Chapter 7: Triangles

Exercise:7.1(Multiple Choice questions)

Question 1: Which of the following is not a criterion for congruence of triangles?
(a) SAS (b) ASA (c) SSA (d) SSS
Thinking Process
For a triangle to be congruent the equal angles must be included between the pairs of equal sides. So, the SAS congruence rule holds but not ASS or SSA rule.

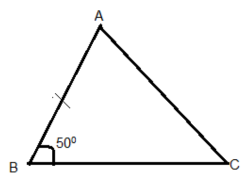
Answer: (c) We know that two triangles are congruent if the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.
Also, the criterion for congruence of triangles is SAS (Side-Angle-Side), ASA (Angle-Side-Angle), SSS (Side-Side-Side) and RHS (right angle-hypotenuse-side).
So, SSA is not a criterion for the congruence of triangles.

Question 2:
If AB = QR, BC = PR and CA = PQ, then
(a) ΔABC ≅ ΔQRP (b) ΔCBA ≅ ΔPRQ
(c) ΔBAC ≅ ΔRQP (d) ΔPQR ≅ ΔBCA

Answer: (b) We know that, if ΔRST is congruent to ΔUVW i.e., ΔRST = ΔUVW, then sides of ΔRST fall on corresponding equal sides of ΔUVW and angles of ΔRST fall on corresponding equal angles of ΔUVW.
Here, given AB = QR, BC = PR and CA = PQ, which shows that AB covers QR, BC covers PR and CA covers PQ i.e., A corresponds to Q, B corresponds to R and C correspond to P.
or A↔Q, B↔R, C↔P
Under this correspondence,
ΔABC ≅ ΔQRP, so option (a) is incorrect,
or ΔBAC ≅ ΔRQP, so option (c) is incorrect,
or ΔCBA ≅ ΔPRQ, so option (b) is correct,
or ΔBCA ≅ ΔRPQ, so option (d) is incorrect.

Question 3: In ΔABC,if AB = AC and ∠B = 50°, then ∠C is equal to
(a) 40° (b) 50° (c) 80° (d)130°

Answer: (b)
Given, triangle ABC, such that, AB = AC and ∠B = 50^0
In triangle ABC, AB = AC



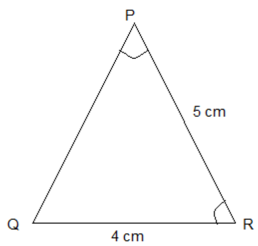
∠C = ∠D ………………[angles opposite to equal sides are equal]
or, ∠C = 50^0 [as, ∠B = 50^0 , given]

Question 4: In ΔABC, if BC = AB and ∠B = 80°, then ∠A is equal to
(a) 80° (b) 40° (c) 50° (d) 100°

Answer: (c) is the correct option.
Given, triangle ABC such that BC = AB and ∠80^0
In triangle ABC AB = BC and ∠C = ∠D. ……………………………(1)[angles opposite to equal sides are equal]
We know that the sum of all angles of a triangle is 180^0
Thus, ∠A + ∠B + ∠C = 180^0
or, ∠A + 80^0 + ∠A = 180^0
or, 2∠A = $180^0 – 80^0 = 100^0$
or, ∠A = 50^0

Question 5: In ΔPQR, if ∠R = ∠P and QR = 4 cm and PR = 5 cm. Then, the length of PQ is
(a) 4 cm (b) 5 cm (c) 2 cm (d) 2.5 cm

Answer: The correct answer is (a)
Given, triangle PQR, such that ∠R = ∠P and QR = 4 cm and PR = 5cm

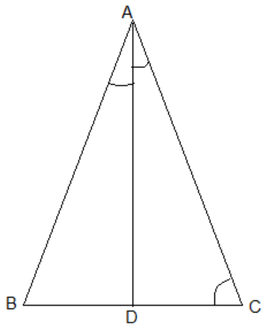


In a triangle, PQR, ∠R = ∠P
or, PQ = QP [sides opposite to equal angles are equal]
or, PQ = 4 cm [As, QR = 4 cm]

Question 6: If D is a point on the side BC of a ΔABC such that AD bisects ∠BAC. Then,
(a) BD = CD (b) BA > BD (c) BD > BA (d)CD > CA
Thinking Process
(i) Firstly, use the property, exterior angle of a triangle is greater than the interior opposite angle.
(ii) Secondly, use the property that in a triangle, the side opposite to the greater angle is longer.

Answer: (b) is the correct option.
Given, triangle ABC such that AB bisects ∠BAC.

∠BAD = ∠CAD…………………………………………………..(1)



In triangle ABC, ∠BDA = is an exterior angle.
Thus, ∠BDA > ∠CAD……………………………………………(1)
or, ∠BDA > ∠BAD [eq (1)]
or, BA > BD

Question 7: It is given that ΔABC = ΔFDE and AB = 5 cm, ∠B = 40° and ∠A = 80°, then Which of the following is true?
(a) DF = 5 cm, ∠F = 60° (b) DF = 5 cm, ∠E =60°
(c) DE = 5 cm, ∠E = 60° (d) DE = 5 cm, ∠D = 40°

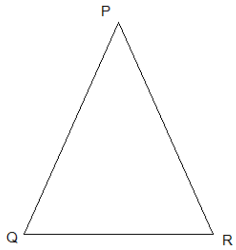
Answer: (b)

Question 8: If two sides of a triangle are of lengths 5 cm and 1.5 cm, then the length of the third side of the triangle cannot be
(a) 3.6 cm (b) 4.1 cm (c) 3.8 cm (d) 3.4 cm
Thinking Process
Use the condition that, sum of any two sides of a triangle is greater than the third side and the difference of any two sides is less than the third side.

Answer: (d) Given, the length of the two sides of a triangle is 5 cm and 1.5 cm, respectively.
Let sides AB = 5 cm and CA = 1.5 cm
We know that a closed figure formed by three intersecting lines (or sides) is called a triangle, if the difference of two sides < third side and sum of two sides > third side
∴ 5-1.5 < BC and 5+1.5 > BC
or, 3.5 < BC and 6.5 > BC
Here, we see that options (a), (b) and (c) satisfy the above inequality but option (d) does not satisfy the above inequality.

Question 9: In ΔPQR, if ∠R > ∠Q, then
(a) QR > PR (b) PQ > PR (c) PQ < PR (d) QR < PR

Answer: (b)
Given, ∠R > ∠Q



Or, PQ > PR [side opposite to greater angle is longer]

Question 10: In ΔABC and ΔPQR, if AB = AC, ∠C =∠P and ∠B = ∠Q, then the two triangles are

(a) isosceles but not congruent **(b) isosceles and congruent**

(c) congruent but not isosceles **(d) Neither congruent nor isosceles**

Answer: (a)

Question 11: In ΔABC and ΔDEF, AB = FD and ∠A = ∠D. The two triangles will be congruent by SAS axiom, if

(a) BC = EF **(b) AC = DE** **(c)AC=EF** **(d) BC = DE**

Answer: (b) Given, in ΔABC and ΔDEF, AB = DF and ∠A = ∠D

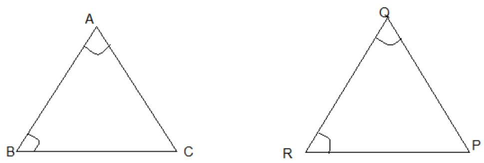
We know that, two triangles will be congruent by ASA rule if two angles and the included side of one triangle are equal to the two angles and the included side of another triangle.

∴ AC = DE

Exercise 7.2 (Very short Answer type question)

Question 1: In ΔABC and ΔPQR, ∠A = ∠Q and ∠B =∠R. Which side of ΔPQR should be equal to side AB of ΔABC so that the two triangles are congruent? Give a reason for your answer.

