## Chapter 12: Heron's Formula
## Exercise 12.1

**Question 1: A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side a. Find the area of the signal board, using Heron's formula.If its perimeter is 180 cm, what will be the area of the signal board?**

Answer: Let each side of the equilateral triangle be a.
Semi-perimeter of the triangle,

s = $\frac{a+a+a}{2} = \frac{3a}{2}$

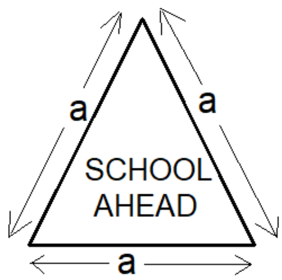
Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

=$\sqrt{s(s-a)(s-a)(s-a)} = \sqrt{s(s-a)^3}$

= $\sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)^3}$

= $\sqrt{\frac{3a}{2}\times\left(\frac{a}{2}\right)^3}$

= $\sqrt{\frac{3a^4}{2^4}} = \frac{\sqrt{3}}{4}a^2$



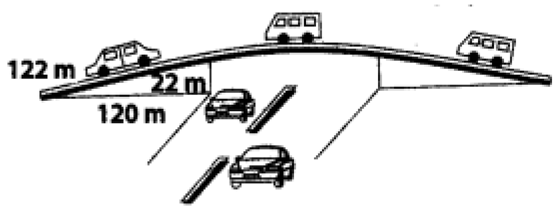

Now, its perimeter is 180cm, therefore,
a + a + a = 180cm
or, 3a = 180cm
or, a = $\frac{180}{3}$ cm = 60cm

Therefore, area of the triangle = $\frac{\sqrt{3}}{4}a^2$

= $\frac{\sqrt{3}}{4}(60)^2$ $cm^2$

=$900\sqrt{3}$ $cm^2$

**Question 2:** The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see figure). The advertisements yield an earning of ₹5000 per m² per year. A company hired one of its walls for 3 months. How much rent did it pay?




Answer: Let the sides of the triangular will be a = 122m, b = 120m, c = 22m

Semi-perimeter = $\frac{a+b+c}{2}$ = $(\frac{122+120+22}{2})$ m = $\frac{264}{2}$ m = 132m

The area of the triangular side wall

= $\sqrt{s(s-a)(s-b)(s-c)}$

=$\sqrt{132(132-122)(132-120)(132-22)}$ $m^2$

=$\sqrt{132 \times 10 \times 12 \times 110}$ $m^2$

=$\sqrt{12 \times 11 \times 10 \times 12 \times 11 \times 10}$ $m^2$

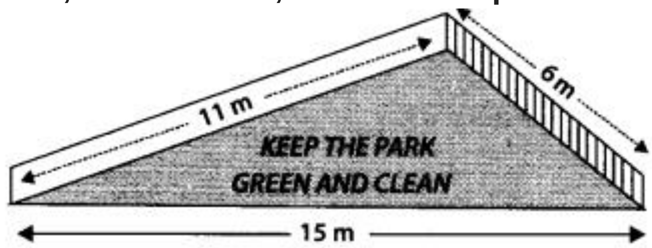
=1320 $m^2$

Rent for 1 year (i.e, 12 months) per $m^2$ = Rs. 5000

Rent for 3 months per $m^2$ = Rs. 5000 x $\frac{3}{12}$

= Rent for 3 months for 1320 $m^2$ = Rs. 5000 x $\frac{3}{12}$ x 1320 = Rs. 16,50,000.

