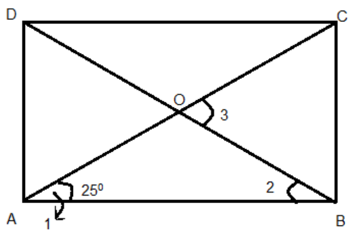
Chapter 8: Quadrilaterals
Exercise 8.1 (MCQ)

Question 1: Question 1:
Three angles of a quadrilateral are 75°, 90° and 75°, then the fourth angle is
(a) 90°
(b) **95°**
(c) **105°**
(d) **120°**

Answer: **(d)** Given, ∠A = 75°, ∠B = 90° and ∠C = 75°
We know that the sum of all the1 angles of a quadrilateral is 360°.
∠A+ ∠B + ∠C + ∠D = 360°
=> 75° + 90°+ 75°+ ∠D = 360°
∠D = 360° – (75° + 90° + 75°)
= 360° - 240° = 120°
Hence, the fourth angle of a quadrilateral is 120°.

Question 2:
A diagonal of a rectangle is inclined to one side of the rectangle at 25°. The acute angle between the diagonals is
(a) 55°
(b) **50°**
(c) **40°**
(d) **25**°
Answer: (b)
We know that the diagonals of a rectangle are equal in length.



AC = BD
or, $\frac{1}{2}$AC = $\frac{1}{2}$BD [Dividing both sides by 2]
or, OA = OB [O is the mid-point of AC and BD]
∠2 = ∠1 = 25^0 [Angle opposite to equal sides are equal]

hence, ∠3 = ∠1 + ∠2 = 25^0 + 25^0 = 50^0

Question 3:
ABCD is a rhombus such that ∠ACB = 40°, then ∠ADB is

(a) 40°
(b) **45°**
(c) **50°**
(d) **60°**
Solution:

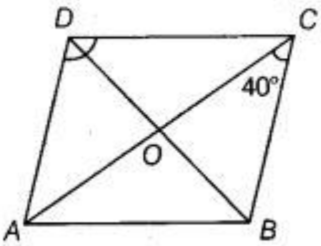
(c) Given, *ABCD* is a rhombus such that $\angle ACB = 40° \Rightarrow \angle OCB = 40°$

Since, $AD \parallel BC$

$\angle DAC = \angle BCA = 40°$ [alternate interior angles]

Also, $\angle AOD = 90°$

[diagonals of a rhombus are perpendicular to each other]



We know that, sum of all angles of a triangle *ADO* is 180°.

$\therefore \quad \angle ADO + \angle DOA + \angle OAD = 180°$

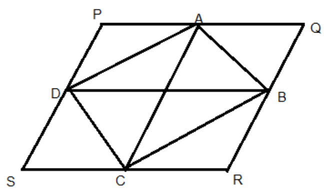
$\therefore \quad \angle ADO = 180° - (40° + 90°)$

$= 180° - 130° = 50°$

$\Rightarrow \quad \angle ADB = 50°$

Question 4:
The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rectangle, if
(a) PQRS is a rectangle
(b) PQRS is a parallelogram
(c) diagonals of PQRS are perpendicular
(d) diagonals of PQRS are equal

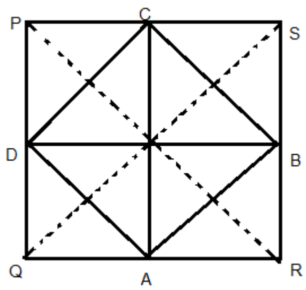
Answer: **(c)** Since the quadrilateral ABCD formed by joining the mid-points of quadrilateral PQRS is a rectangle.
AC = BD [since diagonals of a rectangle are equal]
or, PQ = QR
Thus, quadrilateral PQRS is a rhombus.



Hence, diagonals of PQRS i.e., PR and QS are perpendiculars [Since diagonals of a rhombus are perpendicular to each other]

Question 5:
The quadrilateral formed by joining the mid-points of the side of quadrilateral PQRS, taken in order, is a rhombus, if
(a) PQRS is a rhombus
(b) PQRS is a parallelogram
(c) diagonals of PQRS are perpendicular
(d) diagonals of PQRS are equal

Answer: **(d)** Given, the quadrilateral ABCD is a rhombus. So, sides AB, BC, CD and AD are equal.

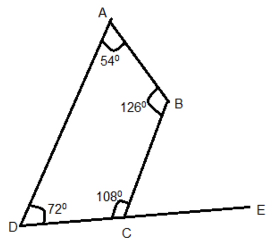


Now, in triangle PQS, we have
D and C are the mid-points of PQ and PS.
So, DC = $\frac{1}{2}$QS [by mid-point theorem] ……(1)
Similarly, in Triangle PSR, BC = $\frac{1}{2}$PR ………..(2)
As, BC = DC [ABCD, is a rhombus]
Hence, $\frac{1}{2}$QS = $\frac{1}{2}$PR
or, QS = PR
Diagonals are equal.

Question 6:
If angles A, B, C and D of the quadrilateral ABCD, taken in order are in the ratio 3:7:6:4, then ABCD is a
(a) rhombus
(b) parallelogram
(c) trapezium
(d) kite

Answer: **(c)** Given, the ratio of angles of quadrilateral ABCD is 3: 7 : 6: 4.
Let angles of quadrilateral ABCD be 3x, 7x, 6x and 4x, respectively. We know that the sum of all angles of a quadrilateral is 360°.
3x + 7x + 6x + 4x = 360°
or, 20x = 360°
or, x=360°/20° = 18°

Angles of the quad. are
$\angle A = 3 \times 18 = 54^0$
$\angle B = 7 \times 18 = 126^0$
$\angle C = 6 \times 18 = 108^0$
$\angle D = 4 \times 18 = 72^0$



from figure, $\angle BCE = 180^0 - \angle BCD$
$\angle BCE = 180^0 – 108^0 = 72^0$

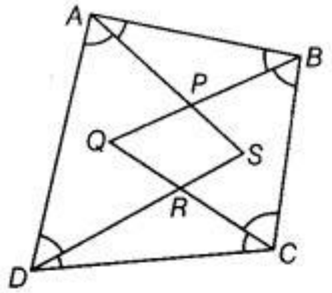
As the corresponding angles are equal, BC ∥AD
now, the sum of interior angles,
$\angle A + \angle B = 126^0 + 54^0 = 180^0$
$\angle C + \angle D = 108^0 + 72^0 = 180^0$

Hence, ABCD is a trapezium

Question 7:
If bisectors of ∠A and ∠B of a quadrilateral ABCD intersect each other at P, of ∠B and ∠C at Q, of ∠C and ∠D at R and of ∠D and ∠A at S, then PQRS is a
(a) rectangle
(b) rhombus
(c) parallelogram
(d) quadrilateral whose opposite angles are supplementary

Answer:

(d) Given, $ABCD$ is a quadrilateral and all angles bisectors form a quadrilateral $PQRS$.



We know that, sum of all angles in a quadrilateral is 360°.

$$\therefore \quad \angle A + \angle B + \angle C + \angle D = 360^\circ$$

On dividing both sides by 2, we get

$$\frac{1}{2}(\angle A + \angle B + \angle C + \angle D) = \frac{360^\circ}{2}$$

$$\Rightarrow \quad \angle PAB + \angle PBA + \angle RCD + \angle RDC = 180^\circ \quad \ldots(i)$$

[since, AP and PB are the bisectors of $\angle A$ and $\angle B$ respectively also RC and RD are the bisectors of $\angle C$ and $\angle D$ respectively]

Now, in ΔAPB,

$$\angle PAB + \angle ABP + \angle BPA = 180^\circ$$

[by angle sum property of a triangle]

$$\Rightarrow \quad \angle PAB + \angle ABP = 180^\circ - \angle BPA \quad \ldots(ii)$$

Similarly in ΔRDC,

$$\angle RDC + \angle DCR + \angle CRD = 180^\circ$$

[by angle sum property of a triangle]

$$\Rightarrow \quad \angle RDC + \angle DCR = 180^\circ - \angle CRD \quad \ldots(iii)$$

On substituting the value Eqs. (ii) and (iii) in Eq. (i), we get

$$180^\circ - \angle BPA + 180^\circ - \angle DRC = 180^\circ$$

$$\Rightarrow \quad \angle BPA + \angle DRC = 180^\circ$$

$$\Rightarrow \quad \angle SPQ + \angle SRQ = 180^\circ$$

[$\because \angle BPA = \angle SPQ$ and $\angle DRC = \angle SRQ$ vertically opposite angles]

Hence, $PQRS$ is a quadrilateral whose opposite angles are supplementary.

