

Important Questions
Chapter 1: Number Systems

Q.1: Find five rational numbers between 1 and 2.

Solution: We have to find five rational numbers between 1 and 2.

So, let us write the numbers with denominator $5 + 1 = 6$

Thus, $6/6 = 1$, $12/6 = 2$

From this, we can write the five rational numbers between $6/6$ and $12/6$ as:

$7/6, 8/6, 9/6, 10/6, 11/6$

Q.2: Find five rational numbers between $3/5$ and $4/5$.

Solution: We have to find five rational numbers between $3/5$ and $4/5$.

So, let us write the given numbers by multiplying with $6/6$, (here $6 = 5 + 1$)

Now,

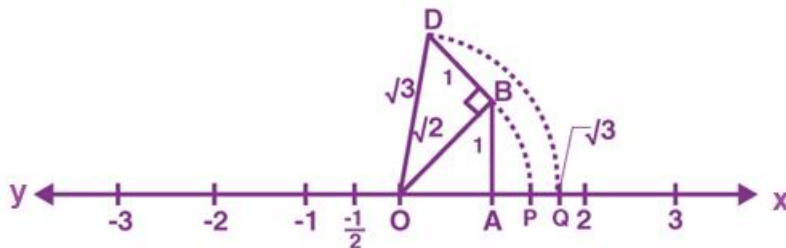
$$3/5 = (3/5) \times (6/6) = 18/30$$

$$4/5 = (4/5) \times (6/6) = 24/30$$

Thus, the required five rational numbers will be: $19/30, 20/30, 21/30, 22/30, 23/30$

Q.3: Locate $\sqrt{3}$ on the number line.

Solution:



Construct BD of unit length perpendicular to OB (here, $OA = AB = 1$ unit) as shown in the figure.

By Pythagoras theorem,

$$OD = \sqrt{(2 + 1)} = \sqrt{3}$$

Taking O as the centre and OD as radius, draw an arc that intersects the number line at point Q using a compass. Therefore, Q corresponds to the value of $\sqrt{3}$ on the number line.

Q.4: Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is rational.

Solution: No, since the square root of a positive integer 16 is equal to 4. Here, 4 is a rational number.

Q.5: Find the decimal expansions of $10/3$, $7/8$ and $1/7$.

Solution:

3	3.333..
	10
	9
	10
	9
	10
	9
	10
	9
	1

8	0.875
	7.0
	64
	60
	56
	40
	40
	0

7	0.142857
	1.0
	7
	30
	28
	20
	14
	60
	56
	40
	35
	50
	49
	1

Therefore, $10/3 = 3.3333\dots$

$7/8 = 0.875$

$1/7 = 0.1428571\dots$

Q.6: Show that $0.3333\dots = 1/3$ can be expressed in the form p/q , where p and q are integers and $q \neq 0$.

Solution:

Let $x = 0.3333\dots$

Multiply with 10,

$10x = 3.3333\dots$

Now, $3.3333\dots = 3 + x$ (as we assumed $x = 0.3333\dots$)

Thus, $10x = 3 + x$

$10x - x = 3$

$9x = 3$

$x = 1/3$

Therefore, $0.3333\dots = 1/3$. Here, $1/3$ is in the form of p/q and $q \neq 0$.

Q.7: What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $1/17$? Perform the division to check your answer.

Solution:

$$\begin{array}{r}
 0.058823529411764705\dots \\
 \hline
 17 \overline{) 1.00} \\
 \underline{85} \\
 150 \\
 \underline{136} \\
 140 \\
 \underline{136} \\
 40 \\
 \underline{34} \\
 60 \\
 \underline{51} \\
 90 \\
 \underline{85} \\
 50 \\
 \underline{34} \\
 160 \\
 \underline{153} \\
 70 \\
 \underline{68} \\
 20 \\
 \underline{17} \\
 20 \\
 \underline{17} \\
 30 \\
 \underline{17} \\
 130 \\
 \underline{119} \\
 110 \\
 \underline{102} \\
 80 \\
 \underline{68} \\
 120 \\
 \underline{119} \\
 100 \\
 \underline{85} \\
 15
 \end{array}$$

Thus, $1/17 = 0.0588235294117647\dots$

Therefore, $1/17$ has 16 digits in the repeating block of digits in the decimal expansion.

Q.8: Find three different irrational numbers between the rational numbers $5/7$ and $9/11$.

Solution: The given two rational numbers are $5/7$ and $9/11$.

$$5/7 = 0.714285714\dots$$

$$9/11 = 0.81818181\dots$$

Hence, the three irrational numbers between $5/7$ and $9/11$ can be:

$$0.720720072000\dots$$

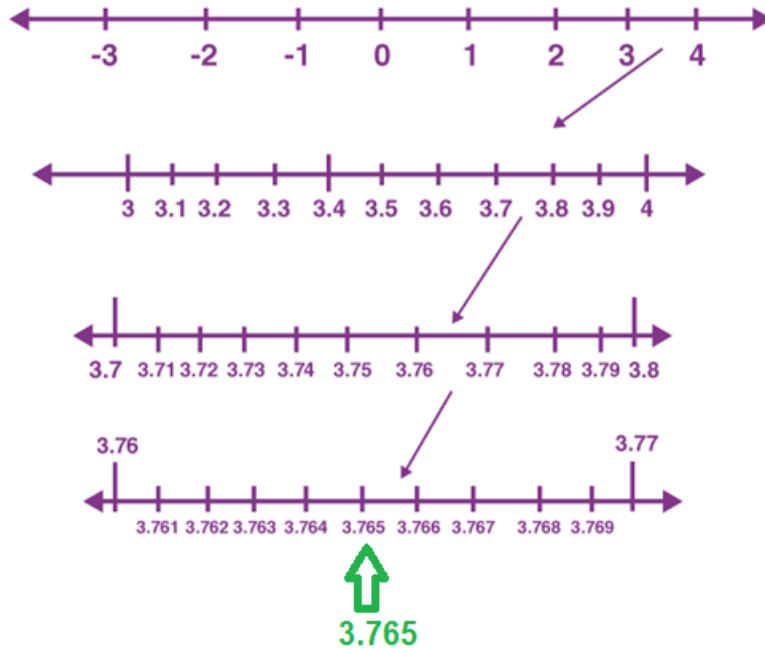
$$0.730730073000\dots$$

$$0.808008000\dots$$

Q.9: Visualise 3.765 on the number line, using successive magnification.