**Question 3: There is a slide in a park. One of its side Company hired one of its walls for 3 months.walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see figure). If the sides of the wall are 15 m, 11 m and 6m, find the area painted in colour.**




Answer: Let the sides of the wall be a = 15m, b = 11m, c = 6m

Semi-perimeter = $\frac{a+b+c}{2}$ = $\frac{15+11+6}{2}$ m = $\frac{32}{2}$ m = 16m

Now, area of the triangular surface of the wall,

= $\sqrt{s(s-a)(s-b)(s-c)}$

=$\sqrt{16(16-15)(16-11)(16-6)}$ $m^2$

$=\sqrt{16 \times 1 \times 5 \times 10}$ m$^2$

$=\sqrt{80 \times 10}$ m$^2$

$=20\sqrt{2}$ m$^2$

Thus, the required area painted in colour = $20\sqrt{2}$ m$^2$

**Question 4: Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.**

Answer: Let the sides of the triangle be a =18 cm, b = 10 cm and c = x cm
Since, perimeter of the triangle = 42 cm
therefore, 8cm + 10 cm + x cm = 42 cm
x = [42 – (18 + 10)cm = 14cm

Now, semi-perimeter, s = $\frac{42}{2}$ cm = 21 cm

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{21(21-18)(21-10)(21-14)}$ cm$^2$

$= \sqrt{21 \times 3 \times 11 \times 7}$ cm$^2$

$=\sqrt{3 \times 7 \times 3 \times 11 \times 7}$ cm$^2$

$= 21\sqrt{11}$ cm$^2$

Thus, the required area of the triangle = $21\sqrt{11}$ cm$^2$

**Question 5: Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540 cm. Find its area.**

Answer: Let the sides of the triangle be
a = 12x cm, b = 17x cm, c = 25x cm
Perimeter of the triangle = 540 cm
Now, 12x + 17x + 25x = 540
or, 54x = 540⇒ x = 10
therefore, a = (12 x10)cm = 120cm,
b = (17 x 10) cm = 170 cm
and c = (25 x 10)cm = 250 cm

Now, semi-perimeter, s = $\frac{540}{2}$cm = 270 cm

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{270(270-120)(270-170)(270-250)}$ cm$^2$

$= \sqrt{270 \times 150 \times 100 \times 20}$ cm$^2$

$= \sqrt{3^2 \times 3^2 \times 5^2 \times 10^2 \times 10^2 \times 2^2}$ cm$^2$

$= (10 \times 10 \times 3 \times 3 \times 5 \times 2)$ cm$^2$

= 9000 cm$^2$

**Question 6: An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.**

Answer: Let the sides of an isosceles triangle be a = 12cm, b = 12cm,c = x cm
Since, perimeter of the triangle = 30 cm
12cm + 12cm + x cm = 30 cm
or, x = (30 – 24) = 6
Now, semi-perimeter, s = $\frac{30}{2}$cm =15 cm

Therefore, Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-12)(15-12)(15-6)}$ cm$^2$

$= \sqrt{15 \times 3 \times 3 \times 9}$ cm$^2$

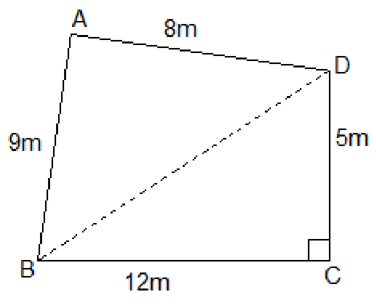
$= 9\sqrt{15}$ cm$^2$

Thus, the required area of the triangle = $9\sqrt{15}$ cm$^2$

## Exercise 12.2

**Question 1: A park, in the shape of a quadrilateral ABCD, has ∠C = 90°, AB = 9m,BC = 12m,CD = 5m and AD = 8 m. How much area does it occupy?**

Answer: Given, a quadrilateral ABCD with ZC = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. Let us join B and D, such that ABCD is a right angled triangle.



