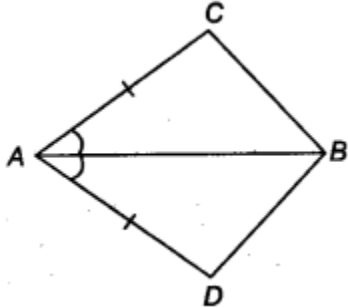


Chapter 7 – Triangles

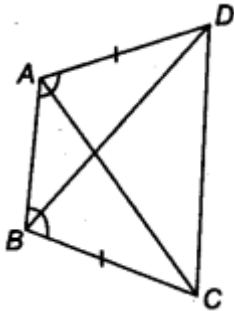
Exercise 7.1

Question 1: In quadrilateral ACBD, $AC = AD$ and AB bisects $\angle A$ (see figure). Show that $\triangle ABC \cong \triangle ABD$. What can you say about BC and BD ?



Answer: In quadrilateral ACBD, we have $AC = AD$ and AB being the bisector of $\angle A$.
Now, In $\triangle ABC$ and $\triangle ABD$,
 $AC = AD$ (Given)
 $\angle CAB = \angle DAB$ (AB bisects $\angle CAB$)
and $AB = AB$ (Common)
therefore, $\triangle ABC \cong \triangle ABD$ (By SAS congruence axiom)
Hence, $BC = BD$ (By CPCT)

Question 2: ABCD is a quadrilateral in which $AD = BC$ and $\angle DAB = \angle CBA$ (see figure). Prove that



- (i) $\triangle ABD \cong \triangle BAC$
- (ii) $BD = AC$
- (iii) $\angle ABD = \angle BAC$

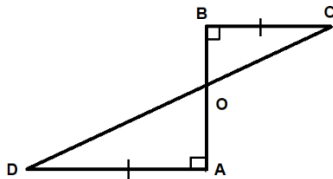
Answer: In quadrilateral ACBD, we have $AD = BC$ and $\angle DAB = \angle CBA$

(i) In $\triangle ABC$ and $\triangle BAC$,
 $AD = BC$ (Given)
 $\angle DAB = \angle CBA$ (Given)
 $AB = AB$ (Common)
therefore, $\triangle ABD \cong \triangle BAC$ (By SAS congruence)

(ii) Since $\triangle ABD \cong \triangle BAC$
 $BD = AC$ [By C.P.C.T.]

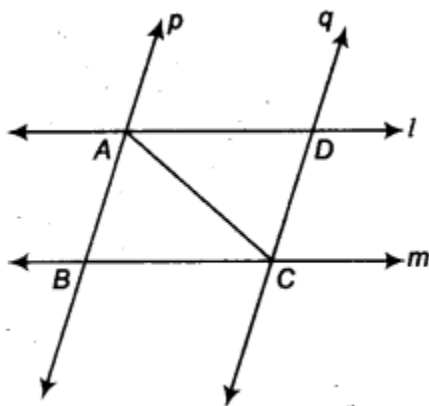
(iii) Since $\triangle ABD \cong \triangle BAC$
 $\angle ABD = \angle BAC$ [By C.P.C.T.]

Question 3: AD and BC are equal perpendiculars to a line segment AB (see figure). Show that CD bisects AB.



Answer: In $\triangle BOC$ and $\triangle AOD$, we have
 $\angle BOC = \angle AOD$
 $BC = AD$ [Given]
 $\angle BOC = \angle AOD$ [Vertically opposite angles]
therefore, $\triangle OBC \cong \triangle OAD$ [By AAS congruency]
or, $OB = OA$ [By C.P.C.T.]
i.e., O is the mid-point of AB.
Thus, CD bisects AB.

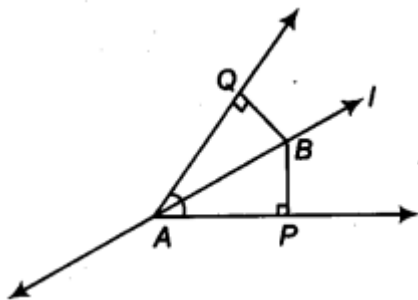
Question 4: l and m are two parallel lines intersected by another pair of parallel lines p and q (see figure). Show that $\triangle ABC = \triangle CDA$.



Answer: Given, $p \parallel q$ and AC is a transversal,
 Thus, $\angle BAC = \angle DCA$ (1) [Alternate interior angles]
 Also, $l \parallel m$ and AC is a transversal,
 Therefore, $\angle BCA = \angle DAC$ (2) [Alternate interior angles]

Now, in $\triangle ABC$ and $\triangle CDA$, we have
 $\angle BAC = \angle DCA$ [From (1)]
 $CA = AC$ [Common]
 $\angle BCA = \angle DAC$ [From (2)]
 Hence, $\triangle ABC \cong \triangle CDA$ [By ASA congruency]

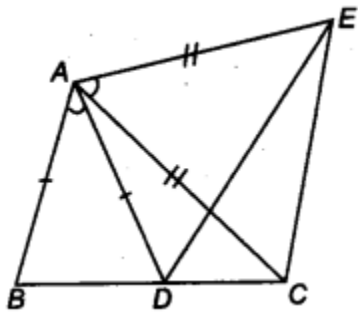
Question 5: Line l is the bisector of an $\angle A$ and B is any point on l . BP and BQ are perpendiculars from B to the arms of $\angle A$ (see figure). Show that
 (i) $\triangle APB \cong \triangle AQB$
 (ii) $BP = BQ$ or B is equidistant from the arms at $\angle A$.



