## Chapter 12:Surface area and Volume

## Exercise: 12.1

Question 1: Choose the correct answer from the given four options:

1. A cylindrical pencil sharpened at one edge is the combination of (A) a cone and a cylinder (B) frustum of a cone and a cylinder
(C) a hemisphere and a cylinder (D) two cylinders.

Solution: (A) a cone and a cylinder


The Nib of a sharpened pencil = conical shape
The rest of the part of a sharpened pencil = cylindrical
Therefore, a pencil is a combination of a cylinder and a cone.
Question 2. $A$ is the combination of
(A) a sphere and a cylinder (B) a hemisphere and a cylinder
(C) two hemispheres (D) a cylinder and a cone.

Solution: (A) a sphere and a cylinder


The top part of surah = cylindrical shape
The bottom part of surah = spherical shape
Therefore, is a combination of a Sphere and a cylinder.
Question 3. A plumbline (Sahul) is the combination of (see Fig. 12.2)

(A) a cone and a cylinder (B) a hemisphere and a cone
(C) frustum of a cone and a cylinder (D) sphere and cylinder

Solution: (B) a hemisphere and a cone

The upper part of plumbline $=$ hemispherical,
The bottom part of plumbline = conical
Therefore, it is a combination of hemisphere and cone.
Question 4. The shape of a glass (tumbler) (see Fig. 12.3) is usually in the form of


Fig. 12.3
(A) a cone (B) frustum of a cone
(C) a cylinder (D) a sphere

Solution: (B) frustum of a cone


The shape of the glass is a frustum or specifically, an inverted frustum.
Question 5. The shape of a girl, in the Gilli-danda game (see Fig. 12.4), is a combination of


Fig. 12.4
(A) two cylinders (B) a cone and a cylinder
(C) two cones and a cylinder (D) two cylinders and a cone

Solution: (C) two cones and a cylinder


The left and right part of a gilli = conical
The central part of a gilli = cylindrical
Therefore, it is a combination of a cylinder and two cones.

Question 6. A shuttlecock used for playing badminton has the shape of the combination of
(A) a cylinder and a sphere (B) a cylinder and a hemisphere
(C) a sphere and a cone (D) frustum of a cone and a hemisphere

Solution: (D) frustum of a cone and a hemisphere


The cork of a shuttle = hemispherical shapes
The upper part of a shuttle = shape of the frustum of a cone.
Therefore, it is a combination of a frustum of a cone and a hemisphere.
Question 7. A cone is cut through a plane parallel to its base and then the cone that is formed on one side of that plane is removed. The new part that is leftover on the other side of the plane is called
(A) a frustum of a cone (B) cone
(C) cylinder (D) sphere

Solution: (A) a frustum of a cone


When a cone is divided into two parts by a plane through any point on its axis parallel to its base, the upper and lower parts obtained are a cone and a frustum respectively.
Question 8. A hollow cube of internal edge 22 cm is filled with spherical marbles of diameter 0.5 cm and it is assumed that $1 / 8$ space of the cube remains unfilled. Then the number of marbles that the cube can accommodate is
(A) 142296
(B) 142396
(C) 142496
(D) 142596

Solution:
(A) 142296

According to the question,
The volume of cube $=22^{3}=10648 \mathrm{~cm}^{\text {The volume }}$ of the cube that remains unfilled $=\frac{1}{8} \times$ $10648=1331 \mathrm{~cm}^{3}$
volume occupied by spherical marbles $=10648-1331=9317 \mathrm{~cm}^{3}$
Radius of the spherical marble $=\frac{0.5}{2}=0.25 \mathrm{~cm}=\frac{1}{4} \mathrm{~cm}$
Volume of 1 spherical marble $=\frac{4}{3} \times \frac{22}{7} \times(1 / 4)^{3}=\frac{11}{168} \mathrm{~cm}^{3}$
Numbers of spherical marbles, $\mathrm{n}=9317 \times \frac{11}{168}=142296$
Question 9. A metallic spherical shell of internal and external diameters $4 \mathbf{c m}$ and 8 cm , respectively is melted and recast into the form of a cone of base diameter 8 cm . The height of the cone is
(A) 12 cm
(B) 14 cm
(C) 15 cm
(D) 18 cm