We have, area of ∆BCD = $\frac{1}{2} \times BC \times CD$ = $\left(\frac{1}{2} \times 12 \times 5\right)$ $m^2$ = $30m^2$

Now, to find the area of ∆ABD, we need the length of BD.
In right-angled ∆BCD, by Pythagoras theorem
$BD^2 = 50^2 + CD^2$
or, $BD^2 = 12^2 + 5^2$
or, $BD^2 = 144 + 25 = 169$
or, BD = 13 m
Now, for ∆ABD, we have
a = AB = 9 m, b = AD = 8 m, c = BD = 13 m

Semi-perimeter, s = $\frac{a+b+c}{2} = \frac{9+8+13}{2}$ m = $\frac{30}{2}$ m = 15m

Area of ∆ABD = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{15(15-9)(15-8)(15-13)}$ $m^2$

= $\sqrt{15 \times 6 \times 7 \times 2}$ $m^2$

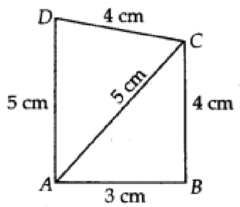
= $3 \times 2\sqrt{35}$ $m^2$

= 35.5 $m^2$ (approx.)

Therefore, Area of quadrilateral ABCD = area of ∆BCD + area of ∆ABD = 30 $m^2$ + 35.5 $m^2$
= 65.5 $m^2$ (approx.)

**Question 2: Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm,**
**CD = 4 cm, DA = 5 cm and AC = 5 cm**

Answer: Given a quadrilateral ABCD with AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.




For ∆ABC, a = AB = 3 cm, b = BC = 4 cm and c = AC = 5 cm

Semi-perimeter, s = $\frac{a+b+c}{2} = \frac{3+4+5}{2}$ cm = $\frac{12}{2}$ cm = 6cm

Area of ∆ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{6(6-3)(6-4)(6-5)}$ $cm^2$

= $\sqrt{6 \times 3 \times 2 \times 1}$ $cm^2$

= $6cm^2$

For $\Delta ACD,$ a = AD = 5cm
b = CD = 4cm
c = AC = 5cm

Semi-perimeter, s = $\frac{a+b+c}{2}$ = $\frac{5+4+5}{2}$ cm = $\frac{14}{2}$ cm = 7cm

Area of $\Delta ACD$ = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{7(7-5)(7-4)(7-5)}$ $cm^2$

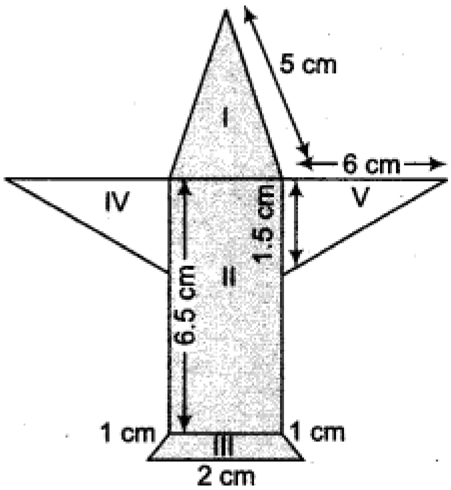
= $\sqrt{7 \times 2 \times 3 \times 2}$ $cm^2$

= $2\sqrt{21}$ $cm^2$

= 9.2 $cm^2$ (approx.)

Now, area of quadrilateral ABCD = area of $\Delta$ABC + area of $\Delta$ACD
= 6 $cm^2$ + 9.2 $cm^2$ = 15.2 $cm^2$ (approx.)

**Question 3: Radha made a picture of an aeroplane with coloured paper as shown in figure. Find the total area of the paper used.**




Answer: For surface I:

It is an isosceles triangle whose sides are a = 5 cm, b = 5 cm, c = 1 cm

Semi perimeter, s = $\frac{a+b+c}{2}$ = $\frac{5+5+1}{2}$ cm = $\frac{11}{2} cm$

Area of Surface I = $\sqrt{s(s-a)(s-b)(s-c)}$

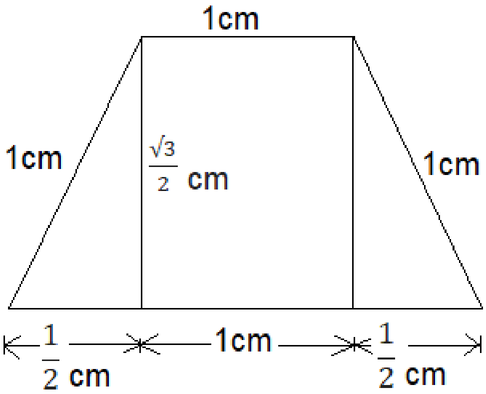
$= \sqrt{\frac{11}{2}\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-1\right)}$

$= \sqrt{\frac{11}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{9}{2}}$ $cm^2$

$= \frac{3}{4}\sqrt{11}$ $cm^2$

= (0.75 x 3.3) $cm^2$
= 2.475 $cm^2$ (approx.)

