Chapter 7 – Coordinate Geometry
Exercise -7.1

Question 1: Find the distance between the following pairs of points:

(i) (2, 3), (4, 1)
(ii) (-5, 7), (-1, 3)
(iii) (a, b), (- a, – b)

Answer: As we know that, the distance between two points (x_1, y_1) and (x_2, y_2) is,
d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ or, $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

(i) d = $\sqrt{(4-2)^2 + (1-3)^2} = \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

(ii) d = $\sqrt{\left((-1)+5\right)^2 + (3-7)^2} = \sqrt{4^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ units

(iii) d = $\sqrt{\left((-a)-a\right)^2 + (-b-b)^2} = \sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2}$ units

Question 2: Find the distance between the points (0, 0) and (36, 15).

Answer: Let the points be A(0,0) and B(36,15)
Distance between the points be,
AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
= $\sqrt{(36-0)^2 + (15-0)^2} = \sqrt{36^2 + 15^2} = \sqrt{1296 + 225} = \sqrt{1529}$ = 39 units.

Question 3: Determine if the points (1, 5), (2, 3) and (-2, -11) are colinear.

Answer: We know that, the sum of the lengths of any two line segments is equal to the length of the third line segment then all three points are collinear.

Let us consider, A = (1, 5) B = (2, 3) and C = (-2, -11)

Hence, AB = $\sqrt{(2-1)^2 + (3-5)^2} = \sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$ units
BC = $\sqrt{(-2-2)^2 + (-11-3)^2} = \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212}$ units,
CA = $\sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$ units

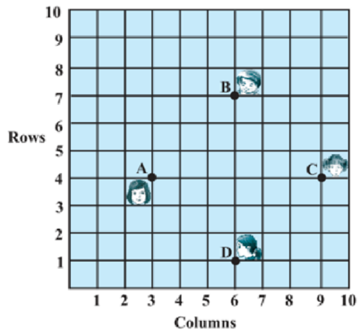
Since AB + BC ≠ CA
Hence, the points (1, 5), (2, 3), and (– 2, – 11) are not collinear.

Question 4: Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.

Answer: For isosceles triangle, two sides need to be equal.
Now, let us consider the points (5, – 2), (6, 4), and (7, – 2) are representing the vertices A, B, and C respectively.

AB = $\sqrt{(6-5)^2+(4+2)^2}$ = $\sqrt{(-1)^2+6^2}$ = $\sqrt{1+36}$ = $\sqrt{37}$ units,
BC = $\sqrt{(7-6)^2+(-2-4)^2}$ = $\sqrt{1^2+(-6)^2}$ = $\sqrt{1+36}$ = $\sqrt{37}$units
CA = $\sqrt{(7-5)^2+(-2+2)^2}$ = $\sqrt{2^2+0^2}$ = $\sqrt{4+0}$ = $\sqrt{4}$ = 2units
Hence, AB = BC = $\sqrt{37}$ units
Therefore, the points formed an isosceles triangle.

Question 5: In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. 7.8. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.



Answer: From the given figure, the coordinates of points A, B, C and D are (3, 4), (6, 7), (9, 4) and (6,1).

AB = $\sqrt{(6-3)^2+(7-4)^2}$ = $\sqrt{3^2+3^2}$ = $\sqrt{9+9}$ = $3\sqrt{2}$
BC = $\sqrt{(9-6)^2+(4-7)^2}$ = $\sqrt{3^2+3^2}$ = $\sqrt{9+9}$ = $3\sqrt{2}$
CD = $\sqrt{(6-9)^2+(1-4)^2}$ = $\sqrt{3^2+3^2}$ = $\sqrt{9+9}$ = $3\sqrt{2}$
DA = $\sqrt{(6-3)^2+(1-4)^2}$ = $\sqrt{3^2+3^2}$ = $\sqrt{9+9}$ = $3\sqrt{2}$

AB = BC = CD = DA = $3\sqrt{2}$ units
All the sides are of equal length.
Hence, ABCD is a square and therefore, Champa was correct.

Question 6: Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) (- 1, – 2), (1, 0), (- 1, 2), (- 3, 0)
(ii) (- 3, 5), (3, 1), (0, 3), (- 1, – 4)
(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Answer: (i) Let the points (- 1, – 2), (1, 0), (– 1, 2), and (– 3, 0) be representing the vertices A, B, C, and D respectively.
AB = $\sqrt{(1+1)^2+(0+2)^2}$ = $\sqrt{2^2+2^2}$ = $\sqrt{4+4}$ = $2\sqrt{2}$ units
BC = $\sqrt{(-1-1)^2+(2-0)^2}$ = $\sqrt{2^2+2^2}$ = $\sqrt{4+4}$ = $2\sqrt{2}$ units
CD = $\sqrt{(-3+1)^2+(0-2)^2}$ = $\sqrt{(-2)^2+(-2)^2}$ = $\sqrt{4+4}$ = $2\sqrt{2}$units
DA = $\sqrt{(-3+1)^2+(0-2)^2}$ = $\sqrt{(-2)^2+(-2)^2}$ = $\sqrt{4+4}$ = $2\sqrt{2}$ units
AC = $\sqrt{(-1+1)^2+(2+2)^2}$ = $\sqrt{0^2+4^2}$ = $\sqrt{0+16}$ = 4units
BD = $\sqrt{(-3-1)^2+(0-0)^2}$ = $\sqrt{4^2+0^2}$ = $\sqrt{16+0}$ = 4units

