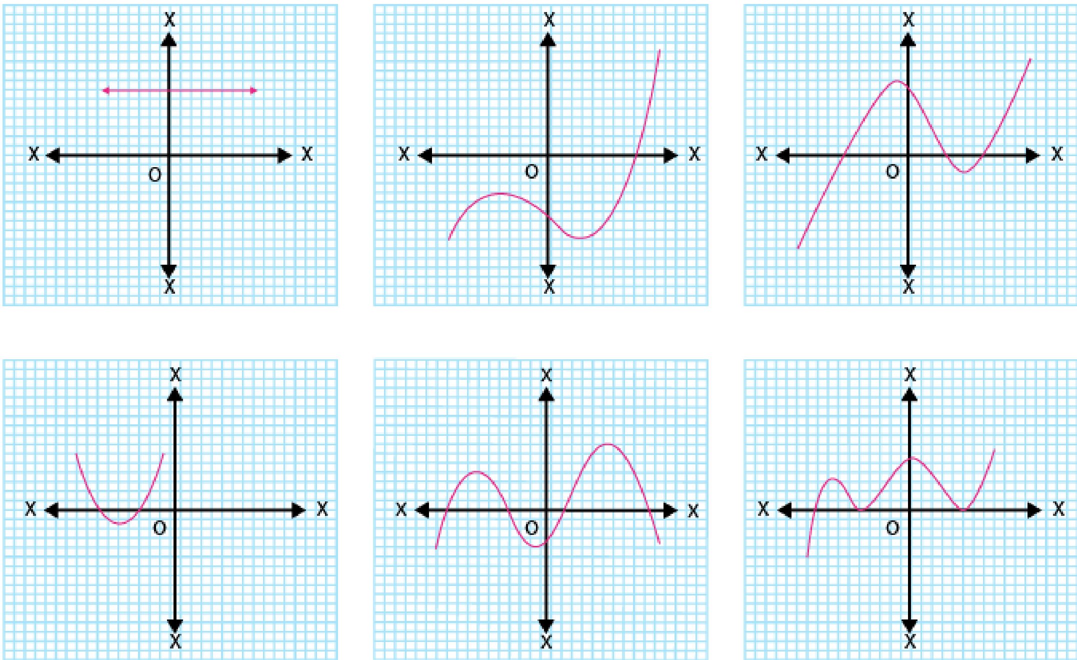
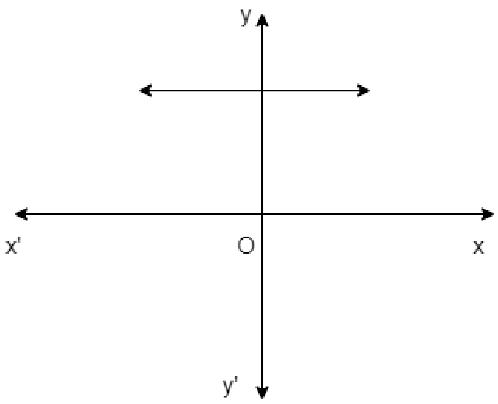
Chapter 2 – POLYNOMIALS
Exercise – 2.1

Question 1: The graphs of y = p(x) are given in Fig. 2.10 below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.



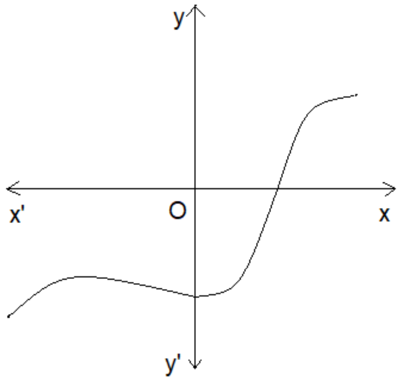
Answer:

(i)



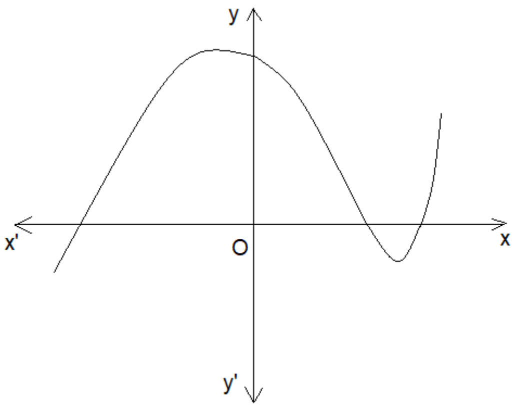
This graph shows p(x) has no zero.

