Chapter 2 – Polynomials
Exercise 2.1 (Multiple choice question)

Question 1: Which one of the following is a polynomial?

(a) $\frac{x^2}{2} - \frac{2}{x^2}$

(b) $\sqrt{2x} - 1$

(c) $x^2 + \frac{3x^{\frac{3}{2}}}{\sqrt{x}}$

(d) $\frac{x-1}{x+1}$

Answer: The correct option is (c).

(a) is not a polynomial, as $\frac{x^2}{2} - \frac{2}{x^2} = \frac{x^2}{2} - 2x^{-2}$, where exponent of x is (-2) which in not a whole number.

(b) is also not a polynomial, as $\sqrt{2x} - 1 = \sqrt{2}x^{\frac{1}{2}} - 1$ where exponent of x is $\left(-\frac{1}{2}\right)$ which is not a whole number.

(c) is a polynomial as exponents of x is a whole number.

(d) is not a polynomial as $\frac{x-1}{x+1}$ is a rational function.

Question 2: √2 is a polynomial of degree

(a) 2 (b) 0 (c) 1 (d)½

Answer: (b) √2 = -√2x°. Hence, √2 is a polynomial of degree 0, because exponent of x is 0.

Question 3: Degree of the polynomial $4x^4 + 0x^3 + 0x^5 + 5x + 7$ is

(a) 4 (b) 5 (c) 3 (d) 7

Answer: (a) Degree of $4x^4 + 0x^3 + 0x^5 + 5x + 7$ is equal to the highest power of variable x. Here, the highest power of x is 4, Hence, the degree of a polynomial is 4.

Question 4: Degree of the zero polynomial is

(a) 0 (b) 1

(c) any natural number (d) not defined

Answer: (d) The degree of zero polynomial is not defined, because in zero polynomial, the coefficient of any variable is zero i.e., $0x^2$ or $0x^5$,etc.
Hence, we cannot exactly determine the degree of variable.

Question 5: If p (x) = x2 – 2√2x + 1, then p (2√2) is equal to

(a) 0 (b) 1 (c) 4√2 (d) 8 √2 +1

Answer: (b) Given, $p(x) = x^2 – 2\sqrt{2}x + 1$ ……………………………….(1)
On putting $x = 2\sqrt{2}$ in Eq. (1), we get,
$P(2\sqrt{2}) = (2\sqrt{2})^2 – (2\sqrt{2})(2\sqrt{2}) + 1 = 8-8+1=1$

Question 6: The value of the polynomial $5x – 4x^2 + 3$, when $x = – 1$ is
(a)-6 (b) 6 (c) 2 (d) -2

Answer: (a) Let $p(x) = 5x – 4x^2 + 3$ ……………………………………….(1)
On putting x = -1 in Eq. (1), we get
$p(-1) = 5(-1) -4(-1)^2 + 3= - 5 – 4 + 3 = -6$

Question 7: If p (x) = x + 3, then p(x)+ p (- x) is equal to
(a) 3 (b) 2x (c) 0 (d) 6

Answer: (d) Given p(x) = x+3, put x = (-x) in the given equation, we get
p(-x) = -x+3
Now, p(x)+ p(-x) = x+ 3+ (-x)+ 3=6

Question 8: Zero of the zero polynomial is
(a) 0 (b) 1 (c) any real number (d) not defined

Answer: (c) Zero of the zero polynomial is any real number.
e.g., Let us consider zero polynomial be 0(x-k), where k is a real number For determining the zero, put (x-k) = 0 we get, x = k
Hence, zero of the zero polynomial be any real number.

Question 9: Zero of the polynomial p(x)=2x+5 is
(a) -2/5 (b) -5/2 (c)2/5 (d)5/2

Answer: (b) Given, p(x) = 2x+5
For zero of the polynomial, put p(x) = 0
∴ 2x + 5 = 0
or, -5/2
Hence, zero of the polynomial p(x) is -5/2.

Question 10: One of the zeroes of the polynomial $2x^2 + 7x – 4$ is
(a) 2 (b)½ (c)-1 (d)-2
Thinking Process
(i) Firstly, determine the factor by using splitting method.
(ii) Further, put the factors equals to zero, then determine the values of x.

Answer: (b) Let $p(x) = 2x^2 + 7x-4$
$= 2x^2 + 8x-x-4$ [by splitting middle term]
= 2x(x+ 4)-1(x+ 4)
= (2x-1)(x+ 4)
For zeroes of p(x), put p(x) = 0
or, (2x -1) (x + 4) = 0
or, 2x-1 = 0 and x + 4 = 0

or, $x = \frac{1}{2}$ and $x = (-4)$
Hence, one of the zeroes of the polynomial p(x) is ½.

Question 11: **If $x^{51} + 51$ is divided by $x + 1$, then the remainder is**
(a) 0 (b) 1 (c) 49 (d) 50

Answer: (d) Let $p(x) = x^{51} + 51$ ……………………………………………..…(1)
When we divide p(x) by x+1, we get the remainder p(-1)
On putting x= -1 in Eq. (1), we get $p(-1) = (-1)^{51} + 51 = -1 + 51 = 50$
Hence, the remainder is 50.