Answer: We have, l is the bisector of $\angle QAP$.
 therefore, $\angle QAB = \angle PAB$
 $\angle Q = \angle P$ [Each 90°]
 $\angle ABQ = \angle ABP$ [By angle sum property of A]

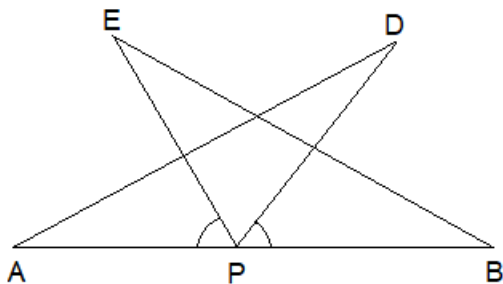
Now, in $\triangle APB$ and $\triangle AQB$, we have
 $\angle ABP = \angle ABQ$ [Proved above]
 $AB = BA$ [Common]
 $\angle PAB = \angle QAB$ [Given]
 therefore, $\triangle APB \cong \triangle AQB$ [By ASA congruency]
 Since $\triangle APB \cong \triangle AQB$
 or, $BP = BQ$ [By C.P.C.T.]
 i. e., Perpendicular distance of B from $AQ =$ Perpendicular distance of B from AP
 Thus, point B is equidistant from the arms of $\angle A$.

Question 6: In figure, $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$.



Answer: We have, $\angle BAD = \angle EAC$
 Adding $\angle DAC$ on both sides, we have
 $\angle BAD + \angle DAC = \angle EAC + \angle DAC$
 or, $\angle BAC = \angle DAE$
 Now, in $\triangle ABC$ and $\triangle ADE$, we have
 $\angle BAC = \angle DAE$ [Proved above]
 $AB = AD$ [Given]
 $AC = AE$ [Given]
 therefore, $\triangle ABC \cong \triangle ADE$ [By SAS congruency]
 or, $BC = DE$ [By C.P.C.T.]

Question 7: AB is a line segment, and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. (see figure). Show that
 (i) $\triangle DAP \cong \triangle EBP$
 (ii) $AD = BE$



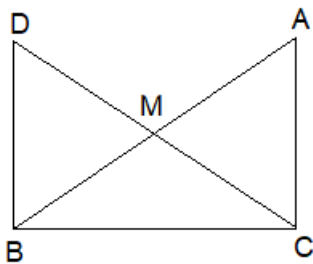
Answer: We have, P is the mid-point of AB .
 Hence, $AP = BP$
 $\angle EPA = \angle DPB$ [Given]
 Adding $\angle EPD$ on both sides, we get
 $\angle EPA + \angle EPD = \angle DPB + \angle EPD$
 or, $\angle APD = \angle BPE$

(i) Now, in $\triangle DAP$ and $\triangle EBP$, we have
 $\angle PAD = \angle PBE$ [As, $\angle BAD = \angle ABE$]
 $AP = BP$ [Proved above]
 $\angle DPA = \angle EPB$ [Proved above]
 or, $\triangle DAP \cong \triangle EBP$ [By ASA congruency]

(ii) Since, $\triangle DAP \cong \triangle EBP$
 or, $AD = BE$ [By C.P.C.T.]

Question 8: In right triangle ABC, right-angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that $DM = CM$. Point D is joined to point B (see figure). Show that

- (i) $\triangle AMC \cong \triangle BMD$
 (ii) $\angle DBC$ is a right angle
 (iii) $\triangle DBC \cong \triangle ACB$
 (iv) $CM = \frac{1}{2}AB$



Answer: Since M is the midpoint of AB, therefore, $BM = AM$.

(i) In $\triangle AMC$ and $\triangle BMD$, we have
 $CM = DM$ [Given]
 $\angle AMC = \angle BMD$ [Vertically opposite angles]
 $AM = BM$ [Proved above]
 Therefore, $\triangle AMC \cong \triangle BMD$ [By SAS congruency]

(ii) Since $\triangle AMC \cong \triangle BMD$
 or, $\angle MAC = \angle MBD$ [By C.P.C.T.]
 But they form a pair of alternate interior angles.
 Hence, $AC \parallel DB$
 Now, BC is a transversal which intersects the parallel lines AC and DB,
 therefore, $\angle BCA + \angle DBC = 180^\circ$ [Co-interior angles]
 But $\angle BCA = 90^\circ$ [As, $\triangle ABC$ is right-angled at C]
 Hence, $90^\circ + \angle DBC = 180^\circ$
 or, $\angle DBC = 90^\circ$

(iii) Again, $\triangle AMC \cong \triangle BMD$ [Proved above]
 therefore, $AC = BD$ [By C.P.C.T.]

