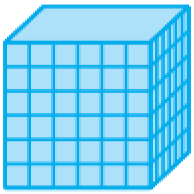
Chapter 13: Surface area and volume

Q.1: Hameed has built a cubical water tank with a lid for his house, with each outer edge 1.5 m long. He gets the outer surface of the tank excluding the base, covered with square tiles of the side 25 cm (see in the figure below). Find how much he would spend on the tiles if the cost of the tiles is Rs.360 per dozen.



Solution: Given,

Edge of the cubical tank (a) = 1.5 m = 150 cm

So, the surface area of the tank = $5 \times 150 \times 150$ cm^2

The measure of the side of a square tile = 25 cm

Area of each square tile = side × side = 25×25 cm^2

Required number of tiles = (Surface area of the tank)/(area of each tile)

= $(5 \times 150 \times 150)/(25 \times 25)$

= 180

Also, given that the cost of the tiles is Rs. 360 per dozen.

Thus, the cost of each tile = Rs. 360/12 = Rs. 30

Hence, the total cost of 180 tiles = 180 × Rs. 30 = Rs. 5400

Q.2: The paint in a certain container is sufficient to paint an area equal to 9.375 sq.m. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?

Solution: Given,

Dimensions of the brick = 22.5 cm × 10 cm × 7.5 cm

Here, l = 22.5 cm, b = 10 cm, h = 7.5 cm

Surface area of 1 brick = $2(lb + bh + hl)$

= $2(22.5 \times 10 + 10 \times 7.5 + 7.5 \times 22.5)$ cm^2

= $2(225 + 75 + 168.75)$ cm^2

= 2×468.75 cm^2

= 937.5 cm^2

Area that can be painted by the container = 9.375 m^2 (given)

$= 9.375 \times 10000\ cm^2$

$= 93750\ cm^2$

Thus, the required number of bricks = (Area that can be painted by the container)/(Surface area of 1 brick)

= 93750/937.5

= 937500/9375

= 100brieks

Q.3: The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of whitewashing the walls of the room and the ceiling at the rate of Rs.7.50 per sq.m.

Solution: Given,

Length of the room (l) = 5 m

Breadth of the room (b) = 4 m

Height of the room (h) = 3 m

Area of walls of the room = Lateral surface area of cuboid

= 2h(l + b)

= 2 × 3(5 + 4)

= 6 × 9

= 54 sq.m

Area of ceiling = Area of base of the cuboid

= lb

= 5 × 4

= 20 sq.m

Area to be whitewashed = (54 + 20) sq.m = 74 sq.m

Given that, the cost of whitewashing 1 sq.m = Rs. 7.50

Therefore, the total cost of whitewashing the walls and ceiling of the room = 74 × Rs. 7.50 = Rs. 555

Q.4: The curved surface area of a right circular cylinder of height 14 cm is 88 sq. cm. Find the diameter of the base of the cylinder.

Solution:

Let d be the diameter and r be the radius of a right circular cylinder.

Given,

Height of cylinder (h) = 14 cm

Curved surface area of right circular cylinder = $88\ cm^2$

⇒ $2\pi rh = 88\ cm^2$

⇒ $\pi dh = 88\ cm^2$ (since $d = 2r$)

⇒ $22/7 \times d \times 14\ cm = 88\ cm^2$

⇒ $d = 2\ cm$

Hence, the diameter of the base of the cylinder is 2 cm.

Q.5: Curved surface area of a right circular cylinder is 4.4 sq.m. If the radius of the base of the cylinder is 0.7 m, find its height.

Solution:

Let h be the height of the cylinder.

Given,

The radius of the base of the cylinder (r) = 0.7 m

The curved surface area of cylinder = 4.4 m^2

Thus,

$2\pi rh = 4.4$

$2 \times 3.14 \times 0.7 \times h = 4.4$

$4.4 \times h = 4.4$

$h = 4.4/4.4$

$h = 1$

Therefore, the height of the cylinder is 1 m.

Q.6: In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.

Solution:

Given,

Length of the cylindrical pipe = h = 28 m

Diameter of the pipe = 5 cm

Now, the radius of piper (r) = 5/ 2 cm = 2.5 cm = 0.025 m

Total radiating surface in the system = Total surface area of the cylinder

$= 2\pi r(h + r)$

$= 2 \times (22/7) \times 0.025\ (28 + 0.025)\ m^2$

$= (44 \times 0.025 \times 28.025)/7\ m^2$

$= 4.4\ m^2$ (approx)

Q.7: The height of a cone is 16 cm and its base radius is 12 cm. Find the curved surface area and the total surface area of the cone. (Take $\pi = 3.14$)

Solution:

Given

Height of a cone (h) = 16 cm

Radius of the base (r) = 12 cm

Now,

Slant height of cone (l) = $\sqrt{(r^2 + h^2)}$

= $\sqrt{(256 + 144)}$

= $\sqrt{400}$

= 20 cm

Curved surface area of cone = πrl

= 3.14 × 12 × 20 cm^2

= 753.6 cm^2

Total surface area = πrl + πr^2

= (753.6 + 3.14 × 12 × 12) cm^2

= (753.6 + 452.16) cm^2

= 1205.76 cm^2

Q.8: Find the total surface area of a cone, if its slant height is 21 m and the diameter of its base is 24 m.