Side length = AB = BC = CD = DA = $2\sqrt{2}$ and the diagonal measures = AC = BD = 4
Therefore, the given points are the vertices of a square.

(ii) Let the points (- 3, 5), (3, 1), (0, 3), and (– 1, – 4) be representing the vertices A, B, C, and D respectively.

AB = $\sqrt{(-3-3)^2+(1-5)^2} = \sqrt{6^2+(-4)^2} = \sqrt{36+16} = 2\sqrt{13}$ units
BC = $\sqrt{(0-3)^2+(3-1)^2} = \sqrt{(-3)^2+2^2} = \sqrt{9+4} = \sqrt{13}$ units
CD = $\sqrt{(-1-0)^2+(-4-3)^2} = \sqrt{(-1)^2+(-7)^2} = \sqrt{1+49} = 5\sqrt{2}$
AD = $\sqrt{(-1+3)^2+(-4-5)^2} = \sqrt{(2)^2+(-9)^2} = \sqrt{4+81} = \sqrt{85}$ units

We can clearly see that, the above points do not form a quadrilateral.

(iii) Let the points (4, 5), (7, 6), (4, 3), and (1, 2) be representing the vertices A, B, C, and D respectively.
AB = $\sqrt{(7-4)^2+(6-5)^2} = \sqrt{3^2+1^2} = \sqrt{4+1} = \sqrt{5}$ units
BC = $\sqrt{(4-7)^2+(3-6)^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18}$units
CD = $\sqrt{(1-4)^2+(2-3)^2} = \sqrt{(-3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$
DA = $\sqrt{(1-4)^2+(2-5)^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18}$ units
AC = $\sqrt{(4-4)^2+(3-5)^2} = \sqrt{(0)^2+(-2)^2} = \sqrt{0+4} = 2$ units
BD = $\sqrt{(1-7)^2+(2-6)^2} = \sqrt{(-6)^2+(-4)^2} = \sqrt{36+16} = 2\sqrt{13}$ units
As, the opposite sides of this quadrilateral are of the same length and the diagonals are of different lengths, therefore, the given points are the vertices of a parallelogram.

Question 7: Find the point on the x-axis which is equidistant from (2, – 5) and (-2, 9).

Answer: Let A(2, -5) and B(-2, 9) be the given points.
Also let P(x, 0) be the point on the x-axis such that,
PA = PB
or, $PA^2 = PB^2$

$(x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$
or, $(x-2)^2 - (x+2)^2 = 81 - 25$
or, (x – 2 + x + 2)(x + 2 – x – 2) = 56
or, (2x)(-4) = 56
or, -8x = 56
or, x = -7
Hence, the required point is (-7, 0)

Question 8: Find the values of y for which the distance between the points P (2, – 3) and Q (10, y) is 10 units.

Answer: Distance between P(2, – 3) and Q(10, y) is 10.
Hence, PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
or, 10 = $\sqrt{(10-2)^2 + (y+3)^2}$
= $\sqrt{8^2 + (y+3)^2}$
or,100 = 64 + y^2 + 9 + 6y
or, 100 = y^2 + 6y + 73
or, y^2 + 6y = 100 – 73
or, y^2 + 6y – 27 = 0
or, y^2 + 9y – 3y – 27 = 0
or, y(y + 9) – 3(y + 9) = 0
or, (y – 3) (y + 9) = 0
or, y – 3 = 0
or, y + 9 = 0

Hence, y = 3 or (-9)

Question 9: If Q (0, 1) is equidistant from P (5, – 3) and R (x, 6), find the values of x. Also find the distance QR and PR.

Answer: Q(0, 1) is equidistant from P(5, -3) and R(x, 6)
Hence, QP = QR
or, $QP^2 = QR^2$
or, $(5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$
or, 25 + 16 = x^2 + 25
or, x^2 = 16
or, x = ± 4

Now, if R(4, 6) then,

QR = $\sqrt{(0-4)^2 + (1-6)^2} = \sqrt{4^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$

PR = $\sqrt{(5-4)^2 + (-3-6)^2} = \sqrt{(1)^2 + 9^2} = \sqrt{1+81} = \sqrt{82}$

And R(-4, 6) then,

QR = $\sqrt{(0+4)^2 + (1-6)^2} = \sqrt{4^2 + (-5)^2} = \sqrt{4^2 + 5^5} = \sqrt{16+25} = \sqrt{41}$ units

PR = $\sqrt{(5+4)^2 + (-3-6)^2} = \sqrt{(9)^2 + (-9)^2} = \sqrt{81+81} = 9\sqrt{2}$ units

Question 10: Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (- 3, 4).