Question 12: If x + 1 is a factor of the polynomial $2x^2 + kx$, then the value of k is
(a) -3 (b) 4 (c) 2 (d)-2

Answer: (c) Let $p(x) = 2x^2 + kx$
Since, (x + 1) is a factor of p(x), then
p(-1)=0
2(-1)2 + k(-1) = 0
or, 2-k = 0
or, k= 2
Hence, the value of k is 2.

Question 13: x + 1 is a factor of the polynomial
(a) $x^3 + x^2 – x +1$ (b) $x^3 + x^2 + x+1$
(c) $x^4 + x^3 + x^2 +1$ (d) $x^4 + 3x^3 + 3x^2 + x +1$

Answer: (b) Let assume (x + 1) is a factor of $x^3 + x^2 + x+1$.
So, x = -1 is zero of $x^3 + x^2 + x+1$
$(-1)^3 + (-1)^2 + (-1) + 1 = 0$
or, -1+1-1 + 1 = 0
or, 0 = 0 Hence, our assumption is true.

Question 14: One of the factors of $(25x^2 – 1) + (1 + 5x)^2$ is
(a) 5 + x (b) 5 – x (c) 5x -1 (d) 10x

Answer: (d) Now, $(25x^2 -1) + (1 + 5x)^2$
$= 25x^2 -1 + 1 + 25x^2 + 10x$ [using identity, $(a + b)^2 = a^2 + b^2 + 2ab$]
$= 50x^2 + 10x = 10x (5x+ 1)$
Hence, one of the factor of given polynomial is 10x.

Question 15: The value of $249^2 – 248^2$ is
(a) 1^2 (b) 477 (c) 487 (d) 497

Answer: (d) Now, $249^2 – 248^2 = (249 + 248) (249 – 248)$ [using identity, $a^2 – b^2 = (a – b)(a + b)$]
= 497 x 1 = 497.

Question 16: The factorization of $4x^2 + 8x + 3$ is
(a) (x +1) (x + 3) (b) (2x+1) (2x + 3)
(c) (2x + 2) (2x + 5) (d) (2x -1) (2x – 3)

Answer: (b) Now, $4x^2 + 8x + 3 = 4x^2 + 6x + 2x + 3$ [by splitting middle term]
= 2x(2x+ 3) + 1 (2x+ 3)
= (2x + 3) (2x + 1)

Question 17: Which of the following is a factor of $(x+ y)^3 – (x^3 + y^3)$?
(a) $x^2 + y^2 + 2xy$ (b) $x^2 + y^2 – xy$ (c) xy^2 (d) 3xy

Answer: (d) Now, (x+ y)3 – $(x^3 + y^3)$ = (x + y) – (x + y)$(x^2 – xy + y^2)$
[using identity, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$] = (x+ y)[$(x+ y)^2$ -(x^2 -xy+ y^2)]
= (x+ y)(x^2+ y^2+ 2xy- x^2+ xy- y^2)
[using identity, $(a + b)^2 = a^2 + b^2 + 2ab$]
= (x + y) (3xy)
Hence, one of the factor of given polynomial is 3xy.

Question 18: The coefficient of x in the expansion of $(x + 3)^3$ is
(a) 1 (b) 9 (c) 18 (d) 27

Answer: (d) Now, $(x + 3)^3 = x^3 + 3^3 + 3x(3)(x + 3)$
[using identity, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$]
= x^3 + 27 + 9x (x + 3)
= x^3 +27 + $9x^2$+27x Hence, the coefficient of x in $(x + 3)^3$ is 27.

Question 19: If $\frac{x}{y} + \frac{y}{x} = -1$ (where x, y ≠ 0) then the value of $x^3 – y^3$ is,

(a) 1 (b) -1 (c) 0 (d) $\frac{1}{2}$

Answer: (c) Given, $\frac{x}{y} + \frac{y}{x} = -1$
or, $\frac{x^2+y^2}{xy}$ = -1

Or, $x^2 + y^2 = -xy$
or, $x^2 + y^2 + xy = 0$
Now, $x^3 – y^3 = (x – y)(x^2 + xy + y^2) = (x – y)\times 0 = 0$

Question 20: If $49x^2 – b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$, then the value of b is,
(a) 0
(b)$\frac{1}{\sqrt{2}}$
(c) $\frac{1}{4}$
(d)$\frac{1}{2}$

Answer: Given, $49x^2 – b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$

Or, $\left[49x^2 - (\sqrt{b})^2\right] = \left[(7x)^2 - \left(\frac{1}{2}\right)^2\right]$

Or, $49x^2 - (\sqrt{b})^2 = 49x^2 - \left(\frac{1}{2}\right)^2$

Or, $(\sqrt{b})^2 = \left(\frac{1}{2}\right)^2$

Or, $b = \frac{1}{4}$

Question 21: If a + b + c =0, then $a^3+b^3 + c^3$ is equal to
(a) 0 (b) abc (c) 3abc (d) 2abc

Answer: (d) Now, $a^3+b^3 + c^3 = (a+ b + c)(a^2 + b^2 + c^2 – ab – be – ca) + 3abc$
[using identity, $a^3+b^3 + c^3 – 3abc = (a + b + c)(a^2 + b^2 + c^2 – ab – be -ca)$] = 0 + 3abc
[∴ a + b + c = 0, given]
$a^3+b^3 + c^3 = 3abc$

Exercise 2.2 (Short answer type question)

Question 1: Which of the following expressions are polynomials? Justify your answer,
(i) 8
(ii) $\sqrt{3}x^2 – 2x$
(iii) $1 - \sqrt{5}x$
(iv) $\frac{1}{5x^{-2}} + 5x + 7$
(v) $\frac{(x-2)(x-4)}{x}$
(vi) $\frac{1}{x+1}$
(vii) $\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$
(viii) $\frac{1}{2x}$

Answer: (i) Polynomial, because the exponent of the variable of 8 or $8x^0$ is 0 which is a whole number.