Solution:

Given,

Diameter of the cone = 24 m

Radius of the cone (r) = 24/2 = 12 m

Slant height of the cone (l) = 21 m

Total surface area of a cone = πr(l + r)

= (22/7) × 12 × (21 + 12)

= (22/7) × 12 × 33

= 1244.57 m^2

Q.9: The slant height and base diameter of a conical tomb is 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of Rs.210 per 100 sq.m.

Solution:

Given,

Slant height of a cone (l) = 25 m

Diameter of the base of cone = 2r = 14 m

∴ Radius = r = 7 m

Curved Surface Area = πrl

= (22/7) x 7 x 25

= 22 × 25

= 550 sq.m

Also, given that the cost of white-washing 100 sq.m = Rs. 210

Hence, the total cost of white-washing for 550 sq.m = (Rs. 210 × 550)/100 = Rs. 1155

Q.10: The hollow sphere, in which the circus motorcyclist performs his stunts, has a diameter of 7 m. Find the area available to the motorcyclist for riding.

Solution:

Given,

Diameter of the sphere = 7 m

Radius (r) = 7/2 = 3.5 m

Now, the riding space available for the motorcyclist = Surface area of the sphere

= $4\pi r^2$

= 4 × (22/7) × 3.5 × 3.5

= 154 m^2

Q.11: The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Solution:

Given,

Radius of balloon = r = 7 cm

Radius of pumped balloon = R = 14 cm

Ratio of surface area = (TSA of balloon with r = 7 cm)/(TSA of balloon with R = 14 cm)

= $(4\pi r^2)/(4\pi R^2)$

= r^2/R^2

= $(7)^2/(14)^2$

= 49/196

= 1/4

Hence, the ratio of surface areas of the balloon in the two cases is 1: 4.

Q.12: A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

Solution:

Given,

Depth of the river (h) = 3 m

Width of the river (w) = 40 m

Flow rate of water = 2 km/hr

i.e. Flow of water in 1 hour = 2 km = 2000 m

Flow of water in 1 minute = 2000/60 = 100/3 m

Thus, length (l) = 100/3 m

The volume of water falling into the sea in 1 minute = Volume of a cuboid with dimension l, w, h

= l × w × h

= (100/3) × 40 × 3

= 4000 m^3

= 4000 x 1000 L

= 4000000 L

Q.13: A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

Solution:

Given,

Diameter of the pencil = 7 mm

Radius of the pencil (R) = 7/2 mm

Diameter of the graphite cylinder = 1 mm

Radius of the graphite (r) = 1/2 mm

Height (h) = 14 cm = 140 mm (since 1 cm = 10 mm)

Volume of a cylinder = $\pi r^2 h$

Volume of graphite cylinder = πr2h

= (22/7) × (1/2) × (1/2) × 140

= 110 mm^3

Volume of pencil = $\pi R^2 h$

= (22/7) × (7/2) × (7/2) × 140

= 490 × 11

= 5390 mm^2

Volume of wood = Volume of pencil – Volume of graphite

$= 5390 - 110 = 5280\ mm^3$

$= 5280/1000$ (since $1\ mm^3 = 1/1000\ cm^3$)

$= 5.28\ cm^3$

Q.14: Meera has a piece of canvas whose area is 551 m^2. She uses it to have a conical tent made, with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting, amounts to approximately 1 m^2, find the volume of the tent that can be made with it.

Solution:

Given,

Area of the canvas = 551 m^2

Area of the canvas lost in wastage = 1 m^2

Thus, the area of canvas available for making the tent = $(551 – 1)\ m^2 = 550\ m^2$

Now, the surface area of the tent = 550 m^2

The required base radius of the conical tent = 7 m

Curved surface area of tent = 550 m^2

That means,

$\pi rl = 550$

$(22/7) \times 7 \times l = 550$

$l = 550/22$

$l = 25$ m

Now, $l^2 = h^2 + r^2$

$h^2 = l^2 – r^2 = (25)^2 – (7)^2 = 625 – 49 = 576$

$h = 24$ m

So, the volume of the conical tent = $(1/3)\pi r^2h$

$= (1/3) \times (22/7) \times 7 \times 7 \times 24$

$= 1232\ m^3$

Q.15: A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

Solution: Given,

Diameter of capsule = 3.5 mm

Radius of capsule = (r) = 3.5/2 = 1.75 mm

Volume of spherical capsule = $(4/3)\pi r^3$

$= (4/3) \times (22/7) \times 1.75 \times 1.75 \times 1.75$

$= 22.458\ mm^3$

Therefore, the volume of the capsule is 22.46 mm^3 approx.

Q.16: Calculate the amount of ice-cream that can be put into a cone with a base radius of 3.5 cm and height 12 cm.

Solution:

Given,

Base radius = r = 3.5 cm

Height = h = 12 cm

The amount of ice-cream that can be put into a cone = Volume of a cone

$= (1/3)\pi r^2 h$

$= (1/3) \times (22/7) \times 3.5 \times 3.5 \times 12$

$= 154\ cm^3$

Q.17: A spherical ball is divided into two halves. Given that the curved surface area of each half is 56.57 cm, what will be the volume of the spherical ball?

Solution:

Given,

Curved surface area of of half of the spherical ball = 56.57 cm^2

$(1/2)\ 4\pi r^2 = 56.57$

$2 \times 3.14 \times r^2 = 56.57$

$r^2 = 56.57/6.28$

$r^2 = 9$ (approx)

r = 3 cm

Now,

Volume of spherical ball = $(4/3)\pi r^3$

$= (4/3) \times 3.14 \times 3 \times 3 \times 3$

$= 113.04\ cm^3$