Question 8:

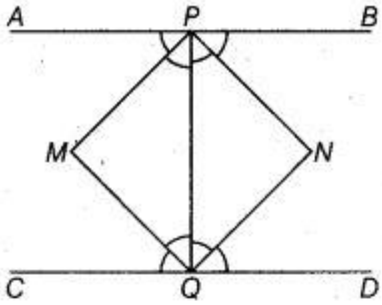
If APB and CQD are two parallel lines, then the bisectors of the angles APQ, BPQ, CQP and PQD form

(a) a square (b) a rhombus

(c) a rectangle (d) any other parallelogram

Answer:

(c) Given, *APB* and *CQD* are two parallel lines.



Let the bisectors of angles *APQ* and *CQP* meet at a point *M* and bisectors of angles *BPQ* and *PQD* meet at a point *N*.

Join *PM*, *MQ*, *QN* and *NP*.

Since, $APB \parallel CQD$

Then, $\angle APQ = \angle PQD$ [alternate interior angles]

$\Rightarrow$ $\angle MPQ = 2\angle NQP$

[since, *PM* and *NQ* are the angle bisectors of $\angle APQ$ and $\angle DQP$ respectively]

$\Rightarrow$ $\angle MPQ = \angle NQP$ [dividing both sides by 2]

[since, alternate interior angles are equal.]

$\therefore$ $PM \parallel QN$

Similarly, $\angle BPQ = \angle CQP$ [alternate interior angles]

$\therefore$ $PN \parallel QM$

So, quadrilateral *PMQN* is a parallelogram.

$\because$ $\angle CQD = 180^\circ$ [since, *CQD* is a line]

$\Rightarrow$ $\angle CQP + \angle DQP = 180^\circ$

$\Rightarrow$ $2\angle MQP + 2\angle NQP = 180^\circ$

[since, *MQ* and *NQ* are the bisectors of the angles *CQP* and *DQP*]

$\Rightarrow$ $2(\angle MQP + \angle NQP) = 180^\circ$

$\Rightarrow$ $\angle MQN = 90^\circ$

Hence, *PMQN* is a rectangle.

Question 9:

The figure obtained by joining the mid-points of the sides of a rhombus, taken in order, is

(a) a rhombus

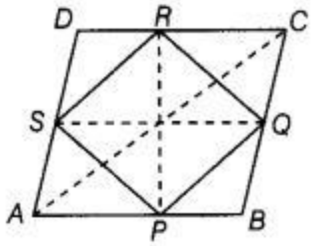
(b) a rectangle

(c) a square

(d) any parallelogram

Solution:

(b) Let $ABCD$ be a rhombus in which P, Q, R and S are the mid-points of sides AB, BC, CD and DA, respectively.



Join AC, PR and SQ

In ΔABC, P is the mid-point of AB and Q is the mid-point of BC.

$\Rightarrow$ $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ [by using mid-point theorem]...(i)

Similarly, in ΔDAC,

$SR \parallel AC$ and $SR = \frac{1}{2}AC$ ...(ii)

From Eqs. (i) and (ii),

$PQ \parallel SR$ and $PQ = SR$

So, $PQRS$ is a parallelogram.

Also, $ABQS$ is a parallelogram.

$\Rightarrow$ $AB = SQ$ ...(iii)

[opposite sides of a parallelogram are equal]

Similarly, $PBCR$ is a parallelogram.

$\Rightarrow$ $BC = PR$ [opposite sides of a parallelogram are equal]

$\Rightarrow$ $AB = PR$ [$\because BC = AB$ sides of a rhombus]

$\Rightarrow$ $SQ = PR$ [from Eq. (iii)]

So, the diagonals of a parallelogram are equal.

Hence, $PQRS$ is a rectangle.

Question 10:

D and E are the mid-points of the sides AB and AC of ΔABC and 0 is any point on side BC. 0 is joined to A. If P and Q are the mid-points of OB and OC respectively, then DEQP is

(a) a square

(b) a rectangle

(c) a rhombus

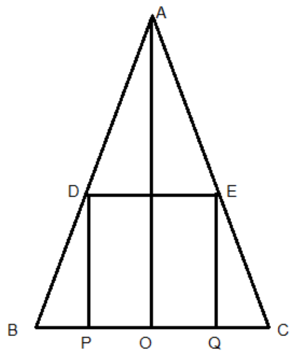
(d) a parallelogram

Answer: **(d)** In ΔABC, D and E are the mid-points of sides AB and AC, respectively.

By mid-point theorem,

DE || BC ……………………………………………(1)

DE = 1/2 BC



Then DE = $\frac{1}{2}$ [BP + PO + OQ + QC]
DE = $\frac{1}{2}$ [2PO + 2OQ]   [since, P and Q are the mid-points of OB and OC respectively]
or, DE = PO + OQ
or, DE = PQ

Now in triangle AOC, Q and E are the mid-points of OC and AC respectively.
Thus, EQ ∥AO and EQ = $\frac{1}{2}$AO   [By midpoints theorem] ……………..(3)
Similarly, in triangle ABO, PD ∥AO and PD = $\frac{1}{2}$AO   [by mid-point theorem]……(4)

From eq(3) and (4), EQ ∥PD and EQ = PD
From eq(1) and (2) DE ∥BC and DE ∥PQ
Hence, DEQP is a parallelogram.

Question 11:
The figure formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order, is a square only, if
(a) ABCD is a rhombus
(b) diagonals of ABCD are equal
(c) diagonals of ABCD are equal and perpendicular
(d) diagonals of ABCD are perpendicular

Answer:

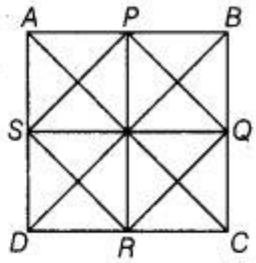
(c) Given, $ABCD$ is a quadrilateral and P, Q, R and S are the mid-points of sides of AB, BC, CD and DA, respectively. Then, $PQRS$ is a square.

$\therefore \quad PQ = QR = RS = PS \quad \dots(i)$

and $\quad PR = SQ$

But $\quad PR = BC$ and $SQ = AB$

$\therefore \quad AB = BC$



Thus, all the sides of quadrilateral $ABCD$ are equal.

Hence, quadrilateral $ABCD$ is either a square or a rhombus.

Now, in ΔADB, use mid-point theorem

$$SP \parallel DB$$

and $\quad SP = \frac{1}{2} DB \quad \dots(ii)$

Similarly in ΔABC (by mid-point theorem) $PQ \parallel AC$ and $PQ = \frac{1}{2} AC \quad \dots(iii)$

From Eq. (i), $\quad PS = PQ$

$\Rightarrow \quad \frac{1}{2} DB = \frac{1}{2} AC \quad$ [from Eqs. (ii) and (iii)]

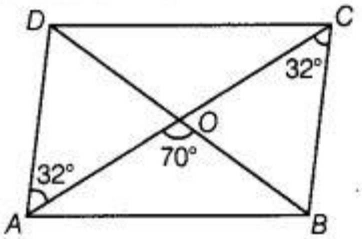
$\Rightarrow \quad DB = AC$

Thus, diagonals of $ABCD$ are equal and therefore quadrilateral $ABCD$ is a square not rhombus. So, diagonals of quadrilateral are also perpendicular.