For surface II:
It is a rectangle with length 6.5 cm and breadth 1 cm.
therefore, Area of surface II = Length x Breadth = (6.5 x 1) $cm^2$ = 6.5 $cm^2$




Its height is given by,
h = $\sqrt{1^2 - \left(\frac{1}{2}\right)^2}$ cm

$= \sqrt{1-\frac{1}{4}}$ cm

$= \sqrt{\frac{3}{4}}$ cm

$= \frac{\sqrt{3}}{2}$ cm

Area of a trapezium = $\frac{1}{2}\left[(sum\ of\ the\ parallel\ sides)\ \times Height\right]$

Therefore, area of Surface III = $\frac{1}{2}\times\left[(2+1)\times\frac{\sqrt{3}}{2}\right]$ $cm^2$

$= \frac{3\sqrt{3}}{2}$ cm$^2$

$= 1.3$ cm$^2$ (approx.)

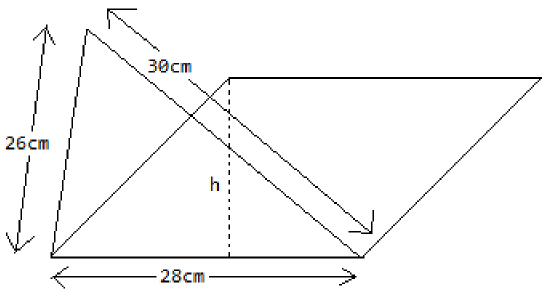
For surface IV and V

Surface V is a right-angled triangle with base 6cm arid height 1.5 cm.
Also, area of surface IV = area of surface V
$= \frac{1}{2}$ x base x height
$= (\frac{1}{2}$ x 6 x 15) cm$^2$ = 4.5 cm$^2$

Thus, the total area of the paper used = (area of surface I) + (area of surface II) + (area of surface III) + (area of surface IV) + (area of surface V) = [2.475 + 6.5 + 1.3 + 4.5 + 4.5] cm$^2$
= 19.275 cm$^2$
= 19.3 cm$^2$ (approx.)

**Question 4: A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.**

Answer: For the given triangle, we have a = 28 cm, b = 30 cm, c = 26 cm




Semi-perimeter, s = $\frac{a+b+c}{2} = \frac{28+30+26}{2}$ cm = $\frac{84}{2}$ cm = 42cm

Area of $\Delta$ = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-28)(42-30)(42-26)}$ cm$^2$

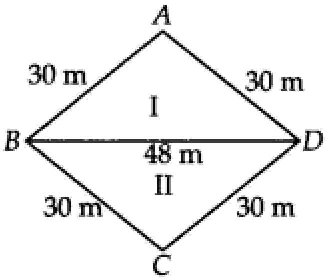
$= \sqrt{42 \times 14 \times 12 \times 16}$ cm$^2$

$= \sqrt{112896}$ cm$^2$

Area of the given parallelogram = Area of the given triangle
therefore, area of the parallelogram = 336 cm$^2$
or, base x height = 336

or, 28 x h = 336, where ‘h’ be the height of the parallelogram.
or, h = $\frac{336}{28}$ = 12
Thus, the required height of the parallelogram = 12 cm

**Question 5: A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?**
Answer: Here, each side of the given rhombus = 30 m.
Let ABCD be the given rhombus and the diagonal, BD = 48 m




