Chapter 3: Linear equations in two variables

2016

Short Answer Type Question I [2 Marks]

Question 1.
Solve the following pair of linear equations:
y -4x= 1
6x- 5y= 9
Solution:

$$\mathbf{y - 4x = 1}$$
$$\mathbf{6x - 5y = 9}$$

equation (1) is $-4x + y = 1$
equation (2) is $6x - 5y = 9$
operate: 5 × equation (1) + equation (2)

$$-20x + 5y = 5$$
$$6x - 5y = 9$$

on adding $-14x = 14$

$$\therefore \quad x = \frac{14}{-14} = -1$$

Put value of $x = -1$ in equation (1), we get

$$-4(-1) + y = 1$$
$$4 + y = 1 \Rightarrow y = -3$$

Short Answer Type Questions II [3 Marks]

Question 2.
A part of the monthly Hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay 13000 as hostel charges whereas, Mansi who takes food for 25 days pays? 3500 as hostel charges. Find the fixed charges and the cost of food per day.
Solution:

Let fixed hostel charges be ₹ x

Charge per day is ₹ y

Charge paid by Swati = ₹ 3000

$\therefore$ 1st condition is $x + 20y = 3000$

Charge paid by Mansi = ₹ 3500

$\therefore$ 2nd condition is $x + 25y = 3500$

Subtracting (*ii*) from (*i*).

$$\begin{array}{r} x + 20y = 3000 \\ x + 25y = 3500 \\ - \quad - \qquad - \\ \hline -5y = -500 \\ \hline \end{array}$$

$$\Rightarrow \qquad y = ₹\ 100$$

Put value of $y = 100$ in equation (*i*), we get

$$\Rightarrow \qquad x + 20\,(100) = 3000$$

$$x = 3000 - 2000 = ₹\ 1000$$

Hence, fixed charges is ₹ 1000 and charges per day is ₹ 100.

Question 3.

Solve using cross multiplication method:

x+y=1

2x – 3y = 11

Solution:

$$x + y = 7$$

$$2x - 3y = 11$$

By cross multiplication method, we have

$$\frac{x}{\begin{matrix} 1 & & 7 \\ -3 & & 11 \end{matrix}} = \frac{y}{\begin{matrix} 7 & & 1 \\ 11 & & 2 \end{matrix}} = \frac{-1}{\begin{matrix} 1 & & 1 \\ 2 & & -3 \end{matrix}}$$

$$\frac{x}{11 + 21} = \frac{y}{14 - 11} = \frac{-1}{-3 - 2}$$

$$\frac{x}{32} = \frac{y}{3} = \frac{-1}{-5}$$

$$\Rightarrow \qquad \frac{x}{32} = \frac{1}{5} \Rightarrow x = \frac{32}{5}$$

and $$\frac{y}{3} = \frac{1}{5} \Rightarrow y = \frac{3}{5}$$

Question 4.

Draw the graphs of the pair of equations x + 2y = 5 and 2x-3y = – 4. Also, find the points where the lines meet the x-axis.

Solution:

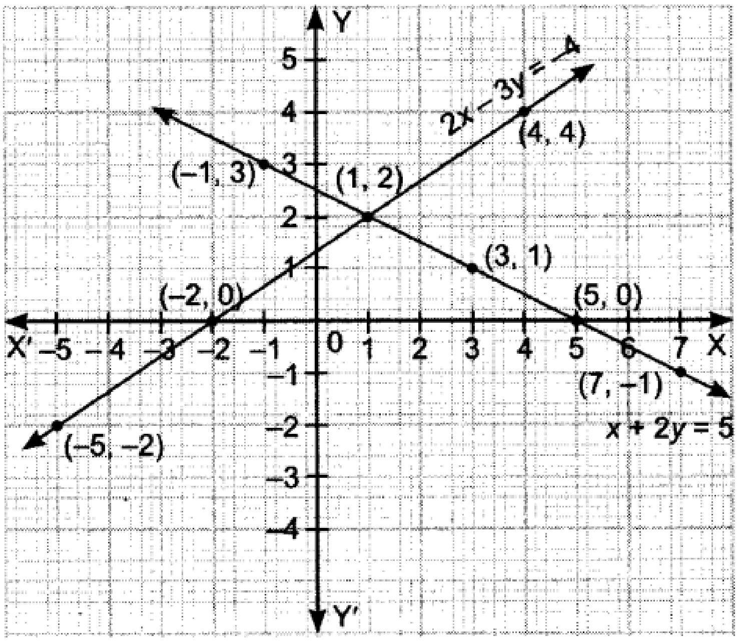
Given equations are $x + 2y = 5$
and $2x - 3y = -4$
Table for $x + 2y = 5$

x	5	3	7	– 1
y	0	1	– 1	3

Plot values of x and y from above table on graph.
Table for $2x - 3y = -4$

x	– 2	1	– 5	4
y	0	2	– 2	4

Plot values of x and y from above table on graph



lines meet the x-axis at (5, 0) and (–2, 0).

2015

Short Answer Type Question I [2 Marks]

Question 5.
Find whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident: 2x – 3y + 6 = 0,4x – 5y + 2 = 0
Solution:
Given system of linear equations is $2x - 3y + 6 = 0,\ 4x - 5y + 2 = 0$

Here $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2};\ \frac{b_1}{b_2} = \frac{-3}{-5} = \frac{3}{5}$

$\because\ \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \left(\text{as } \frac{1}{2} \neq \frac{3}{5}\right)$

So, given system has unique solution and given lines intersect at a point.

Short Answer Type Questions II [3 Marks]

Question 6.
Given a linear equation 3x-5y = 11. Form another linear equation in these variables such that the geometric representation of the pair so formed is:
(i) intersecting lines
(ii) coincident lines
(iii) parallel lines
Solution:
(*i*) $3x - 5y = 11$ (given equation of line)
$5x - 3y = 11$
(System has unique solution, *i.e.* lines intersect each other at one point)
(*ii*) $3x - 5y = 11$ (given equation of line)
$6x - 10y = 22$
(System represents coincident lines having many solutions)
(*iii*) $3x - 5y = 11$ (given equation of line)
$6x - 10y = 12$
(System represents parallel lines having no solution)

Question 7.
Solve for x and y
x + 2y – 3= 0
3x – 2y + 7 = 0
Solution:
Given system of equations is
$$x + 2y = 3$$
$$3x - 2y = -7$$
On adding the equations (*i*) and (*ii*), we get
$$4x = -4 \Rightarrow \boxed{x = -1}$$
Putting $x = -1$ in equation (*i*), we get
$$-1 + 2y = 3 \Rightarrow 2y = 4 \Rightarrow \boxed{y = 2}$$
Hence, solution of the system is $x = -1$ and $y = 2$.