Question 12:
The diagonals AC and BD of a parallelogram ABCD intersect each other at point 0. If ∠DAC = 32° and ∠AOB = 70°, then ∠DBC is equal to
(a) 24°
(b) 86°
(c) 38°
(d) 32°

Answer:

(c) Given, $\angle AOB = 70°$ and $\angle DAC = 32°$



$\therefore \quad \angle ACB = 32° \quad$ [$AD \parallel BC$ and AC is transversal]

Now, $\quad \angle AOB + \angle BOC = 180° \quad$ [linear pair axiom]

$\Rightarrow \quad \angle BOC = 180° - \angle AOB = 180° - 70° = 110°$

Now, in ΔBOC, we have

$\angle BOC + \angle BCO + \angle OBC = 180°$ [by angle sum property of a triangle]

$\Rightarrow \quad 110° + 32° + \angle OBC = 180° \quad$ [$\because \angle BCO = \angle ACB = 32°$]

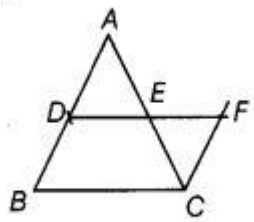
$\Rightarrow \quad \angle OBC = 180° - (110° + 32°) = 38°$

$\therefore \quad \angle DBC = \angle OBC = 38°$

Question 13:
Which of the following is not true for a parallelogram?
(a) Opposite sides are equal
(b) Opposite angles are equal
(c) Opposite angles are bisected by the diagonals
(d) Diagonals bisect each other

Answer: **(c)** We know that, in a parallelogram, opposite sides are equal, opposite angles are equal, opposite angles are not bisected by the diagonals and diagonals bisect each other.

Question 14:
D and E are the mid-points of the sides AB and AC, respectively, of ΔABC. DE is produced to F. To prove that CF is equal and parallel to DA, we need additional information that is
(a) ∠DAE = ∠EFC
(b) AE = EF
(c) DE = EF
(d) ∠ADE = ∠ECF
Solution:
(c) In ΔADE and ΔCFE, suppose $DE = EF$



Now,	$AE = CE$	[since, E is the mid-point of AC]
Suppose	$DE = EF$	
and	$\angle AED = \angle FEC$	[vertically opposite angles]
∴	$\Delta ADE \cong \Delta CFE$	[by SAS congruence rule]
∴	$AD = CF$	[by CPCT rule]
and	$\angle ADE = \angle CFE$	[by CPCT]
Hence,	$AD \parallel CF$	[since, alternate interior angles are equal]

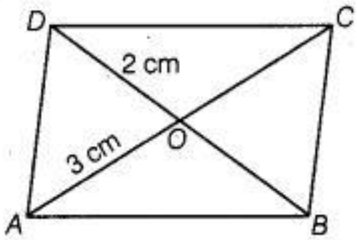
Therefore, we need an additional information which is $DE = EF$

Exercise 8.2: Very Short Answer Type Questions

Question 1:
Diagonals AC and BD of a parallelogram ABCD intersect each other at O. If OA = 3 cm and OD = 2 cm, determine the lengths of AC and BD.

Solution:

Given, $ABCD$ is a parallelogram $OA = 3$ cm and $OD = 2$ cm



We know that, diagonals of a parallelogram bisect each other.

$\therefore$ Diagonal $AC = 2\,OA = 6$ cm $[\because AO = OC]$

and Diagonal $BD = 2\,OD = 4$ cm $[\because BO = OD]$

Hence, the length of the diagonals AC and BD are 6 cm and 4 cm, respectively.

Question 2:
Diagonals of a parallelogram are perpendicular to each other. Is this statement true? Give a reason for your answer.

Answer: No, diagonals of a parallelogram are not perpendicular to each other, because they only bisect each other.

Question 3:
Can the angles 110°, 80°, 70° and 95° be the angles of a quadrilateral? Why or why not?
Answer: No, we know that the sum of all angles of a quadrilateral is 360°.
Here, sum of the angles = 110°+ 80° + 70° + 95° = 355° ≠ 360°
So, these angles cannot be the angles of a quadrilateral.

Question 4: In quadrilateral ABCD, ∠A + ∠D = 180°. What special name can be given to this quadrilateral?
Answer:
It is a trapezium because the sum of the interior angles is 180°.

Question 5: All the angles of a quadrilateral are equal. What special name is given to this quadrilateral?

Answer: We know that the sum of all angles in a quadrilateral is 360°.
If ABCD is a quadrilateral,
∠A+ ∠B+ ∠C + ∠D = 360° ………………………..(1)
But it is given all angles are equal.
∠A = ∠B = ∠C = ∠D From Eq.(1)
∠A + ∠A + ∠A + ∠A = 360°
or, 4 ∠A = 360°
∠A = 90°
So, all angles of a quadrilateral are 90°.
Hence, the given quadrilateral is a rectangle.

Question 6:
The diagonals of a rectangle are equal and perpendicular. Is this statement true? Give a reason for your answer.

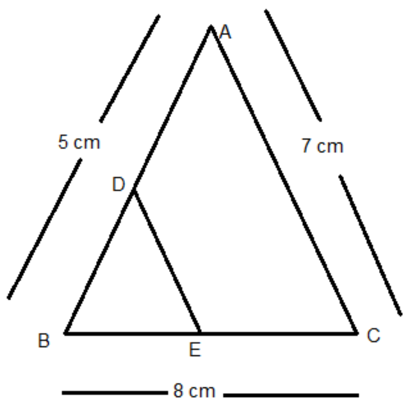
Answer: No, the diagonals of a rectangle are equal but need not be perpendicular.

Question 7:
Can all the four angles of a quadrilateral be obtuse? Give a reason for your answer.

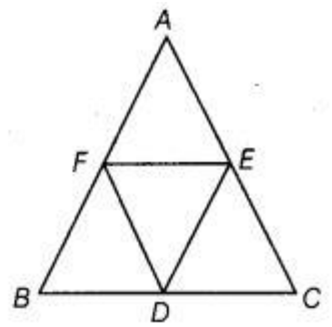
Answer: No, all the four angles of a quadrilateral cannot be obtuse. As the sum of the angles of a quadrilateral is 360°, they may have a maximum of three obtuse angles.

Question 8:
In ΔABC, AB = 5 cm, BC = 8 cm and CA = 7 cm. If D and E are respectively the mid-points of AB and BC, determine the length of DE.

Answer: In ΔABC, we have AB = 5cm, BC = 8 cm and CA = 7 cm. Since, D and E are the mid-points of AB and BC, respectively.
By mid-point theorem, DE || AC
and DE = $\frac{1}{2}$AC = $\frac{7}{2}$ = 3.5cm

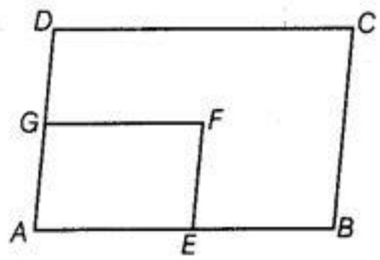


Question 9:
In the figure, it is given that BDEF and FDCE are parallelograms. Can you say that BD = CD? Why or why not?



Answer: Yes, in the given figure, BDEF is a parallelogram
∴ BD || EF and BD = EF ……………………………..(1)
Also, FDCE is a parallelogram.
∴ CD||EF
and CD = EF …………………………………..…(2)
From Eqs. (1) and (2), BD = CD = EF

Question 10:
In the figure, ABCD and AEFG are two parallelograms. If ∠C = 55°, then determine ∠F.



Answer:
We have, ABCD and AEFG are two parallelograms and ∠C = 55°. Since ABCD is a parallelogram, then opposite angles of a parallelogram are equal.
∠A = ∠C = 55° ...(i)
Also, AEFG is a parallelogram.
∴ ∠A=∠F = 55° [from Eq. (i)]

Question 11:
Can all the angles of a quadrilateral be acute? Give a reason for your answer.