(ii)Polynomial because the exponent of the variable of $\sqrt{3}x^2 - 2x$ is a whole number

(iii) Not polynomial, because the exponent of the variable of 1 - $\sqrt{5}$x or 1 - $\sqrt{5}x^{\frac{1}{2}}$ is $\frac{1}{2}$ which is not a whole number

(iv) polynomial because the exponent of the variable of $\frac{1}{5x^2} + 5x + 7 = \frac{1}{5}x^2 + 5x + 7$, is a whole number.

(v) Not polynomial
(vi)Not polynomial
(vii)Polynomial
(viii) not polynomial

Question 2: Write whether the following statements are true or false. Justify your answer. '
(i) A Binomial can have atmost two terms.
(ii) Every polynomial is a Binomial.
(iii) A binomial may have degree 5.
(iv) Zero of a polynomial is always 0.
(v) A polynomial cannot have more than one zero.
(vi) The degree of the sum of two polynomials each of degree 5 is always 5.

Answer: (i) False, because a binomial has exactly two terms.

(ii) False, because every polynomial is not a binomial .
e.g., (a) $3x^2 + 4x + 5$ [polynomial but not a binomial]
(b) $3x^2 + 5$ [polynomial and also a binomial]

(iii) True, because a binomial is a polynomial whose degree is a whole number greater than equal to one. So, it may have degree 5.

(iv) False, because zero of a polynomial can be any real number e.g., p(x) = x – 2, then 2 is a zero of polynomial p(x).

(v) False, because a polynomial can have any number of zeroes. It depends upon the degree of the polynomial
e.g., p(x) = x^2 -2, as degree pf p(x) is 2 ,so it has two degree, so it has two zeroes i.e., $\sqrt{2}$ and —$\sqrt{2}$.

(vi) False, because the sum of any two polynomials of same degree is not always same degree.
e.g., Let f(x) = $x^4 + 2$ and g(x) = $-x^4 + 4x^3 + 2x$
∴ Sum of two polynomials,

f{x) + g(x) = x^4 + 2 + ($-x^4$ + $4x^3$ + 2x)
= $4x^3$ + 2x + 2 which is not a polynomial of degree 4.

Exercise 2.3(Short answer type question)

Question 1: Classify the following polynomials as polynomials in one variable, two variables etc.
(i) $x^2 + x + 1$ (ii) $y^3 - 5y$
(iii) xy+ yz +zx (iv) $x^2 - Zxy + y^2 + 1$

Answer: (i) Polynomial x^2+ x+ 1 is a one variable polynomial, because it contains only one variable i.e., x.
(ii) Polynomial $y^3 - 5y$ is a one variable polynomial, because it contains only one variable i.e., y.
(iii) Polynomial xy+ yz+ zx is a three variables polynomial, because it contains three variables x, y and z.
(iv) Polynomial $x^2 - Zxy + y^2 + 1$ is a two variables pplynomial, because it contains two variables x and y.

Question 2: Determine the degree of each of the following polynomials.
(i) 2x - 1 (ii) -10
(iii) $x^3 - 9x + 3x^5$ (iv) $y^3(1 - y^4)$

Answer: (i) Degree of polynomial 2x-1 is one, Decause the maximum exponent of x is one.
(ii) Degree of polynomial -10 or -10x° is zero, because the exponent of x is zero.
(iii) Degree of polynomial x3 – 9x + 3xs is five, because the maximum exponent of x is five.
(iv) Degree of polynomial $y^3(1-y^4)$ or $y^3 - y^7$ is seven, because the maximum exponent of y is seven.

Question 3: For the polynomial $\frac{x^2+2x+1}{5}$ - $\frac{7}{2}x^2 - x^6$, then write
(i)The degree of the polynomial
(ii) the coefficient of x^3
(iii)the coefficient of x^6
(iv) the constant term

Answer: Given polynomial $\frac{x^2+2x+1}{5}$ - $\frac{7}{2}x^2 - x^6 = \frac{1}{5}x^3 + \frac{2x}{5} + \frac{1}{5} - \frac{7}{2}x^2 - x^6$
(i) Degree of the polynomial is the highest power of the variable i.e. 6
(ii) The coefficient of x^3 in given polynomial is $\frac{1}{5}$
(iii) The coefficient of x^6 in given polynomial is -1
(iv) The constant term is given polynomial is $\frac{1}{5}$

Question 4: Write the coefficient of x^2 in each of the following,

(i) $\frac{\pi}{6}x$ + x^2 – 1

(ii) 3x – 5

(iii) (x – 1)(3x – 4)

(iv) (2x – 5)($2x^2$ – 3x + 1)

Answer: (i) the coefficient of x^2 in $\frac{\pi}{6}x$ + x^2 – 1 is 1

(ii) The coefficient of x^2 in 3x – 5 = 0

(iii) Let p(x) = (x – 1)(3x – 4)