Long Answer Type Question [4 Marks]

Question 8.
4 chairs and 3 tables cost? 2100 and 5 chairs and 2 tables cost? 1750. Find the cost of one chair and one table separately

Solution:

Let the cost of one chair and one table is ₹ x and ₹ y respectively.

According to question,
$$4x + 3y = 2100$$
$$5x + 2y = 1750$$

Multiplying equation (i) by 2 and equation (ii) by 3, we get
$$8x + 6y = 4200$$
$$15x + 6y = 5250$$

Subtracting equation (iii) from (iv), we get
$$\begin{array}{rcl} 15x + 6y &=& 5250 \\ 8x + 6y &=& 4200 \\ - \quad - & & - \\ \hline 7x &=& 1050 \Rightarrow x = 150 \end{array}$$

Putting $x = 150$ in equation (i), we get
$$4 \times 150 + 3y = 2100$$
$$\Rightarrow \quad 3y = 2100 - 600 \Rightarrow 3y = 1500 \Rightarrow y = 500$$

Hence, cost of one chair = ₹ 150 and cost of one table = ₹ 500.

2014

Short Answer Type Questions II [3 Marks]

Question 9.

Solve for x andy:

2x= 5y + 4;

3x-2y + 16 = 0

Solution:

Given system is
$$2x = 5y + 4 \quad \Rightarrow \quad 2x - 5y = 4 \quad \ldots(i)$$
$$3x - 2y + 16 = 0 \quad \Rightarrow \quad 3x - 2y = -16 \quad \ldots(ii)$$

On multiplying equation (i) by 3 and (ii) by 2, we get
$$6x - 15y = 12 \quad \ldots(iii)$$
$$6x - 4y = -32 \quad \ldots(iv)$$

Subtracting equation (iv) from (iii), we get
$$\begin{array}{rcl} 6x - 15y &=& 2 \\ 6x - 4y &=& -32 \\ - \quad + & & + \\ \hline -11y &=& 44 \\ y &=& -4 \end{array}$$

Putting $y = -4$ in equation (i), we get
$$2x = 5(-4) + 4 \Rightarrow 2x = -16 \Rightarrow x = -8$$

Hence, solution of given system is $x = -8$ and $y = -4$.

Question 10.
Solve for x andy:

$$\frac{5}{x-1}+\frac{1}{y-2}=2 \text{ and } \frac{6}{x-1}-\frac{3}{y-2}=1 \qquad [\text{Where } x \neq 1, y \neq 2]$$

Solution:
Given equations are

$$\frac{5}{x-1}+\frac{1}{y-2} = 2 \quad ...(i)$$

$$\frac{6}{x-1}-\frac{3}{y-2} = 1 \quad ...(ii)$$

Let $\frac{1}{x-1}=a$ and $\frac{1}{y-2}=b$ then above system becomes,

$$5a+b = 2 \quad ...(iii)$$

$$6a-3b = 1 \quad ...(iv)$$

On multiplying equation (*iii*) by 3 and then adding with equation (*iv*), we get

$$15a+3b = 6$$

$$6a-3b = 1$$

$$21a = 7 \Rightarrow a=\frac{1}{3}$$

Putting $a=\frac{1}{3}$ in equation (*iii*), we get

$$\frac{5}{3}+b = 2 \Rightarrow b=2-\frac{5}{3} \Rightarrow b=\frac{1}{3}$$

Thus, $a=\frac{1}{3}$ and $b=\frac{1}{3} \Rightarrow \frac{1}{x-1}=\frac{1}{3}$ and $\frac{1}{y-2}=\frac{1}{3}$

$\Rightarrow$ $x-1=3$ and $y-2=3$

$\Rightarrow$ $x = 4$ and $y=5$

Hence, solution of system is $x=4$ and $y=5$.

Question 11.
Solve for x and y:
6(ax + by) = 3a + 2b
6(bx – ay) = 3b -2a

Solution:
Given equations are

$$6ax+6by = 3a+2b \quad ...(i)$$

$$6bx-6ay = -2a+3b \quad ...(ii)$$

Multiplying equation (i) by 'a' and equation (ii) by 'b' and then adding, we get

$$6a^2x + 6aby = 3a^2 + 2ab \quad \text{...}(i)$$
$$6b^2x - 6aby = -2ab + 3b^2 \quad \text{...}(ii)$$
$$\overline{6(a^2 + b^2)x = 3(a^2 + b^2)}$$
$$x = \frac{1}{2}$$

Putting $x = \frac{1}{2}$ in equation (i), we get

$$6a \times \frac{1}{2} + 6by = 3a + 2b \Rightarrow 6by = 2b \Rightarrow y = \frac{1}{3}$$

Hence, solution of the system is $x = \frac{1}{2}$ and $y = \frac{1}{3}$

Question 12.
Solve the following pair of equations by reducing them to a pair of linear equations:

Solution:
Given system is

$$\frac{1}{x} - \frac{4}{y} = 2 \quad \text{...}(i)$$
$$\frac{1}{x} + \frac{3}{y} = 9 \quad \text{...}(ii)$$

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$ then the system becomes,

$$a - 4b = 2 \quad \text{...}(iii)$$
$$a + 3b = 9 \quad \text{...}(iv)$$

On multiplying equation (iii) by 3 and equation (iv) by 4 and then adding, we get

$$3a - 12b = 6$$
$$4a + 12b = 36$$
$$\overline{7a = 42} \quad \Rightarrow a = 6$$

Putting $a = 6$ in equation (iii), we get

$$6 - 4b = 2 \Rightarrow 4b = 4 \Rightarrow b = 1$$

Thus

$$a = 6 \text{ and } b = 1$$
$$\Rightarrow \frac{1}{x} = 6 \text{ and } \frac{1}{y} = 1$$
$$\Rightarrow x = \frac{1}{6} \text{ and } y = 1$$

Hence, solution of the system is $\left(\frac{1}{6}, 1\right)$.