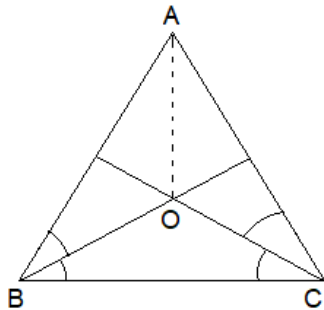
Now, in $\triangle DBC$ and $\triangle ACB$, we have
 $BD = CA$ [Proved above]
 $\angle DBC = \angle ACB$ [Each 90°]
 $BC = CB$ [Common]
Hence, $\triangle DBC \cong \triangle ACB$ [By SAS congruency]

(iv) As $\triangle DBC \cong \triangle ACB$
 $DC = AB$ [By C.P.C.T.]
But $DM = CM$ [Given]
Therefore, $CM = \frac{1}{2}DC = \frac{1}{2}AB$
or, $CM = \frac{1}{2}AB$

Exercise 7.2

Question 1: In an isosceles triangle ABC, with $AB = AC$, the bisectors of $\angle B$ and $\angle C$ intersect each other at O. Join A to O. Show that
(i) $OB = OC$
(ii) AO bisects $\angle A$

Answer:

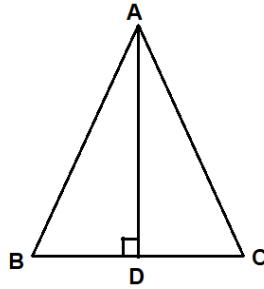


i) In $\triangle ABC$, we have
 $AB = AC$ [Given]
therefore, $\angle ABC = \angle ACB$ [Angles opposite to equal sides of a triangle are equal]
or, $\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$
or $\angle OBC = \angle OCB$
or, $OC = OB$ [Sides opposite to equal angles of a \triangle are equal]

(ii) In $\triangle ABO$ and $\triangle ACO$, we have
 $AB = AC$ [Given]
 $\angle OBA = \angle OCA$ [As, $\frac{1}{2}\angle B = \frac{1}{2}\angle C$]
 $OB = OC$ [Proved above]
 $\triangle ABO \cong \triangle ACO$ [By SAS congruency]

or, $\angle OAB = \angle OAC$ [By C.P.C.T.]
Or, AO bisects $\angle A$.

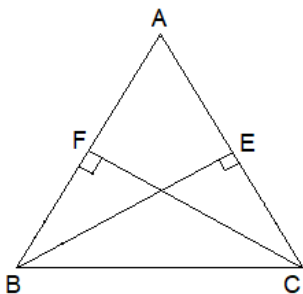
Question 2: In $\triangle ABC$, AD is the perpendicular bisector of BC (see figure). Show that $\triangle ABC$ is an isosceles triangle in which $AB = AC$.



Answer: Since AD is the bisector of BC.
therefore, $BD = CD$

Now, in $\triangle ABD$ and $\triangle ACD$, we have
 $AD = DA$ [Common]
 $\angle ADB = \angle ADC$ [Each 90°]
 $BD = CD$ [As proved above]
Hence, $\triangle ABD \cong \triangle ACD$ [By SAS congruency]
or, $AB = AC$ [By C.P.C.T.]
Thus, $\triangle ABC$ is an isosceles triangle.

Question 3: ABC is an isosceles triangle in which altitudes BE, and CF are drawn to equal sides AC and AB respectively (see figure). Show that these altitudes are equal.



Answer: $\triangle ABC$ is an isosceles triangle.

therefore, $AB = AC$

or, $\angle ACB = \angle ABC$ [Angles opposite to equal sides of a triangle are equal]

or, $\angle BCE = \angle CBF$

Now, in $\triangle BEC$ and $\triangle CFB$

$\angle BCE = \angle CBF$ [As proved above]

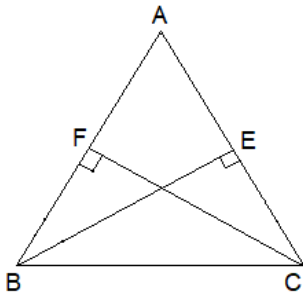
$\angle BEC = \angle CFB$ [Each 90°]

$BC = CB$ [Common]

therefore, $\triangle BEC \cong \triangle CFB$ [By AAS congruency]

So, $BE = CF$ [By C.P.C.T.]

Question 4: ABC is a triangle in which altitudes are, and CF to sides AC and AB are equal (see figure).