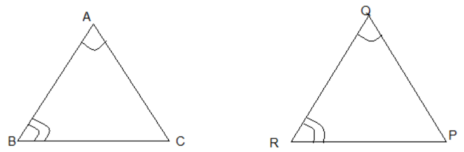
Answer:



Since AB and QR are included between equal angles. Hence, the side of ΔPQR is QR which should be equal to side AB of ΔABC, so that the triangles are congruent by the rule ASA.

Question 2:
In ΔABC and ΔPQR, ∠A = ∠Q and ∠B = ∠R. Which side of ΔPQR should be equal to side BC of ΔABC so that the two triangles are congruent? Give a reason for your answer.

Answer: We have given, in ΔABC and ΔPQR,
∠A = ∠Q and ∠B = ∠R
Since two pairs of angles are equal in two triangles.
We know that, two triangles will be congruent by the AAS rule if two angles and the side of one triangle are equal to the two angles and the side of another triangle.
∴ BC = RP



Question 3:
'If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be. Is the statement true? Why?

Answer: No, because in the congruent rule, the two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle i.e., SAS rule.

Question 4:
'If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent.' Is the statement true? Why?

Answer: No, because sides must be corresponding sides.

Question 5:
Is it possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm? Give a reason for your answer.

Answer: No, it is not possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm because here we see that some of the lengths of two sides are equal

to the third side i.e., 4+3 = 7.
As we know that, the sum of any two sides of a triangle is greater than its third side, so a given statement is not correct.

Question 6:
It is given that ΔABC ≅ ΔRPQ. Is it true to say that BC = QR? Why?

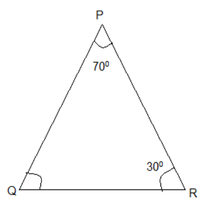
Answer: No, we know that two triangles are congruent if the sides and angles of one triangle are equal to the corresponding side and angles of another triangle.
Here ΔABC ≅ ΔRPQ
AB = RP, BC = PQ and AC = RQ Hence, it is not true to say that BC = QR.

Question 7:
If ΔPQR ≅ ΔEOF, then is it true to say that PR = EF? Give a reason for your answer.

Answer: Yes, if ΔPQR ≅ ΔEDF, then it means that corresponding angles and their sides are equal because we know that, two triangles are congruent, if the sides and angles of one triangle are equal to the corresponding sides and angles of another triangle.
Here, ΔPQR ≅ ΔEDF
∴ PQ = ED, QR = DF and PR = EF
Hence, it is true to say that PR = EF.

Question 8:
In ΔPQR, ∠P = 70° and ∠R = 30°. Which side of this triangle is the longest? Give a reason for your answer.

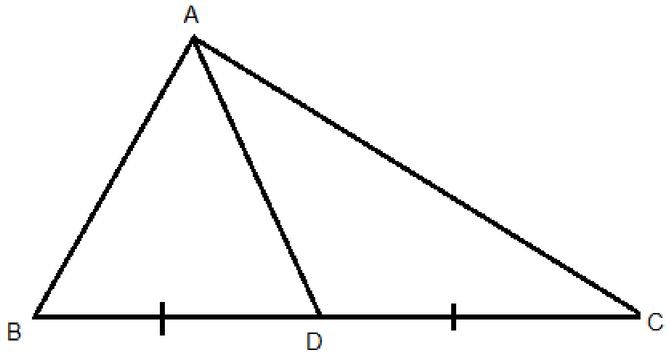
Answer: Given, in ΔPQR, ∠P = 70° and ∠R = 30°.
We know that the sum of all the angles of a triangle is 180°.
∠P + ∠Q + ∠R = 180°



$\angle Q = 180^0 - (70^0 + 30^0) = 80^0$
We know that here ∠Q is the longest, so side PR is the longest.
[∴ since in a triangle, the side opposite to the largest angle is the longest]

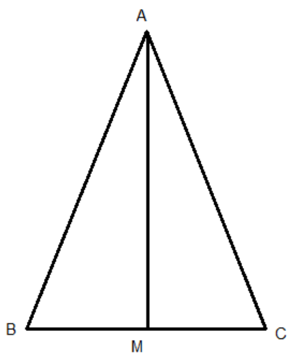
Question 9:
AD is a median of the ΔABC. Is it true that AB + BC +CA > 2AD? Give a reason for your answer.

Answer: yes,



In the triangle ABD, we have
AB + BD > AD ……………………………….(1)
In triangle ACD, we have, AC + CD > AD ……………………..(2)
On adding eq(1) and (2), we get,
(AB + BD + AC + CD) > 2AD
or, (AB + BD + CD + AC) > 2AD
Hence, AB + BC + AC > 2AD [as, BC = BD + CD]

Question 10:
M is a point on side BC of a triangle ABC such that AM is the bisector of ∠BAC. Is it true to say that perimeter of the triangle is greater than 2 AM? Give a reason for your answer?
Answer: Yes in Triangle ABC, M is the point of side BC such that AM is the bisector of ∠BAC.
In triangle ABM, AB + BM > BM …………………………………..(1)
In triangle ACM, AC + CM > AM ………………………………………(2)



On adding eq(1) and eq(2), we get,
(AB + BM + AC + CM) > 2AM
or, (AB + BM + MC + AC) > 2AM

or, AB + BC + AC(perimeter of triangle ABC) > 2AM

Question 11:
Is it possible to construct a triangle with lengths of its sides as 9 cm, 7 cm and 17 cm? Give a reason for your answer.

Answer: No. Here, we see that 9 + 7 =16 < 17
i.e., the sum of two sides of a triangle is less than the third side.
Hence, it contradicts the property that the sum of two sides of a triangle is greater than the third side. Therefore, it is not possible to construct a triangle with given sides.

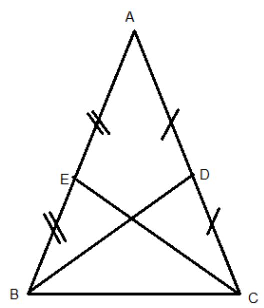
Question 12:
Is it possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm? Give a reason for your answer.

Answer: Yes, because in each case the sum of two sides is greater than the third side. i.e.,
7 + 4 >8,
8+ 4 >7, 7 + 8 >4
Hence, it is possible to construct a triangle with given sides.

Exercise 7.3 (Short type question)

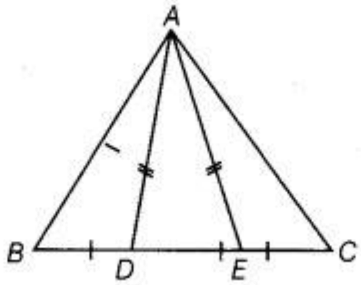
Question 1: ABC is an isosceles triangle with AB = AC and BD, CE are its two medians. Show that BD = CE.

Answer: Given ΔABC is an isosceles triangle in which AB = AC and BD, CE are its two medians.
To show BD = CE.



Proof: In triangle ABC and ACE,
AB = AC [Given]
∠A = ∠A [common angle]
AD = AE
Since, AB = AC
or, $\frac{1}{2}$AB = $\frac{1}{2}$AC
or, AE = AD
As D is the mid-point of AC and E is the mid-point of AB.
thus, ΔABD ≅ ΔACE [by SAS]
Or, BD = CE [CPCT]

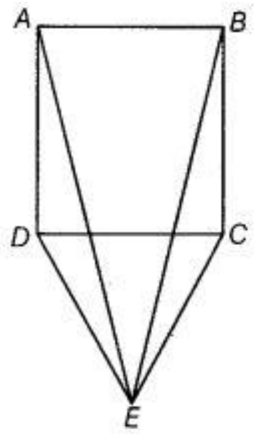
Question 2: In the figure, D and E are points on side BC of a ΔABC such that BD = CE and AD = AE. Show that ΔABD ≅ ΔACE.



