three angles is not equal to 180°. So, no triangle can be drawn with the given angles.

Question 5: How many triangles can be drawn having its angles as 53°, 64° and 63°? Give reason for your answer.

Answer :Infinitely many triangles,
The sum of given angles = 53° + 64° + 63° = 180°

Here, we see that sum of all interior angles of triangle is 180°, so infinitely many triangles can be drawn.

Question 6:
In the figure, find the value of x for which the lines l and m are parallel.



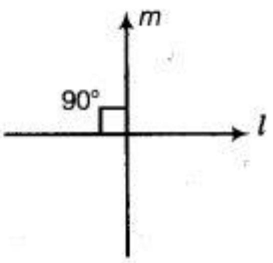
Answer :In the given figure, l || m and we know that, if a transversal intersects two parallel lines, then sum of interior angles on the same side of a transversal is supplementary. $x + 44° = 180°$
$x = 180° - 44°$
or, $x = 136°$.

Question 7: Two adjacent angles are equal. Is it necessary that each of these angles will be a right angle? Justify your answer.

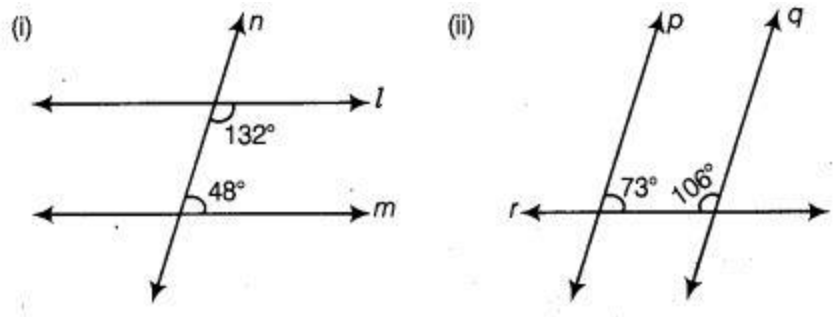
Answer: No, because each of these will be a right angle only when they form a linear pair.

Question 8:
If one of the angles formed by two intersecting lines is a right angle, what can you say about the other three angles? Give reason for your answer.

Answer: Let two intersecting lines l and m makes a one right angle, then it means that lines I and m are perpendicular each other. By using linear pair axiom aniom, other three angles will be a right angle.



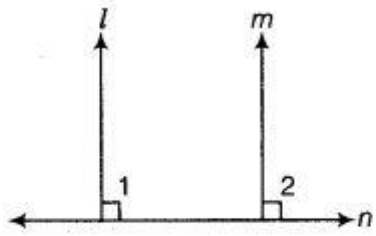
Question 9:
In the figure, which of the two lines are parallel and why?



Answer: In Fig. (i) sum of two interior angles 132° + 48° = 180° [∴ equal to 180°]
Here, we see that the sum of two interior angles on the same side of n is 180°, then they are the parallel lines.
In Fig. (ii), the sum of two interior angles 73° + 106° = 179° ≠ 180°. Here, we see that the sum of two interior angles on same side of r is not equal to 180°, then they are not the parallel lines.

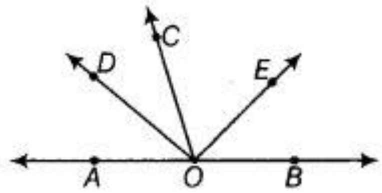
Question 10:
Two lines l and m are perpendicular to the same line n. Are l and m perpendicular to each other? Give reason for your answer.

Answer: No, since, lines l and m are perpendicular to the line n.
∠1 = ∠2 = 90° [∴ l ⊥ n and min]
It implies that these are corresponding angles.
Hence, l|| m.



Exercise 6.3 (Short type Questions)

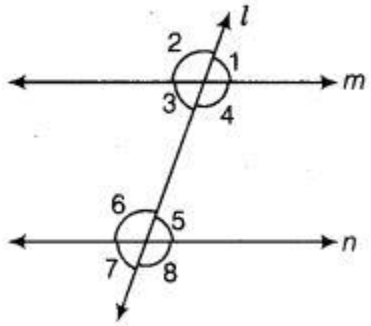
Question 1: In the figure, OD is the bisector of ∠AOC, OE is the bisector of ∠BOC and OD ⊥ OE. Show that the points A, 0 and B are collinear.



Thinking Process
For showing collinearity of A, O and B, we have to show that ∠AOB =180°.