Solution: (B) 14 cm


The volume of spherical shell = Volume of cone recast by melting
For Spherical Shell,
Internal diameter, $\mathrm{d}_{1}=4 \mathrm{~cm}$
Internal radius, $r_{1}=2 \mathrm{~cm}$ [as radius $=1 / 2$ diameter]
External diameter, $\mathrm{d}_{2}=8 \mathrm{~cm}$
External radius, $\mathrm{r}_{2}=4 \mathrm{~cm}$
Now, as the volume of spherical shell $=\frac{4}{3} \pi\left(r_{2}{ }^{3}-r_{1}{ }^{3}\right)$, where $r_{1}$ and $r_{2}$ are internal and external radii respectively.
the volume of given shell $=4 / 3 \pi\left(4^{3}-2^{3}\right)$
$=4 / 3 \pi(56)$
$=(224 / 3) \pi$
We know that, the volume of cone $=224 \mathrm{~m} / 3 \mathrm{~cm}^{3}$
For cone,
Base diameter $=8 \mathrm{~cm}$
Base radius, $\mathrm{r}=4 \mathrm{~cm}$
Let Height of cone = 'he.
Now, volume of cone $=(1 / 3) \pi r^{2} h$,
Where $\mathrm{r}=$ Base radius and $\mathrm{h}=$ height of the cone
The volume of given cone $=(1 / 3) \pi 4^{2} h$
$\Rightarrow 224 \pi / 3=16 \pi h / 3$
$\Rightarrow 16 \mathrm{~h}=224$
$\mathrm{h}=14 \mathrm{~cm}$

So, the Height of the cone is 14 cm .
Question 10. A solid piece of iron in the form of a cuboid of dimensions 49 cm $\times 33 \mathrm{~cm} \times 24 \mathrm{~cm}$, is moulded to form a solid sphere. The radius of the sphere is
(A) 21 cm (B) 23 cm
(C) $25 \mathrm{~cm}(D) 19 \mathrm{~cm}$

Solution: (A) 21 cm
As we know,
The volume of cuboid $=l \mathrm{lbh}$
Where, $\mathrm{I}=$ length, $\mathrm{b}=$ breadth and $\mathrm{h}=$ height
Forgiven cuboid,
Length, I = 49 cm
Breadth, $b=33 \mathrm{~cm}$
Height, $\mathrm{h}=24 \mathrm{~cm}$
Volume of cube $=49 \times(33) \times(24) \mathrm{cm}^{3}$
Now, let the radius of the cube be $r$.
As the volume of sphere $=4 / 3 \pi r^{3}$
The volume of cuboid = volume of sphere moulded
So, 49(33)(24) $=4 / 3 \pi r^{3}$
$\Rightarrow \pi r^{3}=29106$
$\Rightarrow r^{3}=29106 \times 22 / 7$
$\Rightarrow r^{3}=9261$
$\Rightarrow r=\sqrt[3]{9261} \mathrm{~cm}=21 \mathrm{~cm}$
Henthe ce, the radius of the sphere is 21 cm

## Exercise 12.2

Write 'True' or 'False' and justify your answer in the following:
Question 1. Two identical solid hemispheres of equal base radius $r \mathrm{~cm}$ are stuck together along their bases. The total surface area of the combination is $6 \pi r^{2}$.

Solution: False
When two hemispheres are joined together along with their bases, a sphere of the same base radius is formed.

Curved Surface Area of a sphere $=4 \pi r^{2}$.
Question 2. A solid cylinder of radius $r$ and height $h$ is placed over another cylinder of the same height and radius. The total surface area of the shape so formed is $4 \pi r h+4 \pi r^{2}$.

Solution: False

According to the question,
When one cylinder is placed over another, the base of the first cylinder and the top of another cylinder will not be covered in total surface area.
We know that, Total surface area of the cylinder $=2 \pi r h+2 \pi r^{2} h$, where $r=$ base radius and $\mathrm{h}=$ height

The total surface area of the shape formed = 2(Total surface of a single cylinder) -
2(Area of base of the cylinder)
$=2\left(2 \pi r h+2 \pi r^{2}\right)-2\left(\pi r^{2}\right)$
$=4 \pi r h+2 \pi r^{2}$
Question 3. A solid cone of radius $r$ and height $\boldsymbol{h}$ is placed over a solid cylinder having the same base radius and height as that of a cone. The total surface area of the combined solid is $\pi r\left[\sqrt{ }\left(r^{2}+h^{2}+3 r+2 h\right]\right.$.