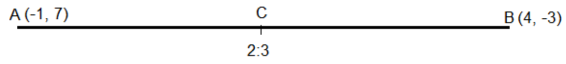
Answer: Point (x, y) is equidistant from (3, 6) and (– 3, 4).
Therefore,
$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-(-3))^2 + (y-4)^2}$
or, $(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$
or, x^2 +9 – 6x + y^2 + 36 – 12y = x^2 + 9 + 6x + y^2 + 16 – 8y
or, 36 – 16 = 6x + 6x + 12y – 8y
or, 20 = 12x + 4y

or, 3x + y = 5
or, 3x + y – 5 = 0

Exercise 7.2

Question 1: Find the coordinates of the point which divides the join of (- 1, 7) and (4, – 3) in the ratio 2:3.

Answer: Let the coordinates of point C be (x, y)



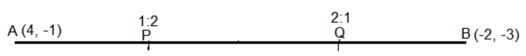
x-coordinate of C = $\frac{mx_2+nx_1}{m+n} = \frac{2\times4+3\times(-1)}{2+3} = \frac{8-3}{5} = 1$

y-coordinate of C = $\frac{my_2+ny_1}{m+n} = \frac{2\times(-3)+3\times7}{2+3} = \frac{-6+21}{5} = 3$

Hence, the coordinates of C are (1, 3).

Question 2: Find the coordinates of the points of trisection of the line segment joining (4, -1) and (-2, -3).

Answer:



Let points P and Q trisect the line AB. Therefore, AP = PQ = QB
hence, the point P divides AB internally in the ratio 1 : 2 and Q divides AB in the ratio 2 : 1.

P(x – coordinate) = $\frac{1\times(-2)+2\times4}{1+2} = \frac{-2+8}{3} = 2$

P(y – coordinate) = $\frac{1\times(-3)+2\times(-1)}{1+2} = \frac{-3-2}{3} = -\frac{5}{3}$

The coordinates of P are $\left(2, -\frac{5}{3}\right)$

Q(x – coordinate) = $\frac{2\times(-2)+1\times4}{1+2} = \frac{-4+4}{3}$ = 0

Q(y – coordinate) = $\frac{2\times(-3)+1\times(-1)}{1+2} = \frac{-6-1}{3} = -\frac{7}{3}$

The coordinates of Q are $\left(0, -\frac{7}{3}\right)$

Question 3: To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the following figure. Niharika runs 1/4 th the distance AD on the 2nd line and posts a green flag. Preet runs 1/5th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

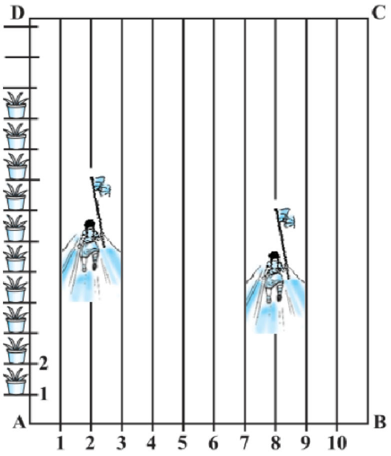


Fig. 7.12

Answer: From the above figure, A as (0,0) x-axis along AB and y-axis along AD, we will obtain the coordinates of the green flag and the red flag.

The green flag is at $\frac{1}{4}$th of the total distance = $\frac{1}{4} \times 100$ = 25 m in second line.
Therefore, the coordinates of the green flag are (2, 25)

The red flag is at $\frac{1}{5}$th of the total distance = $\frac{1}{5} \times 100$ = 20m in 8th line
Therefore, the coordinates of the red flag are (8, 20)

Hence, the distance between the flags
= $\sqrt{(8-2)^2 + (20-25)^2} = \sqrt{(6)^2 + (5)^2} = \sqrt{36+25} = \sqrt{61}$ m

Now, the blue flag is posted at the midpoint of the distance between two flags,
Coordinates of the blue flag = $\left(\frac{2+8}{2}, \frac{25+20}{2}\right)$ = (5, 22.5)

Therefore, the blue flag will be posted in 5^{th} line at a distance of 22.5m

Question 4: Find the ratio in which the line segment joining the points (-3, 10) and (6, -8) is divided by (-1, 6).