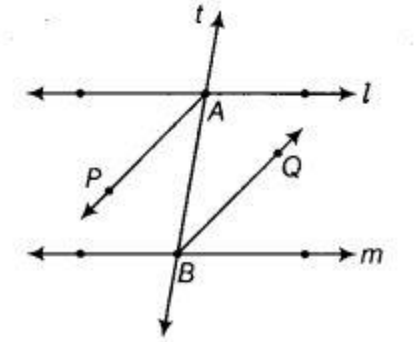
Answer: Given In the figure, OD ⊥ OE, OD and OE are the bisectors of ∠AOC and ∠BOC.
To show Points A, O and B are collinear i.e., AOB is a straight line.
Proof Since, OD and OE bisect angles ∠AOC and ∠BOC, respectively.
∠AOC =2 ∠DOC …(i)
and ∠COB = 2 ∠COE …(ii)
On adding Eqs. (i) and (ii), we get
∠AOC + ∠COB = 2 ∠DOC +2 ∠COE => ∠AOC +∠COB = 2(∠DOC +∠COE)
or, ∠AOC + ∠COB= 2 ∠DOE
or, ∠AOC+ ∠COB = 2 x 90° [∴ OD ⊥ OE]
or, ∠AOC + ∠COB = 180°
∴ ∠AOB = 180°
So, ∠AOC and ∠COB are forming linear pair.
Also, AOB is a straight line.
Hence, points A, O and B are collinear.

Question 2:
In the figure, ∠1 = 60° and ∠6 = 120°. Show that the lines m and n are parallel.

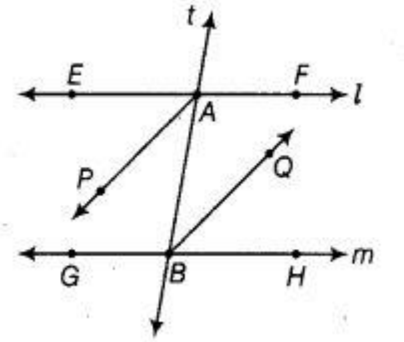


Answer: Given In the figure ∠1 = 60° and ∠6 = 120°
To show m||n
Proof Since, ∠1 = 60° and ∠6 = 120°
Here, ∠1 = ∠3 [vertically opposite angles]
∠3 = ∠1 = 60°
Now, ∠3 + ∠6 = 60° + 120°
or, ∠3 + ∠6 = 180°
We know that, if the sum of two interior angles on same side of l is 180°, then lines are parallel.
Hence, m || n

Question 3:
AP and BQ are the bisectors of the two alternate interior angles formed by the intersection of a transversal t with parallel lines l and m (in the given figure). Show that AP || BQ.

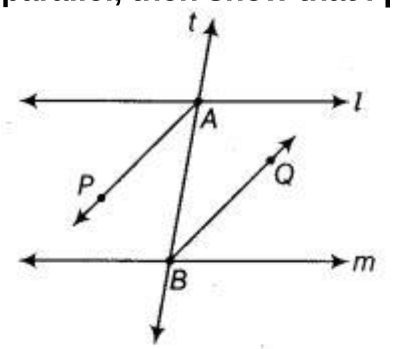


Answer: Given In the figure l || m, AP and BQ are the bisectors of ∠EAB and ∠ABH, respectively.
To prove AP|| BQ
Proof Since, l || m and t is transversal.
Therefore, ∠EAB = ∠ABH [alternate interior angles]



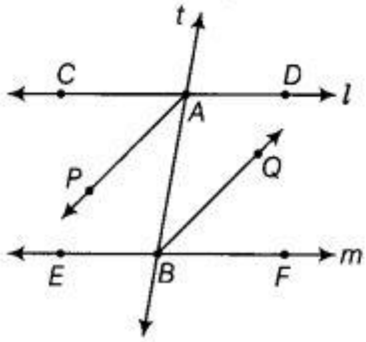
½ ∠EAB =½ ∠ABH [dividing both sides by 2]
∠PAB =∠ABQ
[AP and BQ are the bisectors of ∠EAB and ∠ABH] Since, ∠PAB and ∠ABQ are alternate interior angles with two lines AP and BQ and transversal AB. Hence, AP || BQ.

Question 4:
In the given figure, bisectors AP and BQ of the alternate interior angles are parallel, then show that l ||m.