Question 13.
Determine graphically whether the following pair of linear equations 2x – 3y = 5; 3x + 4y = – 1 has
(i) a unique solution
(ii) infinitely many solutions or
(iii) no solution

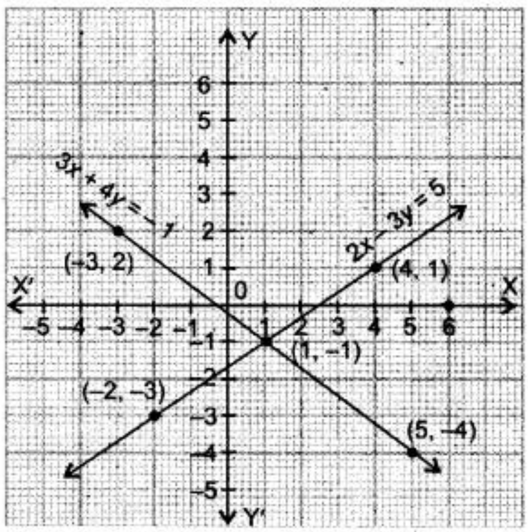
Solution:

$$2x - 3y = 5$$
$$\Rightarrow \quad 3y = 2x - 5$$
$$\Rightarrow \quad y = \frac{2x-5}{3}$$

x	1	4	–2
y	–1	1	–3

$$3x + 4y = -1$$
$$4y = -1 - 3x$$
$$y = \frac{-1-3x}{4}$$

x	1	5	–3
y	–1	–4	2



Since the lines whose equations are given above intersect at one point $(1, -1)$ so, given pair of linear equations have a unique solution.

Question 14.
Find those integral values of m for which the c-coordinate of the point of intersection of lines represented by y = MX + 1 and 3x + 4y = 9 is an integer.

Solution:

Given equations are

$$y = mx + 1 \quad \ldots(i)$$
$$3x + 4y = 9 \quad \ldots(ii)$$

Substitute the value of y from (i) in equation (ii), we get

$$3x + 4(mx + 1) = 9$$
$$\Rightarrow \quad 3x + 4mx + 4 = 9$$
$$\Rightarrow \quad (3 + 4m)x = 5 \Rightarrow x = \frac{5}{3+4m}$$

If $m = -2$ then $x = -1$

Hence, for $m = -2$ the x-coordinate is an integral value equal to -1.

Long Answer Type Questions [4 Marks]

Question 15.
In a two-digit number, the digit in the unit place is twice the digit in the tenth place. If the digits are reversed, the new number is 27 more than the given number. Find the number.

Solution:

Let unit's place digit be 'x' and ten's place digit be 'y'

Then the two digit number $= 10y + x$

According to Ist condition, $\quad x = 2y \quad$ ...(i)

On reversing the digits of two digit number, the number becomes $10x + y$.

According to IInd condition,

$$(10x + y) = (10y + x) + 27$$

$$\Rightarrow \quad 10x + y - x - 10y = 27$$

$$\Rightarrow \quad 9x - 9y = 27$$

$$\Rightarrow \quad x - y = 3 \quad \text{...}(ii)$$

Substituting the value of x from equation (i) in equation (ii), we get

$$2y - y = 3 \Rightarrow y = 3$$

Putting $y = 3$ in equation (i), we get

$$x = 6$$

Hence, the number is 36.

Question 16.

Solve the following system of linear equations graphically.

3x + y – 12 = 0;

x – 3y + 6 = 0

Shade the region bounded by the lines and ii>axis. Also, find the area of the shaded region.

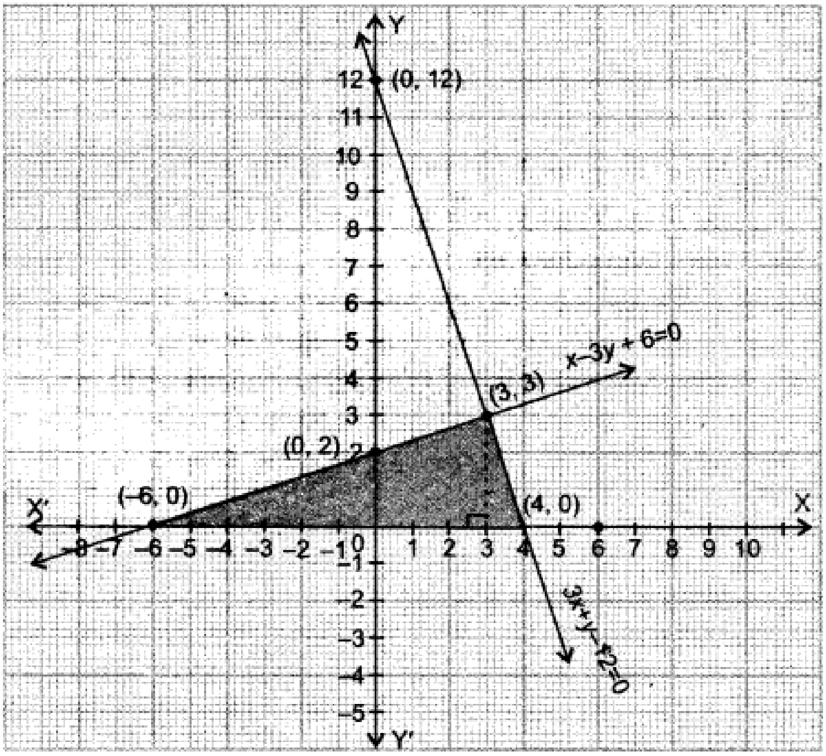
Solution:

Table for line $3x+y-12=0$

x	0	4	3
y	12	0	3

Table for line $x-3y+6=0$

x	0	–6	3
y	2	0	3



Since both lines intersects each other at point (3, 3).

So, solution of the given system is (3, 3)

Now, area of shaded triangle $= \frac{1}{2} \times 10 \times 3$

$= 15$ sq. units

Question 17.

The owner of a taxi company decides to run all the taxi on CNG fuels instead of petrol/ diesel. The taxi charges in the city comprised of fixed charges together with

the charge for the distance covered.
For a journey of 13 km, the charge paid is? 129 and for the journey of 22 km, the charge paid is ^ 210.
(i) What will a person have to pay for travelling a distance of 32 km?
(ii) Why did he decide to use CNG for his taxi as fuel?
Solution:
(*i*) Let the fixed charges be ₹ x and the charge for per km be ₹ y.
According to Ist condition, $x + 13y = 129$ ...(*i*)
According to IInd condition, $x + 22y = 210$ ...(*ii*)
On solving equations (*i*) and (*ii*) we get, $x = 12$ and $y = 9$
Thus, for travelling a distance of 32 km, a person has to pay ₹ $(12 + 32 \times 9)$ i.e. ₹ 300.
(*ii*) He decided to use CNG as it is pollution free. It is good for environment and also cheaper in comparison to petrol/diesel.