Answer: D and E are the points on side BC of a triangle ABC such that BD = CE and AD = AE [Given]
To proof: ΔABD = ΔACE
proof: We have, AD = AE [given]
∠ADE = ∠AED …………………………………..(1)
We have, $\angle ADB + \angle ADE = 180^0$ [Linear pair axiom]
or, $\angle ADB = 180^0 - \angle ADE = 180^0 - \angle AED$ ...[from eq(1)]

In triangle ABD and ACE,
$\angle ADB = \angle AEC$ [As, $\angle AEC + \angle AED = 180^0$, linear pair axiom]
BD = CE
AD = AE
ΔABD ≅ACE [SAS congruency]

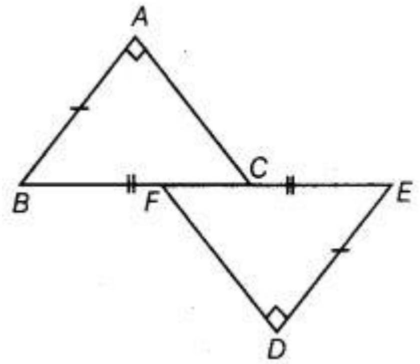
Question 3:
In the given figure, ΔCDE is an equilateral triangle formed on a side CD of a square ABCD. Show that ΔADE ≅ ΔBCE.



Answer: In the figure, triangle CDE is an equilateral triangle formed on a side CD of a square ABCD. [Given]
To proof: ΔADE ≅ ΔBCE
Proof: In triangle ADE and BCE,
DE = CE [sides of an equilateral triangle]
∠ADE = ∠BCE [Since, ∠ADE = ∠BCD = 90^0 and ∠EDC = 60^0,
∠ADE = $90^0 + 60^0 = 150^0$ and, ∠BCE = $90^0 + 60^0 = 150^0$]

AD = BC
ΔADE ≅ ΔBCE [SAS congruence]

Question 4:
In the figure, BA ⊥ AC, DE ⊥ DF such that BA = DE and BF = EC. Show that ΔABC ≅ ΔDEF.

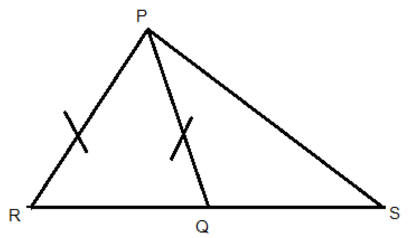


Thinking Process
Use the RHS congruence rule to show the given result

Answer: In the figure BA ⊥ AC, DE ⊥ DF such that BA = DE and BF = EC [Given]
To proof: ΔABC ≅ ΔDEF
Proof: Since, BF = EC
On adding CF on both sides, we get
BF + CF = EC + CF
BC = EF ………………………………………………..(1)
In triangle ABC and DEF,
∠A = ∠D = 90^0 [BA ⊥ AC and DE ⊥ DF]
BC = EF [from eq (1)]
BA = DE [given]
Hence, ΔABC = ΔDEF…………..[RHS congruence]

Question 5:
If Q is a point oh the side SR of a ΔPSR such that PQ = PR, then prove that PS > PQ.
Thinking Process
Use the property of a triangle that if two sides are equal then their opposite angles are also equal. Also, use the property that side opposite to a greater angle is longer.

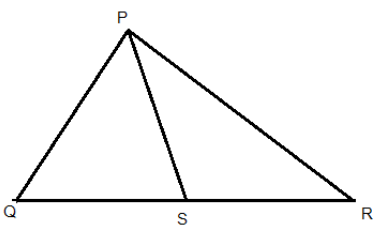
Answer: Given: In triangle PSR, Q is a point on the side SR such that PQ = PR
To prove: PS > PQ



In triangle PQR, PQ = PR [Given]
∠R = ∠PQR ……………………(1) [angles opposite to equal sides are equal]
But, ∠PQR > ∠S …………………(2) [Exterior angle of a triangle is greater than each of the interior angle]
From eq(1) and (2),
∠R = ∠S
or, PS > PR [side opposite to greater angle is longer]
or, PS > PQ [As, PQ = PR]

Question 6:
S is any point on the side QR of a ΔPQR. Show that PQ + QR + RP > 2 PS.
Thinking Process
Use the inequality of a triangle i.e., sum of two sides of a triangle is greater than the third side. Further, show the required result.

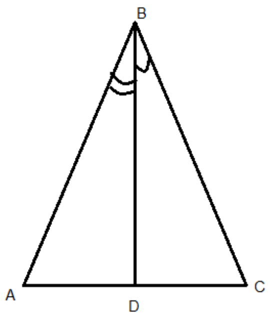
Answer: In triangle QPR, S is any point on side QR.



To proof: PQ + QR + RP > 2PS
Proof: In triangle PQS, PQ + QS > PS…………………..(1) [Sum of two sides of a triangle is greater than the third side]
Similarly, in Triangle PRS, SR + RP > PS ……………. (2) [Sum of two sides of a triangle is greater than the third side]
On adding eq(1) and (2) we get,
PQ + QS + SR + RP > 2PS
or, PQ + (QS + SR) + RP > 2PS
or, PQ + QR + RP > 2PS [as, QR = QS + SR]

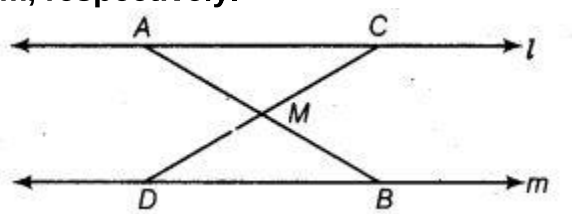
Question 7:
D is any point on side AC of a ΔABC with AB = AC. Show that CD < BD.

Answer: In triangle ABC, D is any point on side AC
To proof: CD > BD or BD > CD



Proof: In triangle ABC, AC = AB [Given]
∠ABC = ∠ACB ………………..(1)[Angles opposite to equal sides are equal]
In triangle ABC and DBC,
$\angle ABC > \angle DBC$ [Since, ∠DBC is an internal angle of ∠B]
or, $\angle ACB > \angle DBC$ …..[From eq (1)]
or, $BD > CD$ [Side opposite to greater angle is longer]
or, $CD < BD$

Question 8:
In the given figure l || m and M is the mid-point of a line segment AB. Show that M is also the mid-point of any line segment CD, having its endpoints on l and m, respectively.



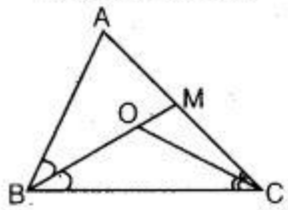
Answer: In the figure, *l* ∥ *m* and M is the mid-point of a line-segment AB i.e., AM = BM
To proof: MC = MD
proof: *l* ∥ *m* [Given]

∠BAC = ∠ABD [Alternate interior angles]
∠AMC = ∠BMD [Vertically opposite angles]
In triangle AMC and BMD,
∠BAC = ∠ABD [proved above]
AM = AM [Given]
and ∠AMC = ∠BMD [Proved above]
Hence, ΔAMC ≅ ΔBMD [SAS congruence]
or, MC = MD [CPCT]

Question 9:
Bisectors of the angles B and C of an isosceles triangle with AB = AC intersect each other at O. BO is produced to a point M. Prove that ∠MOC = ∠ABC.
Answer:

Given Lines, OB and OC are the angle bisectors of $\angle B$ and $\angle C$ of an isosceles ΔABC such that $AB = AC$ which intersect each other at O and BO is produced to M.