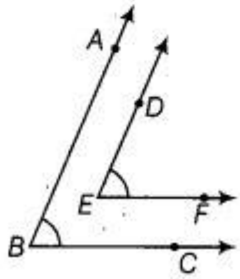
Answer: Given, In the figure AP|| BQ, AP and BQ are the bisectors of alternate interior angles ∠CAB and ∠ABF.
To show l || m
Proof Since, AP|| BQ and t is transversal, therefore ∠PAB = ∠ABQ
[alternate interior angles]

or, 2 ∠PAB = 2 ∠ABQ [multiplying both sides by 2]

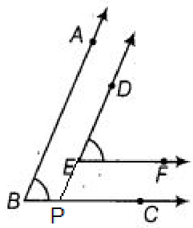


So, alternate interior angles are equal.
We know that, if two alternate interior angles are equal, then lines are parallel.
Hence, l || m.

Question 5:
In the figure, BA || ED and BC || EF. Show that ∠ABC = ∠DEF.

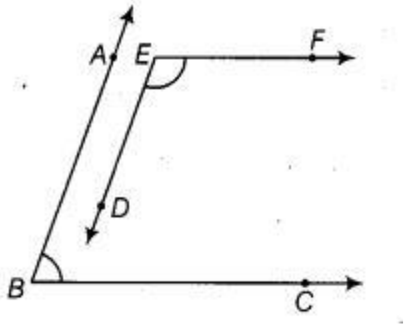


Answer: Given BA || ED and BC || EF.
To show ∠ABC = ∠DEF.
Construction Draw a ray EP opposite to ray ED.

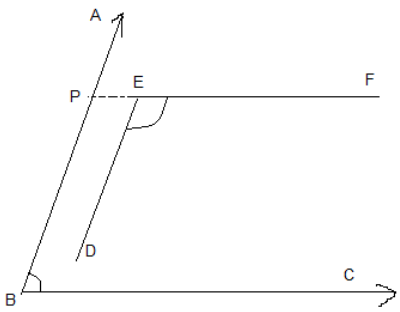


Proof: In the figure, BA || ED or BA || DP
Therefore, ∠ABP = ∠EPC………………….[Corresponding angles]
or, ∠ABC = ∠EPC……………………………………………………….(1)
Again, BC || EF or PC || EF
Therefore, ∠DEF = ∠EPC …………………………..[Corresponding angles] (2)
From eq (1) and (2),
∠ABC = ∠DEF

Question 6: In the figure, BA || ED and BC || EF. Show that ∠ABC + ∠DEF = 180°.

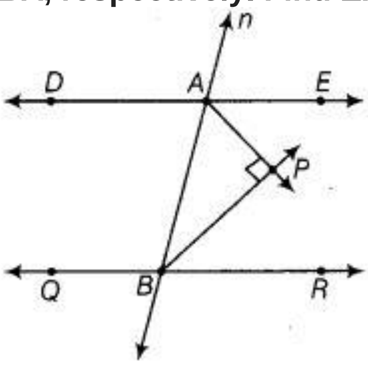


Answer: Given BA || ED and BC || EF
To show, ∠ABC + ∠DEF = 180°
Construction Draw a ray PE opposite to ray EF.



Proof: in the figure, BC || EF
Therefore, ∠EPB + ∠PBC = 180^0 …………(1)..[Sum of the co-interior angles is 180^0]
Now, AB || ED and PE is a transversal line,
Therefore, ∠EPB = ∠DEF …………………..(2) [Corresponding angles]
From eq(1) and eq(2),
∠DEF + ∠PBC = 180^0
or, ∠ABC + ∠DEF = 180^0 [Since, ∠PBC = ∠ABC]

Question 7: In the figure, DE || QR and AP and BP are bisectors of ∠EAB and ∠RBA, respectively. Find ∠APB.



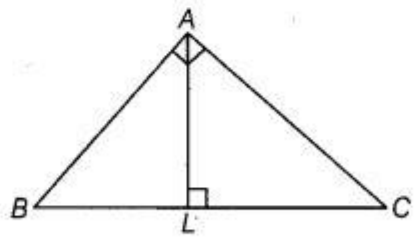
Answer: Given, DE || QR and AP and PB are the bisectors of ∠EAB and ∠RBA, respectively.
We know that, the interior angles on the same sides of transversal are supplementary.
Therefore, $\angle EAB + \angle RBA = 180^0$
or, $\frac{1}{2}\angle EAB + \frac{1}{2}\angle RBA = 90^0$ ….[dividing both sides by 2] ……………………………….(1)
Since, AP and BP are the bisectors of ∠EAB and ∠RBA, respectively,
Therefore, $\angle BAP = \frac{1}{2}\angle EAB$ …………………………………………………….(2)
and, $\angle ABP = \frac{1}{2}\angle RBA$ ……………………………………………..(3)
On adding eq(2) and (3) we get,
$\angle BAP + \angle ABP = \frac{1}{2}\angle EAB + \frac{1}{2}\angle RBA$
from eq(1), $\angle BAP + \angle ABP = 90^0$…………………………………(4)

In triangle APB, $\angle BAP + \angle ABP + \angle APB = 180^0$
or, $90^0 + \angle APB = 180^0$
or, $\angle APB = 90^0$

Question 8 : A ΔABC is right angled at A. L is a point on BC such that AL ⊥ BC. Prove that ∠BAL = ∠ACB.