Answer: let the required ratio be k : 1

x = $\frac{mx_2+nx_1}{m+n}$

or, -1 = $\frac{k\times6+1\times(-3)}{k+1}$

or, -k – 1 = 6k – 3

or, 7k = 2

or, k = $\frac{2}{7}$

y = $\frac{my_2+ny_1}{m+n}$

or, 6 = $\frac{k\times(-8)+1\times10}{k+1}$

or, 6k + 6 = -8k + 10

or, 14k = 4

or, k = $\frac{2}{7}$

Hence, the required ratio is 2 : 7

Question 5: Find the ratio in which the line segment joining A (1, – 5) and B (-4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Answer: Let P(x, 0) be the point which divides the line segment joining A(1,5) and B(-4, 5) in the ratio m : 1

Then using the section formula, we get,

(x, 0) = $\left(\frac{m\times(-4)+1\times1}{m+1}, \frac{m\times5+1\times(-5)}{m+1}\right)$

or, 0 = $\frac{m\times5+1\times(-5)}{m+1}$ …………………………(Taking y-coordinate)

or, 5m – 5 = 0

or, m = 1

or, m : 1 = 1 : 1

Hence, the required ratio is 1 : 1.

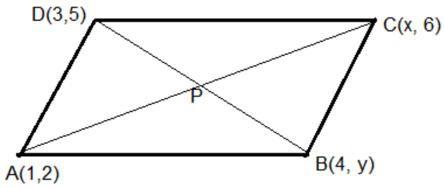
Since, the ratio is 1 : 1, P is the midpoint.

Therefore, x = $\frac{1-4}{2}$ = $\frac{-3}{2}$

Hence, $\left(-\frac{3}{2}, 0\right)$

Question 6: If (1, 2), (4, *y*), (*x*, 6) and (3, 5) are the vertices of a parallelogram taken in order, find *x* and *y*.

Answer:



Midpoint of AC = midpoint of BD.

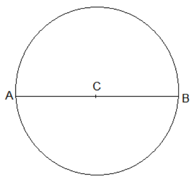
$\frac{x+1}{2}, \frac{6+2}{2} = \frac{4+3}{2}, \frac{y+5}{2}$

$\frac{x+1}{2} = \frac{7}{2}$ and $\frac{6+2}{2} = \frac{y+5}{2}$

or, x + 1 = 7 and 8 = y + 5
or, x = 6 and y = 3

Question 7: Find the coordinates of a point A, where AB is the diameter of circle whose centre is (2, – 3) and B is (1,4).

Answer: Let the coordinates of the point A be (x, y)
Then as C(2, - 3) is the midpoint of diameter AB.



Coordinates of C = $\left(\frac{x+1}{2}, \frac{y+4}{2}\right)$
or, 2 = $\frac{x+1}{2}$; -3 = $\frac{y+4}{2}$
or, x = 3 ; y = -10

Hence, the coordinates of A are (3, -10)

Question 8: If A and B are (-2, -2) and (2, -4), respectively, find the coordinates of P such that AP = 3/7 AB and P lies on the line segment AB.

Answer: AP = $\frac{3}{7}$AB
AB = AB – AP

$= \frac{AB}{1} - \frac{3}{7}AB$

$= \frac{4AB}{7}$

$\frac{AP}{BP} = \frac{\frac{3}{7}AB}{\frac{4}{7}AB} = 3 : 4$

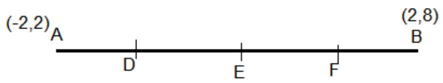
$x = \frac{3\times(2)+4\times(-2)}{3+4} = \frac{6-8}{7} = -\frac{2}{7}$

$y = \frac{3\times(-4)+4\times(-2)}{3+4} = \frac{-12-8}{7} = -\frac{20}{7}$

Hence, the coordinates of $P\left(-\frac{2}{7}, -\frac{20}{7}\right)$

Question 9: Find the coordinates of the points which divide the line segment joining A (- 2, 2) and B (2, 8) into four equal parts.

Answer:



AD = DE = EF = FB [Given]
Hence, E is the midpoint of AB.

Coordinates of E = $\left(\frac{-2+2}{2}, \frac{2+8}{2}\right)$ = (0,5)

D is the midpoint of AE.

Coordinates of E = $\left(\frac{-2+0}{2}, \frac{2+5}{2}\right) = \left(-1, \frac{7}{2}\right)$

F is the midpoint of EB.
Coordinates of F = $\left(\frac{0+2}{2}, \frac{5+8}{2}\right) = \left(1, \frac{13}{2}\right)$

Hence, the required points are $\left(-1, \frac{7}{2}\right)$, $(0,5)$ and $\left(1, \frac{13}{2}\right)$

Question 10: Find the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2,-1) taken in order. [Hint: Area of a rhombus = 1/2(product of its diagonals)

Answer:

Let points be A(0,3) B(4,5) C(-1,4) D(-2,-1)
AC = $\sqrt{(-1-3)^2 + (4-0)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ units