Sides $\Delta$ABC are a = AB = 30m, b = AD = 30m, c = BD = 48m

Semi-perimeter, s = $\frac{a+b+c}{2} = \frac{30+30+48}{2}$ m = $\frac{108}{2}$ m = 54m

Area of triangle I = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{54(54-30)(54-30)(54-48)}$ $m^2$

= $\sqrt{54 \times 24 \times 24 \times 6}$ $m^2$

= $\sqrt{186624}$ $m^2$

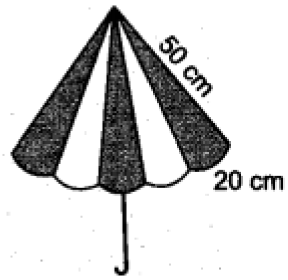
= 432 $m^2$

Since, a diagonal divides the rhombus into two congruent triangles.
therefore, area of triangle II = 432 $m^2$
Now, total area of the rhombus = Area of triangle I + Area of triangle II
= 432 $m^2$ + 432 $m^2$= 864 $m^2$

Area of grass for 18 cows to graze = 864 $m^2$
or, Area of grass for 1 cow to graze = $\frac{864}{18}$ $m^2$ = 48 $m^2$

**Question 6: An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see figure), each piece measuring 20 cm, 50 cm and 50**

**cm. How much cloth of each colour is required for the umbrella?**




Answer: Let the sides of each triangular piece be a = 20 cm, b = 50 cm, c = 50 cm

Semi-perimeter, s = $\frac{a+b+c}{2} = \frac{20+50+50}{2}$ cm = $\frac{120}{2}$ cm = 60cm

Area of each piece of triangle

= $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{60(60-20)(60-50)(60-50)}$ $cm^2$

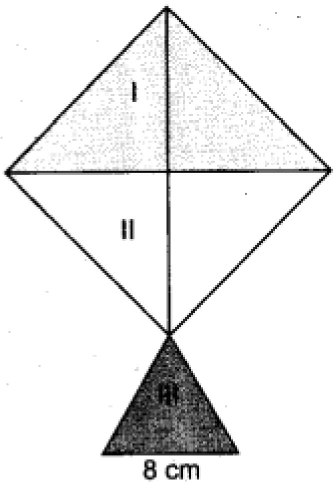
=$\sqrt{60 \times 40 \times 10 \times 10}$ $cm^2$

= $200\sqrt{6}$ $cm^2$

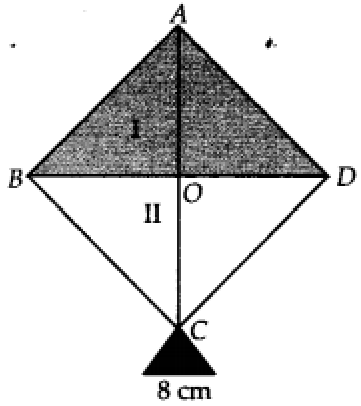
Area of 5 triangular pieces of one colour = $5\times 200\sqrt{6}$ $cm^2$ = $1000\sqrt{6}$ $cm^2$

Therefore, area of 5 triangular pieces of other colour = $1000\sqrt{6}$ $cm^2$

**Question 7: A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used in it?**



Answer: Each shade of paper is divided into 3 triangles i.e., I, II, III




For triangle I:

Given that ABCD is a square. As diagonals of a square are equal and bisect each other therefore, AC = BD = 32 cm
Height of ΔABD = OA = ($\frac{1}{2}$ x 32 )cm = 16 cm
Area of triangle I = ($\frac{1}{2}$ x 32 x 16 ) $cm^2$ = 256$cm^2$

For triangle II:

Since, diagonal of a square divides it into two congruent triangles.
So, area of triangle II = area of triangle I
therefore. area of triangle II = 256 $cm^2$

For triangle III:

The sides are given as a = 8 cm, b = 6 cm and c = 6 cm

Semi-perimeter, s = $\frac{a+b+c}{2} = \frac{8+6+6}{2}$ cm = $\frac{20}{2}$ cm = 10cm

