Chapter 4: Quadratic Equations

2016

Short Answer Type Questions I [2 Marks]

Question 1.
If x= 2/3 and x = – 3 are roots of the quadratic equations $ax^2 + lx + b = 0$, find the values of a and b.

Solution :

Given quadratic equation is $ax^2 + 7x + b = 0$ ...(*i*)

$$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0 \qquad \left[\because x = \frac{2}{3} \text{ is the root of equation } (i)\right]$$

$$\Rightarrow \quad \frac{4}{9}a + \frac{14}{3} + b = 0$$

$$\Rightarrow \quad \frac{4a + 42 + 9b}{9} = 0 \Rightarrow 4a + 9b + 42 = 0 \qquad ...(ii)$$

$$a(-3)^2 + 7(-3) + b = 0 \qquad [\because x = -3 \text{ is the root of equation } (i)]$$

$$\Rightarrow \quad 9a + b - 21 = 0 \qquad ..(iii)$$

Putting the value of b from (*iii*) in (*ii*), we get

$$4a + 9[21 - 9a] + 42 = 0$$

$$\Rightarrow \quad 4a + 189 - 81a + 42 = 0$$

$$\Rightarrow \quad 231 - 77a = 0$$

$$\Rightarrow \quad 77a = 231$$

$$\Rightarrow \quad a = 3$$

Putting $a = 3$ in (*iii*), we have

$$27 + b = 21$$

$$\Rightarrow \quad b = -6$$

$$a = 3, b = -6$$

Question 2.
If- 5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Solution :

$\because -5$ is the root of the quadratic equation $2x^2 + px - 15 = 0$

$$\Rightarrow \quad 2(-5)^2 + p(-5) - 15 = 0$$

$$\Rightarrow \quad 50 - 5p - 15 = 0 \Rightarrow 35 - 5p = 0$$

$$\Rightarrow \quad 5p = 35 \Rightarrow p = 7$$

Now, given that equation $p(x^2 + x) + k = 0$ has equal roots

i.e. $7(x^2 + x) + k = 0$ has equal roots

i.e. $7x^2 + 7x + k = 0$ has equal roots

$$\Rightarrow \quad 7^2 - 4 \times 7 \times k = 0 \qquad [\because \text{For equal roots, D} = 0, \text{ i.e. } b^2 - 4ac = 0]$$

$$\Rightarrow \quad 7(7 - 4k) = 0$$

$$\Rightarrow \quad k = \frac{7}{4}$$

Question 3.

Solve for x: $\sqrt{2x+9}$ + x = 13.

Solution :

$\sqrt{2x+9} + x = 13$

$\Rightarrow \quad \sqrt{2x+9} = (13-x)$

Squaring both sides, we get

$\Rightarrow \quad 2x + 9 = (13-x)^2$

$\Rightarrow \quad 2x + 9 = 169 - 26x + x^2$

$\Rightarrow \quad x^2 - 28x + 160 = 0$

$\Rightarrow \quad (x-20)(x-8) = 0$

$\Rightarrow \quad x = 20 \text{ or } x = 8$

$\Rightarrow \quad x = 8$ [as $x = 20$ does not satisfy the equation]

Question 4.

Solve for x :$\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0 = 0$.

Solution :

$\sqrt{6x+7} - (2x-7) = 0$

$\Rightarrow \quad \sqrt{6x+7} = 2x - 7$

Squaring both sides, we get

$\Rightarrow \quad 6x + 7 = (2x-7)^2$

$\Rightarrow \quad 6x + 7 = 4x^2 - 28x + 49$

$\Rightarrow \quad 4x^2 - 34x + 42 = 0$

$\Rightarrow \quad 2x^2 - 17x + 21 = 0$

$\Rightarrow \quad 2x^2 - 14x - 3x + 21 = 0$

$\Rightarrow \quad 2x(x-7) - 3(x-7) = 0$

$\Rightarrow \quad (2x-3)(x-7) = 0$

$\Rightarrow \quad x = 7 \text{ or } x = \frac{3}{2}$

$\Rightarrow \quad x = 7$ [as $x = \frac{3}{2}$ does not satisfy the equation]

Question 5.

A two digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number.

Solution :

Let ones digit of number $= x$

Let tens digit of number $= y$

$\therefore$ Number will be $= 10y + x$

According to question,

$$\Rightarrow \quad 10y + x = 4(x + y)$$

$$\Rightarrow \quad 10y + x = 4x + 4y$$

$$3x - 6y = 0$$

$$x - 2y = 0 \Rightarrow x = 2y \quad \ldots(i)$$

and $10y + x = 3xy \quad \ldots(ii)$

Putting $x = 2y$ from (i) in (ii), we get,

$$10y + 2y = 3(2y)y \Rightarrow 12y = 6y^2 \Rightarrow y = 2$$

and $x = 2y \Rightarrow x = 4 \quad (\because y = 2)$

The required number $= 10(2) + 4 = 20 + 4 = 24.$

Question 6.

Solve for x:$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

Solution :

$$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0 \Rightarrow \sqrt{3}x^2 + \sqrt{2}x - 3\sqrt{2}x - 2\sqrt{3} = 0$$

$$\Rightarrow x(\sqrt{3}x + \sqrt{2}) - \sqrt{6}(\sqrt{3}x + \sqrt{2}) = 0 \Rightarrow (\sqrt{3}x + \sqrt{2})(x - \sqrt{6}) = 0$$

$$\therefore \quad x = \frac{-\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{6}}{3} \text{ and } x = \sqrt{6}$$

Question 7.

$$\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}, x \neq 3, -5.$$

Solve for x:

Solution :

$$\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}; x \neq 3, -5$$

$$\Rightarrow \quad \frac{(x+5)-(x-3)}{(x-3)(x+5)} = \frac{1}{6} \Rightarrow \frac{x+5-x+3}{x^2+2x-15} = \frac{1}{6}$$

$$\Rightarrow \quad 8 \times 6 = x^2 + 2x - 15 \Rightarrow x^2 + 2x - 15 = 48$$

$$\Rightarrow \quad x^2 + 2x - 63 = 0 \Rightarrow (x + 9)(x - 7) = 0$$

$$\Rightarrow \quad x = 7 \text{ or } x = -9$$

Short Answer Type Questions II [3 Marks]

Question 8.

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0,\quad x\neq 3,-\frac{3}{2}$$

Solve for x:

Solution :

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0 \Rightarrow \frac{2x(2x+3)+(x-3)+(3x+9)}{(x-3)(2x+3)}=0$$

$$\Rightarrow 4x^2+6x+x-3+3x+9=0 \Rightarrow 4x^2+10x+6=0$$

$$\Rightarrow 2x^2+5x+3=0 \Rightarrow 2x^2+2x+3x+3=0$$

$$\Rightarrow 2x(x+1)+3(x+1)=0 \Rightarrow (x+1)(2x+3)=0$$

$$\Rightarrow x+1=0 \text{ or } 2x+3=0$$

$$\Rightarrow x=-1 \text{ or } x=\frac{-3}{2} \quad \left(\because \text{ given, } x\neq\frac{-3}{2}\right)$$

$\because$ It is given that $x\neq\frac{-3}{2}$.

Hence, solution of the given equation is $x=-1$.

Question 9.

$$\frac{x+1}{x-1}+\frac{x-2}{x+2}=4-\frac{2x+3}{x-2};\ x\neq 1,-2,2$$

Solve for x:

Solution :

$$\frac{x+1}{x-1}+\frac{x-2}{x+2} = 4-\frac{2x+3}{x-2}$$

$$\Rightarrow \quad \frac{x+1}{x-1}+\frac{x-2}{x+2}+\frac{2x+3}{x-2} = 4$$

$$\Rightarrow \frac{(x+1)(x+2)(x-2)+(x-2)^2(x-1)+(2x+3)(x-1)(x+2)}{(x-1)(x+2)(x-2)} = 4$$

$$\Rightarrow (x+1)(x^2-4)+(x-1)(x^2+4-4x)+(2x+3)(x^2+x-2) = 4(x-1)(x^2-4)$$

$$\Rightarrow x^3-4x+x^2-4+x^3+4x-4x^2-x^2-4+4x+2x^3+2x^2-4x+3x^2+3x-6 = 4(x^3-4x-x^2+4)$$

$$\Rightarrow x^3+x^2-4x-4+x^3-5x^2+8x-4+2x^3+5x^2-x-6 = 4(x^3-x^2-4x+4)$$

$$\Rightarrow \quad 4x^3+x^2+3x-14 = 4x^3-4x^2-16x+16$$

$$\Rightarrow \quad 5x^2+19x-30 = 0$$

$$\Rightarrow \quad 5x^2+25x-6x-30 = 0$$

$$\Rightarrow \quad 5x(x+5)-6(x+5) = 0$$

$$\Rightarrow \quad (x+5)(5x-6) = 0$$

$$\Rightarrow \quad x+5 = 0 \text{ or } 5x-6 = 0$$

$$\Rightarrow \quad x = -5 \text{ or } x = \frac{6}{5}$$

Thus, solutions of given equation are $x = -5$ and $x = \frac{6}{5}$

Question 10.
Solve the following quadratic equation for x:

$$x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1=0$$

Solution :

$$x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1 = 0$$

$$\Rightarrow \quad x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+\left(\frac{a}{a+b}\right)\left(\frac{a+b}{a}\right) = 0$$

$$\Rightarrow \quad x^2+\left(\frac{a}{a+b}\right)x+\left(\frac{a+b}{a}\right)x+\left(\frac{a}{a+b}\right)\cdot\left(\frac{a+b}{a}\right) = 0$$

$$\Rightarrow \quad x\left[x+\frac{a}{a+b}\right]+\left(\frac{a+b}{a}\right)\left[x+\frac{a}{a+b}\right] = 0$$

$$\Rightarrow \quad \left(x+\frac{a}{a+b}\right)\left(x+\frac{a+b}{a}\right) = 0$$

$$\Rightarrow \quad x+\frac{a}{a+b} = 0 \quad \text{or} \quad x+\frac{a+b}{a} = 0$$

$$\Rightarrow \quad x = -\left(\frac{a}{a+b}\right) \text{ or } x = -\left(\frac{a+b}{a}\right)$$

Question 11.

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3}, \quad x \neq 1, 2, 3$$

Solve for x:

Solution :

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)} = \frac{2}{3}, x \neq 1, 2, 3$$

$$\Rightarrow \quad \frac{x-3+x-1}{(x-1)(x-2)(x-3)} = \frac{2}{3} \quad \Rightarrow \quad \frac{(2x-4)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{2(x-2)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\Rightarrow \quad 3 = (x-1)(x-3) \quad \Rightarrow \quad x^2-4x+3 = 3$$

$$\Rightarrow \quad x^2-4x = 0 \quad \Rightarrow \quad x(x-4) = 0 \quad \Rightarrow \quad x = 0 \text{ or } x = 4$$

Question 12.

If the roots of the quadratic equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that $2a = b + c$.

Solution :

$(a-b)x^2 + (b-c)x + (c-a) = 0$

For equal roots, discriminant, D = 0

$$\Rightarrow \quad (b-c)^2 - 4(a-b)(c-a) = 0$$
$$\Rightarrow \quad b^2 - 2bc + c^2 - 4(ac - a^2 - bc + ab) = 0$$
$$\Rightarrow \quad b^2 - 2bc + c^2 - 4ac + 4a^2 + 4bc - 4ab = 0$$
$$\Rightarrow \quad 4a^2 + b^2 + c^2 - 4ab + 2bc - 4ac = 0$$
$$\Rightarrow \quad (2a - b - c)^2 = 0$$
$$\Rightarrow \quad 2a - b - c = 0$$
$$\Rightarrow \quad 2a = b + c$$

Question 13.

Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers

Solution :

Let three consecutive natural numbers are $x - 1, x$ and $x + 1$.

According to question,

$$x^2 - [(x+1)^2 - (x-1)^2] = 60$$
$$\Rightarrow \quad x^2 - [(x + 1 - x + 1)(x + 1 + x - 1)] = 60$$
$$\Rightarrow \quad x^2 - 4x - 60 = 0$$
$$\Rightarrow \quad x^2 - 10x + 6x - 60 = 0$$
$$\Rightarrow \quad x(x-10) + 6(x-10) = 0$$
$$\Rightarrow \quad (x+6)(x-10) = 0$$
$$\Rightarrow \quad x = 10 \qquad (x = -6, \text{ rejected})$$

Hence, the numbers are 9, 10 and 11.

Question 14.