Answer: Given In ΔABC, ∠A = 90° and AL ⊥ BC
To prove ∠BAL = ∠ACB
Proof In ΔABC and ΔLAC, ∠BAC = ∠ALC [each 90°] …………………………….(1)

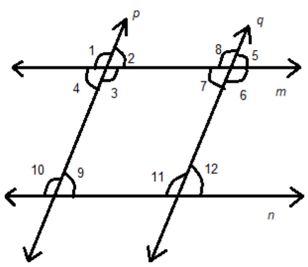
and ∠ABC = ∠ABL [common angle] ………………………………..(2)



On adding Eqs. (1) and (2), we get
∠BAC + ∠ABC = ∠ALC + ∠ABL ………………………………………(3)
Again, in ΔABC,
∠BAC + ∠ACB + ∠ABC = 180°
[sum of all angles of a triangle is 180°] =>∠BAC+∠ABC = 1 80°-∠ACB ……………(4)
In ΔABL,
∠ABL + ∠ALB + ∠BAL = 180° [sum of all angles of a triangle is 180°]
or, ∠ABL+ ∠ALC = 180° – ∠BAL [∴ ∠ALC = ∠ALB= 90°] …………………………(5)
On substituting the value from Eqs. (4) and (5) in Eq. (3), we get
180° – ∠ACS = 180° – ∠SAL
or, ∠ACB = ∠BAL
Hence proved.

Question 9: Two lines are respectively perpendicular to two parallel lines. Show that they are parallel to each other.

Answer: Given Two lines m and n are parallel and another two lines p and q are respectively perpendicular to m and n.
i.e., p ⊥ m, p ⊥ n, q ⊥ m, q ⊥ n
To prove p||g
Proof Since, m || n and p is perpendicular to m and n.

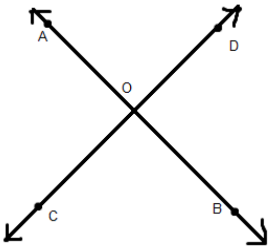


Therefore, ∠1 = ∠10 = 90^0
∠2 = ∠9 = = 90^0
Thus, Similarly, if m ∥ n and q is perpendicular to m and n.
Then, ∠7 = 90^0 and ∠11 = 90^0
Now, ∠3 + ∠7 = 90^0 + 90^0 = 180^0
So, the sum of the two interior angles is supplementary.
We know that, if a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.
Hence, p||g.

Exercise 6.4 (Long answer type question)

Question 1: If two lines intersect prove that the vertically opposite angles are equal.

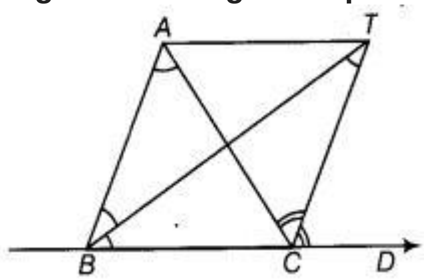
Answer: Given Two lines AB and CD intersect at point O.
To prove: (i) ∠AOC = ∠BOD
(ii) ∠AOD = ∠BOC
Proof : (i) Ray OA stands on line AB.
∠AOC + ∠AOC = 180^0 …………………………..[Linear axiom] (1)
Ray OD stands on line AB.
∠AOD + ∠BOD = 180^0 ……………………………….[Linear axiom] (2)



From eq(1) and eq(2),
∠AOC + ∠AOD = ∠AOD + ∠BOD
or, ∠AOC = ∠BOD

(ii) Since ray OD stands on line AB
Hence, ∠AOD + ∠BOD = 180^0 ………………….[Linear pair axiom].(3)
Since ray OB stands on line CD
Hence, ∠DOB + ∠BOC = 180^0
from eq(3) and eq(4)
∠AOD + ∠BOD = ∠DOB + ∠BOC
or, ∠AOD = ∠BOC

Question 2: Bisectors of interior ∠B and exterior ∠ACD of a ΔABC intersect at the point T. Prove that ∠BTC = ½ ∠BAC.
Thinking Process
For obtaining the interior required result use the property that the exterior angle of a triangle is equal to the sum of the two opposite angles of a triangle.