(ii)



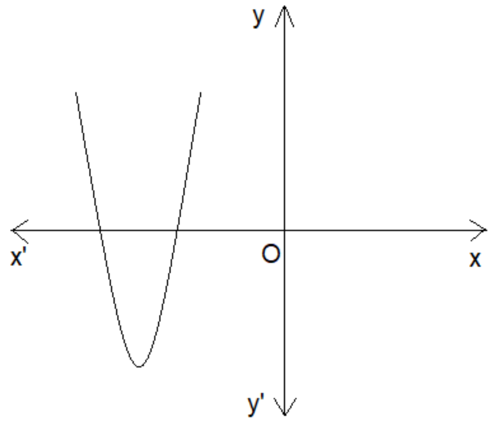
This graph shows p(x) has one zero.

(iii)



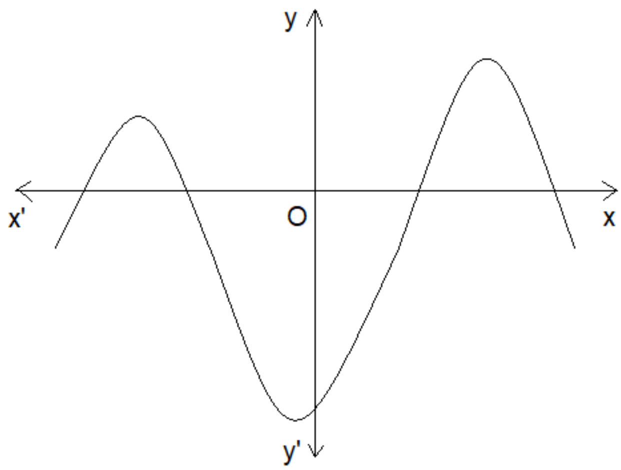
This graph shows p(x) has three zeroes.

(iv)



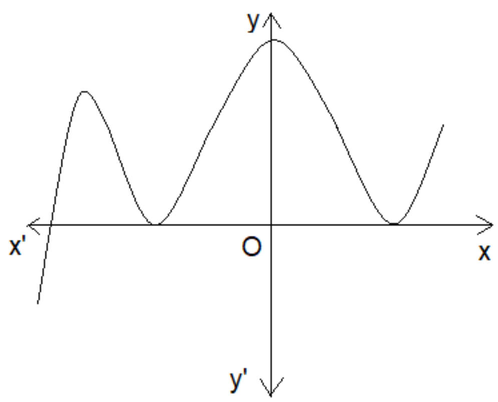
This graph shows p(x) has two zeroes.

(v)



This graph shows p(x) has four zeroes.

(vi)



This graph shows p(x) has three zeroes.

Exercise 2.2

Question 1: Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

(ii) $4s^2 – 4s + 1$
(iii) $6x^2 – 3 – 7x$
(iv) $4u^2 + 8u$
(v) $t^2 – 15$
(vi) $3x^2 – x – 4$

Answer: (i) $x^2 – 2x – 8$

$= x^2 – 4x + 2x – 8$

$=x(x – 4) + 2(x – 4)$

$=(x – 4)(x + 2)$

Zeroes of polynomial equation = (4, -2)

Hence, sum of the zeroes = (4 – 2) = 2 = $\frac{-(-2)}{1} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

And, product of the zeroes = 4 × (-2) = -8 = $-\left(\frac{8}{1}\right) = \frac{constant\ term}{coefficient\ of\ x^2}$

(ii) $4s^2 – 4s + 1$

$= 4s^2 – 2s – 2s + 1$

$= 2s(2s – 1) – 1(2s – 1)$

$= (2s – 1)(2s – 1)$

Zeroes of polynomial equation = $\left(\frac{1}{2}, \frac{1}{2}\right)$

Hence, sum of the zeroes = $\frac{1}{2} + \frac{1}{2} = 1 = -\frac{4}{4} = \frac{-(coefficient\ of\ s)}{(coefficient\ of\ s^2)}$