Two water taps together can fill a tank in 9 hours 36 minutes. The tap of larger diameter takes 8 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution :

Let x be the time taken by larger diameter tap.

x + 8 be the time taken by smaller diameter tap.

According to question,

$$\frac{1}{x}+\frac{1}{x+8} = \frac{10}{96} \qquad \left(\because\ 9 \text{ hrs } 36 \text{ min} = \frac{96}{10} \text{ hrs}\right)$$

$$\Rightarrow \quad \frac{x+8+x}{x(x+8)} = \frac{10}{96}$$

$$\Rightarrow \quad 96(2x+8) = 10(x^2+8x)$$

$$\Rightarrow \quad 10x^2 - 112x - 768 = 0$$

$$\Rightarrow \quad 5x^2 - 56x - 384 = 0$$

$$\Rightarrow \quad x = \frac{56 \pm \sqrt{(56)^2 - 4\times 5\times(-384)}}{2\times 5}$$

$$\Rightarrow \quad x = \frac{56+104}{10} \text{ or } \frac{56-104}{10}$$

$$\Rightarrow \quad x = 16 \text{ or } x = -4.8 \qquad \text{(Rejected)}$$

Hence, time taken by larger and smaller taps are 16 hrs and 24 hrs respectively.

Question 15.

Solve the given quadratic equation for x: $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$.

Solution :

$9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$

Roots of above quadratic equation are given by quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-[-9(a+b)] \pm \sqrt{[-9(a+b)]^2 - 4(9)(2a^2+5ab+2b^2)}}{2(9)}$$

$$x = \frac{9(a+b) \pm 3\sqrt{9a^2+9b^2+18ab-8a^2-8b^2-20ab}}{18}$$

$$x = \frac{9(a+b) \pm 3\sqrt{a^2+b^2-2ab}}{18}$$

$$x = \frac{9(a+b) \pm 3\sqrt{(a-b)^2}}{18}$$

$$x = \frac{9(a+b) \pm 3(a-b)}{18}$$

$$\therefore \quad x = \frac{9a+9b+3a-3b}{18} \text{ and } x = \frac{9a+9b-3a+3b}{18}$$

$$x = \frac{12a+6b}{18} \text{ and } x = \frac{6a+12b}{18}$$

$$\therefore \quad x = \frac{2a+b}{3} \text{ and } x = \frac{a+2b}{3}$$

Question 16.

$$\frac{1}{x}+\frac{2}{2x-3}=\frac{1}{x-2}, \quad x \neq 0, \frac{3}{2}, 2$$

Solve for x:

Solution :

$$\frac{1}{x}+\frac{2}{2x-3} = \frac{1}{x-2}$$

$$\Rightarrow \quad \frac{2x-3+2x}{x(2x-3)} = \frac{1}{x-2} \quad \Rightarrow \quad \frac{(4x-3)}{2x^2-3x} = \frac{1}{x-2}$$

$$\Rightarrow \quad (x-2)(4x-3) = 2x^2-3x \quad \Rightarrow \quad 4x^2-3x-8x+6 = 2x^2-3x$$

$$\Rightarrow \quad 4x^2-3x-8x+6-2x^2+3x = 0 \quad \Rightarrow \quad 2x^2-8x+6 = 0$$

$$\Rightarrow \quad x^2-4x+3 = 0 \quad \Rightarrow \quad x^2-3x-x+3 = 0$$

$$\Rightarrow \quad x(x-3)-1(x-3) = 0 \quad \Rightarrow \quad (x-3)(x-1) = 0$$

$$\Rightarrow \quad x = 3, x = 1$$

Question 17.

$$\frac{a}{x-b}+\frac{b}{x-a}=2, x \neq a, b$$

Solve for x (in terms of a and b):

Solution :

$$\frac{a}{x-b}+\frac{b}{x-a} = 2; x \neq a, b$$

$$\Rightarrow \quad \frac{a(x-a)+b(x-b)}{(x-a)(x-b)} = 2$$

$$\Rightarrow \quad ax-a^2+bx-b^2 = 2(x^2-ax-bx+ab)$$

$$\Rightarrow \quad ax+bx-a^2-b^2 = 2x^2-2ax-2bx+2ab$$

$$\Rightarrow \quad 2x^2-3ax-3bx+a^2+b^2+2ab = 0$$

$$\Rightarrow \quad 2x^2-3(a+b)x+(a+b)^2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{3(a+b) \pm \sqrt{9(a+b)^2-4\times 2(a+b)^2}}{2 \cdot 2}$$

$$= \frac{3(a+b) \pm \sqrt{(a+b)^2}}{4} = \frac{3(a+b) \pm (a+b)}{4}$$

$$\Rightarrow \quad x = \frac{3(a+b)+(a+b)}{4} \text{ or } x = \frac{3(a+b)-(a+b)}{4}$$

$$\Rightarrow \quad x = \frac{4(a+b)}{4} \text{ or } x = \frac{2(a+b)}{4}$$

$$\Rightarrow \quad x = a+b \text{ or } x = \frac{a+b}{2}$$

Long Answer Type Questions [4 Marks]

Question 18.
A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/hour than the usual speed. Find the usual speed of the plane. What value is depicted in this question?

Solution :

Let the usual speed of plane be x km/hr.

Time taken to cover 1500 km with usual speed $= \frac{1500}{x}$ hours.

When the speed of plane is increased, then new speed $= (x + 250)$ km/hr.

Time taken to cover 1500 km with the new speed $(x + 250)$ km/hr $= \frac{1500}{x + 250}$

According to question, $\frac{1500}{x} = \frac{1500}{x+250} + \frac{1}{2} \Rightarrow \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$

$$\Rightarrow \quad \frac{1500x + 1500 \times 250 - 1500x}{x(x+250)} = \frac{1}{2} \Rightarrow \frac{1500 \times 250}{x^2 + 250x} = \frac{1}{2}$$

$$\Rightarrow \quad x^2 + 250x = 750000 \Rightarrow x^2 + 250x - 750000 = 0$$

$$\Rightarrow \quad x^2 + 1000x - 750x - 750000 = 0 \Rightarrow x(x + 1000) - 750(x + 1000) = 0$$

$$\Rightarrow \quad (x + 1000)(x - 750) = 0 \Rightarrow x + 1000 = 0 \quad \text{or} \quad x - 750 = 0$$

$$\Rightarrow \quad x = -1000 \quad \text{or} \quad x = 750$$

$$\Rightarrow \quad x = 750 \qquad (\because \text{Speed cannot be negative})$$

Hence, the usual speed of plane is 750 km/hr.

In this question, pilot's caring behaviour toward passengers is shown as well as other side pilot is cautious and alert for his duty to reach destination point at scheduled time.

Question 19.
Find x in terms of a, b and c:

$$\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c},\ x\neq a,b,c$$

Solution :

$$\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}\Rightarrow\frac{a(x-b)+b(x-a)}{(x-a)(x-b)}=\frac{2c}{x-c}$$

$$\Rightarrow\quad\frac{(ax+bx-2ab)}{(x^2-ax-bx+ab)}=\frac{2c}{x-c}$$

$$\Rightarrow\quad(ax+bx-2ab)(x-c)=2c(x^2-ax-bx+ab)$$

$$\Rightarrow\quad ax^2+bx^2-2abx-acx-bcx+2abc=2cx^2-2acx-2bcx+2abc$$

$$\Rightarrow\quad(a+b-2c)x^2+(ac+bc-2ab)x=0$$

$$\Rightarrow\quad x\{(a+b-2c)x+(ac+bc-2ab)\}=0$$

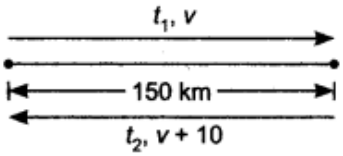
$$\Rightarrow\quad x=0\quad\text{or}\quad x=\frac{2ab-ac-bc}{a+b-2c}$$

Question 20.

The time taken by a person to cover 150 km was 2. 1/2 hours more than the time taken in the return journey. If he returned at a speed of 10 km/hour more than the speed while going, find the speed per hour in each direction.

Solution :

Let t_1 and t_2 be the time taken in going and returning respectively.
Also, v and v + 10 be the speed in going and returning respectively.



As, $\text{Velocity}=\dfrac{\text{Distance}}{\text{Time}}$

$\Rightarrow\quad\text{Time}=\dfrac{\text{Distance}}{\text{Velocity}}$

$\therefore\quad t_1=\dfrac{150}{v}$ and $t_2=\dfrac{150}{v+10}$

According to question, $t_1-t_2=2\dfrac{1}{2}$

$$\frac{150}{v}-\frac{150}{v+10}=\frac{5}{2}\Rightarrow 150\left[\frac{1}{v}-\frac{1}{v+10}\right]=\frac{5}{2}$$

$$\frac{1}{v}-\frac{1}{v+10}=\frac{1}{60}$$

$$60[v+10-v]=v(v+10)$$

$$v^2+10v-600=0$$

$$v^2+30v-20v-600=0$$

$$v(v+30)-20(v+30)=0$$

$$v=20\quad\text{or}\quad v=-30\ \text{(rejected)}$$

Hence, velocity in going 20 km/hour and in returning 30 km/hour.

Question 21.

Solve for x:

$$\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4},\quad x\neq -1,-2,-4$$

Solution :

$$\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4}\Rightarrow\frac{x+2+2x+2}{(x+1)(x+2)}=\frac{4}{x+4}$$

$$\Rightarrow\quad (3x+4)(x+4)=4(x^2+3x+2)\Rightarrow 3x^2+16x+16=4x^2+12x+8$$

$$\Rightarrow\quad x^2-4x-8=0\Rightarrow x=\frac{4\pm\sqrt{16+32}}{2}\Rightarrow x=\frac{4\pm4\sqrt{3}}{2}=2\pm2\sqrt{3}$$

Question 22.

A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution :

Let the speed of the stream be x km/h

Upstream case: Speed of boat $= (24-x)$ km/h

Time taken for going 32 km upstream $=\frac{32}{24-x}$ hours

Downstream case: Speed of boat $= (24+x)$ km/h

Time taken for going 32 km downstream $=\frac{32}{24+x}$ hours

According to question,

$$\frac{32}{24-x}-\frac{32}{24+x}=1\Rightarrow 32\left[\frac{24+x-24+x}{(24-x)(24+x)}\right]=1\quad\left[\because\ \text{Time}=\frac{\text{Distance}}{\text{Velocity}}\right]$$

$$\Rightarrow\quad 32\left(\frac{2x}{576-x^2}\right)=1\Rightarrow 64x=576-x^2$$

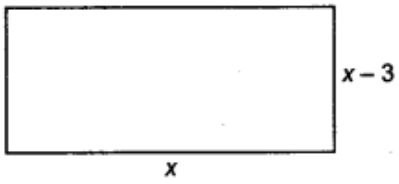
$$\Rightarrow\quad x^2+64x-576=0\Rightarrow (x+72)(x-8)=0$$

Question 23.

A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park.

Solution :

Let length of rectangular park be x m and breadth be $(x-3)$ m



$$\text{Base of isosceles } \Delta = (x-3) \text{ m}$$

$$\text{Altitude of } \Delta = 12 \text{ m}$$

According to question,

$$\text{Area of rectangular park} = \text{Area of } \Delta + 4$$

$$\Rightarrow \quad x(x-3) = \frac{1}{2}(x-3) \times 12 + 4$$

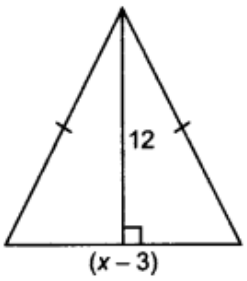
$$\Rightarrow \quad x(x-3) = 6(x-3) + 4$$

$$\Rightarrow \quad x^2 - 3x = 6x - 18 + 4$$

$$\Rightarrow \quad x^2 - 9x + 14 = 0$$

$$\Rightarrow \quad (x-7)(x-2) = 0$$

$$\Rightarrow \quad x = 7 \quad \text{or} \quad x = 2$$



But $x = 2$ is rejected otherwise breadth will be –ve which is not possible.

$\therefore$ Length of rectangular park is 7 m and breadth is 4 m.

Question 24.

Two pipes running together can fill a tank in 11. 1/9 minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

Solution :

Let one pipe takes x minutes to fill the tank.

Then, another pipe takes $x + 5$ to fill the tank.