Question 18.
The area of a rectangle reduces by 160 m if its length is increased by 5 m and breadth is reduced by 4 m. However, if the length is decreased by 10 m and breadth is increased by 2 m, then its area is decreased by 100 m2. Find the dimensions of the rectangle.
Solution:
Let the length of rectangle be x metre and the breadth of rectangle be y metre.
Then area of rectangle $= xy$ m^2.
According to Ist condition, $(x + 5)(y - 4) + 160 = xy$
$\Rightarrow$ $4x - 5y = 140$ ...(*i*)
According to IInd condition, $(x - 10)(y + 2) + 100 = xy$
$\Rightarrow$ $x - 5y = -40$ ...(*ii*)
On solving equation (*i*) and (*ii*), we get
$$x = 60 \text{ and } y = 20$$
Hence, length and breadth of rectangle are 60 m and 20 m respectively.

Question 19.
At a certain time in a zoo, the number of heads and the number of legs of tiger and peacocks was counted and it was found that there were 47 heads and 152 legs. Find the number of tigers and peacocks in the zoo:
Why it is necessary to conserve these animals?
Solution:
Let x be the number of tigers and y be the number of peacocks.
According to conditions given,
$$x + y = 47 \quad \text{...}(i)$$
$$4x + 2y = 152 \quad \text{...}(ii)$$
On solving the above equations, we get
$$x = 29 \text{ and } y = 18$$
Hence, number of tigers = 29 and number of peacocks = 18
It is necessary to conserve each species of animals because all the animals play an important role in balancing the eco-system.

2013

Short Answer Type Question I [2 Marks]

Question 20.
If the system of equations
6x + 2y = 3 and kx + y = 2 has a unique solution, find the value of k.
Solution:
Given equations are $6x + 2y = 3$
$kx + y = 2$
For unique solution, $\frac{6}{k} \neq \frac{2}{1} \Rightarrow k \neq 3$
Thus, k may have any real value except 3.

Short Answer Type Questions II [3 Marks]

Question 21.
Determine the value of m and n so that the following pair of linear equations have an infinite number of solutions.
(2m – l)x + 3y = 5;
3x + (n- l)y = 2
Solution:
Given equations are $(2m-1)x + 3y = 5$ ...(i)
$3x + (n-1)y = 2$ ...(ii)
For infinite number of solutions,
$$\frac{2m-1}{3} = \frac{3}{n-1} = \frac{5}{2} \Rightarrow \frac{2m-1}{3} = \frac{5}{2} \text{ and } \frac{3}{n-1} = \frac{5}{2}$$
$$\Rightarrow 4m - 2 = 15 \text{ and } 6 = 5n - 5 \Rightarrow 4m = 17 \text{ and } 5n = 11$$
$$\Rightarrow m = \frac{17}{4} \text{ and } n = \frac{11}{5}$$

Question 22.
For what values of p and q will the following pair of linear equations has infinitely many solutions?
4x + 5y = 2;
(2p + 7q)x + (p + 8q)y = 2q-p + 1

Solution:

Given equations are $4x + 5y = 2$ ...(i)

$(2p + 7q)x + (p + 8q)y = 2q - p + 1$ ...(ii)

For infinitely many solutions,

$$\frac{4}{2p+7q} = \frac{5}{p+8q} = \frac{2}{2q-p+1} \Rightarrow \frac{4}{2p+7q} = \frac{5}{p+8q} \text{ and } \frac{5}{p+8q} = \frac{2}{2q-p+1}$$

$\Rightarrow$ $4p + 32q = 10p + 35q$ and $10q - 5p + 5 = 2p + 16q$

$\Rightarrow$ $6p + 3q = 0$ and $-5p - 2p + 10q - 16q + 5 = 0$

$\Rightarrow$ $2p + q = 0$ ...(iii) and $7p + 6q = 5$ ...(iv)

On solving the equations (iii) and (iv), we get

$p = -1$ and $q = 2$

Hence, for $p = -1$ and $q = 2$ the given system has infinitely many solutions.

Question 23.

Solve the following pair of equations for x and y

$$\frac{ax}{b} - \frac{by}{a} = a + b; \quad ax - by = 2ab$$

Solution:

Given equations are $\frac{ax}{b} - \frac{by}{a} = a + b$

$\Rightarrow$ $a^2x - b^2y = a^2b + ab^2$ ...(i)

and $ax - by = 2ab$ ...(ii)

On multiplying equation (ii) by 'a' and then subtracting from (i), we get

$$\begin{aligned} a^2x - b^2y &= a^2b + ab^2 \\ a^2x - aby &= 2a^2b \\ - \quad + \quad &\quad - \\ \hline (ab - b^2)y &= ab^2 - a^2b \\ b(a-b)y &= ab(b-a) \\ y &= -a \end{aligned}$$

Putting $y = -a$ in equation (ii), we get

$ax + ab = 2ab$

$\Rightarrow$ $ax = ab$

$\Rightarrow$ $x = b$

Hence, solution of given system is $x = b$ and $y = -a$.

Question 24.

8 men and 12 boys can finish a piece of work in 10 days, while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.

Solution:

Let a man alone takes x days to finish the work and a boy alone takes y days to finish the work.

$\therefore$ 1 man's one day's work $= \frac{1}{x}$ and 1 boy's one day's work $= \frac{1}{y}$.

According to Ist condition, $\frac{8}{x}+\frac{12}{y} = \frac{1}{10}$ ...(*i*)

According to IInd condition, $\frac{6}{x}+\frac{8}{y} = \frac{1}{14}$ ...(*ii*)

On solving equations (*i*) and (*ii*), we get

$$x = 140 \text{ and } y = 280$$

Hence, one man alone can finish the work in 140 days and
one boy alone can finish the work in 280 days.

Long Answer Type Questions [4 Marks]

Question 25.
A two-digit number is equal to 7 times the sum of its digits. The number formed by reversing its digits is less than the original number by 18. Find the original number.
Solution:

According to Ist condition,

$10y + x = 7(x + y) \Rightarrow 10y + x = 7x + 7y \Rightarrow 6x - 3y = 0$

$\Rightarrow \quad 2x - y = 0$ ...(*i*)

Now, on reversing the digits of two digit number, the number becomes $10x + y$.

According to IInd condition,

$$10y + x = 10x + y + 18 \Rightarrow -9x + 9y = 18$$

$\Rightarrow \quad -x + y = 2$ ...(*ii*)

On solving equations (*i*) and (*ii*), we get

$$x = 2 \text{ and } y = 4.$$

Hence, required two digit number is 42.

Question 26.
The age of the father is twice the sum of the ages of his 2 children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

Solution:

Let the age of father be 'x' years and sum of ages of his 2 children be 'y' years.