And, product of the zeroes = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{constant\ term}{(coefficient\ of\ s^2)}$

(iii) $6x^2 – 3 – 7x$

$= 6x^2 – 7x – 3$

$= 6x^2 – 9x + 2x – 3$

$= 3x(2x – 3) +1(2x – 3)$

$= (3x + 1)(2x – 3)$

Zeroes of polynomial equation = $\left(-\frac{1}{3}, \frac{3}{2}\right)$

Hence, sum of the zeroes = $-\frac{1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

And, product of the zeroes = $-\frac{1}{3} \times \frac{3}{2} = -\left(\frac{1}{2}\right) = -\left(\frac{3}{6}\right) = \frac{constant\ term}{coefficient\ of\ x^2}$

(iv) $4u^2 + 8u = 4u\ (u + 2)$

Zeroes of polynomial equation = (0, -2)

Hence, sum of the zeroes = 0 +(– 2) = - 2 = $-\left(\frac{8}{4}\right) = \frac{-(coefficient\ of\ u)}{(coefficient\ of\ u^2)}$

And, product of the zeroes = 0 × (-2) = 0 = $\frac{0}{4} = \frac{constant\ term}{coefficient\ of\ u^2}$

(v) $t^2 – 15$
or, t = ±√15
Zeroes of polynomial equation = $(\sqrt{15}, -\sqrt{15})$
Hence, sum of the zeroes = $\sqrt{15} + (-\sqrt{15})$ = 0 = $-\left(\frac{0}{1}\right) = \frac{-(coefficient\ of\ t)}{(coefficient\ of\ t^2)}$

And, product of the zeroes = $\sqrt{15} \times (-\sqrt{15})$ = (-15) = $-\left(\frac{15}{1}\right) = \frac{constant\ term}{coefficient\ of\ t^2}$

(vi) $3x^2 – x – 4$
= $3x^2 – 4x + 3x – 4$
= x(3x – 4) + 1(3x – 4)
= (3x – 4)(x + 1)

Zeroes of polynomial equation = $\left(\frac{4}{3}, -1\right)$

Hence, sum of the zeroes = $\frac{4}{3} + (-1) = \frac{1}{3} = -\left(-\frac{1}{3}\right)\ \frac{-(coefficient\ of\ x)}{(coefficient\ of\ x^2)}$

And, product of the zeroes = $\frac{4}{3} \times (-1) = -\left(\frac{4}{3}\right) = \ = \frac{constant\ term}{coefficient\ of\ x^2}$

Question 2: Find a quadratic polynomial each with the given numbers as the sum and product of zeroes respectively:

(i) $\frac{1}{4}, -1$

(ii) $\sqrt{2}, \frac{1}{3}$

(iii) 0, $\sqrt{5}$

(iv) 1, 1
(v) - $\frac{1}{4}, \frac{1}{4}$

(vi) 4, 1

Answer:
(i) Let the zeroes of the polynomial be α and β , then, $\alpha + \beta = \frac{1}{4}$ and $\alpha\beta$ = -1

Therefore, the required polynomial will be,
$x^2 – (\alpha + \beta)x + \alpha\beta$
= $x^2 - \frac{1}{4}x + (-1)$
=$x^2 - \frac{1}{4}x – 1$
= $4x^2 – x – 4$

(ii) Let the zeroes of the polynomial be α and β , then, $\alpha + \beta = \sqrt{2}$ and $\alpha\beta = \frac{1}{3}$

Therefore, the required polynomial will be,

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - \sqrt{2}x + \frac{1}{3}$

$= 3x^2 - 3\sqrt{2}x + 1$

(iii) Let the zeroes of the polynomial be α and β , then, $\alpha + \beta = 0$ and $\alpha\beta = \sqrt{5}$

Therefore, the required polynomial will be,

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 0x + \sqrt{5}$

$= x^2 + \sqrt{5}$

(iv) Let the zeroes of the polynomial be α and β , then, $\alpha + \beta = 1$ and $\alpha\beta = 1$

Therefore, the required polynomial will be,

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 1x + 1$

$= x^2 - x + 1$

(v) Let the zeroes of the polynomial be α and β , then, $\alpha + \beta = -\left(\frac{1}{4}\right)$ and $\alpha\beta = \frac{1}{4}$