BD = $\sqrt{(4+2)^2+(5+1)^2}$ = $\sqrt{36+36}$ = $6\sqrt{2}$units

Area of the rhombus
= $\frac{1}{2} \times AC \times BD$

= $\frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$
= 24 sq. units

Exercise 7.3

Question 1: Find the area of the triangle whose vertices are:

(i) (2, 3), (-1, 0), (2, -4)
(ii) (-5, -1), (3, -5), (5, 2)

Answer: (i) Area of triangle
= $\frac{1}{2}$ [2 {0- (-4)} + (-1) {(-4) – (3)} + 2 (3 – 0)]

= $\frac{1}{2}$ {8 + 7 + 6}

= $\frac{21}{2}$sq. units

(ii) Area of the triangle
= $\frac{1}{2}$ [-5 { (-5)- (2)} + 3(2-(-1)) + 5{-1 – (-5)}]

= $\frac{1}{2}${35 + 9 + 20}

= 32sq.units

Question 2: In each of the following find the value of 'k', for which the points are collinear.

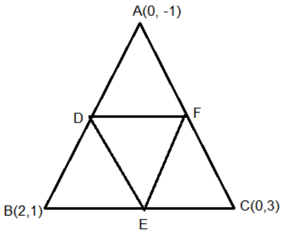
(i) (7, -2), (5, 1), (3, -k)
(ii) (8, 1), (k, -4), (2, -5)

Answer: i) For collinear points, area of triangle formed by them is always zero.
Therefore, area of triangle
$\frac{1}{2}$ [7 { 1- k} + 5(k-(-2)) + 3{(-2) – 1}] = 0
or, 7 – 7k + 5k +10 – 9 = 0
or, -2k + 8 = 0
or, k = 4

(ii)$\frac{1}{2}$ [8 { -4- (-5)} + k{(-5)-(1)} + 2{1 -(-4)}] = 0
or, 8 – 6k + 10 = 0
or, 6k = 18
or, k = 3

Question 3: Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Answer:



Coordinates of D = $\left(\frac{0+2}{2}, \frac{+1-1}{2}\right)$ = (1,0)

Coordinates of E = $\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$ = (1,2)

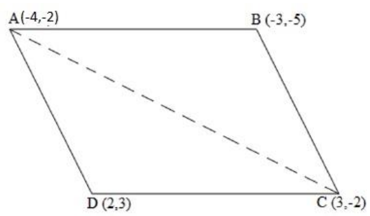
Coordinates of F = $\left(\frac{0+0}{2}, \frac{3-1}{2}\right)$ = (0,1)

Area of Triangle DEF = $\frac{1}{2}$ {1(2 – 1) + 1(1 – 0) + 0(0 – 2)} = $\frac{1}{2}$ (1+1) = 1 sq. units
Area of Triangle ABC = $\frac{1}{2}$ [0(1 – 3) + 2{3 -(- 1) } + 0(- 1 – 1)] = $\frac{1}{2}$ {8} = 4 sq. units.

Therefore, the required ratio is 1:4.

Question 4: Find the area of the quadrilateral whose vertices, taken in order, are
(-4, -2), (-3, -5), (3, -2) and (2, 3).

Answer: Let the vertices of the quadrilateral be A (- 4, – 2), B (– 3, – 5), C (3, – 2), and D (2, 3). Join AC and divide quadrilateral into two triangles.



Area of ΔABC

$= \frac{1}{2}$ [(-4) {(-5) – (-2)} + (-3) {(-2) – (-2)} + 3 {(-2) – (-5)}]

$= \frac{1}{2}$ (12 + 0 + 9)

$= \frac{21}{2}$ sq. units

Area of ΔACD

$= \frac{1}{2}$ [(-4) {(-2) – (3)} + 3{(3) – (-2)} + 2 {(-2) – (-2)}]

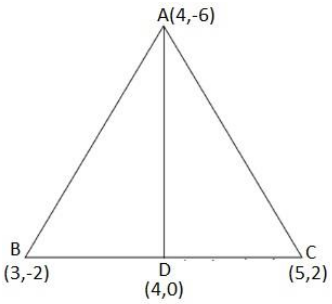
$= \frac{1}{2}$ (20 + 15 + 0)

$= \frac{35}{2}$ square units

Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD = $(\frac{21}{2} + \frac{35}{2})$ sq. units = 28 sq. units

Question 5: You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4, – 6), B (3, – 2) and C (5, 2).

Answer: Let the vertices of the triangle be A (4, -6), B (3, -2), and C (5, 2).



Let D be the mid-point of side BC of ΔABC and hence, AD is the median in ΔABC.

Coordinates of point D = Midpoint of BC = (4, 0)

Area of ΔABD

$= \frac{1}{2}$ [(4) {(-2) – (0)} + 3{(0) – (-6)} + (4) {(-6) – (-2)}]

$= \frac{1}{2}$ (-8 + 18 – 16)

= -3 sq. units

As, area cannot be negative hence, area of ΔABD is 3 sq. units.