According to question,

$$\frac{1}{x} + \frac{1}{x+5} = \frac{9}{100} \qquad \left(\because 11\frac{1}{9} = \frac{100}{9}\right)$$

$$100[x + 5 + x] = 9[x^2 + 5x]$$

$$200x + 500 = 9x^2 + 45x$$

$$9x^2 - 155x - 500 = 0$$

$$9x^2 - 180x + 25x - 500 = 0$$

$$9x(x-20) + 25(x-20) = 0$$

$$(9x + 25)(x - 20) = 0$$

$$x = 20 \quad \text{or} \quad x = -\frac{25}{9} \text{ (Rejected)}$$

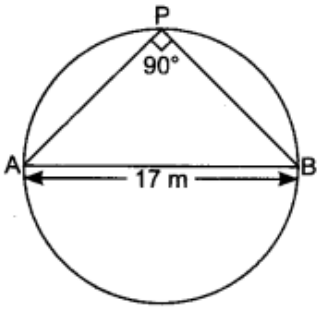
Hence, one pipe takes 20 minutes and another takes 25 minutes to fill the tank.

Question 25.

A pole has to be erected at a point on the boundary of a circular park of diameter 17 m in such a way that the differences of its distances from two diametrically opposite fixed gates where the pole is to be erected.

Solution :

Let P be the position of the pole.



$\angle APB = 90°$ (angle in a semi circle)

By Pythagoras Theorem, $AB^2 = AP^2 + PB^2$

$17^2 = AP^2 + PB^2$ ...(*i*)

$AP - PB = 7$ ...(*ii*)

Squaring both sides, we get

$(AP - PB)^2 = 49$

$AP^2 + PB^2 - 2AP \cdot PB = 49$ ...(*iii*)

From (*i*) and (*iii*), we have

$17^2 - 2AP \cdot PB = 49$

$\Rightarrow \quad 2AP \cdot PB = 289 - 49 \Rightarrow AP \cdot PB = \frac{240}{2}$

$\Rightarrow \quad AP \cdot PB = 120$ ...(*iv*)

From (*ii*) and (*iv*), we have

$120 = PB(7 + PB)$

Let $PB = x$

$120 = x(7 + x) \Rightarrow x^2 + 7x - 120 = 0$

$\Rightarrow \quad x^2 + 15x - 8x - 120 = 0 \Rightarrow x(x + 15) - 8(x + 15) = 0$

$\Rightarrow \quad (x - 8)(x + 15) = 0$

$x = 8$ or $x = -15$ (Rejected)

$PB = x = 8$ m

$\therefore \quad AP = 7 + PB = 7 + 8 = 15$ m

Question 26.

Find the positive value(s) of k for which quadratic equations $x^2 + kx + 64 = 0$ and $x^2 - 8X + k = 0$ both will have real roots.

Solution :

It is given that quadratic equation, $x^2 + kx + 64 = 0$ has real roots.

For real roots $D \geq 0$

$\therefore \quad k^2 - 4(1)(64) \geq 0 \Rightarrow k^2 - 256 \geq 0$

$\Rightarrow \quad k^2 \geq 256 \Rightarrow k \geq \pm 16$

Again, it is given that quadratic equation, $x^2 - 8x + k = 0$ has real roots.

$\therefore \quad (-8)^2 - 4(1)(k) \geq 0 \Rightarrow 64 - 4k \geq 0$

$\Rightarrow \quad 64 \geq 4k \Rightarrow k \leq 16$

Question 27.

If roots of the quadratic equation $x^2 + 2px + mn = 0$ are real and equal, show that the roots of the quadratic equation $x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0$ are also equal.

Solution :

It is given that quadratic equation, $x^2 + 2px + mn = 0$ has real and equal roots.

$\therefore \quad (2p)^2 - 4(mn)(1) = 0 \quad [\because$ For real and equal roots $b^2 - 4ac = 0]$

$\Rightarrow \quad 4p^2 - 4mn = 0 \Rightarrow p^2 - mn = 0$

$p^2 = mn \quad ...(i)$

Now, for quadratic equation, $x^2 - 2(m+n)x + (m^2 + n^2 + 2p^2) = 0$

$D = b^2 - 4ac$

$D = [-2(m+n)]^2 - 4(1)(m^2 + n^2 + 2p^2)$

$D = 4(m+n)^2 - 4(m^2 + n^2 + 2mn)$ [From (i)]

$D = 4(m+n)^2 - 4(m+n)^2$

$D = 0$

$\therefore$ Roots of the quadratic equation $x^2 - 2(m+n)x + (m^2 + n^2 + 2p^2) = 0$, also has equal roots.

Question 28.

The denominator of a fraction is one more than twice its numerator. If the sum of the fraction and its reciprocal 2. 16/21 find the fraction

Solution :

Let the fraction be $\frac{x}{2x+1}$

According to question,

$$\frac{x}{2x+1} + \frac{2x+1}{x} = 2\frac{16}{21}$$

Consider $\frac{x}{2x+1} = y$, then above equation becomes,

$$y + \frac{1}{y} = 2\frac{16}{21} \Rightarrow \frac{y^2+1}{y} = \frac{58}{21} \Rightarrow 21y^2 + 21 = 58y$$

$$\Rightarrow \quad 21y^2 - 58y + 21 = 0 \Rightarrow y = \frac{58 \pm \sqrt{58^2 - 4 \times 21 \times 21}}{2 \times 21}$$

$$\Rightarrow \quad y = \frac{58 \pm \sqrt{3364 - 1764}}{42} \Rightarrow y = \frac{58 \pm \sqrt{1600}}{42} \Rightarrow y = \frac{58 \pm 40}{42}$$

$\Rightarrow \quad y = \frac{58+40}{42}$ or $y = \frac{58-40}{42}$

$\Rightarrow \quad y = \frac{98}{42} \Rightarrow y = \frac{7}{3}$ | $\Rightarrow \quad y = \frac{18}{42} \Rightarrow y = \frac{3}{7}$

$\Rightarrow \quad \frac{x}{2x+1} = \frac{7}{3}$ | $\Rightarrow \quad \frac{x}{2x+1} = \frac{3}{7}$

$\Rightarrow \quad 3x = 14x + 7$ | $\Rightarrow \quad 7x = 6x + 3$

$\Rightarrow \quad -11x = 7$ | $\Rightarrow \quad x = 3$

$\Rightarrow \quad x = \frac{-7}{11}$ (Rejected) | $\Rightarrow \quad \frac{x}{2x+1} = \frac{3}{2 \times 3 + 1} = \frac{3}{7}$

$\therefore$ The required fraction is $\frac{3}{7}$.

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Very Short Answer Type Questions [1 Mark]

Question 29.
If x =-1/2 , is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of K .

Solution :

$\because x = \frac{-1}{2}$ is the solution of $3x^2 + 2kx - 3 = 0$

So, $3\left(\frac{-1}{2}\right)^2 + 2k\left(\frac{-1}{2}\right) - 3 = 0 \Rightarrow \frac{3}{4} - k - 3 = 0 \Rightarrow k = \frac{3}{4} - 3 \Rightarrow k = \frac{-9}{4}$

Question 30.
If the quadratic equation $px^2 - 2\sqrt{5}px + 15 = 0$ has two equal roots, then find the value of p.

Solution :

For equal roots, $D = 0 \Rightarrow (-2\sqrt{5}p)^2 - 4 \times p \times 15 = 0$

$20p^2 - 60p = 0 \Rightarrow 20p(p-3) = 0$

$\Rightarrow p = 3$ or $p = 0$ (Rejected)

Hence, value of p is 3.

Short Answer Type Questions I [2 Marks]

Question 31.
Solve the following quadratic equation for x: $4x^2 - 4a^2 x + (a^4 - b^4) = 0$

Solution :

$4x^2 - 4a^2x + (a^4 - b^4) = 0$

$\Rightarrow x = \frac{4a^2 \pm \sqrt{16a^4 - 4\times4\times(a^4 - b^4)}}{2\times4}$ $\left[\because x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right]$

$\Rightarrow x = \frac{4a^2 \pm \sqrt{16b^4}}{2\times4} \Rightarrow x = \frac{4a^2 \pm 4b^2}{2\times4} = \frac{a^2 \pm b^2}{2}$

$\Rightarrow x = \frac{a^2+b^2}{2}, \frac{a^2-b^2}{2}$

Question 32.
Solve the following quadratic equation for x: $9x^2 - 6b^2 x - (a^4 - b^4) = 0$

Solution :

$$9x^2 - 6b^2x - (a^4 - b^4) = 0$$

$$\Rightarrow \quad x = \frac{6b^2 \pm \sqrt{36b^4 + 4\times 9\times (a^4 - b^4)}}{2\times 9} \qquad \left[\because x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right]$$

$$\Rightarrow \quad x = \frac{6b^2 \pm \sqrt{36b^4 + 36a^4 - 36b^4}}{2\times 9}$$

$$\Rightarrow \quad x = \frac{6b^2 \pm \sqrt{36a^4}}{2\times 9} \quad \Rightarrow \quad x = \frac{6b^2 \pm 6a^2}{2\times 3\times 3}$$

$$\Rightarrow \quad x = \frac{b^2 \pm a^2}{3} \quad \Rightarrow \quad x = \frac{b^2 + a^2}{3}, \frac{b^2 - a^2}{3}$$

Question 33.

Solve the following quadratic equation for x: $4x^2 + 4bx - (a^2 - b^2) = 0$

Solution :

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

$$D = (4b)^2 - 4 \times 4 \times [-(a^2 - b^2)] = 16b^2 + 16a^2 - 16b^2 = 16a^2$$

$$x = \frac{-4b \pm \sqrt{16a^2}}{2\times 4} = \frac{-4b \pm 4a}{8} = \frac{4(-b \pm a)}{8} = \frac{-b \pm a}{2}$$

$$\Rightarrow \quad x = \frac{-b + a}{2}, \frac{-b - a}{2}$$

Question 34.

Solve the following quadratic equation for x: $x^2 - 2ax - (4b^2 - a^2) = 0$

Solution :

$$x^2 - 2ax - (4b^2 - a^2) = 0$$

$$D = (-2a)^2 - 4 \times [-(4b^2 - a^2)] = 4a^2 + 16b^2 - 4a^2 = 16b^2$$

$$\therefore \quad x = \frac{-(-2a) \pm \sqrt{16b^2}}{2\times 1} = \frac{2a \pm 4b}{2} = a \pm 2b$$

$$\Rightarrow \quad x = a + 2b, a - 2b$$

Question 35.

Solve for x .$x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

Solution :

$$x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0 \quad \Rightarrow \quad x^2 - \sqrt{3}x - x + \sqrt{3} = 0$$

$$x(x - \sqrt{3}) - 1(x - \sqrt{3}) = 0 \quad \Rightarrow \quad (x - \sqrt{3})(x - 1) = 0$$

$$x = 1 \text{ or } x = \sqrt{3}.$$

Short Answer Type Questions II [3 Marks]

Question 36.

Find the value ofp for which the quadratic equation $(p + l)x^2 - 6(p + 1)x + 3(p + 9) = 0, p \neq 1$ has equal roots. Hence, find the roots of the equation.

Solution :

$(p+1)x^2 - 6(p+1)x + 3(p+9) = 0, p \neq -1$

$\because$ Above equation has equal roots,

So, discriminant, $D = 0 \Rightarrow \{-6(p+1)\}^2 - 4 \times (p+1).3(p+9) = 0$

$\Rightarrow 36(p^2 + 2p + 1) - 12(p^2 + 10p + 9) = 0 \Rightarrow 24p^2 - 48p - 72 = 0$

$\Rightarrow p^2 - 2p - 3 = 0 \Rightarrow (p-3)(p+1) = 0 \Rightarrow p = 3 \text{ (as } p \neq -1)$

The given quadratic equation becomes,

$4x^2 - 24x + 36 = 0$

$\Rightarrow x^2 - 6x + 9 = 0 \Rightarrow (x-3)^2 = 0 \Rightarrow x = 3, 3.$

Hence, roots of given quadratic equation are 3 and 3.

Question 37.

Find that non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k-1)x + x^2 = 0$ has equal roots. Hence, find the roots of the equation.

Solution :

Now, the given quadratic equation becomes,

$$4x^2 - 4x + 1 = 0 \Rightarrow (2x-1)^2 = 0 \Rightarrow x = \frac{1}{2}, \frac{1}{2}.$$

Hence, the roots of quadratic equation are $\frac{1}{2}, \frac{1}{2}$.

Question 38.

Solve for x: $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

Solution :

$$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$$

$$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$$

$$\sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$$

$$\sqrt{3}x(x - \sqrt{6}) + \sqrt{2}(x - \sqrt{6}) = 0$$

$$(x - \sqrt{6})(\sqrt{3}x + \sqrt{2}) = 0$$

$$x = \sqrt{6} \text{ and } x = \frac{-\sqrt{2}}{\sqrt{3}}$$

Question 39.