Therefore, the required polynomial will be,

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (-\frac{1}{4})x + \frac{1}{4}$

or, $4x^2 + x + 1 = 0$

(vi) Let the zeroes of the polynomial be α and β , then, $\alpha + \beta = 4$ and $\alpha\beta = 1$

Therefore, the required polynomial will be,

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 4x + 1$

Exercise 2.3

Question 1: Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

Answer: (i)Given , dividend = p(x) = x^3-3x^2+5x-3

Divisor = $x^2 - 2$

$$x-3$$
$$x^2-2 \,\big)\, x^3-3x^2+5x-3$$
$$-$$
$$x^3+0x^2-2x$$
$$-3x^2+7x-3$$
$$-$$
$$-3x^2+0x+6$$
$$7x-9$$

Therefore , on division we get:

Quotient = x-3

Remainder = 7x-9

(ii) Given, Dividend =$x^4 - 3x^2 + 4x + 5$

Divisor = $x^2 + 1 - x$

$$x^2 + x - 3$$
$$x^2+1-x \,\big)\, x^4+0x^3-3x^2+4x+5$$
$$-$$
$$x^4 - x^3 + x^2$$
$$x^3 - 4x^2+4x+5$$
$$-$$
$$x^3 - x^2 + x$$
$$-3x^2+3x+5$$
$$-$$
$$-3x^2+3x-3$$
$$8$$

Therefore, on division, we get :

Quotient: $x^2 + x - 3$

Remainder: 8

(iii) Given, Dividend = p(x) = $x^4 - 5x + 6 = x^4 + 0x^3 + 0x^2 - 5x + 6$

Divisor = $2 - x^2 = -x^2 - 2$ $x^4 + 0x^3$

$$-x^2-2$$
$$-x^2+2 \,\big)\, x^4+0x^3+0x^2-5x+6$$
$$-$$
$$x^4+0x^3-2x^2$$
$$2x^2-5x+6$$
$$-$$
$$2x^2+0x-4$$
$$-5x+10$$

Question 2: Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:
(i) $t^2 - 3$, $2t^4 + 3t^3 - 2t^2 - 9t - 12$
(ii) $x^2 + 3x + 1$, $3x^4 + 5x^3 - 7x^2 + 2x + 2$
(iii) $x^2 + 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

Answer:

(i) Given , first polynomial = t^2-3

Second polynomial = $2t^4 +3t^3-2t^2 -9t-12$

$2t^2 + 3t + 4$

$t^2-3 \overline{)\, 2t^4 + 3t^3 - 2t^2 - 9t - 12}$

$-\;2t^4 + 0t^3 - 6t^2$

$3t^3 + 4t^2 - 9t - 12$

$-\;3t^3 + 0t^2 - 9t$

$4t^2 + 0t - 12$

$-\;4t^2 + 0t - 12$

0

We see, the remainder is 0. Therefore, t^2-3 is a factor of $2t^2$+3t+4.

(ii) Given , first polynomial = x^2+3x+1

Second polynomial = $3x^4+5x^3-7x^2+2x+2$

$3x^2 - 4x + 2$

$x^2+3x+1 \overline{)\, 3x^4 + 5x^3 - 7x^2 + 2x + 2}$

$-\;3x^4 + 9x^3 + 3x^2$

$-4x^3 - 10x^2 + 2x + 2$

$-\;-4x^3 - 12x^2 - 4x$

$2x^2 + 6x + 2$

$-\;2x^2 + 6x + 2$

0

We see , the remainder is 0. Therefore , x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$

(iii) Given, first polynomial = x^3-3x+1

Second Polynomial = $x^5-4x^3+x^2+3x+1$

$$
\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x^3-3x+1 & x^5 + 0x^4 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 + 0x^4 - 3x^3 + x^2 \\
\hline
 & \quad - x^3 + 0x^2 + 3x + 1 \\
 & \quad - x^3 + 0x^2 + 3x - 1 \\
\hline
 & \qquad\qquad 2
\end{array}
$$

We see, the remainder is 2 ($\neq 0$). Therefore x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$

Question 3: Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Answer: As the given polynomial equation has degree 4, hence there will be total 4 roots.