Area of ΔACD = $\frac{1}{2}$ [(4) {0 – (2)} + 4{(2) – (-6)} + (5) {(-6) – (0)}] = $\frac{1}{2}$ (-8 + 32 – 30) = -3 sq. units

As, area cannot be negative, hence, area of ΔACD is 3 sq. units.

The area of both sides is same. Thus, median AD has divided ΔABC in two triangles of equal areas.

Exercise 7.4

Question 1: Determine the ratio in which the line 2x + y – 4 = 0 divides the line segment joining the points A(2, –2) and B(3, 7).

Answer: If the ratio in which P divides AB id k : 1 , the coordinates of the P will be, $\frac{kx_2+x_1}{k+1}, \frac{ky_2+y_1}{k+1}$

Let the given line divides the line-segment joining the points A(2, -2) and B(3, 7) in a ratio k : 1

The coordinates of the division = $\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

The point also lies on 2x + y – 4 = 0

Hence, $2\left(\frac{3k+2}{k+1}\right) + \left(\frac{7k-2}{k+1}\right) - 4 = 0$

or, $\frac{6k+4+7k-2-4k-4}{k+1} = 0$

or, 9k – 2 = 0

or, k = $\frac{2}{9}$

Question 2. Find the relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear.

Answer: Let (x, y), (1, 2) and (7, 0) are vertices of a triangle and area of the triangle, should be 0, as the points are collinear.

Therefore, $\frac{1}{2}$[x(2 – 0) + 1 (0 – y) + 7(y – 2)] = 0

or, 2x – y + 7y – 14 = 0

or, 2x + 6y – 14 = 0

or, x + 3y – 7 = 0.

Question 3: Find the centre of a circle passing through points (6, -6), (3, -7) and (3, 3).

Answer: Let O(x, y) be the centre of the circle and let (6, -6), (3, -7) and (3, 3) are A, B, C points on the circumference of the circle.

OA = $\sqrt{(x-6)^2 + (y+6)^2}$

OB = $\sqrt{(x-3)^2 + (y+7)^2}$

OC = $\sqrt{(x-3)^2 + (y-3)^2}$

OA = OB [radius of the circle]

Therefore, $\sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y+7)^2}$

or, $x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 49 + 14y$

or, - 6x – 2y + 14 = 0

or, 3x + y = 7……………………………………………………………………(1)

Again, OA = OC [radius of the circle]

Therefore, $\sqrt{(x-6)^2+(y+6)^2} = \sqrt{(x-3)^2+(y-3)^2}$
or, $x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 9 - 6y$
or, -6x + 18y + 54 = 0
or, -3x + 9y = - 27 ……………………………………………………….(2)

Adding eq. (1) and (2)
10y = -20
or, y = -2

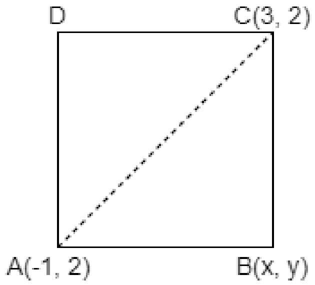
From the eq. (1), 3x – 2 = 7
or, 3x = 9
or, x = 3

Hence, the centre of the circle is (3, -2).

Question 4: The two opposite vertices of a square are (-1, 2) and (3, 2). Find the coordinates of the other two vertices.

Answer:

Let ABCD be the given square whose two opposite vertices are A(-1, 2) and C(3, 2). Let B(x, y) be its third vertex.



Then, AB = BC

or, $AB^2 = BC^2$
or, $(x+1)^2 + (y-2)^2 = (x-3)^2 + (y-2)^2$
or, $x^2+1+2x+y^2+4-4y = x^2+9-6x+y^2+4-4y$
or, $2x-4y+5 = -6x-4y+13$
or, $8x = 8$
or, $x = 1$

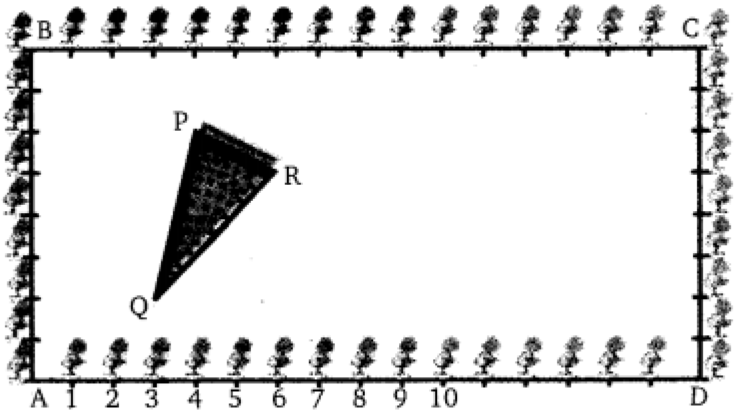
In Rthe ight-angled triangle ABC,

$AB^2 + BC^2 = AC^2$ [Pythagoras theorem]
or, $(x+1)^2 + (y-2)^2 + (x-3)^2 + (y-2)^2 = (3+1)^2 + (2-2)^2$
or, $4 + (y-2)^2 + 4 + (y-2)^2 = 16$ [putting the value of x=1]
or, $8 + 2(y-2)^2 = 16$
or, $(y-2)^2 = 4$

or, $y - 2 = \pm 2$
or, y = 4 or 0

Therefore the coordinates of the other vertices are (1, 4) and (1, 0)