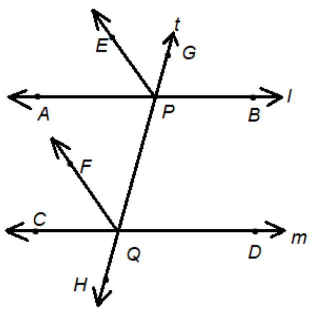
Answer: Given In AABC, produce SC to D and the bisectors of ∠ABC and ∠ACD meet at point T. To prove ∠BTC = ½ ∠BAC.
Proof: In triangle ABC, ∠ACD = ∠ABC + ∠CAB [exterior angle of a triangle is equal to the sum of two opposite angles]
or, $\frac{1}{2}$∠ACD = $\frac{1}{2}$∠CAB + $\frac{1}{2}$∠ABC [dividing both sides by 2]
or, ∠TCD = $\frac{1}{2}$∠CAB + $\frac{1}{2}$∠ABC ……………….(1) [CT is the bisector od $\frac{1}{2}$∠ACD, or, $\frac{1}{2}$∠ACD = ∠TCD]

In triangle BTC, ∠TCD = ∠BTC + ∠CBT
or, ∠TCD = ∠BTC + $\frac{1}{2}$∠ABC ……………………………………(2)[BT bisects of ∠ABC, or, ∠CBT = $\frac{1}{2}$∠ABC]

From eq(1) and eq(2),
$\frac{1}{2}$∠CAB + $\frac{1}{2}$∠ABC = ∠BTC + $\frac{1}{2}$∠ABC
or, ∠BTC = $\frac{1}{2}$∠CAB
or, ∠BTC = $\frac{1}{2}$∠BAC.

Question 3: A transversal intersects two parallel lines. Prove that the bisectors of any pair of corresponding angles so formed are parallel.

Answer: Given Two lines AB and CD are parallel and intersected by transversal t at P and O, respectively. Also, EP and FQ are the bisectors of angles ∠APG and ∠CQP, respectively.

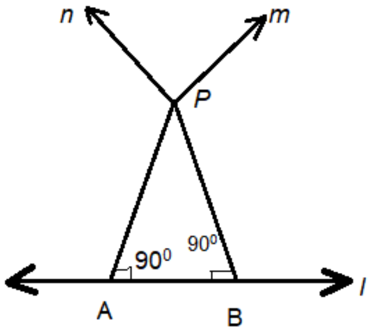


To prove EP ∥ FQ
Proof: Given, AB ∥ CD

∠APG = ∠CQP ……………..[corresponding angles]
or, $\frac{1}{2}$∠APG = $\frac{1}{2}$∠CQP ………………….[dividing both sides by 2]
or, ∠EPG = ∠FQP
Therefore, EP ∥ FQ.

Question 4: Prove that through a given point, we can draw only one perpendicular to a given line.

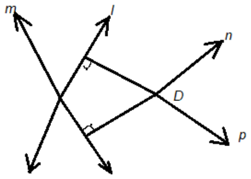
Answer: Given Consider a line l and a point P.



Construction: draw two intersecting lines passing through the point P and which is perpendicular to *l.*
To prove: only one perpendicular line can be drawn through a given point i.e., to prove $\angle P = 0^0$
Proof: In triangle APB, $\angle A + \angle P + \angle B = 180^0$ [by angle sum property of a triangle is 180^0]
or, $90^0 + \angle P + 90^0 = 180^0$
or, $\angle P = 0^0$
So, lines *n* and *m* coincide.
Hence, only one perpendicular line can be drawn through a given point.

Question 5: Prove that two lines that are respectively perpendicular to two intersecting lines intersect each other.

Answer: Given Let lines, l and m are two intersecting lines. Again, let n and p be another two lines that are perpendicular to the intersecting lines meet at point D.



To prove Two lines n and p intersecting at a point.
Proof Suppose we consider lines n and p are not intersecting, then it means they are parallel to each other i.e., n || p …(i)
Since, lines n and pare perpendicular to m and l, respectively.
But from Eq. (i) n || p it implies that l || m.
Hence, it is a contradiction.
Thus, our assumption is wrong.
Therefore, lines n and p intersect at a point.