According to Ist condition, $\qquad x = 2y \qquad$ ...(i)

After 20 years, the age of father = $(x + 20)$ years

and sum of ages of his 2 children = $(y + 40)$ years

According to IInd condition, $x + 20 = y + 40$

$\Rightarrow \qquad x + 20 = \frac{x}{2} + 40 \qquad [\because x = 2y \text{ from } (i) \text{ so, } y = \frac{x}{2}]$

$\Rightarrow \qquad x - \frac{x}{2} = 20 \Rightarrow \frac{x}{2} = 20$

$\Rightarrow \qquad x = 40$

Thus, age of father is 40 years.

Question 27.

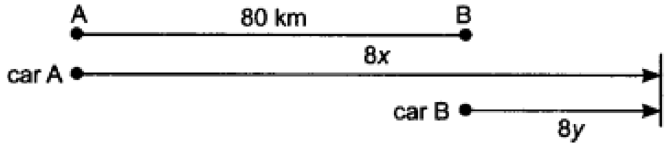
Places A and B are 80 km apart from each other on a highway. A car starts from A and another from B at the same time. If they move in the same direction they meet in 8 hrs and if they move in opposite directions they meet in 1 hr 20 minutes. Find speeds of the cars.

Solution:

Let the speed of car starts from A or car A = x km/hr

and the speed of car starts from B or car B = y km/hr

Case I:



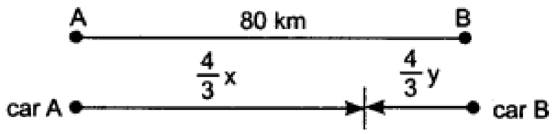
After 8 hours,

distance covered by car A = $8x \qquad \left[\text{as speed} = \frac{\text{distance}}{\text{time}}\right]$

and distance covered by car B = $8y$

So, $\qquad 8x - 8y = 80 \Rightarrow x - y = 10 \qquad$ ...(i)

Case II:



After 1 hr 20 minutes, i.e. $\frac{4}{3}$hrs, distance covered by car A = $\frac{4}{3}x$

and distance covered by car B = $\frac{4}{3}y$

So, $\qquad \frac{4}{3}x + \frac{4}{3}y = 80$

$\Rightarrow \qquad x + y = 60 \qquad$ ...(ii)

On solving equations (i) and (ii), we get

$x = 35$ and $y = 25$

Hence, speed of car A = 35 km/hr and speed of car B = 25 km/hr.

Long Answer Type Questions [4 Marks]

Question 28.
For what value of k will the pair of equations have no solution?
3x + y = 1
(2k-l)x+ (k-l)y = 2k+l
Solution:

Given equations are

$$3x + y = 1$$
$$(2k-1)x + (k-1)y = 2k+1$$

For no solution,

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1} \Rightarrow \frac{3}{2k-1} = \frac{1}{k-1}$$
$$3k-3 = 2k-1$$
$$k = 2$$

Hence, for $k = 2$ the system has no solution.

Question 29.
Solve for x and y:

$$\frac{5}{x-1} + \frac{1}{y-2} = 2; \quad \frac{6}{x-1} - \frac{3}{y-2} = 1$$

Solution:

Let $\frac{1}{x-1} = a$ and $\frac{1}{y-2} = b$

Then given equations become

$$5a + b = 2 \quad \text{... (i)}$$

and

$$6a - 3b = 1 \quad \text{... (ii)}$$

Solving (*i*) and (*ii*), we get

$$a = \frac{1}{3} \text{ and } b = \frac{1}{3}$$

$$\Rightarrow \quad \frac{1}{x-1} = \frac{1}{3} \quad \text{and} \quad \frac{1}{y-2} = \frac{1}{3}$$

$$\Rightarrow \quad x-1 = 3 \quad \text{and} \quad y-2 = 3$$

$$x = 4, \quad y = 5$$

Question 30.
Solve the following pair of linear equations graphically, x + 3y = 6;2x-3y = 12
Also, find the area of the triangle formed by the lines representing the given equations withy-axis.

Solution:

$x + 3y = 6$

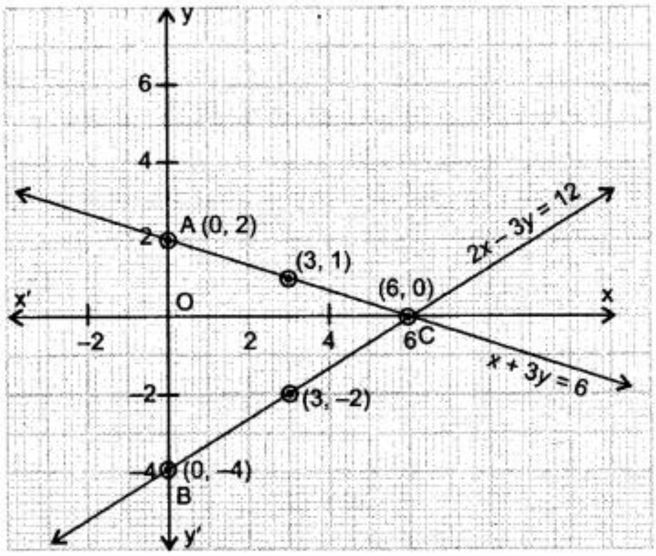
or $\quad x = 6 - 3y$

x	0	3	6
y	2	1	0

$2x - 3y = 12$

or $\quad 2x = 12 + 3y$

x	0	3	6
y	-4	-2	0



ΔABC is formed by the lines with y-axis.

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times AB \times OC$

(Here AB = 6 units, OC = 6 units)

$$= \frac{1}{2} \times 6 \times 6 = 18 \text{ sq. units}$$

2011

Short Answer Type Questions I [2 Marks]

Question 31.
Solve: 99x + lOly = 499
lOlx + 99y = 501
Solution:

Given equations are

$$99x + 101y = 499 \quad ...(i)$$
$$101x + 99y = 501 \quad ...(ii)$$

Adding equations (i) and (ii), we get

$$200x + 200y = 1000$$
$$\Rightarrow \quad x + y = 5 \quad ...(iii)$$

Now, subtracting equation (ii) from (i), we get

$$-2x + 2y = -2$$
$$\Rightarrow \quad x - y = 1 \quad ...(iv)$$

On solving (iii) and (iv), we get

$x = 3$ and $y = 2$

Question 32.
For what value of p will the following system of equations has no solution;
(2p -1) x + (p -1) y = 2p + 1;
y + 3x – 1 = 0

Solution:
Given equations are

$$(2p-1)x+(p-1)y = 2p+1 \quad ...(i)$$
$$3x+y = 1 \quad ...(ii)$$

For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{2p-1}{3} = \frac{p-1}{1} \neq \frac{(2p+1)}{1}$

$\Rightarrow \quad \frac{2p-1}{3} = \frac{p-1}{1} \Rightarrow 2p-1 = 3p-3$

$\Rightarrow \quad 3p-2p = 3-1 \Rightarrow p=2$

$\therefore$ For $p=2$ the system has no solution.

