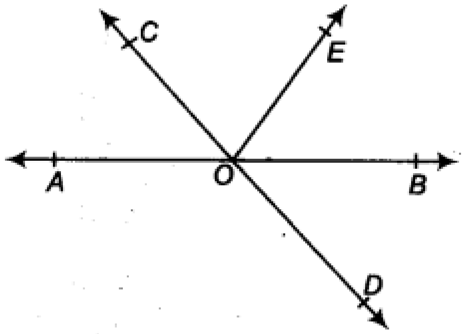
Chapter 6: Lines and Angles
Exercise: 6.1

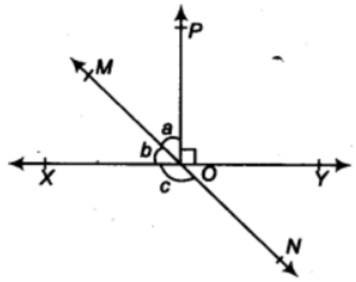
Question 1: In the figure, lines AB and CD intersect at 0. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.



Answer: Since AB is a straight line,
therefore, ∠AOC + ∠COE + ∠EOB = 180°
or (∠AOC + ∠BOE) + ∠COE = 180° or 70° + ∠COE = 180° [∠AOC + ∠BOE = 70° (Given)]
or ∠COE = 180° – 70° = 110°

therefore, Reflex ∠COE = 360° – 110° = 250°
Also, AB and CD intersect at O.
thus, ∠COA = ∠BOD [Vertically opposite angles]
But ∠BOD = 40° [Given]
therefore, ∠COA = 40°
Also, ∠AOC + ∠BOE = 70°
hence, 40° + ∠BOE = 70° or ∠BOE = 70° - 40° = 30°
Thus, ∠BOE = 30° and reflex ∠COE = 250°.

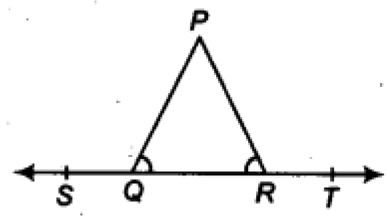
Question 2: In the figure, lines XY and MN intersect at 0. If ∠POY = 90° , and a : b = 2 : 3. find c.



Answer: Since XOY is a straight line.
thus, b+a+∠POY= 180°

But ∠POY = 90° [Given]
therefore, b + a = 180° – 90° = 90° ……………………(1)
Also a : b = 2 : 3 ⇒ b = $\frac{3a}{2}$ ……………………..…(2)
Now from (1) and (2), we get
$\frac{3a}{2}$ + a = 90°
or, $\frac{5a}{2}$ = 90°
or, a = $\frac{90°\times 2}{5}$= 36°
From (2), we get
b = $\frac{3}{2}$ x 36° = 54°
Since XY and MN intersect at O,
therefore, c = [a + ∠POY] [Vertically opposite angles]
or c = 36° + 90° = 126°
Thus, the required measure of c = 126°.

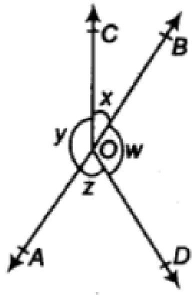
Question 3: In the figure, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.



Answer: ST is a straight line.
therefore, ∠PQR + ∠PQS = 180° ………………………………………(1) [Linear pair]
Similarly, ∠PRT + ∠PRQ = 180° …………………………...…(2) [Linear Pair]

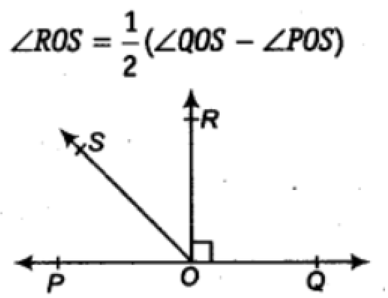
From (1) and (2), we have
∠PQS + ∠PQR = ∠PRT + ∠PRQ
But ∠PQR = ∠PRQ [Given]
thus, ∠PQS = ∠PRT

Question 4: In figure, if x + y = w + z, then prove that AOB is a line



Answer: Sum of all the angles at a point = 360°
therefore, x + y + z + w = 360° or, (x + y) + (z + w) = 360°
But (x + y) = (z + w) [Given]
thus, (x + y) + (x + y) = 360°
or, 2(x + y) = 360°
or, (x + y) = $\frac{360°}{2}$ = 180°
Therefore, AOB is a straight line.