To prove $\angle MOC = \angle ABC$.



Proof In ΔABC, $AB = AC$ [given]

$\Rightarrow$ $\angle ACB = \angle ABC$ [angles opposite to equal sides are equal]

$\frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$ [dividing both sides by 2]

$\Rightarrow$ $\angle OCB = \angle OBC$ ...(i)

[since, OB and OC are the bisector of $\angle B$ and $\angle C$]

Now, $\angle MOC = \angle OBC + \angle OCB$

[exterior angle of a triangle is equal to the sum of two interior angles]

$\Rightarrow$ $\angle MOC = \angle OBC + \angle OBC$ [from Eq. (i)]

$\Rightarrow$ $\angle MOC = 2\angle OBC$

$\Rightarrow$ $\angle MOC = \angle ABC$ [since, OB is the bisector of $\angle B$]

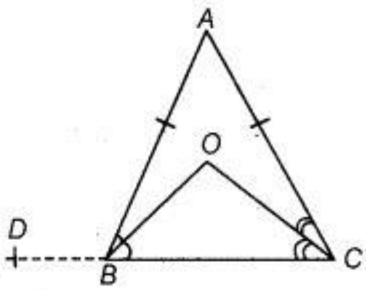
Hence proved.

Question 10:
Bisectors of the angles B and C of an isosceles ΔABC with AB = AC intersect each other at O. Show that the external angle adjacent to ∠ABC is equal to ∠BOC.
Answer:

Given ΔABC is an isosceles triangle in which $AB = AC$, BO and CO are the bisectors of $\angle ABC$ and $\angle ACB$ respectively intersect at O.

To show $\angle DBA = \angle BOC$

Construction Line CB produced to D.



Proof In ΔABC, $AB = AC$ [given]

$\angle ACB = \angle ABC$

[angles opposite to equal sides are equal]

$\Rightarrow$ $\frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$ [on dividing both sides by 2]

$\Rightarrow$ $\angle OCB = \angle OBC$ ...(i)

[$\because$ BO and CO are the bisectors of $\angle ABC$ and $\angle ACB$]

In ΔBOC, $\angle OBC + \angle OCB + \angle BOC = 180°$ [by angle sum property of a triangle]

$\Rightarrow$ $\angle OBC + \angle OBC + \angle BOC = 180°$ [from Eq. (i)]

$\Rightarrow$ $2\angle OBC + \angle BOC = 180°$

$\Rightarrow$ $\angle ABC + \angle BOC = 180°$ [$\because$ BO is the bisector of $\angle ABC$]

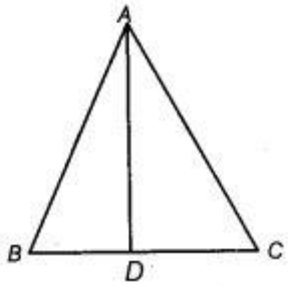
$\Rightarrow$ $180° - \angle DBA + \angle BOC = 180°$ [$\because$ DBC is a straight line]

$\Rightarrow$ $-\angle DBA + \angle BOC = 0$

$\Rightarrow$ $\angle DBA = \angle BOC$

Question 11:

In the following figure if AD is the bisector of ∠BAC, then prove that AB > BD.



Solution:

Given ABC is a triangle such that AD is the bisector of ∠BAC. To prove AB > BD.

Proof Since, AD is the bisector of ∠BAC.

But ∠BAD = CAD …(i)

∴ ∠ADB > ∠CAD

[exterior angle of a triangle is greater than each of the opposite interior angle]

∴ ∠ADB > ∠BAD [from Eq. (i)]

AB > BD [side opposite to greater angle is longer]

Hence proved.

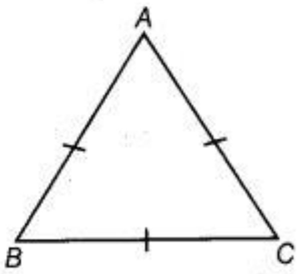
<u>Exercise 7.4: (Long Answer Type Questions)</u>

Question 1:

Find all the angles of an equilateral triangle.

Solution:

Let ABC be an equilateral triangle such that $AB = BC = CA$



We have, $AB = AC \Rightarrow \angle C = \angle B$

[angles opposite to equal sides are equal]

Let $\angle C = \angle B = x°$ ...(i)

Now, $BC = BA$

$\Rightarrow$ $\angle A = \angle C$ ...(ii)

[angles opposite to equal sides are equal]

From Eqs. (i) and (ii),

$\angle A = \angle B = \angle C = x$

Now, in ΔABC, $\angle A + \angle B + \angle C = 180°$ [by angle sum property of a triangle]

$\Rightarrow$ $x + x + x = 180°$

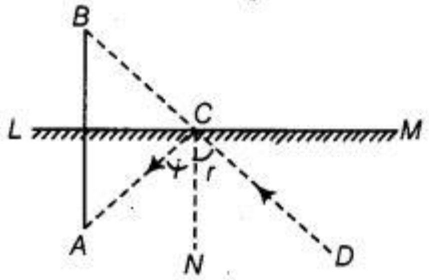
$\Rightarrow$ $3x = 180°$

$\therefore$ $x = 60°$

Hence, $\angle A = \angle B = \angle C = 60°$

Question 2:

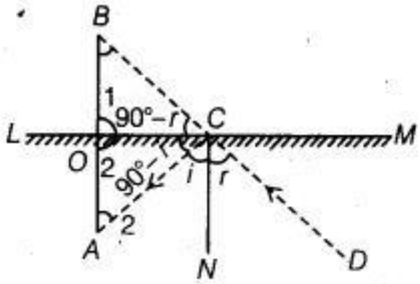
The image of an object placed at point A before a plane mirror LM is seen at point B by an observer at D as shown in the figure. Prove that the image is as far behind the mirror as the object is in front of the mirror.



Solution:

Given An object *OA* placed at a point *A*, *LM* be a plane mirror, *D* be an observer and *OB* is the image.

To prove The image is as far behind the mirror as the object is in front of the mirror *i.e.*, $OB = OA$.



Proof ∵ $CN \perp LM$ and $AB \perp LM$

⇒	$AB \parallel CN$	
	$\angle A = \angle i$	[alternate interior angles]...(i)
	$\angle B = \angle r$	[corresponding angles]...(ii)
Also,	$\angle i = \angle r$	[∵ incident angle = reflected angle]...(iii)
From Eqs. (i), (ii) and (iii),	$\angle A = \angle B$	
In ΔCOB and ΔCOA,	$\angle B = \angle A$	[proved above]
	$\angle 1 = \angle 2$	[each 90°]
and	$CO = CO$	[common side]
∴	$\Delta COB \cong \Delta COA$	[by AAS congruence rule]
⇒	$OB = OA$	[by CPCT]

Hence proved.

Alternate Method

In ΔOBC and ΔOAC,	$\angle 1 = \angle 2$	[each 90°]
Also,	$\angle i = \angle r$	[∵ incident angle = reflected angle]...(i)

On multiplying both sides of Eq. (i) by −1 and then adding 90° both sides, we get

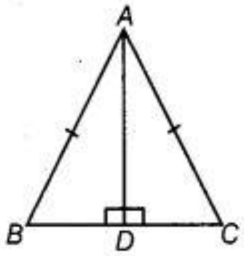
$$90° - \angle i = 90° - \angle r$$

⇒	$\angle ACO = \angle BCO$	
and	$OC = OC$	[common side]
∴	$\Delta OBC \cong \Delta OAC$	[by ASA congruence rule]
⇒	$OB = OA$	[by CPCT]

Hence, the image is as far behind the mirror as the object is in front of the mirror.