Short Answer Type Questions II [3 Marks]

Question 33.
The sum of the digits of a two-digit number is 12. The number obtained by interchanging the two digits exceeds the given number by 18. Find the number.
Solution:
Let digit at ten's place be x and digit at unit's place be y.
$\therefore$ Number is $10x+y$.
According to Ist condition

$$x+y = 12 \quad ...(i)$$

According to IInd condition

$$10y+x = 10x+y+18$$
$\Rightarrow \quad 9y-9x = 18$
$$y-x = 2 \quad ...(ii)$$

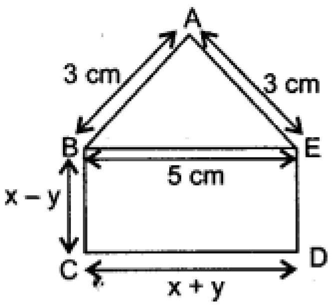
Solving equations (i) and (ii), we get
$x=5, y=7$
$\therefore$ Number $= 57$

Question 34.
In the figure, ABCDE is a pentagon with BE 11 CD and BC 11 DE. BC is perpendicular to CD. If the perimeter of ABCDE is 21 cm, find the value of x andy.
Solution:
$\because$ CD = BE

$\Rightarrow \quad x+y = 5$

Also, $AB+BC+CD+DE+AE = 21$
$$3+x-y+x+y+x-y+3 = 21$$
$$3x-y = 15$$

On solving equations (i) and (ii) we get $x=5, y=0$



Question 35.
The sum of the numerator and denominator of a fraction is 12. If 1 is added to both the numerator and the denominator the fraction becomes 3/4. Find the fraction.
Solution:
Let numerator $= x$ and denominator $= y$
According to Ist condition,
$$x + y = 12 \quad ...(i)$$
According to IInd condition,
$$\frac{x+1}{y+1} = \frac{3}{4} \Rightarrow 4x + 4 = 3y + 3$$
$$\Rightarrow \quad 4x - 3y = -1 \quad ...(ii)$$
Solving equations (i) and (ii), we get $x = 5, y = 7$
$\therefore$ Fraction is $\frac{5}{7}$.

Question 36.
4 men and 6 boys can finish a piece of work in 5 days while 3 men and 4 boys can finish it in 7 days. Find the time taken by 1 man alone or that by 1 boy alone.
Solution:
Let a man takes x days to finish the work and a boy takes y days to finish the work.
$\therefore$ One man's one day's work $= \frac{1}{x}$ and one boy's one day's work $= \frac{1}{y}$
According to Ist condition,
$$4 \times \frac{1}{x} + 6 \times \frac{1}{y} = \frac{1}{5}$$
$$\Rightarrow \quad \frac{4}{x} + \frac{6}{y} = \frac{1}{5} \quad ...(i)$$
According to IInd condition,
$$\frac{3}{x} + \frac{4}{y} = \frac{1}{7} \quad ...(ii)$$
Solving equations (i) and (ii), we get
$$x = 35, y = 70$$
Hence, time taken by one man alone to complete the work is 35 days and by one boy alone is 70 days.

Question 37.
A man travels 600 km partly by train and partly by car. It takes 8 hours and 40 minutes if he travels 320 km by train and the rest by car. It would take 30 minutes more if he travels 200 km by train and the rest by car. Find the speed of the train and the car separately.

Solution:

Let speed of train be x km/hr and speed of car be y km/hr.

According to Ist condition,

$$\frac{320}{x} + \frac{280}{y} = \frac{26}{3} \quad \ldots(i)$$

According to IInd condition,

$$\frac{200}{x} + \frac{400}{y} = \frac{55}{6} \quad \ldots(ii)$$

Solving equations (i) and (ii) for x and y, we get

$$x = 80 \text{ and } y = 60$$

Hence, speed of train is 80 km/hr and speed of the car is 60 km/h.

Long Answer Type Questions [4 Marks]

Question 38.

Solve the equations graphically:

2x + y = 2;

2y-x = 4

What is the area of the triangle formed by the two lines and the lines = 0?

Solution:

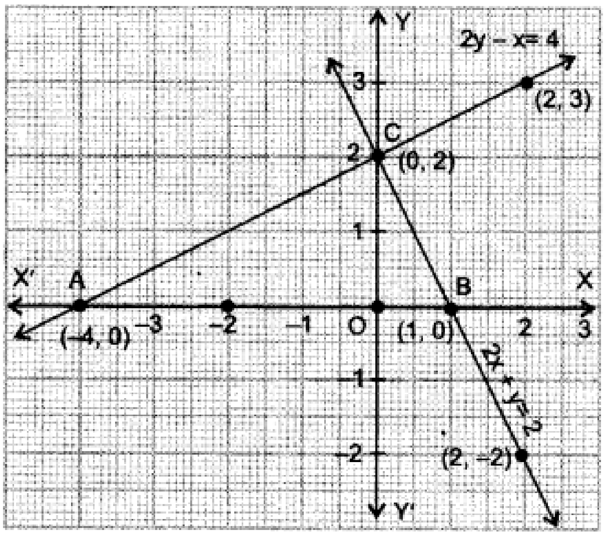
$$2x + y = 2 \quad \ldots(i)$$
$$2y - x = 4 \quad \ldots(ii)$$

From (i), $2x + y = 2$

x	1	0	2
y	0	2	-2

From (ii), $2y - x = 4$

x	0	-4	2
y	2	0	3



From graph, we observe that solution of equations is (0, 2).

$$\text{Area of } \Delta ABC = \frac{1}{2} AB \times CO = \frac{1}{2} \times 5 \times 2$$
$$= 5 \text{ square units}$$

Question 39.

Draw the graphs of the following equations: x + y — 5; x-y = 5

(i) Find the solution of the equations from the graph.

(ii) Shade the triangular region formed by the lines and the y-axis.