Answer: No, all the angles of a quadrilateral cannot be acute. As the sum of the angles of a quadrilateral is 360°. So, a maximum of three acute angles will be possible.

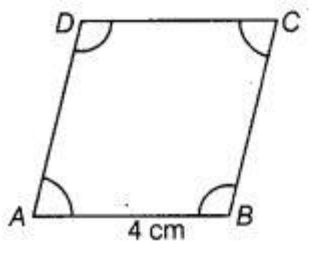
Question 12:
Can all the angles of a quadrilateral be right angles? Give a reason for your answer.

Answer: Yes, all the angles of a quadrilateral can be right angles. In this case, the quadrilateral becomes a rectangle or square.

Question 13:
Diagonals of a quadrilateral ABCD bisect each other. If ∠A= 35°, determine ∠B.
Answer: Since diagonals of a quadrilateral bisect each other, so it is a parallelogram.
Therefore, the sum of interior angles between two parallel lines is 180° i.e.,
∠A+∠B = 180°
or, ∠B = 180° – ∠A = 180°- 35° [∴ ∠A = 35°, given]
or, ∠B = 145°

Question 14:
Opposite angles of a quadrilateral ABCD are equal. If AB = 4 cm, determine CD.

Answer: Given, the opposite angles of a quadrilateral are equal. So, ABCD is a parallelogram and we know that in a parallelogram opposite sides are also equal.
∴ CD = AB = 4cm



Exercise 8.3: Short Answer Type Questions

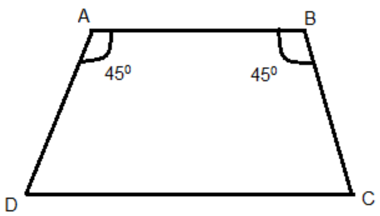
Question 1:
One angle of a quadrilateral is 108° and the remaining three angles are equal. Find each of the three equal angles.
Thinking Process
The sum of all the angles In a quadrilateral is 360°, use this result and simplify it.

Answer: Let each of the three equal angles be x°.
Now, the sum of angles of a quadrilateral = 360°
or, 108° + x° + x° + x° = 360° => 3x° = 360° – 108°
x° = 252°/3
or, x° = 84°
hence, x° = 84°
Hence, each of the three equal angles is 84°.

Question 2:
ABCD is a trapezium in which AB || DC and ∠A = ∠B = 45°. Find angles C and D of the trapezium.

Answer: Given, ABCD is a trapezium and whose parallel sides in the figure are AB and DC.

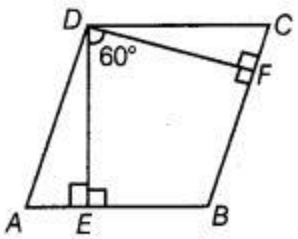
Since AB || CD and BC is transversal, then the sum of two co-interior angles is 180°.



$\angle B + \angle C = 180^0$
or, $\angle C = 180^0 - \angle B = 180^0 – 45^0$ [$\angle B = 45^0$, Given]
$\angle C = 135^0$
Similarly, $\angle A + \angle D = 180^0$ [Sum of the co-interior angles ins 180^0]
or, $\angle D = 180^0 - 45^0$
or, $\angle D = 135^0$

Question 3:
The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60°. Find the angles of the parallelogram.

Answer: Let the parallelogram be ABCD, in which ∠ADC and ∠ABC are obtuse angles. Now, DE and DF are two altitudes of a parallelogram and the angle between them is 60°.



Now, *BEDF* is a quadrilateral, in which $\angle BED = \angle BFD = 90^\circ$

$\therefore \quad \angle FBE = 360^\circ - (\angle FDE + \angle BED + \angle BFD)$
$= 360^\circ - (60^\circ + 90^\circ + 90^\circ)$
$= 360^\circ - 240^\circ = 120^\circ$

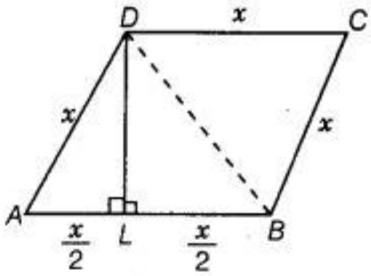
Since, *ABCD* is a parallelogram.

$\therefore \quad \angle ADC = 120^\circ$
Now, $\quad \angle A + \angle B = 180^\circ$ [sum of two cointerior angles is 180°]
$\therefore \quad \angle A = 180^\circ - \angle B$
$5 = 180^\circ - 120^\circ$ [$\because \angle FBE = \angle B$]
$\Rightarrow \quad \angle A = 60^\circ$
Also, $\quad \angle C = \angle A = 60^\circ$
Hence, angles of the parallelogram are 60°, 120°, 60° and 120°, respectively.

Question 4:
ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.
Answer:

Let sides of a rhombus be $AB = BC = CD = DA = x$

Now, join DB.



In ΔALD and ΔBLD, $\angle DLA = \angle DLB = 90°$ [since, DL is a perpendicular bisector of AB]

$AL = BL = \frac{x}{2}$

and $DL = DL$ [common side]

$\therefore$ $\Delta ALD \cong \Delta BLD$ [by SAS congruence rule]

$AD = BD$ [by CPCT]

Now, in ΔADB, $AD = AB = DB = x$

Then, ΔADB is an equilateral triangle.

$\therefore$ $\angle A = \angle ADB = \angle ABD = 60°$

Similarly, ΔDBC is an equilateral triangle.

$\therefore$ $\angle C = \angle BDC = \angle DBC = 60°$

Also, $\angle A = \angle C$

$\therefore$ $\angle D = \angle B = 180° - 60° = 120°$ [since, sum of interior angles is 180°]

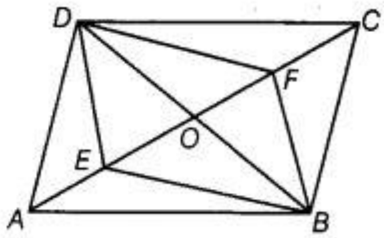
Question 5:
E and F are points on diagonal AC of a parallelogram ABCD such that AE = CF. Show that BFDE is a parallelogram.

Answer:

Given $ABCD$ is a parallelogram and $AE = CF$

To show $OE = OF$

Construction Join BD, meet AC at point O.



Proof Since, diagonals of a parallelogram bisect each other.

$\therefore$ $OA = OC$

and $OD = OB$

Now, $OA = OC$

and $AE = CF$ [given]

$\Rightarrow$ $OA - AE = OC - CF$

$\Rightarrow$ $OE = OF$

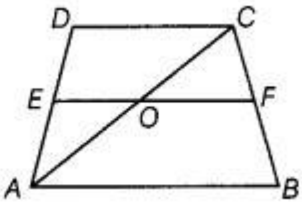
Thus, $BFDE$ is a quadrilateral whose diagonals bisect each other.

Hence, $BFDE$ is a parallelogram. **Hence proved.**

Question 6:
E is the mid-point of the side AD of the trapezium ABCD with AB || DC. A line

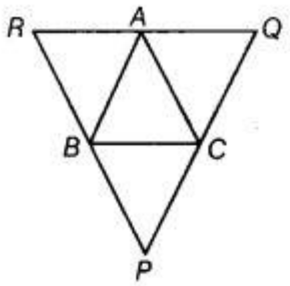
through E drawn parallel to AB intersects BC at F. Show is the mid-point of BC.
Thinking Process
Use the mid-point theorem i.e., the line segment joining the mid-points of two sides of a triangle is parallel to the third side and half of it. Further shown the required result.

Answer: Given ABCD is a trapezium in which AB || DC and EF||AB|| CD.
Construction Join, the diagonal AC which intersects EF at O.
To show F is the mid-point of BC.



Proof Now, in ΔADC, E is the mid-point of AD and OE || CD. Thus, by mid-point theorem, O is the mid-point of AC.
Now, in ΔCBA, 0 is the mid-point of AC and OF || AB.
So, by mid-point theorem, F is the mid-point of BC.