Given, $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are zeroes of polynomial f(x).

Hence, $(x - \sqrt{\frac{5}{3}})\ (x + \sqrt{\frac{5}{3}}) = x^2 - \frac{5}{3} = 0$

$(3x^2-5)=0$, is a factor of given polynomial f(x). $-6x^3+3x^2-510x-5\ 2x+1$

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
3x^2-5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\
 & 3x^4 \qquad - 5x^2 \\
 & (-) \qquad (+) \\
\hline
 & +6x^3 + 3x^2 - 10x - 5 \\
 & -6x^3 \qquad - 10x \\
 & (+) \qquad (+) \\
\hline
 & 3x^2 \qquad -5 \\
 & 3x^2 \qquad -5 \\
 & (-) \qquad (+) \\
\hline
 & \qquad 0 \\
\hline
\end{array}
$$

Therefore, $3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)(x^2 + 2x + 1)$

And on further factorizing $(x^2 + 2x + 1)$ we get,

$x^2 + 2x + 1 = x^2 + x + x + 1 = 0$

Or, $x(x + 1) + 1(x + 1) = 0$

or, $(x + 1)(x + 1) = 0$

So, its zeroes are: x = (−1) and x = (−1).

Hence, all the four zeroes are (-1), (-1), $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

Question 4: On dividing x^3 - $3x^2$ + x + 2 by a polynomial g(x), the quotient and remainder were x – 2 and –2x + 4, respectively. Find g(x).

Answer: Given that, dividend, p(x) = $x^3 - 3x^2 + x + 2$
Quotient = (x – 2) and Remainder is – 2x + 4

So, as we know that,

Dividend = Divisor × Quotient + Remainder

or, $x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$

or, $x^3 - 3x^2 + x + 2 - (-2x + 4) = g(x) \times (x - 2)$

or, $x^3 - 3x^2 + x + 2 + 2x - 4 = g(x) \times (x - 2)$

or, $\frac{x^3 - 3x^2 + 3x - 2}{x-2} = g(x)$

Question 5: Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and
(i) deg p(x) = deg q(x)
(ii) deg q(x) = deg r(x)
(iii) deg r(x) = 0

Answer: According to the division algorithm, dividend p(x) and divisor g(x) are two polynomials, where g(x)≠0.

(i) Let, p(x) = $2x^2 + 2x + 8$
g(x) = 2
q(x) = $x^2 + x + 4$
r(x) = 0
clearly, p(x) is divisible by q(x) and remainder r(x) = 0
And we can see that, the degree of quotient is equal to the degree of dividend.
Hence, division algorithm is satisfied.

(ii) Let, p(x) = $x^2 + x$
g(x) = x + 1
q(x) = x
r(x) = 0
Clearly, p(x) is divisible by q(x) and remainder r(x) = 0
And we can see that, the degree of quotient is equal to the degree of remainder.
Hence, division algorithm is satisfied.

(iii) Let, p(x) = $x^2 + 1$
g(x) = x
So, Clearly, q(x) = x
r(x) = 1

Hence, we can see that, the degree of remainder here is 0.
Therefore, division algorithm is satisfied here.

Exercise 2.4

Question 1: Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 – 5x + 2$; $\frac{1}{2}$, 1, -2
(ii) $x^3 – 4x^2 + 5x – 2$; 2, 1, 1

Answer: (i) $p(x) = 2x^3 + x^2 – 5x + 2$

So, $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5x + 2$
$= \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right) - \left(\frac{5}{2}\right) + 2$
$= -2 + 2 = 0$

$p(1) = 2(1)^3 + (1)^2 – 5(1) + 2$
$= 2 + 1 – 5 + 2 = 0$

$p(-2) = 2(-2)^3 + (-2)^2 -5(-2) + 2$
$= -16 + 4 + 10 + 2 = 0$

Hence, it is proved that $\frac{1}{2}$, 1, -2 are the zeroes of the given eq.

Now, comparing with $ax^3 + bx^2 + cx + d = 0$, we get,
a = 2
b = 1
c = (-5)
d = 2

As we know, if α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then;
$\alpha + \beta + \gamma = -\left(\frac{b}{a}\right)$
$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
$\alpha\beta\gamma = -\left(\frac{d}{a}\right)$.