Solution:

$x + y = 5$

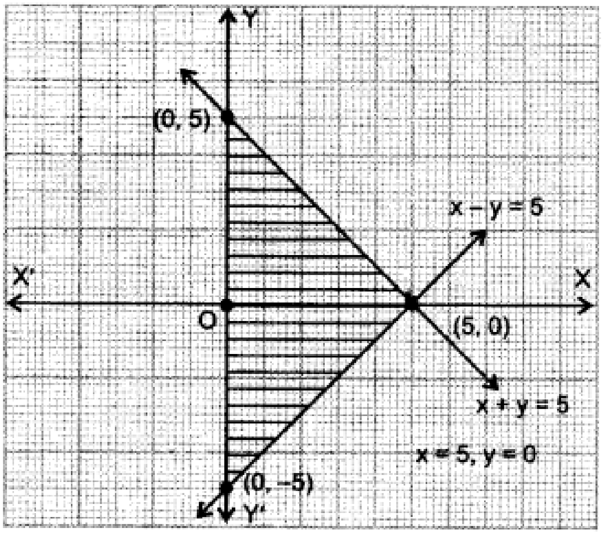
x	5	0
y	0	5

$x - y = 5$

x	0	5
y	–5	0

(*i*) Both lines intersect at point (5, 0). Hence, solution is $x = 5, y = 0$

(*ii*) Required portion is shaded in the graph.



2010

Short Answer Type Questions I [2 Marks]

Question 40.
Find the value of k for which the following pair of linear equations have infinitely many solutions : 2x + 3y = 7; (k – l)x + (k + 2)y = 3k

Solution:

$2x + 3y = 7, (k-1)x + (k+2)y = 3k$

For infinitely many solutions the condition is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \Rightarrow \quad \frac{2}{k-1} = \frac{3}{k+2} = \frac{7}{3k}$$

$$\frac{2}{k-1} = \frac{3}{k+2} \text{ and } \frac{3}{k+2} = \frac{7}{3k}$$

$$2k + 4 = 3k - 3 \Rightarrow k = 7 \text{ and } \quad 9k = 7k + 14 \Rightarrow k = 7$$

Hence, the value of k is 7.

Question 41.
Find the value of m for which the pair of linear equations 2x + 3y – 7 = 0 and (m – l)x + (m + l)y = (3m – 1) has infinitely many solutions.

Solution:

$2x + 3y = 7$

$(m - 1)x + (m + 1)y = (3m - 1)$

For infinitely many solutions the condition is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{m-1} = \frac{3}{m+1} = \frac{7}{3m-1}$$

From first and second

$$\frac{2}{m-1} = \frac{3}{m+1}$$

$$2(m + 1) = 3(m - 1) \Rightarrow 2m + 2 = 3m - 3$$

$$2 + 3 = 3m - 2m \Rightarrow m = 5$$

From second and third

$$\frac{3}{m+1} = \frac{7}{3m-1} \Rightarrow 3(3m - 1) = 7(m + 1)$$

$$9m - 3 = 7m + 7 \Rightarrow 9m - 7m = 7 + 3$$

$$2m = 10 \Rightarrow m = 5$$

Hence, for $m = 5$, the system has infinitely many solutions.

Question 42.

For what value of k will the following pair of linear equations have no solution?

2x + 3y = 9;

6x + (k – 2)y = (3k – 2).

Solution:

Given equations are $2x + 3y = 9$ and $6x + (k - 2)y = (3k - 2)$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Question 43.

For what value of p will the following pair of linear equations have infinitely many solutions?

(p – 3)x + 3y = p;

px + py = 12

Solution:

Given equations are $(p - 3)x + 3y = p$ and $px + py = 12$

For infinitely many solutions,

$$\frac{p-3}{p} = \frac{3}{p} = \frac{p}{12}$$

$$\frac{p-3}{p} = \frac{3}{p} \text{ and } \frac{3}{p} = \frac{p}{12}$$

$\Rightarrow p - 3 = 3$ and $p^2 = 36 \Rightarrow p = 6$ and $p = \pm 6$

Common value of $p = 6$

Hence, for $p = 6$, system has infinitely many solutions.

Question 44.
Find the values of a and b for which the following pair of linear equations has infinitely many solutions: 2x + 3y = 7; 3/4(a + b)x + (2a – b)y = 21
Solution:
Given equations are $2x + 3y = 7$ and $(a + b)x + (2a - b)y = 21$
For infinitely many solutions

$$\frac{2}{a+b} = \frac{3}{2a-b} = \frac{7}{21} \quad ...(i)$$

$$\Rightarrow \quad \frac{2}{a+b} = \frac{1}{3} \text{ and } \frac{3}{2a-b} = \frac{1}{3} \quad \Rightarrow a + b = 6 \text{ and } 2a - b = 9$$

Solving for a and b, we get

$$a = 5, b = 1$$

Hence, for $a = 5, b = 1$, system has infinitely many solutions.

Question 45.
Solve the following pair of equations:

$$\mathbf{\frac{4}{x} + 3y = 8; \frac{6}{x} - 4y = -5.}$$

Solution:
$\frac{4}{x} + 3y = 8, \frac{6}{x} - 4y = -5$

$$4 + 3xy = 8x \quad ..(i)$$

$$6 - 4xy = -5x \quad ...(ii)$$

Multiply (i) by 4 and (ii) by 3 and then adding

$$\begin{array}{r} 16 + 12xy = 32x \\ 18 - 12xy = -15x \\ \hline 34 = 17x \end{array}$$

$$x = \frac{34}{17} = 2$$

From (i)

$$4 + 3 \times 2y = 8 \times 2 \Rightarrow 4 + 6y = 16$$

$$6y = 12 \Rightarrow y = 2$$

Hence, $\quad x = 2, y = 2$

Question 46.
The sum of the numerator and the denominator of a fraction is 4 more than twice the numerator. If 3 is added to each of the numerator and denominator, their ratio becomes 2 : 3. Find the fraction. [All India]

Solution:

Let fraction be $= \frac{x}{y}$.

According to Ist condition, $x + y = 2x + 4$

$$-x + y = 4 \qquad ...(i)$$

According to IInd condition,

$$\frac{x+3}{y+3} = \frac{2}{3}$$

$$\Rightarrow \quad -3x + 2y = 3 \qquad ...(ii)$$

Multiplying equation (i) by 2 and then subtracting from equation (ii)

$$\begin{array}{rcl} -3x + 2y & = & 3 \\ -2x + 2y & = & 8 \\ + \quad - & & - \\ \hline -x & = & -5 \Rightarrow x = 5 \end{array}$$

Putting $x = 5$ in equation (i), we get

$$-5 + y = 4 \quad \Rightarrow y = 9$$

$\therefore$ Required fraction $= \frac{5}{9}$.

Question 47.