Question 5: In the figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2}\,(\angle QOS - \angle POS)$

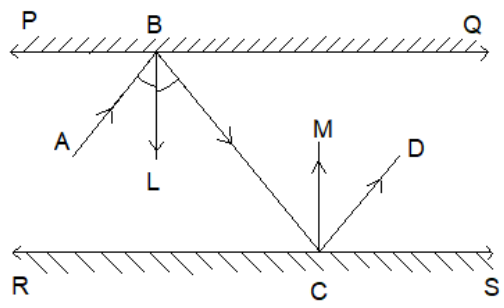


Answer: POQ is a straight line. [Given]
thus, ∠POS + ∠ROS + ∠ROQ = 180°
But OR ⊥ PQ and ∠ROQ = 90°
or, ∠POS + ∠ROS + 90° = 180°
or, ∠POS + ∠ROS = 90°
or, ∠ROS = 90° – ∠POS ……………………………. (1)
Now, we have ∠ROS + ∠ROQ = ∠QOS
or, ∠ROS + 90° = ∠QOS
or, ∠ROS = ∠QOS – 90° ………………………………..(2)

Adding (1) and (2), we have
2 ∠ROS = (∠QOS – ∠POS)
therefore, ∠ROS = $\frac{1}{2}$(∠QOS−∠POS)

Question 6: it exists ∠XYZ = 64°, and XY is produced to point P. Draw a figure from the provided information. If ray YQ bisects ∠ZYP, find ∠XYQ and reflex ∠QYP.

Answer:

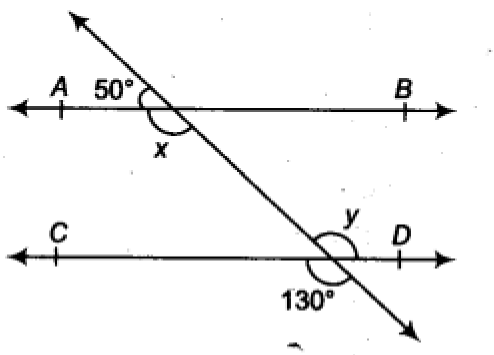


XYP is a straight line.
Therefore, ∠XYZ + ∠ZYQ + ∠QYP = 180°
or, 64° + ∠ZYQ + ∠QYP = 180° [∠XYZ = 64° (given)]
or, 64° + 2∠QYP = 180° [YQ bisects ∠ZYP so, ∠QYP = ∠ZYQ]
or, 2∠QYP = 180° – 64° = 116°
or, ∠QYP = 116∘2 = 58°

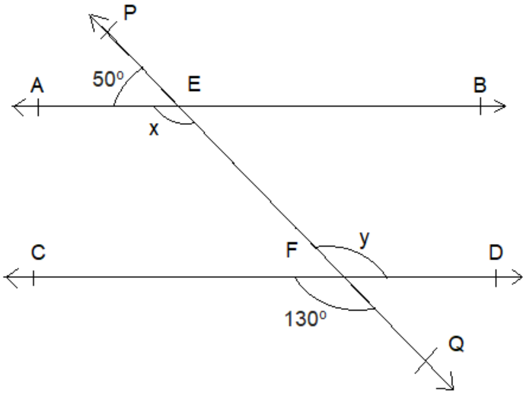
therefore, Reflex ∠QYP = 360° – 58° = 302°
Since ∠XYQ = ∠XYZ + ∠ZYQ
or, ∠XYQ = 64° + ∠QYP [∠XYZ = 64°(Given) and ∠ZYQ = ∠QYP]
or, ∠XYQ = 64° + 58° = 122° [∠QYP = 58°]
Thus, ∠XYQ = 122° and reflex ∠QYP = 302°.

Exercise 6.2

Question 1: In the figure, find the values of x and y and then show that AB || CD.

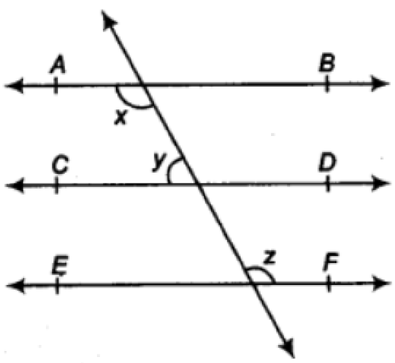


Answer:



In the figure, we have CD and PQ intersect at F.
therefore, y = 130° ………………………………………………(1) [Vertically opposite angles]
Again, PQ is a straight line, and EA stands on it.
∠AEP + ∠AEQ = 180° [Linear pair]
or 50° + x = 180°
or, x = 180° – 50° = 130° ……………………………….(2)
From (1) and (2), x = y
As they are a pair of alternate interior angles.
thus, AB || CD

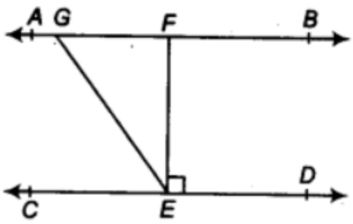
Question 2: In figure, if AB || CD, CD || EF and y : z = 3 : 7, find x.