Solution: False
When a solid cone is placed over a solid cylinder of the same base radius, the base of the cone and top of the cylinder will not be covered in total surface area.
Since the height of the cone and cylinder is the same,
We get,
Total surface area of cone $=\pi r l+\pi r^{2}$, where $r=$ base radius and $I=$ slant height
The total surface area of the shape formed = Total surface area of cone + Total Surface area of cylinder - 2(Area of base)


Total surface area of cylinder $=2 \pi r h+2 \pi r^{2} h$, where $r=$ base radius and $h=$ height
$=\pi r(r+I)+\left(2 \pi r h+2 \pi r^{2}\right)-2\left(\pi r^{2}\right)$
$=\pi r^{2}+\pi r l+2 \pi r h+2 \pi r^{2}-2 \pi r^{2}$
$=\pi r(r+1+h)$
$=\pi r\left(r+\sqrt{r^{2}+h^{2}}+2 h\right)$
Question 4. A solid ball is exactly fitted inside the cubical box of side $a$. The volume of the ball is $4 / 3 \pi \mathrm{ma}^{3}$.

Solution: False
Let the radius of sphere $=r$
When a solid ball is exactly fitted inside the cubical box of side a,
We get,
The diameter of ball = Edge length of the cube
$2 \mathrm{r}=\mathrm{a}$
Radius, $r=a / 2$

We also know that,
The volume of sphere $=4 / 3 \pi r^{3}$
Volume of ball $=4 / 3 \pi(a / 2)^{3}=4 / 3 \pi\left(a^{3} / 8\right)=1 / 6 \pi a^{3}$

## Exercise 12.3

Question 1. Three metallic solid cubes whose edges are $3 \mathrm{~cm}, 4 \mathrm{~cm}$ and 5 cm are melted and formed into a single cube. Find the edge of the cube so formed.

## Solution:

We know that,
The volume of cube $=a^{3}$, where $a=$ side of the cube
According to the question,
Side of the first cube, $a_{1}=3 \mathrm{~cm}$
Side of the second cube, $a_{2}=4 \mathrm{~cm}$
Side of the third cube, $a_{3}=5 \mathrm{~cm}$
Let us assume that the side of cube recast from melting these cubes $=\mathrm{a}$
We know that the total volume of the 3 cubes will be the same as the volume of the newly formed cube,
Volume of new cube $=\left(\right.$ volume of $1^{\text {st }}+2^{\text {nd }}+3^{\text {rd }}$ cube $)$
$\Rightarrow a^{3}=a_{1}{ }^{3}+a_{2}{ }^{3}+a_{3}{ }^{3}$
$\Rightarrow \mathrm{a}^{3}=(3)^{3}+(4)^{3}+(5)^{3}$
$\Rightarrow a^{3}=27+64+125=216$
$\Rightarrow \mathrm{a}=6 \mathrm{~cm}$
Therefore, the side of the cube so formed is 6 cm .

## Question 2. How many shots each having a diameter of 3 cm can be made from a cuboidal lead solid of dimensions $9 \mathrm{~cm} \times 11 \mathrm{~cm} \times 12 \mathrm{~cm}$ ?

Solution: Volume of cuboid $=\mathrm{lbh}$, where, $\mathrm{I}=$ length, $\mathrm{b}=$ breadth and $\mathrm{h}=$ height
Cuboidal lead:
Length, $I=9 \mathrm{~cm}$
Breadth, $b=11 \mathrm{~cm}$
Height, $h=12 \mathrm{~cm}$
Volume of lead $=9(11)(12)=1188 \mathrm{~cm}^{3}$
Volume of sphere $=4 / 3 \pi r^{3}$, where $r=$ radius of sphere
Spherical shots,
Diameter $=3 \mathrm{~cm}$
Radius, $r=1.5 \mathrm{~cm}$
Volume of one shot $=\frac{4}{3} \times \frac{22}{7} \times(1.5)^{3}=\frac{99}{7} \mathrm{~cm}^{3}$
Number of shots can be made $=\frac{\text { volume of lead }}{\text { volume of one shot }}=\frac{1188}{\frac{99}{7}}=\frac{1188 \times 7}{99}=84$
Hence, the number of bullets that can be made from lead $=84$.
Question 3. A bucket is in the form of a frustum of a cone and holds 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm , respectively. Find the height of the bucket.