Question 7:
Through A, B and C lines RQ, PR and QP have been drawn, respectively parallel to sides BC, CA and AB of a ΔABC as shown in the figure. Show that BC = ½ QR

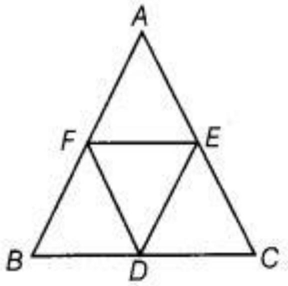


Answer:
Given In ΔABC, PQ || AB and PR || AC and RQ || BC.
To show BC = ½ QR
Proof In quadrilateral BCAR, BR || CA and BC|| RA
So, quadrilateral, BCAR is a parallelogram.
BC = AR ………………………………………………….(1)
Now, in quadrilateral BCQA, BC || AQ
and AB||QC
So, quadrilateral BCQA is a parallelogram,
BC = AQ ………………………………………………..(2)
On adding Eqs. (1) and (2), we get
2 BC = AR+ AQ
or, 2 BC = RQ
or, BC = ½ QR
Now, BEDF is a quadrilateral, in which ∠BED = ∠BFD = 90°

∠FSE = 360° – (∠FDE + ∠BED + ∠BFD) = 360° – (60° + 90° + 90°)
= 360°- 240° =120°

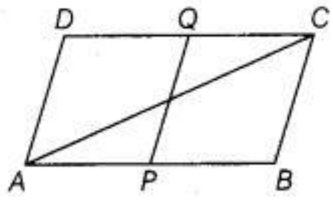
Question 8:
D, E and F are the mid-points of the sides BC, CA and AB, respectively of an equilateral ΔABC. Show that ΔDEF is also an equilateral triangle.

Answer: Given In equilateral ΔABC, D, E and F are the mid-points of sides BC, CA and AB, respectively.
To show ΔDEF is an equilateral triangle.



Proof Since in ΔABC, E and F are the mid-points of AC and AB respectively, then EF || BC and
EF =½ BC ………………………..(1)
DF || AC, DE || AB
DE = ½ AB and FD = ½ AC [by mid-point theorem]………………….(2)
since ΔABC is an equilateral triangle
AB = BC = CA
or, ½ AB = ½ BC = ½ CA [dividing by 2]
or, DE = EF = FD [from Eqs. (1) and (2)]
Thus, all sides of ADEF are equal.
Hence, ΔDEF is an equilateral triangle.
Hence proved.

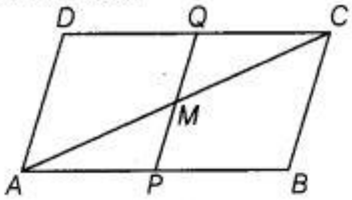
Question 9:
Points P and Q have been taken on opposite sides AB and CD, respectively of a parallelogram ABCD such that AP = CQ. Show that AC and PQ bisect each other.



Solution:

Given $ABCD$ is a parallelogram and $AP = CQ$

To show AC and PQ bisect each other.



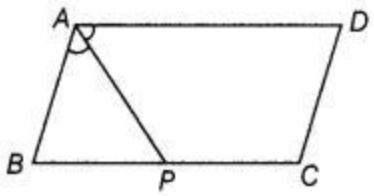
Proof In ΔAMP and ΔCMQ,

	$\angle MAP = \angle MCQ$	[alternate interior angles]
	$AP = CQ$	[given]
and	$\angle APM = \angle CQM$	[alternate interior angles]
$\therefore$	$\Delta AMP \cong \Delta CMQ$	[by ASA congruence rule]
$\Rightarrow$	$AM = CM$	[by CPCT rule]
and	$PM = MQ$	[by CPCT rule]

Hence, AC and PQ bisect each other. **Hence proved.**

Question 10:

In the figure, P is the mid-point of side BC of a parallelogram ABCD such that ∠BAP = ∠DAP. Prove that AD = 2 CD.



answer:

Given In a parallelogram $ABCD$, P is a mid-point of BC such that $\angle BAP = \angle DAP$.

To prove $AD = 2CD$

Proof Since, $ABCD$ is a parallelogram.

So, $AD \parallel BC$ and AB is transversal, then

$$\angle A + \angle B = 180^\circ \quad \text{[sum of cointerior angles is } 180^\circ\text{]}$$

$\Rightarrow$ $$\angle B = 180^\circ - \angle A \quad \text{...(i)}$$

In ΔABP, $$\angle PAB + \angle B + \angle BPA = 180^\circ \quad \text{[by angle sum property of a triangle]}$$

$\Rightarrow$ $$\frac{1}{2}\angle A + 180^\circ - \angle A + \angle BPA = 180^\circ \quad \text{[from Eq. (i)]}$$

$\Rightarrow$ $$\angle BPA - \frac{\angle A}{2} = 0$$

$\Rightarrow$ $$\angle BPA = \frac{\angle A}{2} \quad \text{...(ii)}$$

$\Rightarrow$ $$\angle BPA = \angle BAP$$

$\Rightarrow$ $$AB = BP \quad \text{[opposite sides of equal angles are equal]}$$

On multiplying both sides by 2, we get

$$2AB = 2BP$$

$\Rightarrow$ $$2AB = BC \quad \text{[since } P \text{ is the mid-point of } BC\text{]}$$

$\Rightarrow$ $$2CD = AD$$

[since, $ABCD$ is a parallelogram, then $AB = CD$ and $BC = AD$]

Exercise 8.4: Long Answer Type Questions

Question 1:
A square is inscribed in an isosceles right triangle so that the square and the triangle have one angle common. Show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.

Answer: Given In isosceles triangle ABC, a square ΔDEF is inscribed.
To prove CE = BE
Proof In an isosceles ΔABC, ∠A = 90°
and AB = AC ………………………………………..(1)
Since ΔDEF is a square.
AD = AF [all sides of the square are equal] …………………………..(2)
On subtracting Eq. (2) from Eq. (1), we get
AB – AD = AC- AF

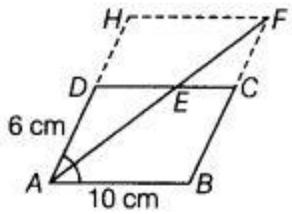
BD = CF ……………………………………(3)
Now in triangle CFE and BDE,

BD = CF [from eq (3)]
DE = EF [sides of an sq.]
and ∠CFE = ∠EDB [each 90^0]
ΔCFE ≅ ΔBDE [SAS]
CE = BE [CPCT]
Hence, vertex E of the square bisects the hypotenuse BC.

Question 2:
In a parallelogram ABCD, AB = 10 cm and AD = 6 cm. The bisector of ∠A meets DC in E. AE and BC produced meet at F. Find the length of CF.

Answer: Given, a parallelogram ABCD in which AB = 10 cm and AD = 6 cm.
Now, draw a bisector of ∠A meets DC in E and produce it to F and produce BC to

meet at F.



Also, produce AD to H and join HF, so that $ABFH$, is a parallelogram.

Since, $HF \parallel AB$

$\therefore$ $\angle AFH = \angle FAB$ [alternate interior angles]

$\angle HAF = \angle FAB$ [since, AF is the bisector of $\angle A$]

$\Rightarrow$ $\angle HAF = \angle AFH$ [from Eq. (i)]

$\Rightarrow$ $HF = AH$ [sides opposite to equal angles are equal]

But $HF = AB = 10$ cm

$\therefore$ $AH = HF = 10$ cm

$\Rightarrow$ $AD + DH = 10$ cm

$\Rightarrow$ $DH = (10 - 6)$ cm

$\therefore$ $DH = 4$ cm

Since, $CFHD$ is a parallelogram.