= $3x^2$ – 7x + 4

= $3x^2$ – 4x – 3x + 4

Hence, the coefficient of x^2 in p(x) is 3

(iv) Let p(x) = (2x – 5)($2x^2$ – 3x +1)

= 2x ($2x^2$ – 3x + 1) – 5($2x^2$ – 3x +1)

= $4x^3$ – $6x^2$ + 2x – $10x^2$ + 15x – 5

= $4x^3$ – $16x^2$ + 17x – 5

Hence, the coefficient of x^2 in p(x) is -16

Question 15: Classify the following as a constant, linear, quadratic and cubic polynomials,

(i)2 – x^2 + x^3

(ii) $3x^3$

(iii) 5t - $\sqrt{7}$

(iv) 4 – $5y^2$

(v)3

(vi) 2 + x

(vii) y^3 - y

(viii) 1 + x + x^2

(ix) t^2

(x) $\sqrt{2}x$ – 1

Answer: (i) Polynomial 2 – x^2 + x^3 is a cubic polynomial, because maximum exponent of x is 3.

(ii) Polynomial $3x^3$ is a cublic polynomial, because maximum exponent of x is 3.

(iii) Polynomial 5t -√7 is a linear polynomial, because maximum exponent of t is 1.

(iv) Polynomial 4- $5y^2$ is a quadratic polynomial, because maximum exponent of y is 2.

(v) Polynomial 3 is a constant polynomial, because the exponent of variable is 0. ’
(vi) Polynomial 2 + x is a linear polynomial, because maximum exponent of x is 1.
(vii) Polynomial $y^3 - y$ is a cubic polynomial, because maximum exponent of y is 3.
(viii) Polynomial $1 + x + x^2$ is a quadratic polynomial, because maximum exponent of x is 2.
(ix) Polynomial t^2 is a quadratic polynomial, because maximum exponent of t is 2.
(x) Polynomial $\sqrt{2}x-1$ is a linear polynomial, because maximum exponent of is 1.

Question 6: Give an example of a polynomial, which is
(i) monomial of degree 1.
(ii) -binomial of degree 20.
(iii) trinomial of degree 2.

Answer: (i) The example of monomial of degree 1 is 5y or 10x.
(ii) The example of binomial of degree 20 is $6x^{20} + x^{11}$ or $x^{20} + 1$
(iii) The example of trinomial of degree 2 is $x^2 - 5x + 4$ or $2x^2 - x - 1$

Question 7: Find the value of the polynomial $3x^3 - 4x^2 + 7x - 5$, when x = 3 and also when x = -3.

Answer: Let $p(x) = 3x^3 - 4x^2 + 7x - 5$
At x= 3, $p(3) = 3(3)^3 - 4(3)^2 + 7(3) - 5$
$= 3×27-4×9 + 21-5 = 81-36+21-5$ P(3) =61
At x = -3, $p(-3)= 3(-3)^3 - 4(-3)^2 + 7(-3) - 5$
$= 3(-27) - 4 × 9 - 21 - 5 = - 81 - 36 - 21 - 5 = -143$ p(-3) = -143
Hence, the value of the given polynomial at x = 3 and x = -3 are 61 and -143, respectively.

Question 8: If p (x) = $x^2 - 4x + 3$, then evaluate p(2) – p (-1) + p (½).

Answer: Given, p (x) = $x^2 - 4x + 3$
Now, p(2) = $2^2 - 4 \times 2 + 3 = 4 - 8 + 3$ = -1
p(-1) = $(-1)^2 - 4(-1) + 3 = 1 + 4 + 3 = 8$

and $p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 4 \times \frac{1}{2} + 3$
$= \frac{1}{4} - 2 + 3$
$= \frac{1-8+12}{4}$
$= \frac{5}{4}$

Therefore, $p(2) - p(-1) + p\left(\frac{1}{2}\right) = -1 - 8 + \frac{5}{4} = -9 + \frac{5}{4} = \frac{-36+5}{4} = \frac{-31}{4}$

Question 9: Find p(0), p(1) and p(-2) for the following polynomials
(i) p(x) = $10x - 4x^2 - 3$ (ii) p(y) = (y + 2)(y – 2)

Answer: (i) Given, polynomial is
$p(x) = 10x – 4x^2 – 3$
On putting x = 0,1 and – 2, respectively in Eq. (i), we get $p(0) = 10(0)-4(0)^2 -3 = 0-0-3= -3$
$p(1) = 10 (1) - 4 (1)^2 -3$
$= 10-4-3= 10-7= 3$
and $p(-2) =10 (-2)- 4 (-2)^2 – 3$
$= -20-4×4-3 =-20-16-3=-39$
Hence, the values of p(0), p(1) and p(-2) are respectively, -3,3 and – 39.

(ii) Given, polynomial is p(y) = (y+2)(y-2)
On putting y =0,1 and -2, respectively in Eq. (i), we get p(0) =(0+2)(0-2)= -4
p(1) = (1 + 2)(1-2) = 3 x (-1) = -3
and p(-2) = (-2 + 2)(-2 -2) =0 (-4) = 0
Hence, the values of p(0),p(1) and p(-2) are respectively,-4,-3 and 0.