Question 5: The class X students school in krishnagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot, as shown in the figure. The students are to sow seeds of flowering plants on the remaining area of the plot.
(i) Taking A as origin, find the coordinates of the vertices of the triangle.
(ii) What will be the coordinates of the vertices of ∆PQR, if C is the origin?
Also, calculate the areas of the triangles in these cases. What do you observe?



Answer:

(i) They are taking A as the origin and taking AD as the x-axis and AB as the y-axis. Now the coordinates of the points P, Q and R are (4, 6), (3, 2) and (6, 5).
Therefore, area of triangle = $\frac{1}{2}[\, x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\,]$
= $\frac{1}{2}[4(2 - 5) + 3(5 - 6) + 6(6 - 2)]$
= $\frac{1}{2}(-12 - 3 + 24) = \frac{9}{2}$ sq. units

(ii) They are taking C as the origin and taking CB as the x-axis and CD as the y-axis. Now the coordinates of the points P, Q and R are (12, 2), (13, 6) and (10, 3).
Therefore, area of triangle = $\frac{1}{2}[\, x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\,]$
= $\frac{1}{2}[12(6 - 3) + 13(3 - 2) + 10(2 - 6)]$
= $\frac{1}{2}(36 + 13 - 40) = \frac{9}{2}$ sq. units

Therefore, the area of the triangle is the same in both cases.

Question 6: The vertices of an ΔABC are A(4, 6), B(1, 5) and C(7, 2). A line is drawn to intersect sides AB and AC at D and E, respectively. $\frac{AD}{AB} = \frac{AE}{AC}$ =. Calculate the area of the ΔADE and compare it with the area of ΔABC.

Answer:

We have $AD/AB = \frac{1}{4}$

or, $\frac{AB}{AD} = 4$

or, $\frac{AD+DB}{AD} = 4$

or, $\frac{AD}{AD} + \frac{DB}{AD} = 4$

or, $1 + \frac{DB}{AD} = 4$

or, $\frac{AD}{DB} = \frac{1}{3}$

Thus D and E divides AB and AC respectively in the ratio 1:3

Therefore, the coordinates of D =

$$= \left(\frac{(1 \times 1 + 3 \times 4}{1+3}, \frac{1 \times 5 + 3 \times 6}{1+3}\right)$$

$$= \left(\frac{1+12}{4}, \frac{5+18}{4}\right) = \left(\frac{13}{4}, \frac{23}{4}\right)$$

Therefore, the coordinates of D =

$$= \left(\frac{(1 \times 7 + 3 \times 4}{1+3}, \frac{1 \times 2 + 3 \times 6}{1+3}\right)$$

$$= \left(\frac{7+12}{4}, \frac{2+18}{4}\right) = \left(\frac{19}{4}, 5\right)$$

Therefore area of ΔADE

$=\frac{1}{2}\left[4\left(\frac{23}{4} - 5\right) + \frac{13}{4}(5-6) + \frac{19}{4}\left(6 - \frac{23}{4}\right)\right]$

$=\frac{1}{2}\left[4\left(\frac{23-20}{4}\right) + \frac{13}{4}(-1) + \frac{19}{4}\left(\frac{24-23}{4}\right)\right]$

$= \frac{1}{2}\left[4 \times \frac{3}{4} - \frac{13}{4} + \frac{19}{4} \times \frac{1}{4}\right]$

$= \frac{1}{2}\left[3 - \frac{13}{4} + \frac{19}{16}\right] = \frac{1}{2} \times \frac{15}{16} = \frac{15}{32}$ sq. units

Therefore area of ΔABC = $\frac{1}{2}[4(5-2) + 1(2-6) + 7(6-5)]$

$= \frac{1}{2}[4 \times 3 + 1 \times (-4) + 7 \times 1] = \frac{1}{2}[12 - 4 + 7] = \frac{15}{2}$

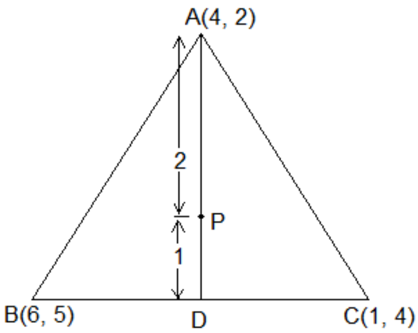
Therefore, $\frac{Area\ of\, \Delta\mathrm{ADE}}{Area\ of\, \Delta\mathrm{ABC}} = \frac{15}{32} \div \frac{15}{2}$

$= \frac{15}{32} \times \frac{2}{15} = \frac{1}{16}$

or, area of ΔADE : area of ΔABC = 1 : 16.