Solution: According to the question,
The bucket is in the form of the frustum of a cone.
We know that,
Volume of frustum of a cone $=1 / 3 \pi h\left(r_{1}^{2}+r_{2}^{2}+r_{1} r_{2}\right)$, where, $h=$ height, $r_{1}$ and $r_{2}$ are the radii $\left(r_{1}>r_{2}\right)$

For bucket,
The volume of bucket $=28.490 \mathrm{~L}$
$1 \mathrm{~L}=1000 \mathrm{~cm}^{3}$
The volume of bucket $=28490$ The radius dies of the top, $r_{1}=28$ radius diestrus of the bottom, $\mathrm{r}_{2}=21 \mathrm{~cm}$
Let the height $=h$.
Substituting these values in the equation to find the volume of the bucket,
We have, volume of bucket $=\frac{1}{3} \pi h\left[28^{2}+21^{2}+28(21)\right]$

$$
\begin{aligned}
28490 & =\frac{1}{3} \times \frac{22}{7} \times \mathrm{h}(784+441+588) \\
& =\frac{22}{7} \times \mathrm{h} \times 1813 \\
\Rightarrow \mathrm{~h} & =\frac{28490 \times 21}{22 \times 1813} \\
\Rightarrow \mathrm{~h} & =15
\end{aligned}
$$

Question 4. A cone of radius 8 cm and height 12 cm is divided into two parts by a plane through the mid-point of its axis parallel to its base. Find the ratio of the volumes of two parts.

## Solution:



According to the question,
Height of cone $=\mathrm{OM}=12 \mathrm{~cm}$
The cone is divided from mid-point hence, the mid-point of cone be $=P$
$\mathrm{OP}=\mathrm{PM}=6 \mathrm{~cm}$
From $\triangle$ OPD and $\triangle O M N$
$\angle \mathrm{POD}=\angle \mathrm{POD}$ [Common]
$\angle O P D=\angle O M N$ [Both $90^{\circ}$ ]
Hence, by the Angle-Angle similarity criterion, we get $\triangle \mathrm{OPD} \sim \triangle \mathrm{OMN}$ and similar triangles have corresponding sides in equal ratio,
So, we have,
PD/MN = OP/OM
$\mathrm{PD} / 8=6 / 12$
$P D=4 \mathrm{~cm}$
[ $\mathrm{MN}=8 \mathrm{~cm}=$ radius of base of cone]
For the First part i.e. cone
Base Radius, $r=P D=4 \mathrm{~cm}$
Height, $h=O P=6 \mathrm{~cm}$
We know that volume of the cone for radius $r$ and height $h, V=1 / 3 \pi r^{2} h$
Volume of first part $=1 / 3 \pi(4)^{2} 6=32 \pi$
For the second part, i.e. Frustum
Bottom radius, $\mathrm{r}_{1}=\mathrm{MN}=8 \mathrm{~cm}$
Top Radius, $r_{2}=P D=4 \mathrm{~cm}$
Height, $h=P M=6 \mathrm{~cm}$
We know that, volume of frustum of a cone $=1 / 3 \pi h\left(r_{1}^{2}+r_{2}^{2}+r_{1} r_{2}\right)$, where, $h=$ height, $r_{1}$ and $r_{2}$ are radii, $\left(r_{1}>r_{2}\right)$

Volume of second part $=1 / 3 \pi(6)\left[8^{2}+4^{2}+8(4)\right]=2 \pi(112)=224 \pi$
Therefore, we get the ratio,
Volume of first part : Volume of second part $=32 \pi: 224 \pi=1: 7$
Question 5. Two identical cubes each of volume 64 cm 3 are joined together end to end. What is the surface area of the resulting cuboid?