A number consists of two digits. When the number is divided by the sum of its digits, the quotient is 7. If 27 is subtracted from the number, the digits interchange their places, find the number

Solution:

Let digit at unit's place be x, and at ten's place be y

$\therefore$ Number $= 10y + x$

According to Ist condition,

$$\frac{10y+x}{y+x} = 7 \Rightarrow 6x - 3y = 0$$

$$\Rightarrow \quad 2x - y = 0 \qquad ...(i)$$

Again according to IInd condition,

$$(10y + x) - 27 = 10x + y$$

$$9x - 9y = -27 \Rightarrow x - y = -3 \qquad ...(ii)$$

Solving for x and y, we get

$$x = 3 \text{ and } y = 6$$

$\therefore$ Number is 63.

2009

Very Short Answer Type Questions [1 Mark]

Question 48.

Find the value of a so that the point (3, a), lies on the line represented by 2x-3y = 5

Solution:

Since point $(3, a)$ lies on line $2x - 3y = 5$

then $2 \times 3 - 3 \times a = 5 \Rightarrow 6 - 5 = 3a$

$$\Rightarrow \quad a = \frac{1}{3}$$

Question 49.

Find the number of solutions of the following pair of linear equations.

x + 2y – 8 = 0

2x + 4y = 16

Solution:

$$x + 2y - 8 = 0 \quad \text{... (i)}$$

$$2x + 4y - 16 = 0 \quad \text{... (ii)}$$

Here, $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore$ Given pair of linear equations has infinitely many solutions.

Question 50.

Write whether the following pair of linear equations is consistent or not

x +y = 14,

x-y = 4

Solution:

$$x + y = 14$$

$$x - y = 4$$

Here, $\frac{a_1}{a_2} = 1, \frac{b_1}{b_2} = -1$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, given system is consistent having unique solution.

Short Answer Type Questions I [2 Marks]

Question 51.

Find the value of k for which the pair of linear equations

kx + 3y = k – 2 and 12x + ky = k has no solution.

Solution:

Since pair of linear equations has no solution

then $\frac{k}{12} = \frac{3}{k} \neq \frac{k-2}{k}$ *i.e.*, $k^2 = 36 \Rightarrow k = \pm 6$

Question 52.

Solve for x and y:

$$\frac{ax}{b} - \frac{by}{a} = a + b; \quad ax - by = 2ab.$$

Solution:

Given equations are

$$\frac{ax}{b} - \frac{by}{a} = a + b$$

$\Rightarrow$ $$a^2x - b^2y = a^2y + ab^2 \quad ...(i)$$

and $$ax - by = 2ab \quad ...(ii)$$

On multiplying equation (*ii*) by a and then subtracting from (*i*)

$$\begin{array}{l} a^2x - b^2y = a^2b + ab^2 \\ a^2x - aby = 2a^2b \\ - \quad + \qquad - \\ \hline (ab - b^2)y = ab^2 - a^2b \\ b(a - b)y = ab(b - a) \Rightarrow y = -a \end{array}$$

Putting $y = -a$ in equation (*ii*), we get

$$ax - b(-a) = 2ab \Rightarrow ax + ab = 2ab \Rightarrow ax = ab \Rightarrow x = b$$

$\therefore$ Solution of the system is $x = b$ and $y = -a$.

Question 53.

Without drawing the graph, find out the lines representing the following pair of linear equations intersect at a point, are parallel or coincident.

$$18x - 7y = 24; \frac{9}{5}x - \frac{7}{10}y = \frac{9}{10}$$

Solution:

$$18x - 7y = 24 \quad ...(i)$$

$$\frac{9}{5}x - \frac{7}{10}y = \frac{9}{10} \quad \text{or} \quad \frac{18x - 7y}{10} = \frac{9}{10} \Rightarrow 18x - 7y = 9 \quad ...(ii)$$

Here, $$\frac{a_1}{a_2} = \frac{18}{18} = 1$$

$$\frac{b_1}{b_2} = \frac{-7}{-7} = 1 \quad \text{and} \quad \frac{c_1}{c_2} = \frac{24}{9} = \frac{8}{3}$$

Clearly, $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the lines are parallel.

Question 54.

Solve the following system of equations for x and y

$$\frac{5}{x-1} + \frac{1}{y-2} = 2, \frac{6}{x-1} - \frac{3}{y-2} = 1$$

Solution:

$$\frac{5}{x-1}+\frac{1}{y-2}=2 \quad \text{...}(i)$$

$$\frac{6}{x-1}-\frac{3}{y-2}=1 \quad \text{...}(ii)$$

Let $\frac{1}{x-1}=a$ and $\frac{1}{y-2}=b$

Then equations (*i*) and (*ii*) become

$$5a+b=2 \quad \ldots(iii)$$

$$6a-3b=1 \quad \ldots(iv)$$

Multiplying (*iii*) by 3 then adding with (*iv*)

$$15a+3b=6$$

$$6a-3b=1$$

$$21a=7 \Rightarrow a=\frac{1}{3}$$

i.e., $\frac{1}{x-1}=\frac{1}{3}$ or $x-1=3 \Rightarrow x=4$

Putting $a=\frac{1}{3}$ in equation (*iii*), we get

$$5\times\frac{1}{3}+b=2 \Rightarrow b=2-\frac{5}{3} \Rightarrow b=\frac{1}{3} \Rightarrow \frac{1}{y-2}=\frac{1}{3}$$

or $y-2=3 \Rightarrow y=5$

Hence, solution of system is $x=4, y=5$.

Question 55.

Solve the following pair of equations

$$\frac{10}{x+y}+\frac{2}{x-y}=4; \frac{15}{x+y}-\frac{5}{x-y}=-2$$

Solution:

$\frac{10}{x+y}+\frac{2}{x-y}=4,\ \frac{15}{x+y}-\frac{5}{x-y}=-2,$

Let $\frac{1}{x+y}=a$ and $\frac{1}{x-y}=b$

Then above equations become,

$$10a+2b=4 \qquad \ldots(i)\times 3$$

$$15a-5b=-2 \qquad \ldots(ii)\times 2$$

Multiplying equation (i) by 3 and equation (ii) by 2, we get

$$30a+6b=12$$

$$30a-10b=-4$$

On subtracting

$$\begin{array}{r} -\quad + \qquad + \\ \hline 16b = 16 \end{array} \Rightarrow b=1$$

Putting $b=1$ in equation (i)

$$10a=4-2=2 \Rightarrow a=\frac{1}{5}$$

Now, $\frac{1}{x-y}=b=1$ or $x-y=1$ $\ldots(iii)$

and $\frac{1}{x+y}=\text{a}=\frac{1}{5}$ or $x+y=5$ $\ldots(iv)$

On adding (iii) and (iv)

$$2x=6$$

$\Rightarrow$ $x=3$ and $y=2$

Hence, solution of system is $x=3$ and $y=2$.