Solve for x: $2x^2 + 6\sqrt{3} - 60 = 0$

Solution :

$$2x^2 + 6\sqrt{3}x - 60 = 0$$

$$2x^2 + 10\sqrt{3}x - 4\sqrt{3}x - 60 = 0$$

$$2x(x + 5\sqrt{3}) - 4\sqrt{3}(x + 5\sqrt{3}) = 0$$

$$(x + 5\sqrt{3})(2x - 4\sqrt{3}) = 0$$

$$x + 5\sqrt{3} = 0 \text{ or } 2x - 4\sqrt{3} = 0$$

$$x = -5\sqrt{3} \text{ or } x = \frac{4\sqrt{3}}{2} = 2\sqrt{3}.$$

Question 40.
Solve for x: $x^2 + 5x – (a^2 + a – 6) = 0$
Solution :

$$x^2 + 5x - (a^2 + a - 6) = 0$$

$$D = (5)^2 - 4 \times 1 \times [-(a^2 + a - 6)]$$

$$= 25 + 4a^2 + 4a - 24$$

$$= 4a^2 + 4a + 1 = (2a + 1)^2$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm \sqrt{(2a+1)^2}}{2 \times 1} = \frac{-5 \pm (2a+1)}{2}$$

$$x = \frac{-4+2a}{2}, \frac{-6-2a}{2} = -2 + a, -3 - a$$

Question 41.
Solve for x:
$x^2 -(2b- l)x + (b^2 -b-20) = 0$
Solution :

$$x^2 - (2b - 1)x + (b^2 - b - 20) = 0$$

$$x^2 - (2b - 1)x + (b^2 - b - 20) = 0$$

$$D = [-(2b - 1)]^2 - 4 \times 1 \times (b^2 - b - 20)$$

$$D = 4b^2 - 4b + 1 - 4b^2 + 4b + 80 = 81$$

$$\therefore \quad x = \frac{(2b-1) \pm \sqrt{81}}{2 \times 1} = \frac{2b - 1 \pm 9}{2}$$

$$x = \frac{2b-10}{2}, \frac{2b+8}{2}$$

$$x = b - 5, b + 4$$

Question 42.
Solve for a:
$x^2 + 6x – (a^2 + 2a – 8) = 0$
Solution :

$$x^2 + 6x - (a^2 + 2a - 8) = 0$$

$$D = (6)^2 - 4 \times 1 \times [-(a^2 + 2a - 8)] = 36 + 4a^2 + 8a - 32$$

$$= 4a^2 + 8a + 4 = 4(a^2 + 2a + 1) = 4(a + 1)^2$$

$$\therefore \quad x = \frac{-6 \pm \sqrt{D}}{2 \times 1} = \frac{-6 + \sqrt{4(a+1)^2}}{2}, \frac{-6 - \sqrt{4(a+1)^2}}{2}$$

$$x = \frac{-6+2(a+1)}{2}, \frac{-6-2(a+1)}{2} = -2 + a, -4 - a$$

Long Answer Type Questions [4 Marks]

Question 43.

$$\frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{23}{5x}, x \neq 0, -1, 2.$$

Solve for x:

Solution :

$$\frac{2}{x+1}+\frac{3}{2(x-2)} = \frac{23}{5x} \Rightarrow \frac{4(x-2)+3(x+1)}{2(x+1)(x-2)} = \frac{23}{5x}$$

$$\Rightarrow \frac{7x-5}{2x^2-2x-4} = \frac{23}{5x} \Rightarrow 35x^2-25x = 46x^2-46x-92$$

$$\Rightarrow 11x^2-21x-92 = 0 \Rightarrow x = \frac{21 \pm \sqrt{(-21)^2-4\times11\times(-92)}}{2\times11}$$

$$\Rightarrow x = \frac{21 \pm \sqrt{441+4048}}{2\times11} \Rightarrow x = \frac{21 \pm \sqrt{4489}}{22}$$

$$\Rightarrow x = \frac{21 \pm 67}{22} \Rightarrow x = \frac{21+67}{22}, \frac{21-67}{22}$$

$$\Rightarrow x = 4, \frac{-23}{11}$$

Hence, $x = 4$ or $\frac{-23}{11}$

Question 44.

$$\frac{3}{x+1}+\frac{4}{x-1}=\frac{29}{4x-1}, x \neq 1, -1, \frac{1}{4}.$$

Solve for x

Solution :

$$\frac{3}{x+1}+\frac{4}{(x-1)} = \frac{29}{4x-1} \Rightarrow \frac{3x-3+4x+4}{x^2-1} = \frac{29}{4x-1}$$

$$\Rightarrow (7x+1)(4x-1) = 29(x^2-1)$$

$$\Rightarrow 28x^2-7x+4x-1 = 29x^2-29 \Rightarrow 29x^2-28x^2+7x-4x+1-29 = 0$$

$$\Rightarrow x^2+3x-28 = 0 \Rightarrow (x+7)(x-4) = 0$$

Hence, $x = -7$ or 4

Question 45.

The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is 29/20. Find the original fraction.

Solution :

Let denominator of fraction $= x$

Then, numerator $= x - 3$

Fraction becomes $\frac{x-3}{x}$

Now, $\frac{x-3+2}{x+2} = \frac{x-1}{x+2}$

According to question,

$$\frac{x-1}{x+2} + \frac{x-3}{x} = \frac{29}{20}$$

$$\Rightarrow \frac{x(x-1)+(x+2)(x-3)}{(x+2)(x)} = \frac{29}{20}$$

$$\Rightarrow \frac{x^2-x+x^2-3x+2x-6}{x^2+2x} = \frac{29}{20}$$

$$\Rightarrow \frac{2x^2-2x-6}{x^2+2x} = \frac{29}{20}$$

$$\Rightarrow 40x^2 - 40x - 120 = 29x^2 + 58x$$

$$\Rightarrow 40x^2 - 40x - 120 - 29x^2 - 58x = 0$$

$$\Rightarrow 11x^2 - 98x - 120 = 0$$

$$\Rightarrow 11x^2 - 110x + 12x - 120 = 0$$

$$\Rightarrow 11x(x-10) + 12(x-10) = 0$$

$$\Rightarrow (x-10)(11x+12) = 0$$

$$\Rightarrow x - 10 = 0 \quad \Rightarrow x = 10$$

$$\Rightarrow 11x + 12 = 0 \quad \Rightarrow x = \frac{-12}{11} \text{ (rejected)}$$

Hence, fraction is $\frac{x-3}{x} = \frac{10-3}{10} = \frac{7}{10}$

Question 46.

larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool.

Solution :

Let time taken by the pipe of larger diameter = T hr

$\therefore$ Time taken by the pipe of smaller diameter = (T + 10)hr

According to question,

$$\frac{4}{T}+\frac{9}{T+10}=\frac{1}{2}$$

$$\frac{4T+40+9T}{T(T+10)}=\frac{1}{2}$$

$$26T+80=T^2+10T$$

$$T^2-16T-80=0$$

$$T^2-(20-4)T-80=0$$

$$T^2-20T+4T-80=0$$

$$T(T-20)+(T-20)=0$$

$$(T-20)(T+4)=0$$

$$T=20 \text{ or } T=-4 \text{ (rejected)}$$

Hence, time taken by larger pipe is 20 hours and time taken by smaller pipe is 30 hours.

Question 47.

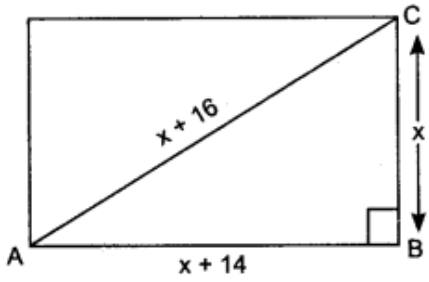
The diagonal of a rectangular field is 16 metres more than the shorter side. If the longer side is 14 metres more than the shorter side, then find the lengths of the sides of the field.

Solution :

Let shorter side = x m

$\therefore$ Diagonal = $(x + 16)$ m

and longer side = $(x + 14)$ m



Now, by Pythagoras Theorem,

$$(x+16)^2=(x+14)^2+x^2$$

$$x^2+32x+256=x^2+28x+196+x^2$$

$$\Rightarrow x^2-4x-60=0$$

$$\Rightarrow (x-10)(x+6)=0$$

$$\Rightarrow x=10 \text{ or } x=-6 \text{ (Rejected)}$$

$\therefore$ Shorter side = 10 m, diagonal = 26 m and longer side = 24 m

Question 48.

A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

Solution :

Let first speed of the train $= x$ km/hr

Distance $= 54$ km

$\therefore$ Time taken $= \frac{54}{x}$ hours

Also distance $= 63$ km

and speed $= (x + 6)$ km/h

$\therefore$ Time taken $= \frac{63}{x+6}$ hours

According to question,

$$\frac{54}{x} + \frac{63}{x+6} = 3 \Rightarrow \frac{54(x+6)+63x}{(x+6)x} = 3$$

$\Rightarrow$ $54x + 324 + 63x = 3(x^2 + 6x) \Rightarrow 117x + 324 = 3x^2 + 18x$

$\Rightarrow$ $3x^2 - 99x - 324 = 0 \Rightarrow x^2 - 33x - 108 = 0$

$(x - 36)(x + 3) = 0 \Rightarrow x = 36$ or $x = -3$ (Rejected)

$\therefore$ First speed $= 36$ km/h

Question 49.

A bus travels at a certain average speed for a distance of 75 km and then travels a distance of 90 km at an average speed of 10 km/h more than the first speed. If it takes 3 hours to complete the total journey, find its first speed.

Solution :

Let first speed of the bus $= x$ km/h

Distance $= 75$ km

Time taken $= \frac{75}{x}$ hours

Speed of bus for a distance of 90 km $= (x + 10)$ km/h

$\therefore$ Time taken $= \frac{90}{x+10}$ hours

According to question, $\frac{75}{x} + \frac{90}{x+10} = 3$

$\Rightarrow$ $\frac{75(x+10)+90x}{x(x+10)} = 3$

$\Rightarrow$ $75x + 750 + 90x = 3x^2 + 30x \Rightarrow 3x^2 - 135x - 750 = 0$

$\Rightarrow$ $x^2 - 45x - 250 = 0$

$(x - 50)(x + 5) = 0 \Rightarrow x = 50$ and $x = -5$ (Rejected)

$\therefore$ First speed $= 50$ km/h

Question 50.

A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at an average speed which is 20 km per hour more than the first speed. If the truck covers the toal distance in 5 hours, find the first speed of the truck

Solution :

Let first speed of truck = x km/h

$$\text{Distance} = 150 \text{ km}$$

$$\therefore \quad \text{Time taken} = \frac{150}{x} \text{ hours}$$

$$\text{Speed to cover 200 km} = (x + 20) \text{ km/h}$$

$$\therefore \quad \text{Time taken} = \frac{200}{x + 20} \text{ hours}$$

According to question,

$$\frac{150}{x} + \frac{200}{x + 20} = 5$$

$$\Rightarrow \quad \frac{150(x + 20) + 200x}{x(x + 20)} = 5$$

$$\Rightarrow \quad 150x + 3000 + 200x = 5(x^2 + 20x)$$

$$\Rightarrow \quad 350x + 3000 = 5x^2 + 100x$$

$$\Rightarrow \quad 5x^2 - 250x - 3000 = 0$$

$$\Rightarrow \quad x^2 - 50x - 600 = 0$$

$$\Rightarrow \quad (x - 60)(x + 10) = 0 \Rightarrow x = 60 \text{ or } x = -10$$

$$\therefore \quad \text{First speed} = 60 \text{ km/h}$$

Question 51.

Ifx = – 2 is a root of the equation $3x^2 + 7x + p = 0$, find the values of k so that the roots of the equation $x^2 + k(4x + k\text{-}1) + p = 0$ are equal.

Solution :

$$\because \quad x = -2 \text{ is a root of } 3x^2 + 7x + p = 0$$

$$\Rightarrow \quad 3(-2)^2 + 7 \times (-2) + p = 0$$

$$\Rightarrow \quad 12 - 14 + p = 0 \Rightarrow p = 2$$

$$\therefore \quad x^2 + k(4x + k - 1) + p = 0$$

$$\Rightarrow \quad x^2 + 4kx + k^2 - k + 2 = 0$$

Now, $\quad D = (4k)^2 - 4 \times 1 \times (k^2 - k + 2) = 16k^2 - 4k^2 + 4k - 8$

$$= 12k^2 + 4k - 8$$

If roots are equal, then $\quad D = 0$

$$\therefore \quad 12k^2 + 4k - 8 = 0 \Rightarrow 3k^2 + k - 2 = 0$$

$$\Rightarrow \quad 3k^2 + 3k - 2k - 2 = 0 \Rightarrow 3k(k + 1) - 2(k + 1) = 0$$

$$\Rightarrow \quad (k + 1)(3k - 2) = 0$$

$$\Rightarrow \quad k = -1 \text{ and } k = \frac{2}{3}$$

Question 52.