## Solution:



Let the side of one cube $=\mathrm{a}$
Surfaces area of resulting cuboid $=2$ (Total surface area of a cube) -2 (area of a single surface)

We know that,
The total surface area of cube $=6 \mathrm{a}^{2}$, where $\mathrm{a}=$ side of the cube
$\Rightarrow$ Surfaces area of resulting cuboid $=2\left(6 \mathrm{a}^{2}\right)-2\left(\mathrm{a}^{2}\right)=10 \mathrm{a}^{2}$
Also, according to the question,
Volume of cube $=64 \mathrm{~cm}^{3}$
The volume of cube $=a^{3}$
$64=a^{3}$
$a=4 \mathrm{~cm}$
Therefore, surface area of resulting cuboid $=10 \mathrm{a}^{2}=10(4)^{2}=160 \mathrm{~cm}^{2}$

Question 6. From a solid cube of side 7 cm , a conical cavity of height 7 cm and radius $\mathbf{3 ~ c m}$ is hollowed out. Find the volume of the remaining solid.

## Solution:



From the figure, we get,
The volume of remaining solid = volume of the cube - the volume of a cone
For Cube
Side, $a=7 \mathrm{~cm}$
We know that, the volume of cube $=a^{3}$, where $a=$ side of the cube
Volume of cube $=(7)^{3}=343 \mathrm{~cm}^{3}$
For cone radius, $r=3 \mathrm{~cm}$
Height, $\mathrm{h}=7 \mathrm{~cm}$
Volume of cone $=\frac{1}{3} \pi r^{2} h=\frac{1}{3} \pi(3)^{27}=3 \times \frac{22}{7} \times 7=66 \mathrm{~cm}^{3}$
The volume of remaining solid $=$ volume of the cube - the volume of a cone
$=343-66$
$=277 \mathrm{~cm}^{3}$
Question 7. Two cones with the same base radius of 8 cm and height of 15 cm are joined together along with their bases. Find the surface area of the shape so formed.

Solution:


We know that,
The total surface area of the shape formed = Curved area of first cone + Curved surface area of the second cone

Since both cones are identical, we have,
The total surface area of the shape formed = Curved area of first cone + Curved surface area of the second cone $=2$ (Surface area of the cone)
We also know that,
The surface area of cone $=\pi r l$, where $r=$ radius and $\mathrm{I}=$ slant height and, the total Surface area of shape so formed $=2 \pi r l$

Given in the question that,
Radius, $\mathrm{r}=8 \mathrm{~cm}$
Height, $h=15 \mathrm{~cm}$
Therefore,
Area $=$ Curved area of first cone + Curved surface area of the second cone
$=2$ (Surface area of the cone)
$=2 \times \pi r$
$=2 \times \pi \times r \times \sqrt{ }\left(r^{2}+h^{2}\right)$
$=2 \times(22 / 7) \times 8 \times \sqrt{ }\left(8^{2}+15^{2}\right)$
$=50.28 \times \sqrt{ } 289$
$=854.85 \mathrm{~cm}^{2}$
$=855 \mathrm{~cm}^{2}$ (approx)
Hence, the surface area of shape so formed is $855 \mathrm{~cm}^{2}$.

## Exercise 12.4

Question 1. A solid metallic hemisphere of radius 8 cm is melted and recast into a right circular cone of base radius 6 cm . Determine the height of the cone.

Solution: For hemisphere,
Radius, $\mathrm{r}=8 \mathrm{~cm}$
We know that, the volume of hemisphere $=2 / 3 \pi r^{3}$, where, $r=$ radius of the hemisphere
So, we get,
The volume of given hemisphere $=2 / 3 \times \pi \times 8^{3}=(1024 / 3) \pi \mathrm{cm}^{3}$
Now, for the cone that is recast from a hemisphere,
Base radius, $r=6 \mathrm{~cm}$
We also know that,
The volume of cone $=1 / 3 \pi r^{2} h$, where, $r$ is base radius and $h$ is the height of the cone.

So, Volume of cone $=1 / 3 \pi(6)^{2} h=12 \pi h$
According to the question,
The volume remains the same when a body is reformed to another body
The volume of cylinder = Volume of a cone
$12 \pi h=1024 \pi / 3$
$\mathrm{h}=28.44 \mathrm{~cm}$
Question 2. A rectangular water tank of base $11 \mathrm{~m} \times 6 \mathrm{~m}$ contains water up to a height of 5 m . If the water in the tank is transferred to a cylindrical tank with a radius of 3.5 m , find the height of the water level in the tank.