Answer: AB || CD, and CD || EF [Given]
therefore, AB || EF
thus, x = z [Alternate interior angles] ………………………...(1)
Again, AB || CD

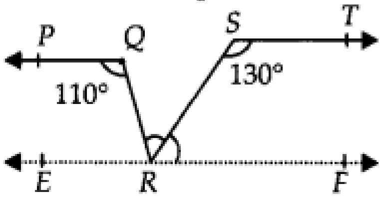
or, x + y = 180° [Co-interior angles]
or, z + y = 180° ……………………………………………… (2) [By (1)]
But y : z = 3 : 7
z = $\frac{7}{3}$ y = $\frac{7}{3}$ (180°- z) [By (2)]
or, 10z = 7 x 180°
or, z = $\frac{7 \times 180°}{10}$ = 126° …………………………..(3)
From (1) and (3), we have
z = x = 126°.

Question 3: In the figure, if AB || CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.

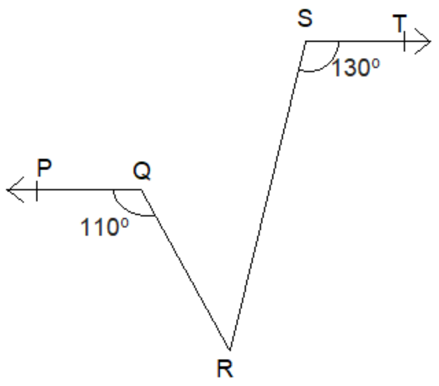


Answer: AB || CD and GE is a transversal.
therefore, ∠AGE = ∠GED [Alternate interior angles]
But ∠GED = 126° [Given]
thus, ∠AGE = 126°
Also, ∠GEF + ∠FED = ∠GED
or ∠GEF + 90° = 126° [EF ⊥ CD (given)]
x = z [Alternate interior angles]
Again, AB || CD
or, x + y = 180° [Co-interior angles]
∠GEF = 126° - 90° = 36°
Now, AB || CD and GE is a transversal.
therefore, ∠FGE + ∠GED = 180° [Co-interior angles]
or ∠FGE + 126° = 180°
or ∠FGE = 180° – 126° = 54°
Thus, ∠AGE = 126°, ∠GEF=36° and ∠FGE = 54°.

Question 4: In figure, if PQ || ST, ∠ PQR = 110° and ∠ RST = 130°, find ∠QRS.



Answer:

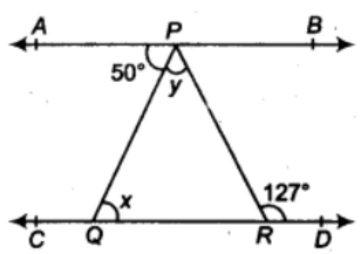


Let us draw a line EF parallel to ST through R.

Since PQ || ST [Given]
and EF || ST [Construction]
therefore, PQ || EF and QR is a transversal
or, ∠PQR = ∠QRF [Alternate interior angles]
But ∠PQR = 110° [Given]

therefore, ∠QRF = ∠QRS + ∠SRF = 110° ……………………….…(1)
Again ST || EF and RS is a transversal
therefore, ∠RST + ∠SRF = 180° [Co-interior angles]
or 130° + ∠SRF = 180°
or, ∠SRF = 180° – 130° = 50°
Now, from (1), we have ∠QRS + 50° = 110°
or, ∠QRS = 110° – 50° = 60°
Thus, ∠QRS = 60°.

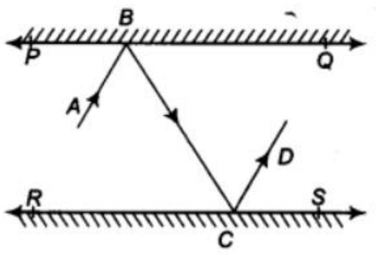
Question 5: In figure, if AB || CD, ∠APQ = 50° and ∠PRD = 127°, find x and y.