Show that

(i) $\triangle ABE \cong \triangle ACF$

(ii) $AB = AC$ i.e., ABC is an isosceles triangle.

Answer: (i) In $\triangle ABE$ and $\triangle ACF$, we have

$\angle AEB = \angle AFC$ [Each 90° as $BE \perp AC$ and $CF \perp AB$]

$\angle A = \angle A$ [Common Angle]

$BE = CF$ [Given]

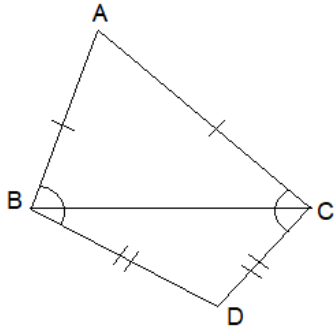
Therefore, $\triangle ABE \cong \triangle ACF$ [By AAS congruency]

(ii) Since, $\triangle ABE \cong \triangle ACF$

Hence, $AB = AC$ [By C.P.C.T.]

Therefore, ABC is an isosceles triangle.

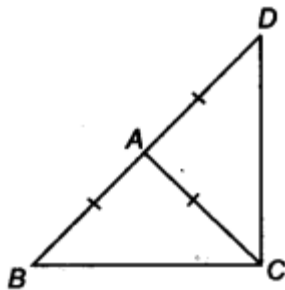
Question 5: ABC and DBC are isosceles triangles on the same base BC (see figure). Show that $\angle ABD = \angle ACD$.



Answer: In $\triangle ABC$, we have
 $AB = AC$ [$\triangle ABC$ is an isosceles triangle]
 therefore, $\angle ABC = \angle ACB$ (1) [Angles opposite to equal sides of a triangle are equal]
 Again, in $\triangle BDC$, we have
 $BD = CD$ [$\triangle BDC$ is an isosceles triangle]
 Therefore, $\angle CBD = \angle BCD$ (2) [Angles opposite to equal sides of a Triangle are equal]

Adding (1) and (2), we have
 $\angle ABC + \angle CBD = \angle ACB + \angle BCD$
 or, $\angle ABD = \angle ACD$.

Question 6: $\triangle ABC$ is an isosceles triangle in which $AB = AC$. Side BA is produced to D such that $AD = AB$ (see figure). Show that $\angle BCD$ is a right angle.



Answer: $AB = AC$ [Given] (1)
 $AB = AD$ [Given](2)
 From (1) and (2), we get $AC = AD$

Now, on $\triangle ABC$, we have
 $\angle ABC + \angle ACB + \angle BAC = 180^\circ$ [Angle sum property of a \triangle]
 or, $2\angle ACB + \angle BAC = 180^\circ$(3) [$\angle ABC = \angle ACB$ (Angles opposite to equal sides of a Triangle are equal)]

Similarly, in $\triangle ACD$,
 $\angle ADC + \angle ACD + \angle CAD = 180^\circ$
 or, $2\angle ACD + \angle CAD = 180^\circ$(4) [$\angle ADC = \angle ACD$ (Angles opposite to equal sides of a Triangle are equal)]

Adding (3) and (4), we have

$$2\angle ACB + \angle BAC + 2\angle ACD + \angle CAD = 180^\circ + 180^\circ$$

$$\text{or, } 2[\angle ACB + \angle ACD] + [\angle BAC + \angle CAD] = 360^\circ$$

$$\text{or, } 2\angle BCD + 180^\circ = 360^\circ \text{ } [\angle BAC \text{ and } \angle CAD \text{ form a linear pair}]$$

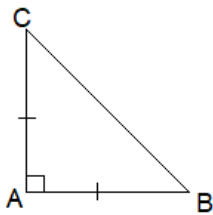
$$\text{or, } 2\angle BCD = 360^\circ - 180^\circ = 180^\circ$$

$$\text{or, } \angle BCD = \frac{180^\circ}{2} = 90^\circ$$

$$\text{Thus, } \angle BCD = 90^\circ$$

Question 7: ABC is a right-angled triangle in which $\angle A = 90^\circ$ and $AB = AC$, find $\angle B$ and $\angle C$.

Answer:



In $\triangle ABC$, we have $AB = AC$ [Given]

Therefore, their opposite angles are equal, hence, $\angle ACB = \angle ABC$

Now, $\angle A + \angle B + \angle C = 180^\circ$ [Angle sum property of a Triangle]

$$\text{or, } 90^\circ + \angle B + \angle C = 180^\circ \text{ } [\angle A = 90^\circ \text{ (Given)}]$$

$$\text{or, } \angle B + \angle C = 180^\circ - 90^\circ = 90^\circ$$

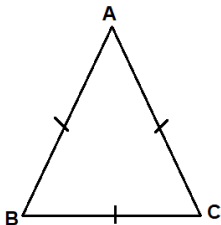
But $\angle B = \angle C$

$$\angle B = \angle C = \frac{90^\circ}{2} = 45^\circ$$

Thus, $\angle B = 45^\circ$ and $\angle C = 45^\circ$

Question 8: Show that the angles of an equilateral triangle are 60° each.