Area of triangle III = $\sqrt{s(s-a)(s-b)(s-c)}$

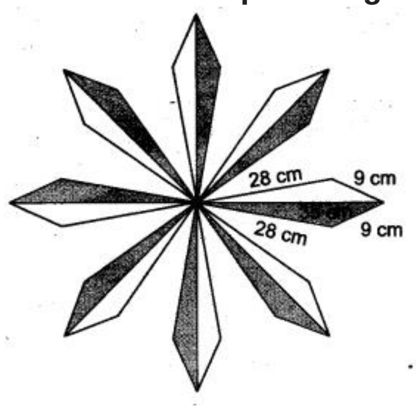
= $\sqrt{10(10-8)(10-6)(10-6)}$ $cm^2$

=$\sqrt{10 \times 2 \times 4 \times 4}$ $cm^2$

=$8\sqrt{5}$ $cm^2$ = 17.92 $cm^2$

Thus, the area of different shades are:
Area of shade I = 256 $cm^2$
Area of shade II = 256 $cm^2$
and area of shade III = 17.92 $cm^2$

**Question 8: A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see figure). Find the cost of polishing the tiles at the rate of 50 paise per cm .**




Answer: Let the sides of the triangle be a = 9 cm, b = 28 cm, c = 35 cm

Semi perimeter, $s = \frac{a+b+C}{2}$

= $\left(\frac{9+28+35}{2}\right) cm$

= $\frac{72}{2} cm = 36\ cm$

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{36(36-9)(36-28)(36-35)}$

=$\sqrt{36 \times 27 \times 8 \times 1}\ cm^2$

= $3 \times 2 \times 3 \times 2\sqrt{3 \times 2}\ cm^2$

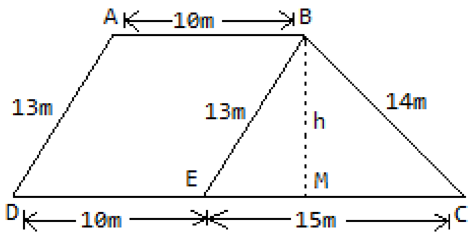
= $36\sqrt{6}\ cm^2 = (36 \times 2.45)\ cm^2$

= 88.2 $cm^2$ (approx.)

Total area of all the 16 triangles = (16 x 88.2) $cm^2$ = 1411.2 $cm^2$ (approx.)
Cost of polishing the tiles = Rs. 0.5 per $cm^2$
therefore, cost of polishing all the tiles = Rs. (0.5 x 1411.2) = Rs. 705.60 (approx.)

**Question 9: A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.**

Answer: Let the given field is in the form of a trapezium ABCD such that parallel sides are AB = 10 m and DC = 25 m
Non-parallel sides are AD = 13 m and BC = 14 m.
We draw BE || AD, such that BE = 13 m.




The given field is divided into two shapes (i) ∆BCE, (ii) parallelogram ABED For ∆BCE:
Sides of the triangle are a = 13 m, b = 14 m, c = 15 m

Semi perimeter, $s = \frac{a+b+C}{2}$

= $\left(\frac{13+14+15}{2}\right) m$

= $\frac{42}{2} cm = 21\ m$

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{21(21-13)(21-14)(21-15)}\ m^2$

= $\sqrt{21 \times 8 \times 7 \times 6}\ m^2 = \sqrt{7056}\ m^2 = 84\ m^2$

For parallelogram ABED:
Let the height of the ∆BCE corresponding to the side EC be h m.
Area of a triangle = $\frac{1}{2}$ x base x height
therefore, $\frac{1}{2}$ x 15 x h = 84sq m
Now, area of a parallelogram = base x height

$= (10 \times \frac{56}{5})\ m^2 = (2 \times 56)\ m^2 = 112\ m^2$

So, area of the field
= area of $\Delta BCE$ + area of parallelogram ABED
$= 84\ m^2 + 112\ m^2 = 196\ m^2$