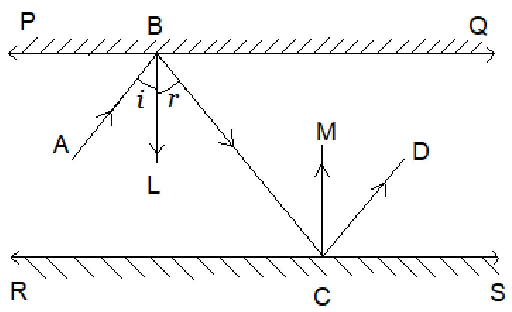
Answer: We have AB || CD and PQ is a transversal.
therefore, ∠APQ = ∠PQR [Alternate interior angles]
or, 50° = x [∵ ∠APQ = 50° (given)]
Again, AB || CD and PR is a transversal.
thus, ∠APR = ∠PRD [Alternate interior angles]
or, ∠APR = 127° [∠PRD = 127° (given)]
or, ∠APQ + ∠QPR = 127°

or, 50° + y = 127° [∠APQ = 50° (given)]
or, y = 127°- 50° = 77°
Thus, x = 50° and y = 77°.

Question 6: In the figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and hits the mirror RS at C and again reflects along CD. Prove that AB || CD.



Answer:

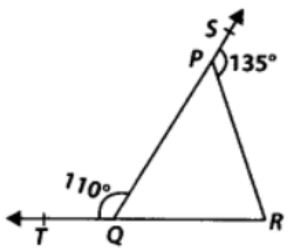


Let us draw ray BL ⊥PQ and CM ⊥ RS
therefore, PQ || RS ⇒ BL || CM   [BL || PQ and CM || RS]
Now, BL || CM and BC is a transversal.
therefore, ∠LBC = ∠MCB ……………………..(1) [Alternate interior angles]
Since the angle of incidence = Angle of reflection
∠ABL = ∠LBC and ∠MCB = ∠MCD
or, ∠ABL = ∠MCD …………………………(2) [from (1)]

Now adding (1) and (2), we get
∠LBC + ∠ABL = ∠MCB + ∠MCD
Or, ∠ABC = ∠BCD   i. e., a pair of alternate interior angles are equal.
therefore, AB || CD.

Exercise 6.3

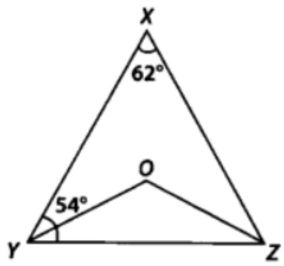
Question 1: In the figure, sides QP and RQ of ∆PQR are produced to points S and T, respectively. If ∠SPR = 135° and ∠PQT = 110°, find ∠PRQ.



Answer: We have, ∠TQP + ∠PQR = 180° [Linear pair]
or, 110° + ∠PQR = 180°
or, ∠PQR = 180° – 110° = 70°

Since the side, QP of ∆PQR is produced to S.
or, ∠PQR + ∠PRQ = 135° [Exterior angle property of a triangle]
or, 70° + ∠PRQ = 135° [∠PQR = 70°]
or, ∠PRQ = 135° – 70°
or, ∠PRQ = 65°

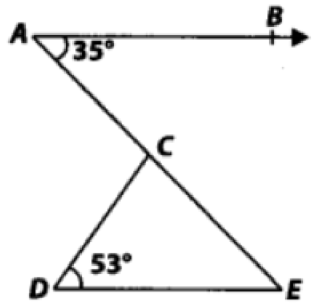
Question 2: In the figure, ∠X = 62°, ∠XYZ = 54°, if YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively of ∆XYZ, find ∠OZY and ∠YOZ.



Answer: In ∆XYZ, we have ∠XYZ + ∠YZX + ∠ZXY = 180° [Angle sum property of a triangle]
But ∠XYZ = 54° and ∠ZXY = 62°
therefore, 54° + ∠YZX + 62° = 180°
or, ∠YZX = 180° – 54° – 62° = 64°
YO and ZO are the bisectors of ∠XYZ and ∠XZY, respectively.
thus, ∠OYZ = $\frac{1}{2}$∠XYZ = $\frac{1}{2}$ (54°) = 27°
and ∠OZY = $\frac{1}{2}$∠YZX = $\frac{1}{2}$ (64°) = 32°

Now, in ∆OYZ, we have
∠YOZ + ∠OYZ + ∠OZY = 180° [Angle sum property of a triangle]
or, ∠YOZ + 27° + 32° = 180°
or, ∠YOZ = 180° - 27° – 32° = 121°
Thus, ∠OZY = 32° and ∠YOZ = 121°

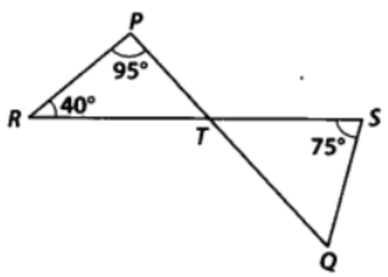
Question 3: In figure, if AB || DE, ∠BAC = 35° and ∠CDE = 539 , find ∠DCE.