Question 10: Verify whether the following are true or false.
(i) -3 is a zero of at – 3
(ii) -1/3 is a zero of 3x + 1
(iii) -4/5 is a zero of 4 – 5y
(iv) 0 and 2 are the zeroes of $t^2 – 2t$
(v) -3 is a zero of $y^2 + y – 6$

Answer: (i) False as zero of x – 3 is 3

(ii)true as zero of 3x + 1 is $-\frac{1}{3}$

(iii)False as zero of 4 – 5y is $\frac{4}{5}$

(iv)true as zeroes of $t^2 – 2t$ are 0 and 2

(v)true.
Now, $y^2 + y – 6$
$= y^2 + 3y – 2y – 6$
$= y(y + 3) – 2(y + 3)$
$= (y – 2)(y + 3)$
Hence, the zeroes of $y^2 + y – 6$ are 2 and -3 .

Question 11:
Find the zeroes of the polynomial in each of the following,
(i) p(x)=x – 4 (ii) g(x)= 3 – 6x
(iii) q(x) = 2x – 7 (iv) h(y) = 2y
Answer:
(i) Given, polynomial is
p(x) = x- 4
For zero of polynomial, put p(x) = x-4 = 0
or, x= 4
Hence, zero of polynomial is 4.
(ii) Given, polynomial is

g(x) = 3-6x
For zero of polynomial, put g(x) = 0
3-6x= 0
or, 6x =3
or, x = $\frac{1}{2}$.
Hence, zero of polynomial is X
(iii) Given, polynomial is q(x) = 2x -7 For zero of polynomial, put q(x) = 2x-7 = 0
2x= 7
or, x = $\frac{7}{2}$
Hence, zero of polynomial is
(iv) Given polynomial h(y) = 2y For zero of polynomial, put h(y) = 0
2y= 0
Hence, the zero of polynomial is 0.

Question 12:
Find the zeroes of the polynomial p(x)= $(x – 2)^2 – (x+ 2)^2$.
Answer:
Given, polynomial is p(x) = $(x – 2)^2 – (x+ 2)^2$
For zeroes of polynomial, put p(x) = 0
$(x – 2)^2 – (x+ 2)^2=0$
(x-2 + x+2)(x-2-x-2) = 0 [using identity, a^2-b^2 =(a-b)(a + b)]
or,(2x)(-4) = 0
or, x = 0.

Question 13:
By actual division, find the quotient and the remainder when the first polynomial is divided by the second polynomial x^4 + 1 and x-1.

Answer:

Using long division method,

$$x-1\overline{)x^4+1}(x^3+x^2+x+1$$

$$x^4-x^3$$
$$\underline{-\quad+}$$
$$x^3+1$$
$$x^3-x^2$$
$$\underline{-\quad+}$$
$$x^2+1$$
$$x^2-x$$
$$\underline{-\quad+}$$
$$x+1$$
$$x-1$$
$$\underline{-\quad+}$$
$$2$$

Hence, Quotient $= x^3+x^2+x+1$ and Remainder $= 2$

Question 14:

By remainder theorem, find the remainder when p(x) is divided by g(x)

(i) p(x) = x^3-2x^2-4x-1, g(x)=x + 1

(ii) p(x) = x^3 -3x^2 + 4x + 50, g(x)= x – 3

(iii) p(x) = x^3 – 12x^2 + 14x -3, g(x)= 2x – 1 – 1

(iv) p(x) = x^3-6x^2+2x-4, g(x) = 1 -(3/2) x

Answer:

(i) Given p(x) = x^3-2x^2-4x-1 and g(x) = x + 1

Here, zero of g(x) is -1.

When we divide p(x) by g(x) by remainder theorem, we get the remainder p(-1).

Therefore, p(-1) = $(-1)^3-2(-1)^2-4(-1)-1$

= -1 - 2 + 4 – 1

= 4 – 4 = 0

Hence, remainder is 0.

(ii) Given, p(x) = $x^3-3x^2+4x+50$ and g(x 0 = x – 3

Hence, zero of g(x) is 3.

When we divide p(x) by g(x) by remainder theorem, we get the remainder p(3).

Therefore, p(3) = $(3)^3-3(3)^2+4(3)+50$

= 27 – 27 + 12 + 50 = 62

Hence, remainder is 62.

(iii) Given, p(x) = $x^3-3x^2+4x+50$ and g(x) = 2x – 1

Hence, Zero of g(x) = $\frac{1}{2}$

When we divide p(x) by g(x) using remainder theorem, we get the remainder p$\left(\frac{1}{2}\right)$

Therefore, p$\left(\frac{1}{2}\right)$ = $4\left(\frac{1}{2}\right)^3-12\left(\frac{1}{2}\right)^2+14\left(\frac{1}{2}\right)-3$

= $4\times\frac{1}{8}-12\times\frac{1}{4}+14\times\frac{1}{2}-3$

= $\frac{1+2}{2}=\frac{3}{2}$

Hence, remainder is $\frac{3}{2}$

(iv) Given, p(x) = $x^3 - 6x^2 + 2x - 4$ and g(x) = $1 - \frac{3}{2}x$

Here, zero of g(x) = $\frac{2}{3}$

When we divide p(x) by g(x) using remainder theorem, we get the remainder $p\left(\frac{2}{3}\right)$

Hence, $\frac{8}{27} - 6 \times \frac{4}{9} + 2 \times \frac{2}{3} - 4 = \frac{8}{27} - \frac{24}{9} + \frac{4}{3} - 4 = \frac{8-72+36-108}{27} = \frac{-136}{27}$

Thus, remainder is $\frac{-136}{27}$.