Answer:



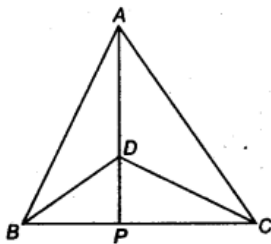
We know that, $AB = BC = CA$ [As, $\triangle ABC$ is an equilateral triangle]
 $AB = BC$
 or, $\angle A = \angle C$ (1) [Angles opposite to equal sides of a Triangle are equal]

Similarly, $AC = BC$
 or, $\angle A = \angle B$ (2)

From (1) and (2), we have
 $\angle A = \angle B = \angle C = x$ (say)
 Since, $\angle A + \angle B + \angle C = 180^\circ$ [Angle sum property of a Triangle]
 therefore, $x + x + x = 180^\circ$
 or, $3x = 180^\circ$
 or, $x = 60^\circ$
 Hence, $\angle A = \angle B = \angle C = 60^\circ$
 Thus, the angles of an equilateral triangle are 60° each.

Exercise 7.3

Question 1: $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see figure). If AD is extended to intersect BC at P , show that



- (i) $\triangle ABD \cong \triangle ACD$
- (ii) $\triangle ABP \cong \triangle ACP$
- (iii) AP bisects $\angle A$ as well as $\angle D$
- (iv) AP is the perpendicular bisector of BC .

Answer: (i) In $\triangle ABD$ and $\triangle ACD$, we have
 $AB = AC$ [Given]
 $BD = CD$ [Given]
 $AD = DA$ [Common]
 Therefore, $\triangle ABD \cong \triangle ACD$ [By SSS congruency]
 $\angle BAD = \angle CAD$ [By C.P.C.T.](1)

(ii) In $\triangle ABP$ and $\triangle ACP$, we have
 $AB = AC$ [Given]
 $\angle BAP = \angle CAP$ [From (1)]
 Hence, $AP = PA$ [Common]
 Therefore, $\triangle ABP \cong \triangle ACP$ [By SAS congruency]

(iii) Since, $\triangle ABP \cong \triangle ACP$
 or, $\angle BAP = \angle CAP$ [By C.P.C.T.]
 Therefore, AP is the bisector of $\angle A$.
 Again, in $\triangle BDP$ and $\triangle CDP$, we have,
 $BD = CD$ [Given]
 $DP = PD$ [Common]
 $BP = CP$ [As, $\triangle ABP \cong \triangle ACP$]
 or, $\triangle BDP \cong \triangle CDP$ [By SSS congruency]
 therefore, $\angle BDP = \angle CDP$ [By C.P.C.T.]
 or, DP (or AP) is the bisector of $\angle BDC$
 Hence, AP is the bisector of $\angle A$ as well as $\angle D$.

(iv) As $\triangle ABP \cong \triangle ACP$
 or, $\angle APS = \angle APC$, $BP = CP$ [By C.P.C.T.]
 But $\angle APB + \angle APC = 180^\circ$ [Linear Pair]
 Hence, $\angle APB = \angle APC = 90^\circ$
 or, $AP \perp BC$, also $BP = CP$
 Hence, AP is the perpendicular bisector of BC.

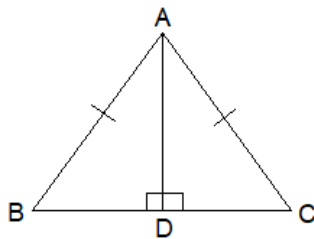
Question 2: AD is an altitude of an isosceles triangle ABC in which $AB = AC$.

Show that

(i) AD bisects BC

(ii) AD bisects $\angle A$

Answer:

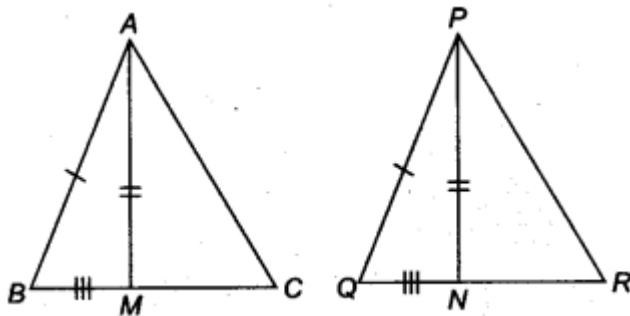


(i) In right $\triangle ABD$ and $\triangle ACD$, we have
 $AB = AC$ [Given]
 $\angle ADB = \angle ADC$ [Each 90°]
 $AD = DA$ [Common]
 Therefore, $\triangle ABD \cong \triangle ACD$ [By RHS congruency]
 So, $BD = CD$ [By C.P.C.T.]
 Hence, D is the mid-point of BC or AD bisects BC.