Question 3:

ABC is an isosceles triangle with AB = AC and D is a point on BC such that AD ⊥BC (see figure). To prove that ∠BAD = ∠CAD, a student proceeded as follows



In ΔABD and ΔACD, we have

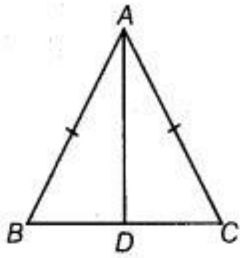
	$AB = AC$	[given]
	$\angle B = \angle C$	[because $AB = AC$]
and	$\angle ADB = \angle ADC$	
Therefore,	$\Delta ABD \cong \Delta ACD$	[by AAS congruence rule]
So,	$\angle BAD = \angle CAD$	[by CPCT]

What is the defect in the above arguments?

Solution:

In ΔABC,	$AB = AC$	
$\Rightarrow$	$\angle ACB = \angle ABC$	[angles oppsoite to the equal sides are equal]

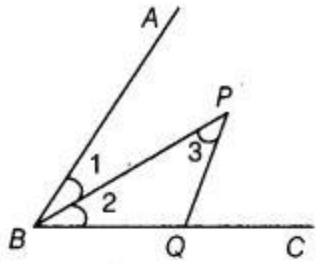
In ΔABD and ΔACD,



	$AB = AC$	[given]
	$\angle ABD = \angle ACD$	[proved above]
	$\angle ADB = \angle ADC$	[each 90°]
$\therefore$	$\Delta ABD \cong \Delta ACD$	[by AAS]
So,	$\angle BAD = \angle CAD$	[by CPCT]

So, the defect in the given argument is that firstly prove $\angle ABD = \angle ACD$
Hence, $\angle ABD = \angle ACD$ is defect.

Question 4:
P is a point on the bisector of ∠ABC. If the line through P, parallel to BA meets BC at Q, prove that BPQ is an isosceles triangle.
Solution:
Given we have P is a point on the bisector of ∠ABC and draw the line through P parallel to BA and meet BC at Q.



To prove ΔBPQ is an isosceles triangle.

Proof $\angle 1 = \angle 2$ [$\because BP$ is bisector of $\angle B$ (given)]

Now, $\angle 1 = \angle 3$ [alternate interior angles as $PQ \parallel AB$]

$\therefore$ $\angle 2 = \angle 3$

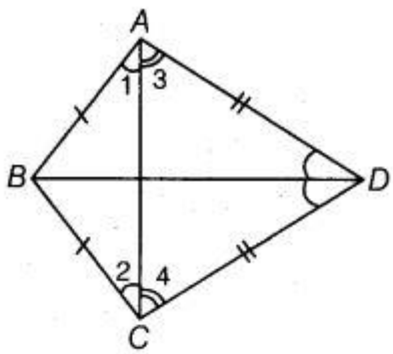
$\Rightarrow$ $PQ = BQ$ [sides opposite to equal angles are equal]

Hence, ΔBPQ is an isosceles triangle. **Hence proved.**

Question 5:
ABCD is a quadrilateral in which AB = BC and AD = CD. Show that BD bisects both the angles ABC and ADC.
Thinking Process
Firstly, use the property that if two sides of a triangle are equal, then their opposite angles are equal. Further, show that ΔBAD and ΔBCD are congruent by the SAS rule.
Solution:

Given $ABCD$ is a quadrilateral in which $AB = BC$ and $AD = CD$.

To show BD bisects both the angles ABC and ADC.



Proof Since, $AB = BC$ (given)

$\therefore$ $\angle 2 = \angle 1$ ...(i)

[angles opposite to equal sides are equal]

and $AD = CD$ [given]

$\Rightarrow$ $\angle 4 = \angle 3$ ...(ii)

[angles opposite to equal sides are equal]

On adding Eqs. (i) and (ii), we get

$$\angle 2 + \angle 4 = \angle 1 + \angle 3$$

$\Rightarrow$ $\angle BCD = \angle BAD$ ...(iii)

In ΔBAD and ΔBCD,

$AB = BC$ [given]

$\angle BAD = \angle BCD$ [from Eq. (iii)]

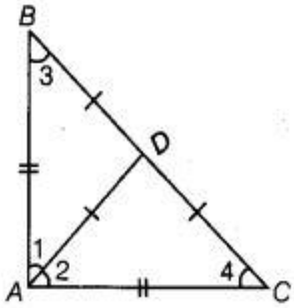
and $AD = CD$ [given]

$\therefore$ $\Delta BAD \cong \Delta BCD$ [by SAS congruence rule]

Hence, $\angle ABD = \angle CBD$ and $\angle ADB = \angle CDB$ *i.e.*, BD bisects the angles ABC and ADC. [by CPCT]

Question 6:
ABC is a right triangle with AB = AC. If bisector of ∠A meets BC at D, then prove that BC = 2AD.
Solution:

Given ΔABC is a right angled triangle with $AB = AC$, AD is the bisector of $\angle A$.



To prove $BC = 2AD$

Proof In ΔABC, $AB = AC$ [given]

$\Rightarrow$ $\angle C = \angle B$ ...(i)

[angles opposite to equal sides are equal]

Now, in right angled ΔABC, $\angle A + \angle B + \angle C = 180°$

[by angle sum property of a triangle in 180°]

$\Rightarrow$ $90° + \angle B + \angle B = 180°$ [from Eq. (i)]

$\Rightarrow$ $2\angle B = 90°$

$\Rightarrow$ $\angle B = 45°$

$\Rightarrow$ $\angle B = \angle C = 45°$

or $\angle 3 = \angle 4 = 45°$

Now, $\angle 1 = \angle 2 = 45°$ [$\because AD$ is bisector of $\angle A$]

$\therefore$ $\angle 1 = \angle 3$, $\angle 2 = \angle 4$

$\Rightarrow$ $BD = AD$, $DC = AD$ ...(ii)

[sides opposite to equal angles are equal]

Hence, $BC = BD + CD = AD + AD \Rightarrow BC = 2\ AD$ [from Eq. (ii)]

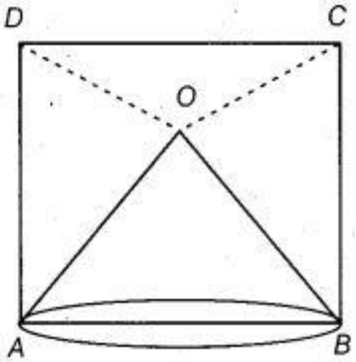
Hence proved.

Question 7:

O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that ΔOCD is an isosceles triangle.

Solution:

Given O is a point in the interior of a square $ABCD$ such that ΔOAB is an equilateral triangle.



Construction Join OC and OD.

To show ΔOCD is an isosceles triangle.

Proof Since, AOB is an equilateral triangle.

$\therefore$ $\angle OAB = \angle OBA = 60°$ ...(i)

Also, $\angle DAB = \angle CBA = 90°$ ...(ii) [each angle of a square is 90°]

[$\because$ $ABCD$ is a square]

On subtracting Eq. (i) from Eq. (ii), we get

$$\angle DAB - \angle OAB = \angle CBA - \angle OBA = 90° - 60°$$

i.e., $\angle DAO = \angle CBO = 30°$

In ΔAOD and ΔBOC,

$AO = BO$ [given]

[all the side of an equilateral triangle are equal]

$\angle DAO = \angle CBO$ [proved above]

and $AD = BC$ [sides of a square are equal]

$\therefore$ $\Delta AOD \cong \Delta BOC$ [by SAS congruence rule]

Hence, $OD = OC$ [by CPCT]

In ΔCOD,

$$OC = OD$$

Hence, ΔCOD is an isosceles triangle. **Hence proved.**

Question 8:

ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, AB = AC and DB = DC. Show that AD is the perpendicular bisector of BC.