Therefore, putting the values of zeroes of the polynomial,
$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\left(\frac{1}{2}\right) = -\left(\frac{b}{a}\right)$

$\alpha\beta + \beta\gamma + \gamma\alpha = (\frac{1}{2} \times 1) + [1 \times (-2)] + (-2 \times \frac{1}{2}) = -\left(\frac{5}{2}\right) = \frac{c}{a}$

$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = -\left(\frac{2}{2}\right) = -\left(\frac{d}{a}\right)$

Hence, the relationship between the zeroes and the coefficients are satisfied.

(ii) $p(x) = x^3 – 4x^2 + 5x – 2$

So, $p(2) = (2)^3 – 4(2)^2 + 5(2) – 2 = 8 – 16 + 10 – 2 = 0$
$p(1) = (1)^3 – 4(1)^2 + 5(1) – 2 = 1 – 4 + 5 – 2 = 0$

Hence, 2, 1, 1 are the zeroes of the eq.

Now, comparing with $ax^3 + bx^2 + cx + d = 0$, we get,
a = 1
b = (-4)
c = 5
d = (-2)
Therefore, putting the values of zeroes of the polynomial,
$\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = -\left(\frac{-4}{1}\right) = -\left(\frac{b}{a}\right)$

$\alpha\beta + \beta\gamma + \gamma\alpha = (2\times 1) + (1 \times 1) + (2 \times 1) = 2 + 1 + 2 = 5 = \left(\frac{5}{1}\right) = \frac{c}{a}$

$\alpha \beta \gamma = 2 \times 1 \times 1 = 2 = -\left(\frac{-2}{1}\right) = -\left(\frac{d}{a}\right)$

Hence, the relationship between the zeroes and the coefficients are satisfied.

Question 2: Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Answer: Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be α, β, γ.
Hence, according to the qs. $\alpha + \beta + \gamma = -\left(\frac{b}{a}\right) = \frac{2}{1}$
$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = -\left(\frac{7}{1}\right)$
$\alpha \beta \gamma = -\left(\frac{d}{a}\right) = -\left(\frac{14}{1}\right)$

Therefore, a = 1, b = (-2), c = (-7), d = 14

Hence, the required cubic polynomial is $x^3 - 2x^2 - 7x + 14$

Question 3: If the zeroes of the polynomial $x^3 – 3x^2 + x + 1$ are a – b, a, a + b, find a and b.

Answer: Let α, β, and γ be the zeroes of polynomial $x^3 – 3x^2 + x + 1$.

Then, α = a – b
β = a
γ = a + b
Hence, sum of the zeroes = α + β + γ
or, 3 = (a – b) + a + (a + b)
or, 3 = 3a
or, a = 1……………………………………………………………(1)

product of the zeroes = α β γ

or, $-1 = (a - b)\,a\,(a + b)$
or, $-1 = a(a^2 - b^2)$
or, $-1 = a^3 - ab^2$……………………………………………………(2)

Putting the value of a form eq.(1) and eq.(2) we get,
$1^3 - 1b^2 = -1$
or, $1 - b^2 = -1$
or, $b^2 = 2$
or, $b = \pm\sqrt{2}$
Hence, $a = 1$ and $b = \pm\sqrt{2}$

Question 4: If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm\sqrt{3}$, find other zeroes.

Answer: The degree is 4, so there will be 4 roots. Two roots are given that, $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

Therefore, $[x - (2 + \sqrt{3})]\,[x - (2 - \sqrt{3})] = 0$
or, $(x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) = 0$
or, $x^2 - 4x + 1 = 0$

Hence,

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & (-)\quad(+)\quad(-) \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & (+)\quad(-)\quad(+) \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & (+)\quad(-)\quad(+) \\
\hline
 & 0 \\
\hline
\end{array}
$$

So, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$
On further factorizing $(x^2 - 2x - 35)$ we get,

$x^2 - (7 - 5)x - 35 = x^2 - 7x + 5x + 35 = 0$
or, $x(x - 7) + 5(x - 7) = 0$
or, $(x + 5)(x - 7) = 0$

So, its zeroes are (-5) and 7.

Therefore, all four zeroes of the given polynomial equation are: $2+\sqrt{3}$, $2-\sqrt{3}$, -5 and 7.