(ii) Since, $\triangle ABD \cong \triangle ACD$,
 or, $\angle BAD = \angle CAD$ [By C.P.C.T.]
 So, AD bisects $\angle A$

Question 3: Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and OR and median PN of $\triangle PQR$ (see figure). Show that

- (i) $\triangle ABC \cong \triangle PQR$
 (ii) $\triangle ABM \cong \triangle PQN$



Answer: In $\triangle ABC$, AM is the median.

therefore, $BM = \frac{1}{2} BC$ (1)

In $\triangle PQR$, PN is the median.

thus, $QN = \frac{1}{2} QR$ (2)

And $BC = QR$ [Given]

or, $\frac{1}{2} BC = \frac{1}{2} QR$

or, $BM = QN$ (3) [From (1) and (2)]

(i) In $\triangle ABM$ and $\triangle PQN$, we have

$AM = PN$ [Given]

$AB = PQ$ [Given]

$BM = QN$ [From (3)]

Hence, $\triangle ABM \cong \triangle PQN$ [By SSS congruency]

(ii) Since $\triangle ABM \cong \triangle PQN$

Hence, $\angle B = \angle Q$ (4) [By C.P.C.T.]

Now, in $\triangle ABC$ and $\triangle PQR$, we have

$\angle B = \angle Q$ [From (4)]

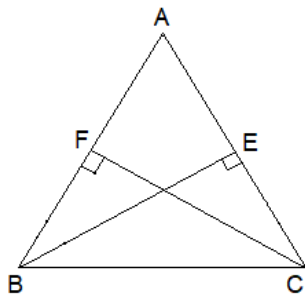
$BC = QR$ [Given]

$AB = PQ$ [Given]

Hence, $\triangle ABC \cong \triangle PQR$ [By SAS congruency]

Question 4: BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Answer:



Since $BE \perp AC$ [Given]

Hence, $\triangle BEC$ is a right triangle such that $\angle BEC = 90^\circ$

Similarly, $\angle CFB = 90^\circ$

Now, in the right $\triangle BEC$ and $\triangle CFB$, we have

$BE = CF$ [Given]

$\angle BEC = \angle CFB$ [Each 90°]

$BC = CB$ [Common hypotenuse]

Hence, $\triangle BEC \cong \triangle CFB$ [By RHS congruency]

So, $\angle BCE = \angle CBF$ [By C.P.C.T.]

or $\angle BCA = \angle CBA$

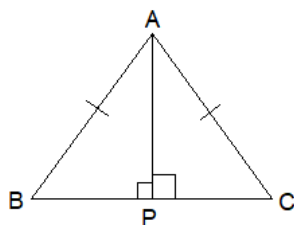
Now, in $\triangle ABC$, $\angle BCA = \angle CBA$

or, $AB = AC$ [Sides opposite to equal angles of a \triangle are equal]

Therefore, $\triangle ABC$ is an isosceles triangle.

Question 5: $\triangle ABC$ is an isosceles triangle with $AB = AC$. Draw $AP \perp BC$ to show that $\angle B = \angle C$.

Answer:



We have, $AP \perp BC$ [Given]

$\angle APB = 90^\circ$ and $\angle APC = 90^\circ$

In $\triangle ABP$ and $\triangle ACP$, we have

$\angle APB = \angle APC$ [Each 90°]

$AB = AC$ [Given]

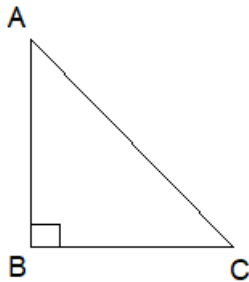
$AP = AP$ [Common]

Therefore, $\triangle ABP \cong \triangle ACP$ [By RHS congruency]
So, $\angle B = \angle C$ [By C.P.C.T.]

Exercise 7.4

Question 1: Show that in a right-angled triangle, the hypotenuse is the longest side.

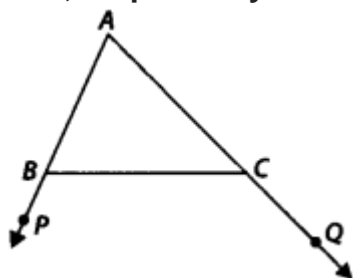
Answer:



Let us consider $\triangle ABC$ such that $\angle B = 90^\circ$
Hence, $\angle A + \angle B + \angle C = 180^\circ$
or, $\angle A + 90^\circ + \angle C = 180^\circ$
or, $\angle A + \angle C = 90^\circ$
or, $\angle A + \angle C = \angle B$
Hence, $\angle B > \angle A$ and $\angle B > \angle C$

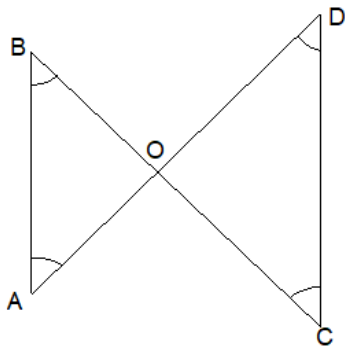
The side opposite to $\angle B$ is longer than the side opposite $\angle A$, i.e., $AC > BC$.
Similarly, $AC > AB$.
Therefore, we get AC is the longest side. But AC is the hypotenuse of the triangle.
Thus, the hypotenuse is the longest side.