Solution:

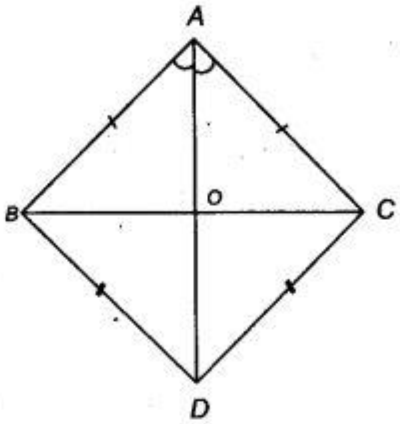
Given Two ΔABC and ΔDBC are formed on the same base BC such that A and D lie on the opposite sides of BC such that $AB = AC$ and $DB = DC$. Also AD intersects BC at O.

To show AD is the perpendicular bisector of BC *i.e.,* $AD \perp BC$ and AD bisects BC.

Proof In ΔABD and ΔACD,

	$AB = AC$	[given]
	$AD = AD$	[common side]
and	$BD = DC$	[given]
$\therefore$	$\Delta ABD \cong \Delta ACD$	[by SSS congruence rule]
$\Rightarrow$	$\angle BAD = \angle CAD$	[by CPCT]
i.e.,	$\angle BAO = \angle CAO$	

In ΔAOB and ΔAOC,



	$AB = AC$	[given]
	$AO = OA$	[common side]
and	$\angle BAO = \angle CAO$	[proved above]
$\therefore$	$\Delta AOB \cong \Delta AOC$	[by SAS congruence rule]
$\Rightarrow$	$BO = OC$	[by CPCT]
and	$\angle AOB = \angle AOC$	[by CPCT]...(i)
But	$\angle AOB + \angle AOC = 180°$	[linear pair axiom]
$\Rightarrow$	$\angle AOB + \angle AOB = 180°$	[from Eq. (i)]
$\Rightarrow$	$2\angle AOB = 180°$	
$\Rightarrow$	$\angle AOB = \frac{180°}{2} = 90°$	

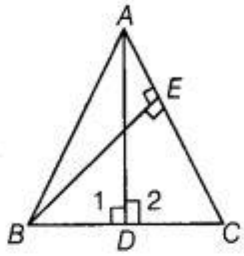
Hence, $AD \perp BC$ and AD bisects BC *i.e.,* AD is the perpendicular bisector of BC.

Question 9:

If ABC is an isosceles triangle in which AC = BC, AD and BE are respectively two altitudes to sides BC and AC, then prove that AE = BD.

Solution:

Given ΔABC is an isosceles triangle in which $AC = BC$. Also, AD and BE are two altitudes to sides BC and AC, respectively.



To prove $AE = BD$.

Proof In ΔABC, $AC = BC$ [given]

$\angle ABC = \angle CAB$

[angles opposite to equal sides are equal]

i.e., $\angle ABD = \angle EAB$ ...(i)

In ΔAEB and ΔBDA,

$\angle AEB = \angle ADB = 90°$ [given, $AD \perp BC$ and $BE \perp AC$]

$\angle EAB = \angle ABD$ [from Eq. (i)]

and $AB = AB$ [common side]

$\therefore$ $\Delta AEB \cong \Delta BDA$ [by AAS congruence rule]

$\Rightarrow$ $AE = BD$ [by CPCT]

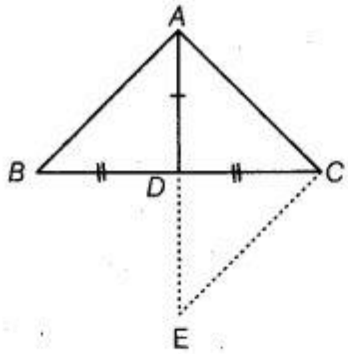
Hence proved.

Question 10:

Prove that sum of any two sides of a triangle is greater than twice the median concerning the third side.

Solution:

Given In ΔABC, AD is a median.



Construction

Produce AD to a point E such that $AD = DE$ and join CE.

To prove $AC + AB > 2AD$

Proof In ΔABD and ΔECD,

$AD = DE$ [by construction]

$BD = CD$ [given AD is the median]

and $\angle ADB = \angle CDE$ [vertically opposite angle]

$\therefore$ $\Delta ABD \cong \Delta ECD$ [by SAS congruence rule]

$\Rightarrow$ $AB = CE$ [by CPCT] ...(i)

Now, in ΔAEC,

$AC + EC > AE$ [sum of two sides of a triangle is greater than the third side]

$\therefore$ $AC + AB > 2AD$ [from Eq. (i) and also taken that $AD = DE$]

Hence proved.

Question 11:

Show that in a quadrilateral ABCD, AB + BC + CD + DA< 2 (BD + AC)

Thinking Process

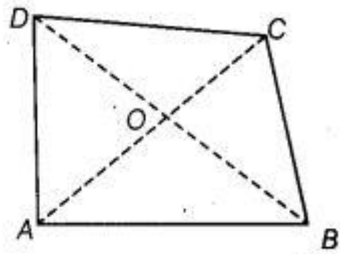
Firstly, draw a quadrilateral ABCD. Further, use the property of a triangle that the sum of two sides of a triangle is greater than the third side and show the required result.

Solution:

Given $ABCD$ is a quadrilateral.

To show $AB + BC + CD + DA < 2(BD + AC)$

Construction Join diagonals AC and BD.



Proof In ΔOAB, $OA + OB > AB$ ...(i)

[sum of two sides of a triangle is greater than the third side]

In ΔOBC, $OB + OC > BC$ ...(ii)

[sum of two sides of a triangle is greater than the third side]

In ΔOCD, $OC + OD > CD$ ...(iii)

[sum of two sides of a triangle is greater than the third side]

In ΔODA, $OD + OA > DA$ ...(iv)

[sum of two sides of a triangle is greater than the third side]

On adding Eqs. (i), (ii), (iii) and (iv), we get

$$2[(OA + OB + OC + OD] > AB + BC + CD + DA$$

$$\Rightarrow \quad 2[(OA + OC) + (OB + OD)] > AB + BC + CD + DA$$

$$\Rightarrow \quad 2(AC + BD) > AB + BC + CD + DA$$

[$\because OA + OC = AC$ and $OB + OD = BD$]

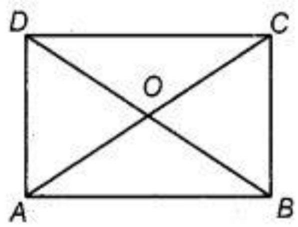
$$\Rightarrow \quad AB + BC + CD + DA < 2(BD + AC)$$

Question 12:

Show that in a quadrilateral ABCD, AB + BC + CD + DA > AC + BD.

Solution:

Given $ABCD$ is a quadrilateral.



Construction

Join diagonals AC and BD.

To show $AB + BC + CD + DA > AC + BD$

In ΔABC, $AB + BC > AC$ ...(i)

[sum of two sides of a triangle is greater than the third side]

In ΔBCD, $BC + CD > BD$ ...(ii)

[sum of two sides of a triangle is greater than the third side]

In ΔCDA, $CD + DA > AC$ ...(iii)

[sum of two sides of a triangle is greater than the third side]

In ΔDAB, $DA + AB > BD$ ...(iv)

[sum of two sides of a triangle is greater than the third side]

On adding Eqs. (i), (ii), (iii) and (iv), we get

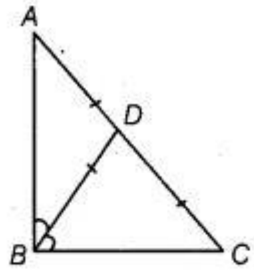
$$2(AB + BC + CD + DA) > 2(AC + BD)$$

$$\Rightarrow \quad AB + BC + CD + DA > AC + BD$$

Question 13:
In A ABC, D is the mid-point of side AC such that BD = ½ AC. Show that ∠ABC is a right angle.
Solution:

Given In ΔABC, D is the mid-point of AC *i.e.*, $AD = CD$ such that $BD = \frac{1}{2} AC$.