## Solution:

The volume of water in tank = volume of the cuboidal tank up to a height of 5 m

According to the question,
For cuboidal tank
Length, $I=11 \mathrm{~m}$
Breadth, $b=6 \mathrm{~m}$
Height, $h=5 \mathrm{~m}$
We know that the equation to find the volume of the tank,
The volume of tank = be, where, I, b and h are the length, breadth and height of the tank respectively
Volume of water $=11(6)(5)=330 \mathrm{~m}^{3}$
We also know that,
Base radius of a cylindrical tank, $r=3.5 \mathrm{~m}$
Let the height till which the cylindrical tank is filled $=\mathrm{hm}$
Hence, using the formula,
The volume of a cylinder $=\pi r^{2} h$, where $r$ is base radius and $h$ is the height of the cylinder
The volume of water in cylindrical tank $=\pi(3.5)^{2} \mathrm{~h}$
$330 \mathrm{~m}^{3}=\frac{22}{7} \times 3.5 \times 3.5 \times \mathrm{h}$
$330 \mathrm{~m}^{3}=\mathrm{h} \times 38.5$
$\mathrm{h}=8.57 \mathrm{~m}$
Hence, the height till which the cylindrical tank is filled $=8.57 \mathrm{~m}$
Question 3. How many cubic centimetres of iron is required to construct an open box whose external dimensions are $36 \mathrm{~cm}, 25 \mathrm{~cm}$ and 16.5 cm provided the thickness of the iron is 1.5 cm . If one cubic $\mathbf{c m}$ of iron weighs 7.5 g , find the weight of the box.

## Solution:

Let the length ( $I$, breath (b), and height (h) be the external dimension of an open box and thickness be x.


The volume of metal used in box = Volume of the external box - Volume of the internal box

Consider an external box,
Length, I = 36 cm
Breadth, $b=25 \mathrm{~cm}$
Height, $\mathrm{h}=16.5 \mathrm{~cm}$
We know that the equation of the volume of the cuboid is given by,

The volume of cuboid = be, where, I, b and h are the length, breadth and height of the tank respectively
Volume of external box $=36(25)(16.5)=14850 \mathrm{~cm}^{3}$
Consider the internal box,
The thickness of the two sides is reduced as follows,
Length, I'm = Length of external box-2(thickness of box) $=36-2(1.5)=33 \mathrm{~cm}$
Breadth, b' = Breadth of external box - 2(thickness of box) $=25-2(1.5)=22 \mathrm{~cm}$
Height, h' = Height of external box - thickness of box $=16.5-1.5=15 \mathrm{~cm}$
Volume of internal box $=33(22)(15)=10890$
Volume of metal in box $=14850-10890=3960 \mathrm{~cm}^{3}$
We know that $1 \mathrm{~cm}^{3}$ weighs 7.5 g
So, $3960 \mathrm{~cm}^{3}$ weighs $3960(7.5)=29,700 \mathrm{~g}$
Therefore, the weight of the box is $29,700 \mathrm{~g}$ i.e. 29.7 kg
Question 4. The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen is used up to writing 3300 words on average. How many words can be written in a bottle of ink containing one-fifth of a litre?

Solution: Let us first calculate the volume of a barrel of the pen that is of cylindrical shape

Consider barrel,
Since $1 \mathrm{~cm}=10 \mathrm{~mm}$
Base diameter $=5 \mathrm{~mm}=0.5 \mathrm{~cm}$
Base radius, $r=0.25 \mathrm{~cm}$
Height, $\mathrm{h}=7 \mathrm{~cm}$
We know that,
Volume of a cylinder $=\pi r^{2} h$
Volume of barrel $=\pi(0.25)^{2} 7$
Volume of barrel $=\frac{22}{7} \times 0.25 \times 0.25 \times 7=1.375 \mathrm{~cm}^{3}$
Hence, according to the question,
$1.375 \mathrm{~cm}^{3}$ of ink can write 3300 words
No of words that can be written by $1 \mathrm{~cm}^{3}$ of ink $=3300 / 1.375=2400$ words $1 / 5^{\text {th }}$ of a litre $=0.2 \mathrm{~L}$
We know that,
$1 \mathrm{~L}=1000 \mathrm{~cm}^{3}$
$0.2 \mathrm{~L}=200 \mathrm{~cm}^{3}$
So, no of words that can be written by $200 \mathrm{~cm}^{3}=2400(200)=480000$ words Therefore, $1 / 5^{\text {th }}$ of a litre of ink can write 480000 words.
Question 5 . Water flows at the rate of $10 \mathrm{~m} /$ minute through a cylindrical pipe 5
mm in diameter. How long would it take to fill a conical vessel whose diameter
at the base is 40 cm and depth 24 cm ?
Solution: Let the time taken by pipe to fill vessel $=\mathrm{t}$ minutes
Since water flows 10 m in 1 minute, it will flow 10t meters in t minutes.