The total cost of a certain length of a piece of cloth is rs 200. If the piece was 5 cm longer and each metre of cloth costs rs 2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre?

Solution :

Let length of piece of cloth $= l$ m

$$\text{Total cost} = ₹\ 200$$

$$\therefore \quad \text{Rate/m} = \frac{200}{l} \qquad ...(i)$$

$$\text{If length} = (l + 5) \text{ m}$$

Then $$\text{rate/m} = \frac{200}{l+5}$$

According to question,

$$\frac{200}{l} - \frac{200}{l+5} = 2$$

$$\frac{200l + 1000 - 200l}{l(l+5)} = 2$$

$$\Rightarrow \quad 1000 = 2l^2 + 10l$$

$$\Rightarrow \quad 2l^2 + 10l - 1000 = 0 \Rightarrow l^2 + 5l - 500 = 0$$

$$(l + 25)(l - 20) = 0 \Rightarrow l = -25 \text{ or } l = 20$$

$\therefore$ The length of the piece of cloth $= 20$ m

$$\text{Original rate} = \frac{200}{20} = ₹\ 10/\text{m} \qquad \text{[From } (i)\text{]}$$

Question 53.

Ifx = 3 is root of the equation x^2 -x + k = 0, find the value of p so that the roots of the equation x^2 + k(2x + k +2)+ p = 0 are equal.

Solution :

$\because$ $x = 3$ is a root of $x^2 - x + k = 0$,

$\therefore$ $3^2 - 3 + k = 0 \Rightarrow k = -6$

Given, $x^2 + k(2x + k + 2) + p = 0$

$$x^2 + (-6)(2x - 6 + 2) + p = 0 \qquad (\because\ k = -6)$$

$$x^2 - 12x + 24 + p = 0$$

$$D = (-12)^2 - 4(24 + p) = 144 - 96 - 4p = 48 - 4p$$

For equal roots, $D = 0 \Rightarrow 48 - 4p = 0 \Rightarrow p = 12$

2014

Short Answer Type Questions I [2 Marks]

Question 54.

Solve the quadratic equation $2x^2$ + ax- a^2= 0 for x.

Solution :

$$2x^2 + ax - a^2 = 0$$

$$\Rightarrow \quad 2x^2 + 2ax - ax - a^2 = 0 \Rightarrow 2x(x + a) - a(x + a) = 0$$

$$\Rightarrow \quad (2x - a)(x + a) = 0 \Rightarrow x = \frac{a}{2} \text{ or } x = -a$$

Question 55.
Find the values of p for which the quadratic equation $4x^2 + px + 3 = 0$ has equal roots.
Solution :

Since quadratic equation $4x^2 + px + 3 = 0$ has equal roots, therefore, $D = 0$.

$$\Rightarrow \quad (p)^2 - 4 \times 4 \times 3 = 0$$

$$\Rightarrow \quad p^2 = 48 \quad \Rightarrow \quad p = \pm\sqrt{48} = \pm 4\sqrt{3}$$

Question 56.
Find the values of k for which the quadratic equation $9x^2 – 3kx + k = 0$ has equal roots.
Solution :

Consider the equation $9x^2 - 3kx + k = 0$

For equal roots, $D = 0$

$$\Rightarrow \quad (-3k)^2 - 4 \times 9 \times k = 0$$

$$\Rightarrow \quad 9k^2 - 36k = 0$$

$$\Rightarrow \quad 9k(k - 4) = 0$$

$$\Rightarrow \quad k = 0 \text{ or } k - 4 = 0$$

$$\Rightarrow \quad k = 0 \text{ or } k = 4$$

Question 57.
Find the value of p so that the quadratic equation $px(x – 3) + 9 = 0$ has equal roots.
Solution :

Consider, $px(x - 3) + 9 = 0$

$$\therefore \quad px^2 - 3px + 9 = 0$$

For equal roots, $D = 0$

$$\Rightarrow \quad (-3p)^2 - 4 \times p \times 9 = 0$$

$$\Rightarrow \quad 9p^2 - 36p = 0$$

$$\Rightarrow \quad 9p(p - 4) = 0$$

$$\Rightarrow \quad p = 0 \text{ or } p - 4 = 0$$

$$\Rightarrow \quad p = 0 \text{ or } p = 4$$

But from (i), we notice $p \neq 0$

$$\therefore \quad p = 4$$

Question 58.
Solve for x: $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

Solution :

Consider, $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$\therefore$ The equation becomes $\sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$

$$\sqrt{3}x(x - \sqrt{6}) + \sqrt{2}(x - \sqrt{6}) = 0$$

$$\Rightarrow \quad (\sqrt{3}x + \sqrt{2})(x - \sqrt{6}) = 0$$

$$\Rightarrow \quad \sqrt{3}x + \sqrt{2} = 0 \text{ or } x - \sqrt{6} = 0 \Rightarrow x = -\frac{\sqrt{2}}{\sqrt{3}} \text{ or } x = \sqrt{6}$$

$$\Rightarrow \quad x = \frac{-\sqrt{6}}{3}, \sqrt{6}$$

Short Answer Type Questions II [3 Marks]

Question 59.

Solve the equation:

$$\frac{4}{x} - 3 = \frac{5}{2x + 3}; x \neq 0, \frac{-3}{2}, \text{ for } x.$$

Solution :

$$\frac{4}{x} - 3 = \frac{5}{2x+3} \Rightarrow \frac{4 - 3x}{x} = \frac{5}{2x+3}$$

$$\Rightarrow \quad (4 - 3x)(2x + 3) = 5x \Rightarrow 8x + 12 - 6x^2 - 9x = 5x$$

$$\Rightarrow \quad 6x^2 + 6x - 12 = 0 \Rightarrow x^2 + x - 2 = 0$$

$$\Rightarrow \quad (x + 2)(x - 1) = 0 \Rightarrow x = -2, 1$$

Question 60.

Solve the equation:

$$\frac{3}{x+1} - \frac{1}{2} = \frac{2}{3x - 1}; x \neq -1, x \neq \frac{1}{3} \text{ for } x.$$

Solution :

$$\frac{3}{x+1} - \frac{1}{2} = \frac{2}{3x-1} \Rightarrow \frac{6 - (x+1)}{2(x+1)} = \frac{2}{3x-1}$$

$$\Rightarrow \quad (5 - x)(3x - 1) = 4(x + 1) \Rightarrow 15x - 3x^2 - 5 + x = 4x + 4$$

$$\Rightarrow \quad 3x^2 - 12x + 9 = 0 \Rightarrow x^2 - 4x + 3 = 0$$

$$\Rightarrow \quad (x - 1)(x - 3) = 0 \Rightarrow x = 1, 3$$

Question 61.

Solve the equation:

$$\frac{14}{x+3}-1=\frac{5}{x+1};\ x\neq -3,-1 \text{ for } x.$$

Solution :

$$\frac{14}{x+3}-1=\frac{5}{x+1} \Rightarrow \frac{14-(x+3)}{(x+3)}=\frac{5}{x+1}$$

$$\Rightarrow (11-x)(x+1)=5(x+3) \Rightarrow 11x+11-x^2-x=5x+15$$

$$\Rightarrow x^2-5x+4=0 \Rightarrow (x-1)(x-4)=0 \Rightarrow x=1,4$$

Question 62.

Solve for x:

$$\frac{16}{x}-1=\frac{15}{x+1};\ x\neq 0,-1$$

Solution :

Consider, $\frac{16}{x}-1=\frac{15}{x+1}$, where $x\neq 0,-1$

$$\Rightarrow \frac{16-x}{x}=\frac{15}{x+1} \Rightarrow (16-x)(x+1)=15x$$

$$\Rightarrow 16x+16-x^2-x=15x \Rightarrow x^2+16x-16x-16=0$$

$$\Rightarrow x^2=16 \Rightarrow x=\pm 4$$

Question 63.

If- 5 is a root of the quadratic equation $2x^2$, +px-15 = 0 and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

Solution :

-5 is a root of the equation $2x^2+px-15=0$

$$\Rightarrow 2(-5)^2+p(-5)-15=0 \Rightarrow 50-5p-15=0$$

$$\Rightarrow 5p=35 \Rightarrow p=7 \quad ...(i)$$

Consider, $p(x^2+x)+k=0$

$$\Rightarrow 7x^2+7x+k=0 \quad [\text{using } (i)]$$

$\because$ This equation has equal roots, therefore, D = 0

$$\Rightarrow (7)^2-4\times 7\times k=0 \Rightarrow 49-28k=0$$

$$\Rightarrow 28k=49 \Rightarrow k=\frac{7}{4}$$

Question 64.

If 2 is a root of the quadratic equation $3x^2 + px – 8 = 0$ and the quadratic equation $4x^2– 2px + k = 0$ has equal roots, find the value of k.

Solution :

Given: 2 is a root of the quadratic equation, $3x^2 + px - 8 = 0$

$\therefore \quad 3(2)^2 + p(2) - 8 = 0$

$\Rightarrow \quad 12 + 2p - 8 = 0 \Rightarrow 2p = -4$

$\Rightarrow \quad p = -2$

Also, equation $4x^2 - 2px + k = 0$ has equal roots.

$\Rightarrow \quad 4x^2 + 4x + k = 0$

For equal roots, $D = 0$

$\Rightarrow \quad (4)^2 - 4 \times 4 \times k = 0$

$\Rightarrow \quad 16 = 16k \Rightarrow k = 1$

Question 65.

If 1 is a root of the quadratic equation $3x^2 + ax - 2 = 0$ and the quadratic equation $a(x^2 + 6x) - b = 0$ has equal roots, find the value of b.

Solution :

$\because$ 1 is a root of the quadratic equation $3x^2 + ax - 2 = 0$,

$\therefore \quad 3 \times (1)^2 + a \times 1 - 2 = 0$

$\Rightarrow \quad 3 + a - 2 = 0 \Rightarrow a = -1 \quad ...(i)$

$\because$ Consider, $a(x^2 + 6x) - b = 0$

$\Rightarrow \quad (-1)(x^2 + 6x) - b = 0 \quad$ [from (i)]

$\Rightarrow \quad x^2 + 6x + b = 0$

$\because$ This equation has equal roots, therefore, $D = 0$

$\Rightarrow \quad (6)^2 - 4 \times 1 \times b = 0$

$\Rightarrow \quad 4b = 36 \Rightarrow b = 9$

Long Answer Type Questions [4 Marks]

Question 66.

Find the value of p for which the quadratic equation $(2p + l)x^2 - (7p + 2)x + (7p - 3) = 0$ has equal roots. Also find these roots.

Solution :

$(2p + 1)x^2 - (7p + 2)x + (7p - 3) = 0$ ...(i)

$\because$ Above quadratic equation has equal roots,

$\therefore$ $D = 0 \Rightarrow \{-(7p + 2)\}^2 - 4(2p + 1)(7p - 3) = 0$

$\Rightarrow 49p^2 + 4 + 28p - 4(14p^2 + p - 3) = 0$

$\Rightarrow 49p^2 + 4 + 28p - 56p^2 - 4p + 12 = 0$

$\Rightarrow -7p^2 + 24p + 16 = 0$

$\Rightarrow 7p^2 - 24p - 16 = 0$

$\Rightarrow (p - 4)(7p + 4) = 0$

$\Rightarrow p = 4$ or $p = -\frac{4}{7}$

When $p = -\frac{4}{7}$, the given quadratic equation (i) becomes

$$\left(-\frac{8}{7}+1\right)x^2 - \left(-\frac{28}{7}+2\right)x + \left(-\frac{28}{7}-3\right) = 0 \Rightarrow -x^2 + 14x - 49 = 0$$

$\Rightarrow x^2 - 14x + 49 = 0 \Rightarrow (x - 7)^2 = 0 \Rightarrow x = 7, 7$

When $p = 4$, then the quadratic equation (i) becomes

$(8 + 1)x^2 - (28 + 2)x + (28 - 3) = 0 \Rightarrow 9x^2 - 30x + 25 = 0$

$\Rightarrow (3x - 5)^2 = 0 \Rightarrow x = \frac{5}{3}, \frac{5}{3}$

Hence, roots are 7, 7 or $\frac{5}{3}, \frac{5}{3}$.