Question 15: Check whether p(x) is a multiple of g(x) or not

(i) p(x) = x – 5x + 4x – 3, g(x) = x – 2.
(ii) p(x) = 2x – 11x – 4x+ 5, g(x) = 2x + l

Answer: (i) g(x) = x – 2
Then zero of the g(x) is 2. [Given] [Since, p(x) = $x^3 - 5x^2 + 4x - 3$]
Now, $p(2) = 2^3 - 5(2)^3 + 4(2) - 3 = 8 - 20 + 8 - 3 = -7 \neq 0$
Since, remainder ≠ 0, so p(x) is not a multiple of g(x).

(ii) Here g(x) = 2x + 1
Then, zero of g(x) = $-\frac{1}{2}$

Now, $p\left(\frac{-1}{2}\right) = 2\left(\frac{-1}{2}\right)^3 - 11\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) + 5$

$= 2\left(\frac{-1}{8}\right) - 11\left(\frac{1}{4}\right) + 2 + 5$

$= \frac{-1}{4} - \frac{11}{4} + 7$

$= \frac{-1-11+28}{4} = \frac{16}{4} = 4$

Since, remainder ≠ 0, so p(x) is not a multiple of g(x).

Question 16: Show that,

(i) x + 3 is a factor of 69 + 11c – x + x
(ii) 2x – 3 is a factor of x + 2x - 9x +12

Answer: (i) Let p(x) = $x^3 - x^2 + 11x + 69$
We have to show that, x + 3 is a factor of p(x) i.e., p(-3) = 0
Now, $p(-3) = (-3)^3 - (-3)^2 + 11(-3) + 69 = -27 - 9 - 33 + 69 = -69 + 69 = 0$
hence, (x + 3) is a factor of p(x)

(ii) let p(x) = $2x^3 - 9x^2 + x + 12$
We have to show that, 2x – 3 is a factor of p(x).
i.e., $p\left(\frac{3}{2}\right) = 0$

Now, $p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \frac{3}{2} + 12$

$= 2\times \frac{27}{8} - 9 \times \frac{9}{4} + \frac{3}{2} + 12$

$= \frac{27-81+6+48}{4} = \frac{81-81}{4} = 0$

hence, (2x – 3) is a factor of p(x).

Question 17: Determine which of the following polynomial has x – 2 a factor (i) 3x + 6x – 24 (ii) 4x + x – 2

Answer: let $p(x) = x^5 – 4a^2x^3 + 2x + 2a + 3$
Since, x + 2a is a factor of p(x), then put p(-2a) = 0
Therefore, $(-2a)^5 – 4a^2(-2a)^3 + 2(-2a) + 2a + 3 = 0$
or, $-32a^5 + 32a^5 – 4a + 2a + 3 = 0$
or, -2a + 3 = 0
or, 2a = 3
or, a = $\frac{3}{2}$

Hence, the value of a is $\frac{3}{2}$

Question 18: Show that p-1 is a factor of p -1 and also of p -1.

Answer: Let g (p) = p -1 and h(p) = p -1.

On putting p=1 in Eq. (i), we get g(1)=1 -1= 1-1=0 Hence, p-1 is a factor of g(p).
Again, putting p = 1 in Eq. (ii), we get,
h(1) = (1) —1 = 1 —1 = 0 Hence, p -1 is a factor of h(p).

Question 19: For what value of m is x -2mx +16 divisible by x + 2?

Answer: Let p(x) = x -2mx +16
Since, p(x) is divisible by (x+2), then remainder = 0
P(-2) = 0
or, (-2) -2m(-2) + 16=0
or, -8 - 8m + 16=0
or, 8 = 8m
or, m = 1
Hence, the value of m is 1.

Question 20: If x + 2a is a factor of a -4a x +2x + 2a +3, then find the value of a.

Answer: Let p(x) =a – 4ax + 2x + 2a + 3
Since, x + 2a is a factor of p(x), then put
p(-2a) = 0 (-2a) – 4a (-2a) + 2(-2a) + 2a + 3 = 0
or, -32a + 32a – 4a + 2a + 3 = 0
or, -2a + 3=0
or, 2a =3
or, a = 3/2.

Hence, the value of a is 3/2.

Question 21: Find the value of m, so that 2x -1 be a factor of 8x + 4x - 16x + 10x + 07.

Answer: Let $p(x) = 8x^4 + 4x^3 - 16x^2 + 10x + m$
Since, $2x - 1$ is a factor of p(x), then put $p\left(\frac{1}{2}\right) = 0$
Therefore, $8\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^3 - 16\left(\frac{1}{2}\right)^2 + 10\left(\frac{1}{2}\right) + m = 0$

Or, $8\times\frac{1}{16} + 4\times\frac{1}{8} - 16\times\frac{1}{4} + 10\left(\frac{1}{2}\right) + m = 0$
or, $\frac{1}{2} + \frac{1}{2} - 4 + 5 + m = 0$
or, 1 + 1 + m = 0
or, m = -2
Hence, the value of the m is -2.

Question 22: If x +1 is a factor of ax +x -2x + 4a – 9 , then find the value of a.