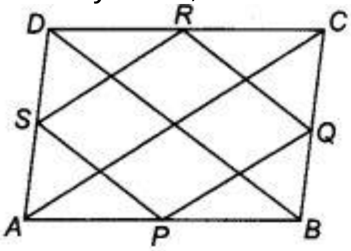
Therefore, opposite sides are equal.

$\therefore$ $DH = CF = 4$ cm

Question 3:
P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which AC = BD. Prove that PQRS is a rhombus,
Thinking Process
Firstly, use the mid-point theorem in various triangles of a quadrilateral. Further show that the line segments formed by joining the mid-points are equal, which prove the required quadrilateral.

Answer: Given In a quadrilateral ABCD, P, Q, R and S are the mid-points of sides

AB, BC, CD and DA, respectively. Also, AC = BD To prove PQRS is a rhombus.



Proof In ΔADC, S and R are the mid-points of AD and DC respectively. Then, by mid-point theorem.

$$SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \ldots(i)$$

In ΔABC, P and Q are the mid-points of AB and BC respectively. Then, by mid-point theorem

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \ldots(ii)$$

From Eqs. (i) and (ii), $$SR = PQ = \frac{1}{2} AC \quad \ldots(iii)$$

Similarly, in ΔBCD, $$RQ \parallel BD \text{ and } RQ = \frac{1}{2} BD \quad \ldots(iv)$$

And in ΔBAD, $$SP \parallel BD \text{ and } SP = \frac{1}{2} BD \quad \ldots(v)$$

From Eqs. (iv) and (v), $$SP = RQ = \frac{1}{2} BD = \frac{1}{2} AC \quad [\text{given, } AC = BD] \ldots(vi)$$

From Eqs. (iii) and (vi), $$SR = PQ = SP = RQ$$

It shows that all sides of a quadrilateral $PQRS$ are equal.

Hence, $PQRS$ is a rhombus. **Hence proved.**

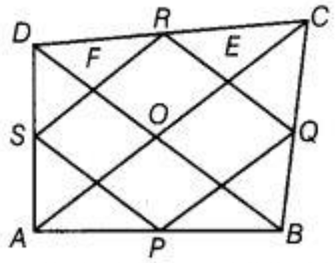
Question 4:
P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD such that AC ⊥ BD. Prove that PQRS is a rectangle.

Answer: Given In quadrilateral ABCD, P, O, S and S are the mid-points of the sides AB, BC, CD and DA, respectively.
Also, AC ⊥ BD
To prove PQRS is a rectangle.
Proof Since, AC ⊥ BD.

∠COD = ∠AOD= ∠AOB= ∠COB = 90°

In ΔADC, S and R are the mid-points of AD and DC respectively, then by mid-point theorem

$$SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \dots(i)$$



In ΔABC, P and Q are the mid-points of AB and BC respectively, then by mid-point theorem

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \dots(ii)$$

From Eqs. (i) and (ii), $PQ \parallel SR$ and $PQ = SR = \frac{1}{2} AC$ ...(iii)

Similarly, $SP \parallel RQ$ and $SP = RQ = \frac{1}{2} BD$ ...(iv)

Now, in quadrilateral $EOFR$, $OE \parallel FR$, $OF \parallel ER$

∴ $\angle EOF = \angle ERF = 90°$ [∵ $\angle COD = 90° \Rightarrow \angle EOF = 90°$] ...(v)

So, $PQRS$ is a rectangle. **Hence proved.**

Question 5:

P, Q, R and S are respectively the mid-points of sides AB, BC, CD and DA of quadrilateral ABCD in which AC = BD and AC ⊥ BD. Prove that PQRS is a square.

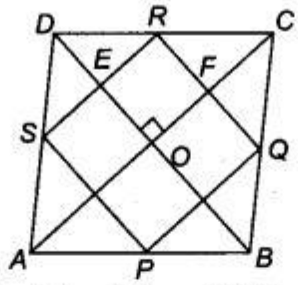
Answer: Given In quadrilateral ABCD, P, Q, R and S are the mid-points of the sides AB, BC, CD and DA, respectively.

Also, AC = BD and AC ⊥ BD.

To prove PQRS is a square.

Proof Now, in ΔADC, S and R are the mid-points of the sides AD and DC respectively, then by mid-point theorem,

$$SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \ldots(i)$$



In ΔABC, P and Q are the mid-points of AB and BC, then by mid-point theorem,

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \ldots(ii)$$

From Eqs. (i) and (ii), $PQ \parallel SR$ and $PQ = SR = \frac{1}{2} AC$ ...(iii)

Similarly, in ΔABD, by mid-point theorem,

$$SP \parallel BD \text{ and } SP = \frac{1}{2} BD = \frac{1}{2} AC \quad [\text{given, } AC = BD] \ldots(iv)$$

and ΔBCD, by mid-point theorem,

$$RQ \parallel BD \text{ and } RQ = \frac{1}{2} BD = \frac{1}{2} AC \quad [\text{given, } BD = AC] \ldots(v)$$

From Eqs. (iv) and (v),

$$SP = RQ = \frac{1}{2} AC \quad \ldots(vi)$$

From Eqs. (iii) and (vi),

$$PQ = SR = SP = RQ$$

Thus, all four sides are equal.

Now, in quadrilateral $OERF$, $OE \parallel FR$ and $OF \parallel ER$

$\therefore$ $\angle EOF = \angle ERF = 90°$

[$\because AC \perp DB \Rightarrow = \angle DOC = \angle EOF = 90°$ as opposite angles of a parallelogram]

$\therefore$ $\angle QRS = 90°$

Similarly, $\angle RQS = 90°$

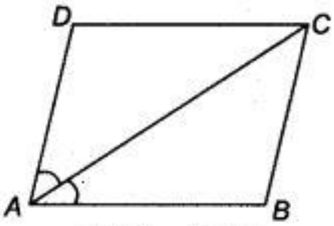
So, $PQRS$ is a square. **Hence proved.**

Question 6:

A diagonal of a parallelogram bisects one of its angles. Show that it is a rhombus.

Solution:

Given Let *ABCD* is a parallelogram and diagonal *AC* bisects the angle *A*.



$\therefore$ $\angle CAB = \angle CAD$ ...(i)

To show *ABCD* is a rhombus.

Proof Since, *ABCD* is a parallelogram, therefore $AB \parallel CD$ and *AC* is a transversal.

$\therefore$ $\angle CAB = \angle ACD$ [alternate interior angles]

Again, $AD \parallel BC$ and *AC* is a transversal.

$\therefore$ $\angle CAD = \angle ACB$ [alternate interior angles]

So, $\angle ACD = \angle ACB$ [$\because \angle CAB = \angle CAD$, given]...(ii)

Also, $\angle A = \angle C$ [opposite angles of parallelogram are equal]

$\Rightarrow$ $\frac{1}{2}\angle A = \frac{1}{2}\angle C$ [dividing both sides by 2]

$\Rightarrow$ $\angle DAC = \angle DCA$ [from Eqs. (i) and (ii)]

$\Rightarrow$ $CD = AD$

[sides opposite to the equal angles are equal]

But $AB = CD$ and $AD = BC$

[opposite sides of parallelogram are equal]

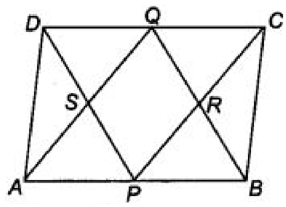
$\therefore$ $AB = BC = CD = AD$

Thus, all sides are equal. So, *ABCD* is a rhombus. **Hence proved.**

Question 7:

P and Q are the mid-points of the opposite sides AB and CD of a parallelogram ABCD. AQ intersects DP at S and BQ intersects CP at R. Show that PRQS is a parallelogram.

Answer: Given In a parallelogram ABCD, P and Q are the mid-points of AS and CD, respectively.



To show PRQS is a parallelogram.

Proof Since, ABCD is a parallelogram.

AB||CD

or, AP || QC

Also, AB = DC

$\frac{1}{2}AB = \frac{1}{2}DC$

or, AP = QC [since P and Q are mid-points of AB ad DC]

Now, AP || QC and AP = QC.