Question 2: In the figure, sides AB and AC of $\triangle ABC$ are extended to points P and Q , respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.



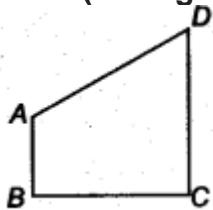
Answer: $\angle ABC + \angle PBC = 180^\circ$ [Linear pair]
 and $\angle ACB + \angle QCB = 180^\circ$ [Linear pair]
 But $\angle PBC < \angle QCB$ [Given] $\Rightarrow 180^\circ - \angle PBC > 180^\circ - \angle QCB$
 or, $\angle ABC > \angle ACB$
 The side opposite to $\angle ABC >$ the side opposite to $\angle ACB$, i.e., $AC > AB$.

Question 3: In the figure, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.

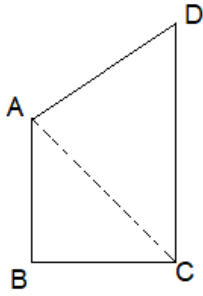


Answer: Since $\angle A > \angle B$ [Given]
 $\therefore OB > OA$ (1)[Side opposite to greater angle is longer]
 Similarly, $OC > OD$ (2)
 Now adding (1) and (2), we have
 $OB + OC > OA + OD$
 Hence, $BC > AD$

Question 4: AB and CD are the smallest and longest sides of a quadrilateral ABCD (see figure). Show that $\angle A > \angle C$ and $\angle B > \angle D$.



Answer:



Let us join AC for the convenience of the solution.

Now, in $\triangle ABC$, $AB < BC$ [As AB is the smallest side of the quadrilateral ABCD]

or, $BC > AB$

or, $\angle BAC > \angle BCA$ (1) [Angle opposite to the longer side of Triangle is greater]

Again, in $\triangle ACD$, $CD > AD$ [As the CD is the longest side of the quadrilateral ABCD]

or, $\angle CAD > \angle ACD$ (2) [Angle opposite to the longer side of Triangle is greater]

Adding (1) and (2), we get

$$\angle BAC + \angle CAD > \angle BCA + \angle ACD$$

or, $\angle A > \angle C$

Now, let us join BD.

So, in $\triangle BCD$, $BC < DC$ [As, the CD is the longest side of the quadrilateral ABCD]

or, $CD > BC$

or, $\angle DBC > \angle BDC$ (3) [Angle opposite to the longer side of Triangle is greater]

Again, in $\triangle ABD$, $AD > AB$ [As AB is the smallest side of the quadrilateral ABCD]

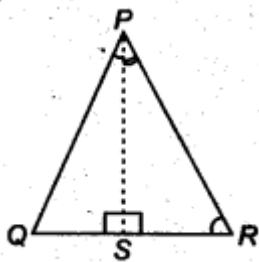
or, $\angle ABD > \angle ADB$ (4) [Angle opposite to the longer side of Triangle is greater]

Adding (3) and (4), we get

$$\angle DBC + \angle ABD = \angle BDC + \angle ADB$$

or, $\angle B > \angle D$

Question 5: In the figure, $PR > PQ$ and PS bisect $\angle QPR$. Prove that $\angle PSR > \angle PSQ$.



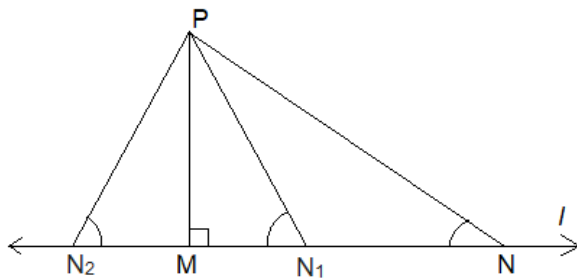
Answer: In $\triangle PQR$, PS bisects $\angle QPR$ [Given]
therefore, $\angle QPS = \angle RPS$

and $PR > PQ$ [Given]
 or, $\angle PQS > \angle PRS$ [Angle opposite to the longer side of Δ is greater]
 or, $\angle PQS + \angle QPS > \angle PRS + \angle RPS$ (1) [$\angle QPS = \angle RPS$]
 As, Exterior $\angle PSR = [\angle PQS + \angle QPS]$
 and exterior $\angle PSQ = [\angle PRS + \angle RPS]$ [An exterior angle is equal to the sum of interior opposite angles]

Now, from (1), we have
 $\angle PSR = \angle PSQ$.

Question 6: Show that of all line segments drawn from a given point, not on it, the vertical line segment is the shortest.