To show $\angle ABC = 90°$



Proof We have, $BD = \frac{1}{2} AC$ ...(i)

Since, D is the mid-point of AC.

$\therefore$ $AD = CD = \frac{1}{2} AC$ ...(ii)

From Eqs. (i) and (ii),

$AD = CD = BD$

In ΔDAB, $AD = BD$ [proved above]

$\therefore$ $\angle ABD = \angle BAD$ ...(iii)

[angles opposite to equal sides are equal]

In ΔDBC, $BD = CD$ [proved above]

$\therefore$ $\angle BCD = \angle CBD$ ...(iv)

[angles opposite to equal sides are equal]

In ΔABC, $\angle ABC + \angle BAC + \angle ACB = 180°$ [by angle sum property of a triangle]

$\Rightarrow$ $\angle ABC + \angle BAD + \angle DCB = 180°$

$\Rightarrow$ $\angle ABC + \angle ABD + \angle CBD = 180°$ [from Eqs.(iii) and (iv)]

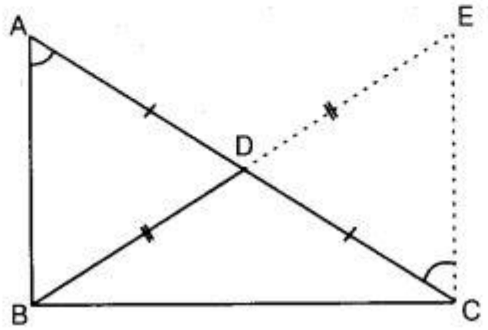
$\Rightarrow$ $\angle ABC + \angle ABC = 180°$

$\Rightarrow$ $2\angle ABC = 180°$

$\Rightarrow$ $\angle ABC = 90°$

Question 14:
In a right triangle, prove that the line-segment joining the mid-point of the hypotenuse to the opposite vertex is half the hypotenuse.
Solution:

Given In ΔABC, $\angle B = 90°$ and D is the mid-point of AC.
Construction Produce BD to E such that $BD = DE$ and join EC.
To prove $BD = \frac{1}{2} AC$



Proof In ΔADB and ΔCDE,

	$AD = DC$	[$\because D$ is mid-point of AC]
	$BD = DE$	[by construction]
and	$\angle ADB = \angle CDE$	[vertically opposite angles]
$\therefore$	$\Delta ADB \cong \Delta CDE$	[by SAS congruence rule]
$\Rightarrow$	$AB = EC$	[by CPCT]
and	$\angle BAD = \angle DCE$	[by CPCT]

But $\angle BAD$ and $\angle DCE$ are alternate angles.
So, $EC \parallel AB$ and BC is a transversal.

$\therefore$	$\angle ABC + \angle BCE = 180°$	[cointerior angles]
$\Rightarrow$	$90° + \angle BCE = 180°$	[$\because \angle ABC = 90°$, given]
$\Rightarrow$	$\angle BCE = 180° - 90°$	
$\Rightarrow$	$\angle BCE = 90°$	

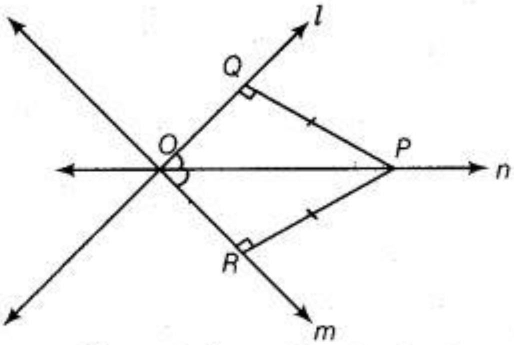
In ΔABC and ΔECB,

	$AB = EC$	[proved above]
	$BC = CB$	[common side]
and	$\angle ABC = \angle ECB$	[each 90°]
$\therefore$	$\Delta ABC \cong \Delta ECB$	[by SAS congruence rule]
$\Rightarrow$	$AC = EB$	[by CPCT]
$\Rightarrow$	$\frac{1}{2} EB = \frac{1}{2} AC$	[dividing both sides by 2]
$\Rightarrow$	$BD = \frac{1}{2} AC$	**Hence proved.**

Question 15:
Two lines l and m intersect at point 0 and P is a point on passing through point 0 such that P is equidistant from l and m. Prove that n is the bisector of the angle formed by l and m.

Solution:

Given Two lines l and m intersect at the point O and P is a point on a line n passing through O such that P is equidistant from l and m. i.e., $PQ = PR$.



To prove n is the bisector of the angle formed by l and m i.e., n is the bisector of $\angle QOR$.

Proof In ΔOQP and ΔORP,

$$\angle PQO = \angle PRO = 90°$$

[since, P in equidistant from l and m, so PQ and PR should be perpendicular to lines l and m respectively]

$OP = OP$ [common side]

$PQ = PR$ [given]

$\therefore$ $\Delta OQP \cong \Delta ORP$ [by RHS congruence rule]

$\Rightarrow$ $\angle POQ = \angle POR$ [by CPCT]

Hence, n is the bisector of $\angle QOR$. **Hence proved.**

Question 16:

The line segment joining the mid-points M and N of parallel sides AB and DC, respectively of a trapezium ABCD is perpendicular to both the sides AB and DC. Prove that AD = BC.

Solution:

Given In trapezium $ABCD$, points M and N are the mid-points of parallel sides AB and DC respectively and join MN, which is perpendicular to AB and DC.

To prove $AD = BC$

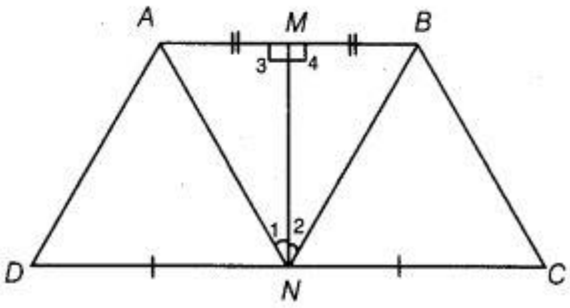
Proof Since, M is the mid-point of AB.

$\therefore$	$AM = MB$	
Now, in ΔAMN and ΔBMN,	$AM = MB$	[proved above]
	$\angle 3 = \angle 4$	[each 90°]
	$MN = MN$	[common side]
$\therefore$	$\Delta AMN \cong BMN$	[by SAS congruence rule]
$\therefore$	$\angle 1 = \angle 2$	[by CPCT]

On multiplying both sides of above equation by −1 and than adding 90° both sides, we get

$$90^\circ - \angle 1 = 90^\circ - \angle 2$$

$\Rightarrow$ $\angle AND = \angle BNC$ ...(i)



Now, in ΔADN and ΔBCN,

	$\angle AND = \angle BNC$	[from Eq. (i)]
	$AN = BN$	[$\because \Delta AMN \cong \Delta BMN$]
and	$DN = NC$	[$\because N$ is the mid-point of CD (given)]
$\therefore$	$\Delta ADN \cong \Delta BCN$	[by SAS congruence rule]
Hence,	$AD = BC$	[by CPCT]

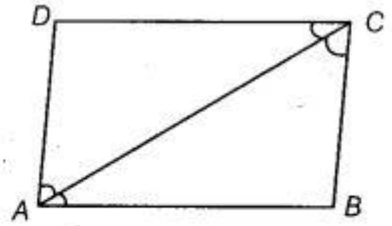
Hence proved.