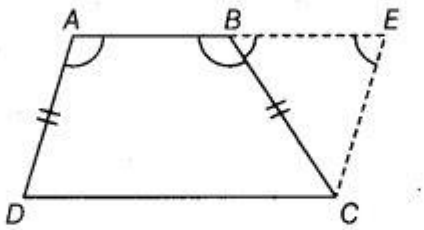
Thus APCQ is a parallelogram.

Therefore, AP || PC or SQ || PR …(i)

Again, AB || DC or BP || DQ

Also, AB = DC
or, $\frac{1}{2}AB = \frac{1}{2}DC$
BP || QD   and BP = QD.
So, BPDQ is a parallelogram.
Therefore, PD || BQ   or PS || QR   …(ii)
From eq (i) and (ii)   SQ || RP   or PS || QR
So, PRQS is a parallelogram.   (hence proved)

Question 8:
ABCD is a quadrilateral in which AB || DC and AD = BC. Prove that ∠A = ∠B and ∠C = ∠D.
Solution:
Given *ABCD* is a quadrilateral such that $AB \parallel DC$ and $AD = BC$

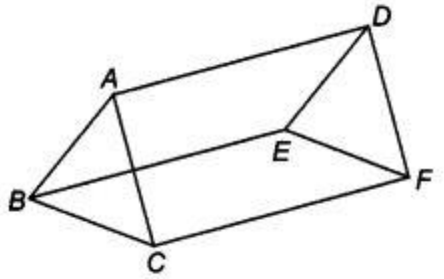


Construction Extend *AB* to *E* and draw a line *CE* parallel to *AD*.
Proof Since, $AD \parallel CE$ and transversal *AE* cuts them at *A* and *E*, respectively.

∴ $\angle A + \angle E = 180^\circ$ [since, sum of cointerior angles is 180°]
⇒ $\angle A = 180^\circ - \angle E$ ...(i)
Since, $AB \parallel CD$ and $AD \parallel CE$
So, quadrilateral *AECD* is a parallelogram.
⇒ $AD = CE \Rightarrow BC = CE$ [∵ $AD = BC$, given]
Now, in ΔBCE $CE = BC$ [proved above]
⇒ $\angle CBE = \angle CEB$
[opposite angles of equal side are equal]
⇒ $180^\circ - \angle B = \angle E$ [∵ $\angle B + \angle CBE = 180^\circ$]
⇒ $180^\circ - \angle E = \angle B$ ...(ii)
From Eqs. (i) and (ii), $\angle A = \angle B$ **Hence proved.**

Question 9:
In figure, AB || DE, AB = DE, AC|| DF and AC = OF. Prove that BC || EF and BC

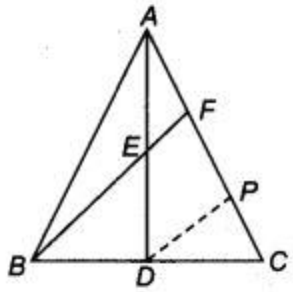
= EF.



Answer: Given In figure AB || DE and AC || DF, also AB = DE and AC = DF
To prove BC ||EF and BC = EF
Proof In quadrilateral ABED, AB||DE and AB = DE
So, ABED is a parallelogram. AD || BE and AD = BE
Now, in quadrilateral ACFD, AC || FD and AC = FD …………………….(1)
Thus, ACFD is a parallelogram.
AD || CF and AD = CF ………………………………………………………(2)
From Eqs.(1) and (2), AD = BE = CF and CF || BE ………………………….(3)
Now, in quadrilateral BCFE, BE = CF
and BE||CF [from Eq. (3)]
So, BCFE is a parallelogram. BC = EF and BC|| EF . Hence proved.

Question 10:
E is the mid-point of a median AD of ΔABC and BE is produced to meet AC at F. Show that AF = 1/3 AC.
Answer:

Given In a ΔABC, AD is a median and E is the mid-point of AD.
Construction Draw $DP \parallel EF$.
Proof In ΔADP, E is the mid-point of AD and $EF \parallel DP$.
So, F is mid-point of AP. [by converse of mid-point theorem]



In ΔFBC, D is mid-point of BC and $DP \parallel BF$.
So, P is mid-point of FC.
Thus, $AF = FP = PC$
$\therefore$ $AF = \frac{1}{3} AC$ **Hence proved.**

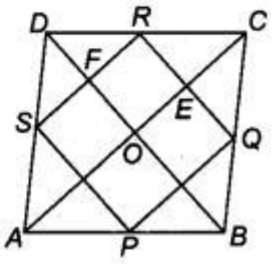
Question 11:
Show that the quadrilateral formed by joining the consecutive sides of a square is also a square.

Solution:

Given In a square $ABCD$, P, Q, R and S are the mid-points of AB, BC, CD and DA, respectively.

To show $PQRS$ is a square.

Construction Join AC and BD.



Proof Since, $ABCD$ is a square.

$\therefore \quad AB = BC = CD = AD$

Also, P, Q, R and S are the mid-points of AB, BC, CD and DA, respectively.

Then, in ΔADC, $\quad SR \parallel AC$

and $\quad SR = \frac{1}{2} AC \quad$ [by mid-point theorem]...(i)

In ΔABC, $\quad PQ \parallel AC$

and $\quad PQ = \frac{1}{2} AC \quad$ ...(ii)

From Eqs. (i) and (ii),

$$SR \parallel PQ \text{ and } SR = PQ = \frac{1}{2} AC \quad \text{...(iii)}$$

Similarly, $\quad SP \parallel BD$ and $BD \parallel RQ$

$\therefore \quad SP \parallel RQ$ and $SP = \frac{1}{2} BD$

and $\quad RQ = \frac{1}{2} BD$

$\therefore \quad SP = RQ = \frac{1}{2} BD$

Since, diagonals of a square bisect each other at right angle.

$\therefore \quad AC = BD$

$\Rightarrow \quad SP = RQ = \frac{1}{2} AC \quad$ ...(iv)

From Eqs. (iii) and (iv), $\quad SR = PQ = SP = RQ \quad$ [all side are equal]

Now, in quadrilateral $OERF$,

$OE \parallel FR$ and $OF \parallel ER$

$\therefore \quad \angle EOF = \angle ERF = 90^\circ$

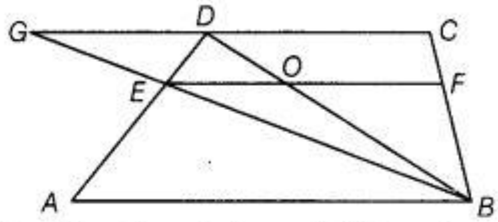
Hence, $PQRS$ is a square. **Hence proved.**

Question 12:

E and F are respectively the mid-points of the non-parallel sides AD and BC of a trapezium ABCD. Prove that EF || AB and EF = ½ (AB + CD).

Answer:

Given *ABCD* is a trapezium in which *AB* || *CD*. Also, *E* and *F* are respectively the mid-points of sides *AD* and *BC*.



Construction Join *BE* and produce it to meet *CD* produced at G, also draw *BD* which intersects *EF* at *O*.

To prove $EF \parallel AB$ and $EF = \frac{1}{2}(AB + CD)$.

Proof In ΔGCB, *E* and *F* are respectively the mid-points of *BG* and *BC*, then by mid-point theorem,

$$EF \parallel GC$$

But $GC \parallel AB$ or $CD \parallel AB$ [given]

$\therefore$ $EF \parallel AB$

In ΔADB, $AB \parallel EO$ and *E* is the mid-point of *AD*.

Therefore by converse of mid-point theorem, *O* is mid-point of *BD*.

Also, $EO = \frac{1}{2} AB$ ...(i)

In ΔBDC, $OF \parallel CD$ and *O* is the mid-point of *BD*.