Answer: Let $p(x) = ax^3 + x^2 - 2x + 4a - 9$
Since, x + 1 is a factor of p(x), then put p(– 1) = 0
Therefore, $a(-1)^3 + (-1)^2 - 2(-1) + 4a - 9 = 0$
or, - a + 1+ 2 + 4a – 9 = 0
or, 3a – 6 = 0
or, 3a = 6
or, a = $\frac{6}{3}$ = 2

Question 23: Factorise

(i) $x^2 + 9x + 18$
(ii) $6x^2 + 7x - 3$
(iii) $2x^2 - 7x - 15$
(iv) $84 - 2r - 2r^2$

Answer: (i) $x^2 + 9x + 18$
$= x^2 + 6x + 3x + 18$
= x(x + 6) + 3(x + 6)
= (x + 3)(x + 6)

(ii) $6x^2 + 7x - 3$
$= 6x^2 + 9x - 2x - 3$
= 3x(2x + 3) -1(2x + 3)
= (3x – 1)(2x + 3)

(iii) $2x^2 - 7x - 15$
$= 2x^2 - 10x + 3x - 15$
= 2x(x – 5) + 3(x – 5)
= (2x + 3)(x – 5)

(iv) $84 - 2r - 2r^2$
$= -2(r^2 + r - 42)$
$= -2(r^2 + 7r - 6r - 42)$
= - 2 [r(r + 7) – 6(r + 7)]
= - 2 (r – 6) (r + 7)
= 2 (6 – r)(r + 7)

Question 24: Factorise

(i) $2x^3 - 3x^2 - 17x + 30$
(ii) $x^3 - 6x^2 + 11x - 6$
(iii) $x^3 + x^2 - 4x - 4$
(iv) $3x^3 - x^2 - 3x + 1$

Answer: (i) Let p(x) = $2x^3 - 3x^2 - 17x + 30$
Constant term p(x) = 30
Therefore, factors of 30 are $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$
By trial, we find that p(2) = 0
so, (x – 2) is a factor of p(x).
Now, we see that $2x^3 - 3x^2 - 17x + 30$
$= 2x^2 - 4x^2 + x^2 - 2x - 15x + 30$
$= 2x^2(x - 2) + x(x - 2) - 15(x - 2)$
$= (x - 2)(2x^2 + x - 15)$

Now, $(2x^2 + x - 15)$ can be factorised either by splitting the middle term or by using the factor theorem.
Now, $(2x^2 + x - 15) = 2x^2 + 6x - 5x - 15$
$= 2x(x + 3) - 5(x + 3)$
$= (x + 3)(2x - 5)$

Therefore, $2x^3 - 3x^2 - 17x + 30 = (x - 2)(x + 3)(2x - 5)$

(ii) Let p(x) = $x^3 - 6x^2 + 11x - 6$
Constant term pf p(x) = -6
factors of -6 are $\pm 1, \pm 2, \pm 3, \pm 6$
By trial, we find that p(1) = 0, so (x – 1) is a factor of p(x).
Now, we see that,
$x^3 - 6x^2 + 11x - 6$
$= x^3 - x^2 - 5x^2 + 5x + 6x - 6$
$= x^2(x - 1) - 5x(x - 1) + 6(x - 1)$
$= (x - 1)(x^2 - 5x + 6)$

Now, $(x^2 - 5x + 6)$
$= x^2 - 3x - 2x + 6$
$= x(x - 3) - 2(x - 3)$
$= (x - 3)(x - 2)$

Therefore, $x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$

(iii) $x^3 + x^2 - 4x - 4 = (x + 1)(x - 2)(x + 2)$
(iv) $3x^3 - x^2 - 3x + 1 = (x - 1)(x + 1)(3x - 1)$

Question 25: Using suitable identity, evaluate the following
(i) 103^3
(ii) 101×102
(iii) 999^2

Answer: (i) $103^3 = (100 + 3)^3$
$= 100^3 + 3^3 + 3\times 100 \times 3(100 + 3)$
$= 1000000 + 27 + 900$
$= 1092727$

(ii) $101 \times 102 = (100 + 1)(100 + 2)$
$= 100^2 + 100\,(1 + 2) + 1\times 2$
$= 10000 + 300 + 2 = 10302$

(iii) $999^2 = (1000 - 1)^2$ [Now proceed by using identity $(a - b)^2 = a^2 + b^2 - 2ab$]

Question 26: Factorize the following:
(i) $4x^2 + 20x + 25$
(ii) $9y^2 - 66yz + 121z^2$
(iii) $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

Answer: (i) proceed by using identity $a^2 + 2ab + b^2 = (a + b)^2$
(ii) proceed by using identity $a^2 - 2ab + b^2 = (a - b)^2$

(iii) $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$
$= \left[\left(2x + \frac{1}{3}\right) - \left(x - \frac{1}{2}\right)\right]\left[\left(2x + \frac{1}{3}\right) - \left(x - \frac{1}{2}\right)\right]$
$= \left(2x - x + \frac{1}{3} + \frac{1}{2}\right)\left(2x + x + \frac{1}{3} - \frac{1}{2}\right)$
$= \left(x + \frac{5}{6}\right)\left(3x - \frac{1}{6}\right)$

Question 27: Factorize the following:
(i) $9x^2 - 12x + 3$
(ii) $9x^2 - 12xy + 4$

Answer: (i) $9x^2 - 12x + 3$
$= 3(3x^2 - 4x + 1)$ [Split the middle term]

(ii) $9x^2 – 12xy + 4$
$= (3x)^2 – 2 \times 3x \times 2 + 2^2$
$= (3x – 2)^2$
$= (3x – 2)(3x – 2)$

Question 28: Expand the following

(i) $(4a-b + 2c)^2$
(ii) $(3a – 5b – c)^2$
(iii) $(-x + 2y-3z)^2$

Answer: (i) [using identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$]
(ii) [using identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$]
(iii) [using identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$]

Exercise 2.4 (Long Answer type question)

Question 1: If the polynomials $az^3 +4z^2 + 3z-4$ and $z^3 -4z + o$ leave the same remainder when divided by z – 3, find the value of a.