Solution:

Visualisation of 3.765 on the number line, using successive magnification is given below:



Q.10: Add $2\sqrt{2} + 5\sqrt{3}$ and $\sqrt{2} - 3\sqrt{3}$.

Solution:

$$\begin{aligned} & (2\sqrt{2} + 5\sqrt{3}) + (\sqrt{2} - 3\sqrt{3}) \\ &= 2\sqrt{2} + 5\sqrt{3} + \sqrt{2} - 3\sqrt{3} \\ &= (2 + 1)\sqrt{2} + (5 - 3)\sqrt{3} \\ &= 3\sqrt{2} + 2\sqrt{3} \end{aligned}$$

Q.11: Simplify: $(\sqrt{3} + \sqrt{7})(\sqrt{3} - \sqrt{7})$.

Solution:

$$\begin{aligned} & (\sqrt{3} + \sqrt{7})(\sqrt{3} - \sqrt{7}) \\ & \text{Using the identity } (a + b)(a - b) = a^2 - b^2, \\ & (\sqrt{3} + \sqrt{7})(\sqrt{3} - \sqrt{7}) = (\sqrt{3})^2 - (\sqrt{7})^2 \\ &= 3 - 7 \\ &= -4 \end{aligned}$$

Q.12: Rationalise the denominator of $1/[7 + 3\sqrt{3}]$.

Solution:

$$1/(7 + 3\sqrt{3})$$

By rationalizing the denominator,

$$= [1/(7 + 3\sqrt{3})] [(7 - 3\sqrt{3})/(7 - 3\sqrt{3})]$$

$$= (7 - 3\sqrt{3})/[(7)^2 - (3\sqrt{3})^2]$$

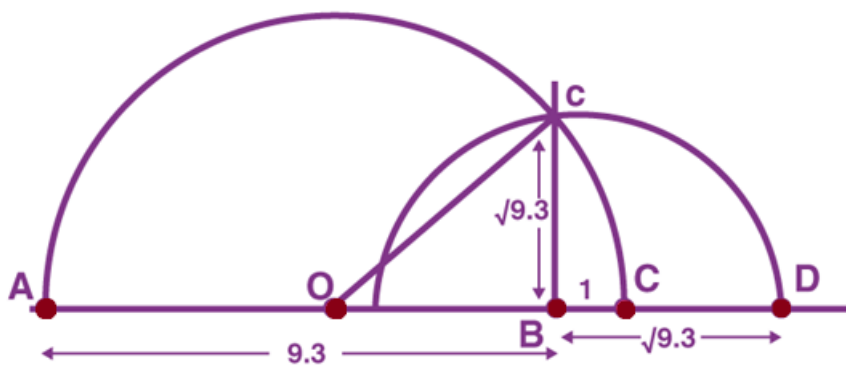
$$= (7 - 3\sqrt{3})/(49 - 27)$$

$$= (7 - 3\sqrt{3})/22$$

Q.13: Represent $\sqrt{9.3}$ on the number line.

Solution:

Representation of $\sqrt{9.3}$ on the number line is given below:



Q.14: Simplify:

- (i) $7^{2/3} \cdot 7^{1/5}$
(ii) $10^{1/2}/10^{1/4}$

Solution:

(i) $7^{2/3} \cdot 7^{1/5}$

Bases are equal, so add the powers.

$$7^{(2/3 + 1/5)}$$

$$= 7^{(10 + 3)/15}$$

$$= 7^{13/15}$$

(ii) $10^{1/2}/10^{1/4}$

Bases are equal, so subtract the powers.

$$= 10^{(1/2 - 1/4)}$$

$$= 10^{1/4}$$

Q.15: What is the product of a rational and an irrational number?

- Always an integer
- Always a rational number
- Always an irrational number
- Sometimes rational and sometimes irrational

Correct Answer: Option (c)

Explanation:

The product of a rational and an irrational number is always an irrational number.

For example, 2 is a rational number and $\sqrt{3}$ is irrational. Thus, $2\sqrt{3}$ is always an irrational number.

Q.16: What is the value of $(256)^{0.16} \times (256)^{0.09}$?

- a) 4
- b) 16
- c) 64
- d) 256.25

Correct answer: Option (a)

$$\begin{aligned}(256)^{0.16} \times (256)^{0.09} &= (256)^{(0.16 + 0.09)} \\ &= (256)^{0.25} \\ &= (256)^{(25/100)} \\ &= (256)^{(1/4)} \\ &= (4^4)^{(1/4)} \\ &= 4^{4(1/4)} \\ &= 4\end{aligned}$$

Q. 17: Are the square roots of all the positive integers irrational? If not, give an example of the square root of a number that is rational.

Solution: We know that the square root of every positive integer will not yield an integer.

We know that, $\sqrt{4}$ is 2, which is an integer. But $\sqrt{7}$ or $\sqrt{10}$ will give an irrational number. Thus, we can conclude that the square roots of every positive number are not an irrational number.