Question 67.

Find the values of k for which the quadratic equation $(3k + 1)x^2 + 2(k + l)x + 1 = 0$ has equal roots. Also find these roots.

Solution :

$(3k + 1)x^2 + 2(k + 1)x + 1 = 0$ ...(i)

$\because$ Above quadratic equation has equal roots,

$\therefore$ $D = 0 \Rightarrow \{2(k + 1)\}^2 - 4(3k + 1).1 = 0$

$\Rightarrow 4(k^2 + 2k + 1) - 12k - 4 = 0 \Rightarrow 4k^2 + 8k + 4 - 12k - 4 = 0$

$\Rightarrow 4k^2 - 4k = 0 \Rightarrow 4k(k - 1) = 0 \Rightarrow k = 0, 1$

When $k = 0$, then equation (i) becomes,

$x^2 + 2x + 1 = 0 \Rightarrow (x + 1)^2 = 0 \Rightarrow x = -1, -1$

When $k = 1$, then equation (i) becomes,

$4x^2 + 4x + 1 = 0 \Rightarrow (2x + 1)^2 = 0 \Rightarrow x = \frac{-1}{2}, \frac{-1}{2}$

Hence, roots are $-1, -1$ or $\frac{-1}{2}, \frac{-1}{2}$.

Question 68.

Find the values of k for which the quadratic equation $(k + 4)x^2 + (k + l)x + 1 = 0$ has equal roots. Also find these roots.

Solution :

$(k+4)x^2 + (k+1)x + 1 = 0$

$\because$ Above quadratic equation has equal roots,

$\therefore \quad D = 0 \Rightarrow (k+1)^2 - 4(k+4).1 = 0$

$\Rightarrow \quad k^2 + 2k + 1 - 4k - 16 = 0 \Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow \quad (k+3)(k-5) = 0 \Rightarrow k = -3, 5$

When $k = -3$, then equation (i) becomes,

$$x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow x = 1, 1$$

When $k = 5$, then equation (i) becomes,

$$9x^2 + 6x + 1 = 0 \Rightarrow (3x+1)^2 = 0 \Rightarrow x = \frac{-1}{3}, \frac{-1}{3}$$

Hence, roots are 1, 1 or $\frac{-1}{3}, \frac{-1}{3}$.

Question 69.

The difference of two natural numbers is 5 and the difference of their reciprocals is 1/10. find the numbers.

Solution :

Let the two required numbers be x and $x + 5$.

According to question,

$$\frac{1}{x} - \frac{1}{x+5} = \frac{1}{10} \Rightarrow \frac{x+5-x}{x(x+5)} = \frac{1}{10}$$

$$\Rightarrow \quad \frac{5}{x^2+5x} = \frac{1}{10} \Rightarrow x^2 + 5x = 50$$

$$\Rightarrow \quad x^2 + 5x - 50 = 0 \Rightarrow (x+10)(x-5) = 0$$

$$\Rightarrow \quad x = 5 \text{ or } x = -10 \text{ (which is rejected)}$$

$\therefore$ Hence, required numbers are 5 and 10.

Question 70.

The difference of two natural numbers is 3 and the difference of their reciprocal is 3/28. find the numbers.

Solution :

Let the two required numbers be x and $x + 3$.

According to question,

$$\frac{1}{x} - \frac{1}{x+3} = \frac{3}{28} \Rightarrow \frac{x+3-x}{x(x+3)} = \frac{3}{28}$$

$$\Rightarrow \quad \frac{3}{x^2+3x} = \frac{3}{28} \Rightarrow x^2 + 3x = 28$$

$$\Rightarrow \quad x^2 + 3x - 28 = 0 \Rightarrow (x+7)(x-4) = 0$$

$$\Rightarrow \quad x = 4 \text{ or } x = -7 \text{ (which is rejected)}$$

$\therefore$ Hence, required numbers are 4 and 7.

Question 71.
The difference of two natural numbers is 5 and the difference of their reciprocal is 5/14. find the numbers.

Solution :

Let the two required numbers be x and $x + 5$.

According to question,

$$\frac{1}{x} - \frac{1}{x+5} = \frac{5}{14} \Rightarrow \frac{x+5-x}{x(x+5)} = \frac{5}{14}$$

$$\Rightarrow \frac{5}{x^2+5x} = \frac{5}{14} \Rightarrow x^2 + 5x = 14$$

$$\Rightarrow x^2 + 5x - 14 = 0 \Rightarrow (x-2)(x+7) = 0$$

$$\Rightarrow x = 2 \text{ or } x = -7 \text{ (which is rejected)}$$

$\therefore$ Hence, required numbers are 2 and 7.

Question 72.
Solve for x

$$\frac{x-2}{x-3} + \frac{x-4}{x-5} = \frac{10}{3}; x \neq 3, 5$$

Solution :

Consider, $\frac{x-2}{x-3} + \frac{x-4}{x-5} = \frac{10}{3}, x \neq 3, 5$

$$\Rightarrow \frac{(x-3)+1}{x-3} + \frac{(x-5)+1}{x-5} = \frac{10}{3} \Rightarrow 1 + \frac{1}{x-3} + 1 + \frac{1}{x-5} = \frac{10}{3}$$

$$\Rightarrow \frac{1}{x-3} + \frac{1}{x-5} = \frac{10}{3} - 2 \Rightarrow \frac{x-5+x-3}{(x-3)(x-5)} = \frac{4}{3}$$

$$\Rightarrow \frac{2x-8}{x^2-8x+15} = \frac{4}{3} \Rightarrow 4x^2 - 32x + 60 = 6x - 24$$

$$\Rightarrow 4x^2 - 38x + 84 = 0 \Rightarrow 2x^2 - 19x + 42 = 0$$

$$\Rightarrow 2x^2 - 12x - 7x + 42 = 0 \Rightarrow 2x(x-6) - 7(x-6) = 0$$

$$\Rightarrow (2x-7)(x-6) = 0$$

$$\Rightarrow 2x - 7 = 0 \text{ or } x - 6 = 0$$

$$\Rightarrow x = \frac{7}{2}, 6.$$

Question 73.
A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution :

Speed of motor boat in still water = 18 km/h

Let speed of the stream = x km/h

$\therefore$ Upstream speed = $(18 - x)$ km/h

Downstream speed = $(18 + x)$ km/h

According to question,

$$\Rightarrow \quad \frac{24}{18-x} - \frac{24}{18+x} = 1 \quad \Rightarrow \quad \frac{24(18+x) - 24(18-x)}{(18-x)(18+x)} = 1$$

$$\Rightarrow \quad \frac{432 + 24x - 432 + 24x}{324 - x^2} = 1 \quad \Rightarrow \quad 48x = 324 - x^2$$

$$\Rightarrow \quad x^2 + 48x - 324 = 0 \quad \Rightarrow \quad x^2 + 54x - 6x - 324 = 0$$

$$\Rightarrow \quad x(x + 54) - 6(x + 54) = 0 \quad \Rightarrow \quad (x - 6)(x + 54) = 0$$

$$\Rightarrow \quad x - 6 = 0 \text{ or } x + 54 = 0$$

$$\Rightarrow \quad x = 6 \text{ or } x = -54 \text{ (rejected)}$$

$\therefore$ Speed of the stream is 6 km/h.

Question 74.

$$\frac{x-3}{x-4} + \frac{x-5}{x-6} = \frac{10}{3}; x \neq 4, 6$$

Solve for x

Solution :

Consider, $\frac{x-3}{x-4} + \frac{x-5}{x-6} = \frac{10}{3}$, where $x \neq 4, 6$

$$\Rightarrow \quad \frac{(x-4)+1}{x-4} + \frac{(x-6)+1}{x-6} = \frac{10}{3} \quad \Rightarrow \quad 1 + \frac{1}{x-4} + 1 + \frac{1}{x-6} = \frac{10}{3}$$

$$\Rightarrow \quad \frac{1}{x-4} + \frac{1}{x-6} = \frac{10}{3} - 2 \quad \Rightarrow \quad \frac{x-6+x-4}{(x-4)(x-6)} = \frac{4}{3}$$

$$\Rightarrow \quad \frac{2x-10}{x^2-10x+24} = \frac{4}{3} \quad \Rightarrow \quad \frac{x-5}{x^2-10x+24} = \frac{2}{3}$$

$$\Rightarrow \quad 2x^2 - 20x + 48 = 3x - 15 \quad \Rightarrow \quad 2x^2 - 23x + 63 = 0$$

$$\Rightarrow \quad 2x^2 - 14x - 9x + 63 = 0 \quad \Rightarrow \quad 2x(x-7) - 9(x-7) = 0$$

$$\Rightarrow \quad (2x - 9)(x - 7) = 0 \quad \Rightarrow \quad 2x - 9 = 0 \text{ or } x - 7 = 0$$

$$\Rightarrow \quad x = \frac{9}{2} \text{ or } x = 7.$$

Question 75.

$$\frac{x-4}{x-5}+\frac{x-6}{x-7}=\frac{10}{3};\ x\neq 5,7$$

Solve for x

Solution :

Consider, $\frac{x-4}{x-5}+\frac{x-6}{x-7}=\frac{10}{3}$, where $x \neq 5, 7$

$$\Rightarrow \quad \frac{(x-5)+1}{x-5}+\frac{(x-7)+1}{x-7} = \frac{10}{3} \Rightarrow 1+\frac{1}{x-5}+1+\frac{1}{x-7}=\frac{10}{3}$$

$$\Rightarrow \quad \frac{1}{x-5}+\frac{1}{x-7} = \frac{10}{3}-2 \Rightarrow \frac{x-7+x-5}{(x-5)(x-7)}=\frac{4}{3}$$

$$\Rightarrow \quad \frac{2x-12}{x^2-12x+35} = \frac{4}{3} \Rightarrow \frac{x-6}{x^2-12x+35}=\frac{2}{3}$$

$$\Rightarrow \quad 2x^2-24x+70 = 3x-18 \Rightarrow 2x^2-27x+88=0$$

$$\Rightarrow \quad 2x^2-11x-16x+88 = 0 \Rightarrow x(2x-11)-8(2x-11)=0$$

$$\Rightarrow \quad (x-8)(2x-11) = 0$$

$$\Rightarrow \quad x-8 = 0 \text{ or } 2x-11=0$$

$$\Rightarrow \quad x = 8, \frac{11}{2}$$

Question 76.

The sum of the squares of two consecutive odd numbers is 394. Find the numbers.

Solution :

Let two consecutive odd numbers be x and $x + 2$.

$$\therefore \quad x^2+(x+2)^2 = 394 \Rightarrow x^2+x^2+4x+4=394$$

$$\Rightarrow \quad 2x^2+4x-390 = 0 \Rightarrow x^2+2x-195=0$$

$$\Rightarrow \quad x^2+15x-13x-195 = 0 \Rightarrow x(x+15)-13(x+15)=0$$

$$\Rightarrow \quad (x-13)(x+15) = 0$$

$$\Rightarrow \quad x-13 = 0 \text{ or } x+15=0$$

$$\Rightarrow \quad x = 13 \text{ or } x=-15$$

$\therefore$ Numbers are 13 and 13 + 2 = 15, i.e. 13 and 15 or –15 and –15 + 3 = –13, i.e. –15 and –13.

Question 77.

$$2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5;\ x\neq -3,\frac{1}{2}.$$

Solve for x:

Solution :

Consider, $2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right) = 5, x \neq -3, \frac{1}{2}$

Let $\frac{2x-1}{x+3} = y \Rightarrow 2y - \frac{3}{y} = 5$

$\Rightarrow \frac{2y^2-3}{y} = 5 \Rightarrow 2y^2 - 3 = 5y$

$\Rightarrow 2y^2 - 5y - 3 = 0 \Rightarrow 2y^2 - 6y + y - 3 = 0$

$\Rightarrow 2y(y-3) + 1(y-3) = 0 \Rightarrow (2y+1)(y-3) = 0$

$\Rightarrow 2y + 1 = 0$ or $y - 3 = 0$

$\Rightarrow y = -\frac{1}{2}$ or $y = 3$

When $y = -\frac{1}{2} \Rightarrow \frac{2x-1}{x+3} = -\frac{1}{2}$

$\Rightarrow 4x - 2 = -x - 3 \Rightarrow 5x = -1$

$\Rightarrow x = -\frac{1}{5}$

When $y = 3$

$\Rightarrow \frac{2x-1}{x+3} = 3 \Rightarrow 2x - 1 = 3x + 9$

$\Rightarrow -x = 10 \Rightarrow x = -10$

$\therefore$ Solution is $x = -\frac{1}{5}, -10$

Question 78.