Answer: Let p (z) = az +4z + 3z-4 and p (z) = z -4z + 0
When we divide p(z) by z – 3, then we get the remainder p(3).
Now, p(3) = a(3)3 + 4(3)2 + 3(3) – 4 = 27a+ 36+ 9-4= 27a+ 41 When we divide p (z) by z – 3 then we get the remainder p(3).

Now, p (3) = (3) -4(3)+a = 27-12 + a = 15+a
According to the question, both the remainders are same.
p (3)= p (3)
or, 27a + 41 = 15 + a
or, 27a - a = 15 – 41 .
or, 26a = 26 a = -1

Question 2: The polynomial $p\{x) = x^4 -2x^3 + 3x^2 – ax + 3a – 7$ when divided by x+1 leaves the remainder 19. Find the values of a. Also, find the remainder when p(x) is divided by x+ 2.

Answer: Given, $p(x) = x^4 -2x^3 + 3x^2 – ax + 3a – 7$
When we divide p(x) by x + 1, then we get the remainder p(-1)
Now, $p(-1) = (-1)^4 – 2 (-1)^3 + 3(-1)^2 – a(-1) + 3(-1) – 7$
According, to the question, p(-1) = 19

or, 4a – 1 = 19
or, 4a = 20
or, a = 5

Required polynomial = $x^4 - 2x^3 + 3x^2 - 5x + 3(5) - 7$
$= x^4 - 2x^3 + 3x^2 - 5x + 15 - 7$
$= x^4 - 2x^3 + 3x^2 - 5x + 8$

When we divide p(x) by x + 2, then we get the remainder p(-2)
Now, $p(-2) = (-2)^4 - 2(-2)^3 + 3(-2) - 5(-2) + 8$
$= 16 + 16 + 12 + 10 + 8 = 62$

Hence, the value of a is 5 and remainder is 62.

Question 3: If both x – 2 and x -(1/2) are factors of px + 5x + r, then show that p = r.

Answer: let $f(x) = px^2 + 5x + r$
Since, x – 2 is factor of f(x), then f(2) = 0
therefore, $p(2)^2 + 5(2) + r = 0$
or, 4p + 10 + r = 0 …………………………………………………(1)
Since, $x - \frac{1}{2}$ is a factor of f(x), then $f\left(\frac{1}{2}\right) = 0$
Therefore, $p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r = 0$
or, p + 10 + 4r = 0 ……………………………………………………………..(2)

Since, x – 2 and $x - \frac{1}{2}$ are factors of $f(x) = px^2 + 5x + r$
From eq(1) and (2),
4p + 10 + r = p + 10 + 4r
or, 3p = 3r
hence, p = r

Question 4: Without actual division, prove that $2x^4 - 5x^3 + 2x^2 - x + 2$ is divisible by $x^2 - 3x + 2$

Answer: Let $p(x) = 2x^4 - 5x^3 + 2x^2 - x + 2$ firstly, factorise $x^2 - 3x + 2$
Now, $x^2 - 3x + 2$ = x - 2x – x + 2 [by splitting middle term]
= x(x – 2) -1(x – 2)
= (x – 1)(x – 2)

Hence, 0 of x^2 - 3x + 2 are l and 2.
We have to prove that, $2x^4 – 5x^3 + 2x^2 – x + 2$ is divisible by $x^2 – 3x + 2$ i.e., to prove that, p (1) =0 and p(2) =0

Now, $p(1) = 2(1)^4 – 5(1)^3 + 2(1)^2 - 1 + 2 = 2 – 5 + 2 – 1 + 2 = 6 – 6 = 0$
and $p(2) = 2(2)^4 – 5(2)^3 + 2(2)^2 – 2 + 2 = 32 – 40 + 8 = 40 – 40 = 0$

Hence, p(x) is divisible by $x^2 - 3x + 2$.

Question 5: Simplify $(2x - 5y)^3 – (2x + 5y)^3$.

Answer: $(2x - 5y)^3 – (2x + 5y)^3$
$= [(2x)^3 – (5y)^3 – 3(2x)(5y)(2x – 5y)] - [(2x)^3 + (5y)^3 + 3(2x)(5y)(2x+5y)]$

[using identity, (a – b) = a -b – 3ab and (a + b) =a +b + 3ab]
$(2x)^3 – (5y)^3 – 30xy(2x – 5y) – (2x)^3 – (5y)^3 – 30xy (2x + 5y)$
$= -2 (5y)^3 – 30xy(2x – 5y + 2x + 5y)$
$= -2 \times 125y^3 – 30xy(4x)$
$= -250y^3 - 120x^2y$