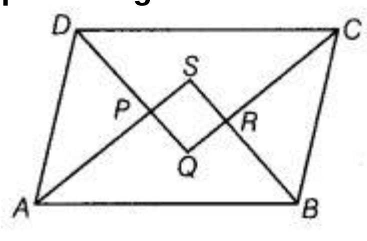
$\therefore$ $OF = \frac{1}{2} CD$ [by converse of mid-point theorem]...(ii)

On adding Eqs. (i) and (ii), we get

$$EO + OF = \frac{1}{2} AB + \frac{1}{2} CD$$

$\Rightarrow$ $EF = \frac{1}{2}(AB + CD)$ **Hence proved.**

Question 13:
Prove that the quadrilateral formed by the bisectors of the angles of a parallelogram is a rectangle.

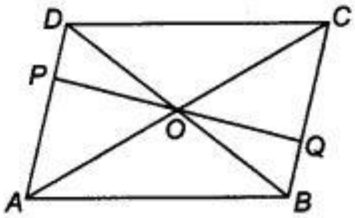


Solution:
Given Let ABCD be a parallelogram and AP, BR, CR, be are the bisectors of ∠A, ∠B, ∠C and ∠D, respectively.
To prove Quadrilateral PQRS is a rectangle.
Proof Since, ABCD is a parallelogram, then DC || AB and DA are transversal.
∠A+∠D= 180°
[sum of interior angles of a parallelogram is 180°]
or, ½ ∠A+ ½ ∠D = 90° [dividing both sides by 2]
∠PAD + ∠PDA = 90°

∠APD = 90° [since the sum of all angles of a triangle is 180°]
∴ ∠SPQ = 90° [vertically opposite angles]
∠PQR = 90°
∠QRS = 90°
and ∠PSR = 90°
Thus, PQRS is a quadrilateral whose each angle is 90°.
Hence, PQRS is a rectangle.

Question 14:
P and O are points on opposite sides AD and BC of a parallelogram ABCD such that PQ passes through the point of intersection O of its diagonals AC and BD. Show that PQ is bisected at O.

Answer:
Given *ABCD* is a parallelogram whose diagonals bisect each other at *O*.
To show *PQ* is bisected at *O*.



In Δ*ODP* and Δ*OBQ*,

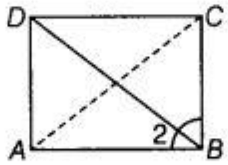
	$\angle BOQ = \angle POD$	[since, vertically opposite angles]
	$\angle OBQ = \angle ODP$	[alternate interior angles]
and	$OB = OD$	[given]
∴	$\Delta ODP \cong \Delta OBQ$	[by ASA congruence rule]
∴	$OP = OQ$	[by CPCT rule]

So, *PQ* is bisected at *O*. **Hence proved.**

Question 15:
ABCD is a rectangle in which diagonal BD bisects ∠B. Show that ABCD is a square.

Solution:

Given In a rectangle *ABCD*, diagonal *BD* bisects $\angle B$.
Construct Join AC.
To show *ABCD* is a square.



Proof

In ΔBAD and ΔBCD, $\angle ABD = \angle CBD$ [given]

$\angle A = \angle C$ [each 90°]

and $BD = BD$ [common side]

$\therefore$ $\Delta BAD \cong \Delta BCD$ [by AAS congruence rule]

$\therefore$ $AB = BC$

and $AD = CD$ [by CPCT rule] ...(i)

But in rectangle *ABCD*, opposite sides are equal.

$\therefore$ $AB = CD$

and $BC = AD$ ...(ii)

From Eqs. (i) and (ii),

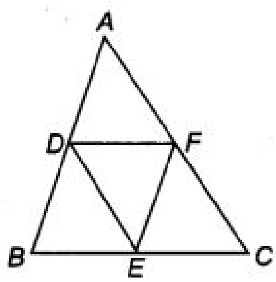
$$AB = BC = CD = DA.$$

So, *ABCD* is a square. **Hence proved.**

Question 16:
D, E and F are respectively the mid-points of the sides AB, BC and CA of a ΔABC. Prove that by joining these mid-points D, E and F, the ΔABC is divided into four congruent triangles.
Solution:
Given In a ΔABC, D, E and F are respectively the mid-points of the sides AB, BC and CA. To prove ΔABC is divided into four congruent triangles.
Proof Since, ABC is a triangle and D, E and F are the mid-points of sides AB, BC and CA, respectively.

Then, AD = BD = $\frac{1}{2}AB$, BE = EC = $\frac{1}{2}BC$, AF = CF = $\frac{1}{2}AC$.

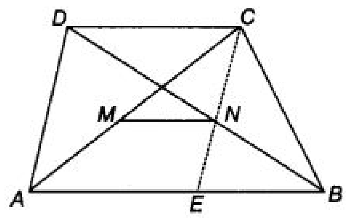
Now using the mid-point theorem,
EF || AB , and EF = $\frac{1}{2}AB$ = AD = BD
ED || AC , and ED = $\frac{1}{2}AC$ = AF = CF
DF || BC , and DF = $\frac{1}{2}BC$ = BE = CE



In ΔADF and ΔEFD, AD = EF
AF = DE
DF = FD [common]
Therefore, $\Delta ADF \cong \Delta EFD$ [by SSS congruence rule]
$\Delta DEF \cong \Delta EDB$
$\Delta DEF \cong \Delta CFE$

So, ΔABC is divided into 4 congruent triangles. (hence proved)

Question 17:
Prove that the line joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium.
Solution:
Given Let ABCD be a trapezium in which AB|| DC and let M and N be the mid-points of the diagonals AC and BD, respectively.
To prove: MN || AB ||CD



We join CN and produce it to meet AB at E.
In ΔCDN and ΔEBN, we have

DN = BN [N is mid-point of BD]
$\angle DCN = \angle BEN$ [alternate interior angles]
$\angle CDN = \angle EBN$ [alternate interior angles]
$\Delta CDN \cong \Delta EBN$ [by AAS congruence rule]

Therefore, $DC = EB$ and $CN = NE$ [CPCT]
Thus, in ΔCAE, the points M and N are mid-points of AC and CE, respectively.
Therefore, MN || AE [mid-point theorem]
or, MN || AB || CD. [Hence proved]

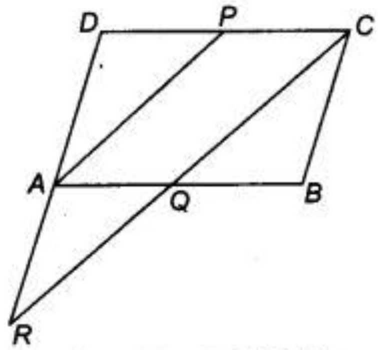
Question 18:
P is the mid-point of the side CD of a parallelogram ABCD. A line through C parallel to PA intersects AB at Q and DA produced at R. Prove that DA = AR and CQ = QR.

Solution:

Given In a parallelogram $ABCD$, P is the mid-point of DC.

To prove $DA = AR$ and $CQ = QR$

Proof $ABCD$ is a parallelogram.



$\therefore$ $BC = AD$ and $BC \parallel AD$

Also, $DC = AB$ and $DC \parallel AB$

Since, P is the mid-point of DC.

$\therefore$ $DP = PC = \frac{1}{2} DC$

Now, $QC \parallel AP$ and $PC \parallel AQ$

So, $APCQ$ is a parallelogram.

$\therefore$ $AQ = PC = \frac{1}{2} DC$

$= \frac{1}{2} AB = BQ$ [$\because DC = AB$] ...(i)

Now, in ΔAQR and ΔBQC, $AQ = BQ$ [from Eq. (i)]

$\angle AQR = \angle BQC$ [vertically opposite angles]

and $\angle ARQ = \angle BCQ$ [Alternate interior angles]

$\therefore$ $\Delta AQR \cong \Delta BQC$ [by AAS congruence rule]

$\therefore$ $AR = BC$ [by CPCT rule]

But $BC = DA$

$\therefore$ $AR = DA$

Also, $CQ = QR$ [by CPCT rule]

Hence proved.