According to the question,
The volume of conical vessel = Volume of water that passes through pipe int minutes
Consider conical pipe
Base Diameter $=40 \mathrm{~cm}$
Base radius, $r=20 \mathrm{~cm}$
Height, $\mathrm{h}=24 \mathrm{~cm}$
We know that the volume of cone $=1 / 3 \pi r^{2} h$
Volume of conical vessel $=1 / 3 \pi(20)^{2}(24)=3200 \pi \mathrm{~cm}^{3}$
Consider cylindrical pipe
Base diameter $=5 \mathrm{~mm}=0.5 \mathrm{~cm}$
Base radius, $r=0.25 \mathrm{~cm}$
Water covers 10 tm distance in the pipe,
Hence, we get,
Height, $h=10 \mathrm{t} m=1000 \mathrm{tcm}$
We also know that,
Volume of a cylinder $=\pi r^{2} h$
Volume of water passed in pipe $=\pi(0.25)^{2}(1000 \mathrm{t})=62.5 \mathrm{t} \pi \mathrm{cm}^{3}$
So, we have
$62.5 \mathrm{t} \pi=3200$
$62.5 \mathrm{t}=3200$
$\mathrm{t}=51.2$ minutes
We know that, 0.2 minutes $=0.2(60)$ seconds $=12$ seconds
Therefore, $\mathrm{t}=51$ minutes 12 seconds
Question 6. A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m . Find the volume of the rice. How much canvas cloth is required to just cover the heap?

Solution: According to the question,
Consider conical heap,
Base Diameter $=9 \mathrm{~cm}$
So, base radius, $r=4.5 \mathrm{~cm}$
Height, $\mathrm{h}=3.5 \mathrm{~cm}$
We know that, Slant height,

$$
\begin{aligned}
I & =\sqrt{r^{2}+h^{2}} \\
& =\sqrt{(4.5)^{2}+(3.5)^{2}} \\
& =\sqrt{20.25+12.25} \\
& =\sqrt{32.5} \\
I & =5.7 \mathrm{~cm}
\end{aligned}
$$

The equation of volume of cone $=\frac{1}{3} \pi r^{2} h$
We know that, the volume of rice = Volume of conical heap

Volume of rice $=\frac{1}{3} \pi(4.5)^{2}(3.5)=74.25 \mathrm{~cm}^{3}$
We also know that canvas requires to just cover heap = Curved surface area of conical heap
And curved surface area of a cone $=\pi r l$
Therefore, the canvas required $=\pi(4.5)(5.7)=80.61 \mathrm{~cm}^{2}$ [appx]
Question 7. A factory manufactures 120000 pencils daily. The pencils are cylindrical each of length 25 cm and circumference of the base as 1.5 cm . Determine the cost of colouring the curved surfaces of the pencils manufactured in one day at Rs 0.05 per dm ${ }^{2}$.

Solution: The shape of pencil = cylinder.
Let the radius of base $=\mathrm{rcm}$
Circumference of base $=1.5 \mathrm{~cm}$
The circumference of a circle is $2 \pi r=1.5 \mathrm{~cm}$
$r=1.5 / 2 \pi \mathrm{~cm}$
According to the question,
Height, $\mathrm{h}=25 \mathrm{~cm}$
We know that, the curved surface area of cylinder $=2 \pi r h$
Curved surface area of pencil $=2 \pi(1.5 / 2 \pi) 25=37.5 \mathrm{~cm}^{2}$
$1 \mathrm{~cm}=0.1 \mathrm{dm}$
$1 \mathrm{~cm}^{2}=0.01 \mathrm{dm}^{2}$
$37.5 \mathrm{~cm}^{2}=0.375 \mathrm{dm}^{2}$
Cost for coloring $1 \mathrm{dm}^{2}=$ Rs. 0.05
Cost for coloring $0.375 \mathrm{dm}^{2}$ (i.e. 1 pencil) $=$ Rs. 0.01875
Cost for coloring 120000 pencils $=120000 \times 0.01875=$ Rs. 2250
Question 8. Water is flowing at the rate of $15 \mathrm{~km} / \mathrm{h}$ through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. At what time will the level of water in the pond rise by 21 cm ?