Answer: AB || DE and AE is a transversal.
So, ∠BAC = ∠AED [Alternate interior angles] and ∠BAC = 35° [Given]
therefore, ∠AED = 35°
Now, in ∆CDE, we have ∠CDE + ∠DEC + ∠DCE = 180° [Angle sum property of a triangle]

thus, 53° + 35° + ∠DCE =180° [∠DEC = ∠AED = 35° and ∠CDE = 53° (Given)]
or, ∠DCE = 180° – 53° – 35° = 92°
Thus, ∠DCE = 92°

Question 4: In the figure, if lines PQ and RS intersect at point T, such that ∠ PRT = 40°, ∠ RPT = 95° and ∠TSQ = 75°, find ∠ SQT.

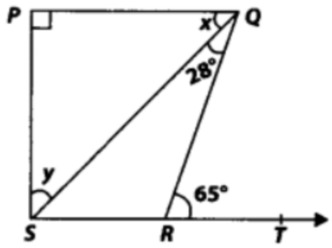


Answer: In ∆PRT, we have ∠P + ∠R + ∠PTR = 180° [Angle sum property of a triangle]
or, 95° + 40° + ∠PTR =180° [∠P = 95°, ∠R = 40° (given)]
or, ∠PTR = 180° – 95° – 40° = 45°

But don't nd RS d at T.
thus, ∠PTR = ∠QTS [Vertically opposite angles]
hence, ∠QTS = 45° [∠PTR = 45°]

Now, in ∆ TQS, we have ∠TSQ + ∠STQ + ∠SQT = 180° [Angle sum property of a triangle]
thus, 75° + 45° + ∠SQT = 180° [∠TSQ = 75° and ∠STQ = 45°]
or, ∠SQT = 180° – 75° – 45° = 60°
Thus, ∠SQT = 60°

Question 5: In the figure, if PQ ⊥ PS, PQ||SR, ∠SQR = 2S° and ∠QRT = 65°, then find the values of x and y.

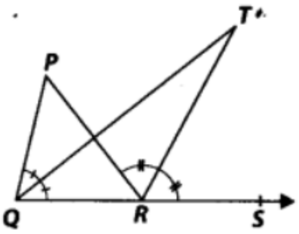


In ∆ QRS, the side SR is produced to T.
Thus, ∠QRT = ∠RQS + ∠RSQ [Exterior angle property of a triangle]

But ∠RQS = 28° and ∠QRT = 65°
So, 28° + ∠RSQ = 65°
or, ∠RSQ = 65° – 28° = 37°
Since PQ || SR and QS is a transversal.
therefore, ∠PQS = ∠RSQ = 37° [Alternate interior angles]
or, x = 37°

Again, PQ ⊥ PS
or, AP = 90°
Now, in ∆PQS,
We have ∠P + ∠PQS + ∠PSQ = 180° [Angle sum property of a triangle]
or, 90° + 37° + y = 180°
or, y = 180° – 90° – 37° = 53°
Thus, x = 37° and y = 53°

Question 6: In the figure, the side QR of ∆PQR is produced to a point S. If the bisectors of ∠PQR and ∠PRS meet at point T, then prove that ∠QTR = $\frac{1}{2}$ ∠QPR



Answer: In ∆PQR, side QR is produced to S, so by exterior angle property,
∠PRS = ∠P + ∠PQR

or, $\frac{1}{2}$∠PRS = $\frac{1}{2}$∠P + $\frac{1}{2}$∠PQR

or, ∠TRS = $\frac{1}{2}$∠P + ∠TQR ……………………(1) [QT and RT are bisectors of ∠QPR and ∠PRS, respectively.]

Now, in ∆QRT, we have
∠TRS = ∠QTR + ∠T ……………………….(2) [Exterior angle property of a triangle]
From (1) and (2),
we have ∠TQR + $\frac{1}{2}$∠P = ∠TQR + ∠T

or, $\frac{1}{2}$∠P = ∠T

or, $\frac{1}{2}$∠QPR = ∠QTR or ∠QTR = $\frac{1}{2}$∠QPR