Answer:



Let us consider the ΔPMN such that $\angle M = 90^\circ$
 Since $\angle M + \angle N + \angle P = 180^\circ$ [Sum of angles of a triangle is 180°]
 Since, $\angle M = 90^\circ$ [$PM \perp l$]
 So, $\angle N + \angle P = \angle M$
 or, $\angle N < \angle M$
 or, $PM < PN$ (1)
 Similarly, $PM < PN_1$ (2)
 and $PM < PN_2$ (3)

Hence, from (1), (2) and (3), we have PM is the smallest line segment drawn from P on the line l . Thus, the vertical line segment is the shortest line segment drawn on a line from a point, not on it.

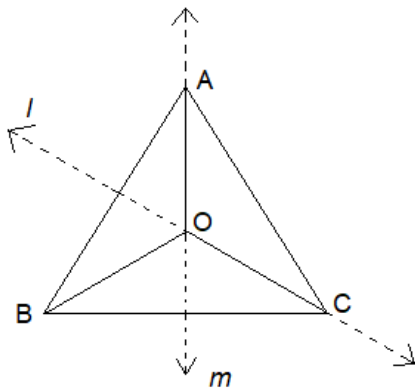
Exercise 7.5

Question 1: ΔABC is a triangle. Locate a point in the interior of ΔABC , which is equidistant from all the vertices of ΔABC .

Answer: STEP 1: Let us take an ΔABC .
STEP 2: Draw a line l (extension of OC) as, the perpendicular bisector of AB .

STE 3: similarly, we need to draw m , the perpendicular bisector of BC .

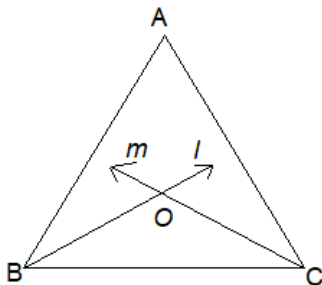
STEP 4: Let the two perpendicular bisectors l and m meet at O .



So, O is the critical point which is equidistant from A , B and C .

Question 2: In a triangle, locate a point in its interior which is equidistant from all the triangle sides.

Answer:



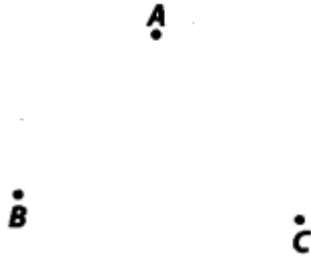
STEP 1: Let us consider an $\triangle ABC$.

STEP 2: Draw m , the bisector of $\angle C$.

STEP 3: Let the two bisectors l and m meet at O .

STEP 4: Thus, O is the critical point which is equidistant from the sides of $\triangle ABC$.

Question 3: In a vast park, people are concentrated at three points (see figure)



A: where these are different slides and swings for children.

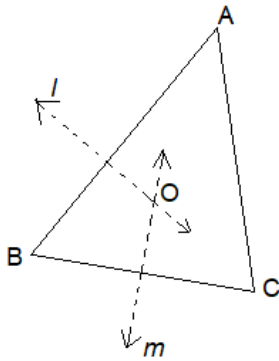
B: near which a human-made lake is situated.

C: which is near to a large parking and exist.

Where should an ice-cream parlour be set? Up so that the maximum number of persons can approach it?

[Hint The parlour should be equidistant from A, B and C.]

Answer:



STEP 1: Let us join A and B, and draw l, the perpendicular bisector of AB.

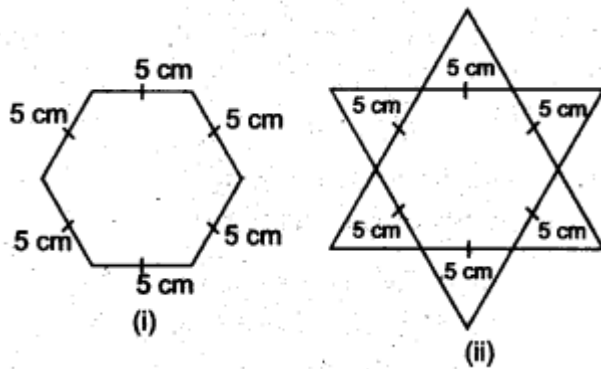
STEP 2: Now, join B and C, and draw m, the perpendicular bisector of BC. Let the perpendicular bisectors l and m meet at O.

STEP 3: The point O is the required point where the ice cream parlour be set up.

Note: If we join A and C and draw the perpendicular bisector, then it will also meet (or pass-through) the point O.

Question 4: Complete the hexagonal and star shaped Rangolies [see Fig. (i) and (ii)] by filling them with as many equilateral triangles of side 1 cm as you

can. Count the number of triangles in each case. Which has more triangles?



Answer: We require 150 equilateral triangles of side 1 cm in the Fig. (i) and 300 equilateral triangles in the Fig. (ii).
Hence, The Fig. (ii) has more triangles.