The sum of the squares of two consecutive even numbers is 340. Find the numbers.

Solution :

Let two consecutive even numbers be x and $x + 2$ respectively.

Given: $x^2 + (x+2)^2 = 340$

$\Rightarrow x^2 + x^2 + 4x + 4 = 340 \Rightarrow 2x^2 + 4x - 336 = 0$

$\Rightarrow x^2 + 2x - 168 = 0 \Rightarrow x^2 + 14x - 12x - 168 = 0$

$\Rightarrow x(x+14) - 12(x+14) = 0 \Rightarrow (x-12)(x+14) = 0$

$\Rightarrow x - 12 = 0$ or $x + 14 = 0$

$\Rightarrow x = 12, -14$

When $x = 12$, then $x + 2 = 14$. So, numbers are 12 and 14.

When $x = -14$, then $x + 2 = -12$. So, numbers are -14 and -12.

Question 79.

Solve for x

Solution :

Consider the equation,

$$3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5; x\neq\frac{1}{3},\frac{-3}{2}$$

Let us consider: $\frac{3x-1}{2x+3}=y$

$\Rightarrow$ From eqn (i), $3y-\frac{2}{y}=5 \Rightarrow \frac{3y^2-2}{y}=5$

$\Rightarrow$ $3y^2-2=5y \Rightarrow 3y^2-5y-2=0$

$\Rightarrow$ $3y^2-6y+y-2=0 \Rightarrow 3y(y-2)+1(y-2)=0$

$\Rightarrow$ $(3y+1)(y-2)=0$

$\Rightarrow$ Either $3y+1=0$ or $y-2=0$

$\Rightarrow$ $y=-\frac{1}{3}$ or 2

When $y=-\frac{1}{3}$, then

$$\frac{3x-1}{2x+3}=-\frac{1}{3} \Rightarrow 9x-3=-2x-3$$

$\Rightarrow$ $11x=0 \Rightarrow x=0$

When $y=2$, then

$$\frac{3x-1}{2x+3}=2$$

$\Rightarrow$ $3x-1=4x+6$

$\Rightarrow$ $-x=7$

$\Rightarrow$ $x=-7$

$\therefore$ Solution is $x=0,-7$.

Question 80.

The sum of the squares of two consecutive multiples of 7 is 637. Find the multiples.

Solution :

Let two consecutive multiples of 7 be x and $x + 7$. As per condition,

$\therefore \quad x^2 + (x + 7)^2 = 637$

$\Rightarrow \quad x^2 + x^2 + 14x + 49 = 637$

$\Rightarrow \quad 2x^2 + 14x + 49 = 637$

$\Rightarrow \quad 2x^2 + 14x - 588 = 0$

$\Rightarrow \quad x^2 + 7x - 294 = 0$

$\Rightarrow \quad x^2 + 21x - 14x - 294 = 0$

$\Rightarrow \quad x(x + 21) - 14(x + 21) = 0$

$\Rightarrow \quad (x - 14)(x + 21) = 0$

$\Rightarrow$ Either $x - 14 = 0$ or $x + 21 = 0 \quad \Rightarrow x = 14$ or -21

When $x = 14, x + 7 = 21$

$\therefore$ Numbers are 14 and 21.

When $x = -21, x + 7 = -21 + 7 = -14,$

$\therefore$ Numbers are -21 and -14

Question 81.

Solve for x :

$$3\left(\frac{7x+1}{5x-3}\right) - 4\left(\frac{5x-3}{7x+1}\right) = 11; x \neq \frac{3}{5}, -\frac{1}{7}.$$

Solution :

Consider the equation: $3\left(\frac{7x+1}{5x-3}\right) - 4\left(\frac{5x-3}{7x+1}\right) = 11, x \neq \frac{3}{5}, -\frac{1}{7}$ ...(*i*)

Let us consider: $\frac{7x+1}{5x-3} = y$

$\Rightarrow$ For eqn (*i*), $3y - \frac{4}{y} = 11 \Rightarrow \frac{3y^2-4}{y} = 11$

$\Rightarrow \quad 3y^2 - 4 = 11y \Rightarrow 3y^2 - 11y - 4 = 0$

$\Rightarrow \quad 3y^2 - 12y + y - 4 = 0 \Rightarrow 3y(y - 4) + 1(y - 4) = 0$

$\Rightarrow \quad (3y + 1)(y - 4) = 0$

$\Rightarrow$ Either $3y + 1 = 0$ or $y - 4 = 0$

$\Rightarrow \quad y = -\frac{1}{3}$ or 4

When $y = -\frac{1}{3}$, then $\frac{7x+1}{5x-3} = -\frac{1}{3}$

$\Rightarrow \quad 21x + 3 = -5x + 3$

$\Rightarrow \quad 26x = 0$

$\Rightarrow \quad x = 0$

When $y = 4$, then $\frac{7x+1}{5x-3} = 4$

$\Rightarrow \quad 7x + 1 = 20x - 12 \Rightarrow -13x = -13 \Rightarrow x = 1$

$\therefore$ Solution is $x = 0, 1$.

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Short Answer Type Questions I [2 Marks]

Question 82.
Solve the following quadratic equation for x; $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$
Solution :

Consider: $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow \quad 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0 \Rightarrow 4x(\sqrt{3x}+2) - \sqrt{3}(\sqrt{3x}+2) = 0$

$\Rightarrow \quad (4x - \sqrt{3})(\sqrt{3x}+2) = 0 \Rightarrow x = \frac{\sqrt{3}}{4}$ and $\frac{-2}{\sqrt{3}}$

Hence, roots of given quadratic equation are $\frac{\sqrt{3}}{4}$ and $\frac{-2}{\sqrt{3}}$.

Question 83.
Solve the following forjr. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
Solution :

Consider: $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0 \Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow \quad x(\sqrt{2}x+5) + \sqrt{2}(\sqrt{2}x+5) = 0 \Rightarrow (\sqrt{2}x+5)(x+\sqrt{2}) = 0$

$\Rightarrow \quad \sqrt{2}x+5 = 0$ or $x+\sqrt{2} = 0 \Rightarrow \sqrt{2}x = -5$ or $x = -\sqrt{2}$

$\Rightarrow \quad x = \frac{-5}{\sqrt{2}}$ or $x = -\sqrt{2}$

Question 84.
Solve for x: $x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$.
Solution :

Consider: $x^2 - (\sqrt{2}+1)x + \sqrt{2} = 0 \Rightarrow x^2 - \sqrt{2}x - x + \sqrt{2} = 0$

$\Rightarrow \quad x(x - \sqrt{2}) - 1(x - \sqrt{2}) = 0 \Rightarrow (x - \sqrt{2})(x - 1) = 0$

$\Rightarrow$ Either $x - \sqrt{2} = 0$ or $x - 1 = 0 \Rightarrow x = \sqrt{2}$ or $x = 1$

Short Answer Type Questions II [3 Marks]

Question 85.
For what value of k, are the roots of the quadratic equation kx(x – 2) + 6 = 0 equal?
Solution :

$\because$ Above equation has equal roots,

so, discriminant, $D = 0 \Rightarrow (-2k)^2 - 4 \times k \times 6 = 0$ $[\because D = b^2 - 4ac]$

$\Rightarrow 4k^2 - 24k = 0 \Rightarrow 4k(k-6) = 0$

$\Rightarrow k = 6$ or $k = 0$(rejected)

{if $k = 0$ then given equation will not be quadratic equation}

$\therefore$ for equal roots, $k = 6$

Question 86.
For what value of k, are the roots of the quadratic equation kx(x – 2√5) + 10 = 0, equal?
Solution :

Given quadratic equation is $kx(x - 2\sqrt{5}) + 10 = 0$

$\Rightarrow kx^2 - 2\sqrt{5}kx + 10 = 0$

$\because$ Above quadratic equation has equal roots,

so, discriminant, $D = 0 \Rightarrow (-2\sqrt{5}k)^2 - 4 \times k \times 10 = 0$ $[\because D = b^2 - 4ac]$

$\Rightarrow 20k^2 - 40k = 0 \Rightarrow 20k(k-2) = 0$

$\Rightarrow k = 2$ or $k = 0$ (rejected)

(as if $k = 0$ then given equation will not be quadratic equation)

$\therefore$ for equal roots, $k = 2$

Question 87.
For what value of k, are the roots of the quadratic equation$(k + 4)x^2 + (k + l)x + 1 = 0$, equal?
Solution :

Given quadratic equation is: $(k + 4)x^2 + (k + 1)x + 1 = 0$

$\because$ Above quadratic equation has equal roots,

so, discriminant, $D = 0 \Rightarrow (k+1)^2 - 4(k+4)1 = 0$ $[\because D = b^2 - 4ac]$

$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0 \Rightarrow k^2 - 2k - 15 = 0 \Rightarrow (k-5)(k+3) = 0$

$\Rightarrow k = 5$ or $k = -3$

Question 88.
For what value of k, are the roots of the quadratic equation $(k – 12)x^2 + 2(k – 12)x + 2 = 0$ equal ?
Solution :

Given quadratic equation is:

$$(k-12)x^2 + 2(k-12)x + 2 = 0$$

For equal roots,

Discriminant $D = 0$, where $D = b^2 - 4ac$

$$\Rightarrow \quad [2(k-12)]^2 - 4(k-12)(2) = 0$$

$$\Rightarrow \quad 4(k-12)^2 - 8(k-12) = 0 \Rightarrow 4(k-12)(k-12) - 8(k-12) = 0$$

$$4[k-12][k-12-2] = 0 \Rightarrow k-12 = 0 \text{ or } k-14 = 0$$

$\therefore$ $k = 12$ (not possible) since quadratic equation becomes zero.

or $k = 14$ (Possible value)

Question 89.

For what value of A, are the roots of the quadratic equation $y^2 + k^2 = 2(k + l)y$ equal?

Solution :

Given quadratic equation is: $y^2 + k^2 = 2(k+1)y$

$$y^2 - 2(k+1)y + k^2 = 0$$

For equal roots, discriminant $D = 0$, where $D = b^2 - 4ac$

$$[-2(k+1)]^2 - 4(1)(k^2) = 0$$

$$4(k^2 + 1 + 2k) - 4k^2 = 0$$

$$4k^2 + 4 + 8k - 4k^2 = 0$$

$$8k + 4 = 0$$

$$8k = -4$$

$$\therefore \quad k = -\frac{1}{2}$$

Question 90.

For what value of k, are the roots of the quadratic equation $(k-4)x^2 + (k-4)x + 4 = 0$ equal ?

Solution :

For equal roots, discriminant $D = 0$, where $D = b^2 - 4ac$.

$$\therefore \quad (k-4)^2 - 4(k-4)(4) = 0$$

$$(k-4)(k-4) - 16(k-4) = 0$$

$$(k-4)[k-4-(4)(4)] = 0$$

$$(k-4)(k-4-16) = 0$$

$$(k-4)(k-20) = 0$$

$\Rightarrow$ $k = 4$ (not possible) since quadratic equation becomes zero.

or $k = 20$ (possible)

Question 91.

For what value of k, are the roots of the quadratic equation $x^2 - (3k - l)x + 2k^2 + 2k - 11 = 0$ equal?

Solution :

Given quadratic equation is:

$x^2 - (3k - 1)x + 2k^2 + 2k - 11 = 0$

Here, $a = 1, b = -(3k - 1), c = 2k^2 + 2k - 11$

$\therefore$ For equal roots, discriminant $D = 0$, where $D = b^2 - 4ac$

$\therefore \quad [-(3k - 1)]^2 - 4[1]\,[2k^2 + 2k - 11] = 0$

$\Rightarrow \quad 9k^2 + 1 - 6k - 4(2k^2 + 2k - 11) = 0$

$\Rightarrow \quad 9k^2 + 1 - 6k - 8k^2 - 8k + 44 = 0$

$\Rightarrow \quad k^2 - 14k + 45 = 0$

$\Rightarrow \quad k^2 - 9k - 5k + 45 = 0$

$\Rightarrow \quad k(k - 9) - 5(k - 9) = 0$

$\Rightarrow \quad (k - 9)(k - 5) = 0$

$\Rightarrow$ Either $k - 9 = 0$ or $k - 5 = 0$

$\Rightarrow \quad k = 9$ or $k = 5$

Question 92.

For what value of m, are the roots of the quadratic equation $x^2 - 2x(1 + 3m) + 7(3 + 2m) = 0$ equal?