## Solution:



Let the time taken by pipe to fill pond $=t$ hours
Water flows 15 km in 1 hour, so, it will flow 15 t meters in t hours.

We know that, the volume of the cuboidal pond up to height $21 \mathrm{~cm}=$ Volume of water that passes through a pipe in " t " hours

Considering cuboidal pond,
Length, $\mathrm{I}=50 \mathrm{~m}$
Breadth, $\mathrm{b}=44 \mathrm{~m}$
Height, $\mathrm{h}=21 \mathrm{~cm}=0.21 \mathrm{~m}$
We know that,
The volume of tank $=\mathrm{lbh}$
Volume of water $=50(44)(0.21)=462 \mathrm{~m}^{3}$
Considering the cylindrical pipe

$$
\text { Base diameter }=14 \mathrm{~cm}
$$

Base radius, $\mathrm{r}=7 \mathrm{~cm}=0.07 \mathrm{~m}$
Height, $h=15 \mathrm{tkm}=15000 \mathrm{t} \mathrm{m}$
We also know that,
Volume of a cylinder $=\pi r^{2} h$
The volume of water passed in pipe $=\pi(0.07)^{2}(15000 \mathrm{t})$
$=\frac{22}{7} \times 0.07 \times 0.07 \times 15000 \mathrm{t}$
$=231 \mathrm{tcm}{ }^{3}$
So, we have
$231 \mathrm{t}=462$
$\mathrm{t}=2$ hours
The time required to fill the tank up to a height of 25 cm is 2 hours.

## Question 9. A solid iron cuboidal block of dimensions $4.4 \mathrm{~m} \times 2.6 \mathrm{~m} \times 1 \mathrm{~m}$ is recast into a hollow cylindrical pipe of internal radius 30 cm and thickness 5 cm . Find the length of the pipe.

Solution: Considering cuboidal block
Length, $\mathrm{I}=4 \mathrm{~m}$
Breadth, $b=2.6 \mathrm{~m}$
Height, $\mathrm{h}=1 \mathrm{~m}$
We know that,
The volume of tank = lbh
Volume of cuboid $=4.4(2.6)(1)=11.44 \mathrm{~m}^{3}$
We know that,
The volume remains the same when a body is recast to another body.
According to the question,
The volume of a cylindrical pipe $=11.44 \mathrm{~m}^{3}$
Considering pipe or the hollow cylinder
Internal radius, $\mathrm{r}_{2}=30 \mathrm{~cm}=0.3 \mathrm{~m}$
Thickness $=5 \mathrm{~cm}$

External radius, $\mathrm{r}_{1}=$ Internal radius + thickness $=30+5=35 \mathrm{~cm}=0.35 \mathrm{~m}$
Let the length of pipe $=h$
We know that, the volume of a hollow cylinder $=\pi h\left(r_{1}{ }^{2}-r_{2}{ }^{2}\right)$


Hence, volume of pipe $=\pi h\left((0.35)^{2}-(0.3)^{2}\right)$
So, the length of the pipe is 112 m .
Question 10. 500 persons are taking a dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is $0.04 \mathrm{~m}^{3}$ ?
Solution: According to the question,
Average displacement by a person $=0.04 \mathrm{~m}^{3}$
Average displacement by 500 persons $=500 \times 0.04=20 \mathrm{~m}^{3}$
Hence, the volume of water raised in the pond $=20 \mathrm{~m}^{3}$


It is also given that,
Length of the pond, $I=80 \mathrm{~m}$
The breadth of the pond, $b=50 \mathrm{~m}$
Height = h
The volume of water raised in pond $=80(50)(\mathrm{h})$
$20 \mathrm{~m}^{3}=4000 \mathrm{~h}$
$\mathrm{h}=0.005 \mathrm{~m}=0.5 \mathrm{~cm}$
Therefore, raise the height of water $=0.5 \mathrm{~cm}$.