Question 7: Let A(4, 2), B(6,5), and C(1, 4) be the vertices of △ABC.
(i) The median from A meters BC at D. Find the coordinates of the point D.
(ii) Find the point P's coordinates on the AD, such that AP: PD = 2: 1.
(iii) Find the coordinates of points Q and R on medians BE and CF, respectively, such that BQ: QE = 2: 1 and CR: RF = 2: 1.
(iv) What do you observe?
[Note: The points which are familiar to all the three medians is called centroid, and this point divides each median in the ratio 2: 1]
(v) If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of △ABC, find the coordinates of the centroid of the triangles.

Answer:



(i) THE median AD of the triangle will divide the side BC into two equal parts.

Therefore, coordinates of D = $\left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$.

(ii) Point P divides the side AD in the ratio of 2:1.

Therefore, coordinates of P = $\left(\frac{2\times\frac{7}{2}+1\times4}{2+1}, \frac{2\times\frac{9}{2}+1\times2}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iii) Median BE will divide the side AC into two equal parts.

Therefore, coordinates of E = $\left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$

Point Q divides the side BE in the ratio 2:1.

Therefore, coordinates of Q = $\left(\frac{2\times\frac{5}{2}+1\times6}{2+1}, \frac{2\times3+1\times5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

Median CF will divide AB into two equal parts.

Therefore, coordinates of F = $\left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$

Point R divides CF in ratio 2:1.

Therefore coordinates of R = $\left(\frac{2\times5+1\times1}{2+1}, \frac{2\times\frac{7}{2}+1\times4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iv) Hence we see the coordinates of P, Q, and R are the same. These all represent the same point on the plane, i.e. the centroid of the triangle.

(v) Now considering a triangle $A(x_1, y_1), B(x_2, y_2)\ and\ C(x_3, y_3)$.
The median AD will divide the side BC into two equal parts.

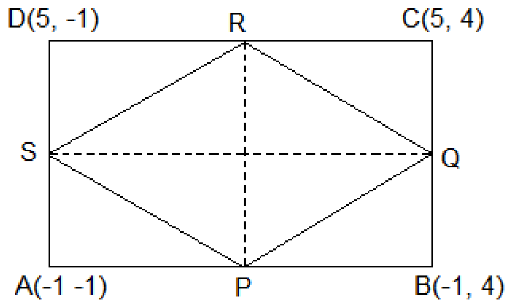
Coordinates of D = $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$
Let the centroid of the triangle be O. Point O divides AD in the ratio 2:1.

Therefore, coordinates of O = $\left(\frac{2\times\frac{x_2+x_3}{2}+1\times x_1}{2+1}, \frac{2\times\frac{y_2+y_3}{2}+1\times y_1}{2+1}\right)$

= $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Question 8: ABCD is a rectangle formed by the points A(-1, -1), B(-1, 4), C(5, 4) and D(5, -1), P, Q, R, and S are the mid-points of Ab, BC, CD and DA respectively. Is the quadrilateral PQRS a square? A rectangle? or a rhombus? Justify your answer.

Answer:



In the figure rectangle, ABCD is shown, where P, Q, R and S are the midpoints of AB, BC, CD and DA, respectively. We join PQ, QR, RS and SP.

Coordinates of P = $\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$

Coordinates of Q = $\left(\frac{-1+5}{2}, \frac{4+4}{2}\right) = (2, 4)$

Coordinates of R = $\left(\frac{5+5}{2}, \frac{4-1}{2}\right) = \left(-1, \frac{3}{2}\right)$

Coordinates of S = $\left(\frac{5-1}{2}, \frac{-1-1}{2}\right) = (2, -1)$

Now, PQ = $\sqrt{(2+1)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2}$

QR = $\sqrt{(5-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2}$

RS = $\sqrt{(2-5)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2}$

SP = $\sqrt{(-1-2)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2}$

We see, PQ = QR = RS = SP

Diagonal PR = $\sqrt{(5+1)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = \sqrt{(6)^2+0^2} = \sqrt{36}$ = 6

Diagonal QS = $\sqrt{(2-2)^2+(-1-4)^2} = \sqrt{(5)^2+0^2} = \sqrt{25} = 5$

Therefore, diagonal PR ≠ QS.

Since four sides of the quadrilateral, PQRS have the same lengths, but diagonals are not equal. Hence it is a rhombus.