Question 17:

If ABCD is a quadrilateral such that diagonal AC bisects the angles A and C, then prove that AB = AD and CB = CD.

Solution:

Given In a quadrilateral $ABCD$, diagonal AC bisects the angles A and C.



To prove $AB = AD$ and $CB = CD$

Proof In ΔADC and ΔABC,

	$\angle DAC = \angle BAC$	[$\because$ AC is the bisector of $\angle A$ and $\angle C$]
	$\angle DCA = \angle BCA$	[$\because$ AC is the bisector of $\angle A$ and $\angle C$]
and	$AC = AC$	[common side]
$\therefore$	$\Delta ADC \cong \Delta ABC$	[by ASA congruence rule]
	$AD = AB$	[by CPCT]
and	$CD = CB$	[by CPCT]

Hence proved.

Question 18:

If ABC is a right-angled triangle such that AB = AC and bisector of angle C intersects the side AB at D, then prove that AC + AD = BC.

Solution:

Given In right angled ΔABC, $AB = AC$ and CD is the bisector of $\angle C$.

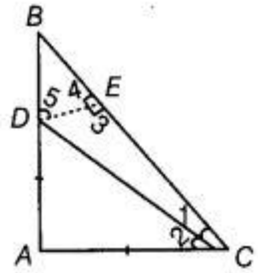
Construction Draw $DE \perp BC$.

To prove $AC + AD = BC$

Proof In right angled ΔABC, $AB = AC$ and BC is a hypotenuse [given]

$\therefore \quad \angle A = 90^\circ$

In ΔDAC and ΔDEC, $\angle A = \angle 3 = 90^\circ$



$\angle 1 = \angle 2$ [given, CD is the bisector of $\angle C$]

$DC = DC$ [common sides]

$\therefore \quad \Delta DAC \cong \Delta DEC$ [by AAS congruence rule]

$\Rightarrow \quad DA = DE.$ [by CPCT]... (i)

and $\quad AC = EC$ ... (ii)

In ΔABC, $AB = AC$

$\angle C = \angle B$

[angles opposite to equal sides are equal]...(iii)

Again, in ΔABC, $\angle A + \angle B + \angle C = 180^\circ$

[by angle sum property of a triangle]

$\Rightarrow \quad 90^\circ + \angle B + \angle B = 180^\circ$ [from Eq. (iii)]

$\Rightarrow \quad 2\angle B = 180^\circ - 90^\circ$

$\Rightarrow \quad 2\angle B = 90^\circ$

$\Rightarrow \quad \angle B = 45^\circ$

In ΔBED, $\angle 5 = 180^\circ - (\angle B + \angle 4)$

[by angle sum property of a triangle]

$= 180^\circ - (45^\circ + 90^\circ)$

$= 180^\circ - 135^\circ = 45^\circ$

$\therefore \quad \angle B = \angle 5$

$\Rightarrow \quad DE = BE$ [$\because$ sides opposite to equal angles are equal] ...(iv)

From Eqs. (i) and (iv),

$DA = DE = BE$ ...(v)

$\because \quad BC = CE + EB$

$= CA + DA$ [from Eqs. (ii) and (v)]

$\therefore \quad AD + AC = BC$ **Hence proved.**

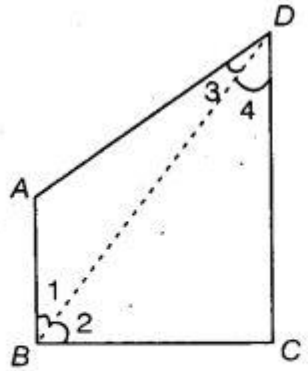
Question 19:

If AB and CD are the smallest and largest sides of a quadrilateral ABCD, out of ∠B and ∠D decide which is greater.

Solution:

Given In quadrilateral $ABCD$, AB is the smallest and CD is the largest side

To find $\angle B > \angle D$ or $\angle D > \angle B$.



Construction Join BD.

Now, in ΔABD, $AD > AB$ [since, AB is the smallest side in $ABCD$]

$\Rightarrow$ $\angle 1 > \angle 3$ [angle opposite to larger side is greater]...(i)

In ΔBCD, $CD > BC$ [since, CD is the largest side in $ABCD$]

$\Rightarrow$ $\angle 2 > \angle 4$ [angle opposite to larger side is greater]...(ii)

On adding Eqs. (i) and (ii), we get

$$\angle 1 + \angle 2 > \angle 3 + \angle 4$$

Hence, $\angle B > \angle D$

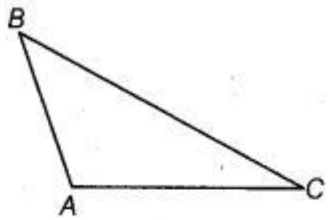
Question 20:

Prove that in a triangle, other than an equilateral triangle, the angle opposite the longest side is greater than 2/3 of a right angle.

Solution:

Consider ΔABC in which BC is the longest side.

To prove $\angle A = \frac{2}{3}$ right angle



Proof In ΔABC, $BC > AB$.

[consider BC is the largest side]

$\Rightarrow$ $\angle A > \angle C$ ...(i)

[angle opposite the longest side is greatest]

and $BC > AC$

$\Rightarrow$ $\angle A > \angle B$ ...(ii)

[angle opposite the longest side is greatest]

On adding Eqs. (i) and (ii), we get

$$2\angle A > \angle B + \angle C$$

$\Rightarrow$ $2\angle A + \angle A > \angle A + \angle B + \angle C$ [adding $\angle A$ both sides]

$\Rightarrow$ $3\angle A > \angle A + \angle B + \angle C$

$\Rightarrow$ $3\angle A > 180°$ [sum of all the angles of a triangle is 180°]

$\Rightarrow$ $\angle A > \frac{2}{3} \times 90°$

i.e., $\angle A > \frac{2}{3}$ of a right angle **Hence proved.**

Question 21:

If ABCD is quadrilateral such that AB = AD and CB = CD, then prove that AC is

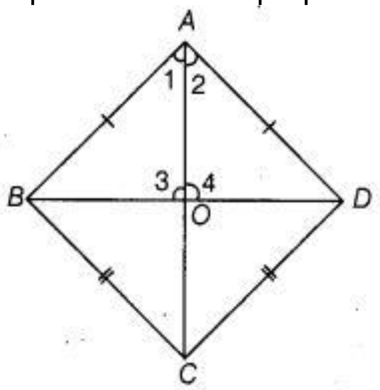
the perpendicular bisector of BD.

Solution:

Given In quadrilateral ABCD, AB= AD and CB = CD.

Construction Join AC and BD.

To prove AC is the perpendicular bisector of BD.



Proof In ΔABC and ΔADC,

	$AB = AD$	[given]
	$BC = CD$	[given]
and	$AC = AC$	[common side]
∴	$\Delta ABC \cong \Delta ADC$	[by SSS congruence rule]
⇒	$\angle 1 = \angle 2$	[by CPCT]
Now, in ΔAOB and ΔAOD,	$AB = AD$	[given]
⇒	$\angle 1 = \angle 2$	[proved above]
and	$AO = AO$	[common side]
∴	$\Delta AOB \cong \Delta AOD$	[by SAS congruence rule]
⇒	$BO = DO$	[by CPCT]
and	$\angle 3 = \angle 4$	[by CPCT]...(i)
But	$\angle 3 + \angle 4 = 180°$	[linear pair axiom]
	$\angle 3 + \angle 3 = 180°$	[from Eq. (i)]
⇒	$2\angle 3 = 180°$	
⇒	$\angle 3 = \frac{180°}{2}$	
∴	$\angle 3 = 90°$	

i.e., *AC* is perpendicular bisector of *BD*.