Solution :

Given quadratic equation, $x^2 - 2x(1 + 3m) + 7(3 + 2m) = 0$

$\Rightarrow \quad x^2 - 2(1 + 3m)x + 7(3 + 2m) = 0$

For equal roots, discriminant $D = 0$, where $D = b^2 - 4ac$

$\Rightarrow \quad [-2(1 + 3m)]^2 - 4(1)(7)(3 + 2m) = 0$

$\Rightarrow \quad 4(1 + 3m)^2 - 28(3 + 2m) = 0$

$\Rightarrow \quad 4(1 + 3m)^2 - 84 - 56m = 0$

$\Rightarrow \quad 4[1 + 9m^2 + 6m - 21 - 14m] = 0$

$\Rightarrow \quad 4[9m^2 - 8m - 20] = 0$

$\Rightarrow \quad 4[9m^2 - 18m + 10m - 20] = 0$

$\Rightarrow \quad 4[9m(m - 2) + 10(m - 2)] = 0$

$4(9m + 10)(m - 2) = 0$

$\Rightarrow \quad 9m + 10 = 0 \quad$ or $\quad m - 2 = 0$

$\Rightarrow \quad m = \frac{-10}{9} \quad$ or $\quad m = 2$

Long Answer Type Questions [4 Marks]

Question 93.

Solve for x:

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

Solution :

Given that: $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

$\Rightarrow \frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$

$\Rightarrow \frac{2x-2a-b-2x}{(2a+b+2x)\,2x} = \frac{b+2a}{2ab} \Rightarrow \frac{-(2a+b)}{2x(2a+b+2x)} = \frac{(2a+b)}{2ab}$

$\Rightarrow \frac{-1}{2x^2+2ax+bx} = \frac{1}{ab} \Rightarrow 2x^2 + 2ax + bx = -ab$

$\Rightarrow 2x^2 + 2ax + bx + ab = 0 \Rightarrow 2x(x+a) + b(x+a) = 0$

$\Rightarrow (x+a)(2x+b) = 0 \Rightarrow x = -a \text{ or } x = -\frac{b}{2}$

Question 94.

Sum of the areas of two squares is 400 cm2. If the difference of their perimeters is 16 cm, find the sides of the two squares.

Solution :

Let, side of one square be $= x$

Side of another square be $= y$

Sum of areas of two squares $= x^2 + y^2$

$\therefore x^2 + y^2 = 400$ ...(*i*)

Difference of their perimeters $= 4x - 4y$

$\therefore 4x - 4y = 16 \Rightarrow x - y = 4$

$\Rightarrow y = x - 4$ [Put in (*i*)]

$\therefore$ Equation (*i*) becomes:

$x^2 + (x-4)^2 = 400$

$2x^2 - 8x + 16 = 400$

$2x^2 - 8x - 384 = 0$

$x^2 - 4x - 192 = 0$

$(x-16)(x+12) = 0$

$\Rightarrow x = 16, x = -12$ (not possible as side cannot be negative)

$\therefore y = 16 - 4 = 12$

Sides of squares are 12 cm and 16 cm.

Question 95.

Solve for x

$$\frac{1}{x-3}+\frac{2}{x-2}=\frac{8}{x};\ x\neq 0, 2, 3.$$

Solution :

Given that: $\frac{1}{x-3}+\frac{2}{x-2}=\frac{8}{x};\ x\neq 0, 2, 3$

$$\Rightarrow \frac{x-2+2(x-3)}{(x-3)(x-2)}=\frac{8}{x} \Rightarrow \frac{x-2+2x-6}{(x-3)(x-2)}=\frac{8}{x}$$

$$\Rightarrow \frac{3x-8}{x^2-5x+6}=\frac{8}{x}$$

$$\Rightarrow 3x^2-8x=8x^2-40x+48$$

$$\Rightarrow 5x^2-32x+48=0$$

$$\Rightarrow 5x^2-20x-12x+48=0$$

$$\Rightarrow 5x(x-4)-12(x-4)=0$$

$$\Rightarrow (x-4)(5x-12)=0$$

$$\Rightarrow x=4 \text{ or } x=\frac{12}{5}$$

$$\therefore x=4 \text{ or } x=\frac{12}{5}$$

Question 96.

Solve for x

$$\frac{4}{x} - 3 = \frac{5}{2x+3}; x \neq 0, -\frac{3}{2}$$

Solution :

Given that: $\frac{4}{x} - 3 = \frac{5}{2x+3}; x \neq 0, \frac{-3}{2}$

$$\frac{4}{x} - \frac{5}{2x+3} = 3$$

$$\frac{4(2x+3) - 5x}{x(2x+3)} = 3$$

$$\frac{8x + 12 - 5x}{2x^2 + 3x} = 3$$

$$3x + 12 = 6x^2 + 9x$$

$$6x^2 + 6x - 12 = 0$$

$$6(x^2 + x - 2) = 0$$

$$x^2 + x - 2 = 0$$

$$x^2 + 2x - x - 2 = 0$$

$$x(x + 2) - 1(x + 2) = 0$$

$$(x + 2)(x - 1) = 0$$

$$x + 2 = 0 \quad \text{or} \quad x - 1 = 0$$

$$\therefore \quad x = -2 \quad \text{or} \quad x = 1$$

Question 97.

Solve for x

$$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}; x \neq 0, 1, 2$$

Solution :

$$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}; x \neq 0, 1, 2$$

Question 98.

$$\frac{1}{2x-3} + \frac{1}{x-5} = 1, x \neq \frac{3}{2}, 5.$$

Solve for x:

Solution :

Given that: $\frac{1}{2x-3}+\frac{1}{x-5} = 1; x \neq \frac{3}{2}, 5$

$\Rightarrow \frac{x-5+2x-3}{(2x-3)(x-5)} = 1$

$\Rightarrow \frac{3x-8}{2x^2-10x-3x+15} = 1$

$\Rightarrow 3x-8 = 2x^2-13x+15$

$\Rightarrow 2x^2-16x+23 = 0$

Now, discriminant, $D = b^2-4ac$

$= (-16)^2-4(2)(23) = 256-184 = 72$

$x = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{16 \pm \sqrt{72}}{2 \times 2} \Rightarrow 4 \pm \frac{\sqrt{72}}{4} \Rightarrow 4 \pm \frac{2\sqrt{18}}{4}$

$\therefore x = 4 \pm \frac{\sqrt{18}}{2}$

$\therefore$ Roots are $\left(4+\frac{\sqrt{18}}{2}, 4-\frac{\sqrt{18}}{2}\right)$.

Question 99.

The present age of a father is equal to the square of the present age of his son. One year ago, the age of the father was 8 times the age of his son. Find their present ages.

Solution :

Let present age of son $= x$

Present age of father $= x^2$

One year ago, age of son $= x-1$

One year ago, age of father $= x^2-1$

As per condition, $x^2-1 = 8(x-1)$

$\Rightarrow x^2-1 = 8x-8$

$\Rightarrow x^2-8x+7 = 0$

$\Rightarrow x^2-7x-x+7 = 0$

$\Rightarrow x(x-7)-1(x-7) = 0$

$\Rightarrow (x-7)(x-1) = 0$

$\Rightarrow$ Either $x-7 = 0$ or $x-1 = 0$

$\Rightarrow x = 7$ or $x = 1$ (not possible)

$\therefore$ Present age of son $= 7$ years

Present age of father $= (7)^2 = 49$ years

Question 100.
Solve the following quadratic equation in variable x : $abx^2 = (a + b)^2 (x - 1)$.
Solution :

Given quadratic equation, $abx^2 = (a + b)^2 (x - 1)$

$abx^2 = (a + b)^2 x - (a + b)^2$

or, $abx^2 - (a + b)^2 x + (a + b)^2 = 0$

Then, discriminant: $D = B^2 - 4AC$

$= [-(a + b)^2]^2 - 4(ab)(a + b)^2$

$= (a + b)^4 - 4ab(a + b)^2$

$= (a + b)^2 [(a + b)^2 - 4ab]$

$= (a + b)^2 [a^2 + b^2 + 2ab - 4ab]$

$= (a + b)^2 [a^2 + b^2 - 2ab] = (a + b)^2 (a - b)^2$

$\therefore$ Solution is: $x = \frac{-B \pm \sqrt{D}}{2A} = \frac{+(a+b)^2 \pm \sqrt{[(a+b)(a-b)]^2}}{2(ab)}$

$= \frac{(a+b)^2 \pm (a^2 - b^2)}{2ab}$

$\therefore$ For positive sign, $x = \frac{(a+b)^2 + (a^2 - b^2)}{2ab}$

$= \frac{a^2 + b^2 + 2ab + a^2 - b^2}{2ab} = \frac{2a^2 + 2ab}{2ab} = \frac{2a(a+b)}{2ab}$

$\therefore$ $x = \frac{a+b}{b}$

or, for negative sign, $x = \frac{(a+b)^2 - (a^2 - b^2)}{2ab} = \frac{a^2 + b^2 + 2ab - a^2 + b^2}{2ab}$

$= \frac{2b^2 + 2ab}{2ab} = \frac{2b(a+b)}{2ab} = \frac{a+b}{a}$

$\therefore$ $x = \frac{a+b}{a}$

$\therefore$ Roots are $\frac{a+b}{a}, \frac{a+b}{b}$

Short Answer Type Questions I [2 Marks]

Question 101.
Find the value(s) of k so that the quadratic equation $2x^2 + kx + 3 = 0$ has equal roots.

Solution :

Given quadratic equation is $2x^2 + kx + 3 = 0$. As the equation has equal roots,

So, discriminant, $D = 0$

$$b^2 - 4ac = 0$$

$$(k)^2 - 4 \times 2 \times 3 = 0$$

$$k^2 - 24 = 0$$

$$k = \pm\sqrt{24}$$

$$\therefore \quad k = \pm 2\sqrt{6}$$

Question 102.

Find the value(s) of so that the quadratic equation $x^2 - 4kx + k = 0$ has equal roots.

Solution :

Given quadratic equation: $x^2 - 4kx + k = 0$ (Given). Then,

$$D = b^2 - 4ac, \text{ where D is discriminant.}$$

$$= (-4k)^2 - 4 \times 1 \times k = 16k^2 - 4k$$

Since given equation has equal roots, so:

$$\therefore \quad D = 0$$

$$16k^2 - 4k = 0$$

$$\Rightarrow \quad 4k(4k - 1) = 0$$

$$\Rightarrow \quad 4k = 0 \text{ and } 4k - 1 = 0$$

$$\Rightarrow \quad k = 0 \text{ and } k = \frac{1}{4}$$

Question 103.

Find the value(s) of k so that the quadratic equation $3x^2 - 2kx + 12 = 0$ has equal roots

Solution :

Quadratic equation is: $3x^2 - 2kx + 12 = 0$ (Given). Then

$$D = b^2 - 4ac, \text{ where D is discriminant.}$$

$$= (-2k)^2 - 4 \times 3 \times 12 = 4k^2 - 144$$

Since given equation has equal roots, so:

$$\therefore \quad D = 0$$

$$4k^2 - 144 = 0$$

$$\Rightarrow \quad 4(k^2 - 36) = 0$$

$$\Rightarrow \quad k^2 - 36 = 0 \text{ and } k^2 = 36$$

$$\Rightarrow \quad k = \pm 6$$

$$\Rightarrow \quad k = 6 \text{ and } k = -6$$

Question 104.

Find the value of m for which the roots of the equation mx(6x + 10) + 25 = 0, are equal.

Solution :

The given equation is

$$mx(6x + 10) + 25 = 0$$

$$\Rightarrow \quad 6mx^2 + 10mx + 25 = 0$$

When roots are equal, then $D = 0$, where D is discriminant.

$$b^2 - 4ac = 0$$

$$(10m)^2 - 4(6m)(25) = 0$$

$$\Rightarrow \quad 100m^2 - 600m = 0 \quad \Rightarrow \quad 100m\,(m - 6) = 0$$

$$\Rightarrow \quad m = 0 \text{ and } m = 6 \quad \Rightarrow \quad m = 6$$

Since, $m = 0$ is rejected because if $m = 0$, then equation will not be quadratic.

Question 105.

Find the value of k for which the roots of the equation kx(3x – 4) + 4 = 0, are equal.

Solution :

Given equation is $kx(3x - 4) + 4 = 0$

$$3kx^2 - 4kx + 4 = 0$$

Compare the equation with $ax^2 + bx + c = 0$, we have, $a = 3k,\ b = -4k,\ c = 4$

As the roots of given equation are equal, so discriminant $D = 0$.

So,

$$b^2 - 4ac = 0$$

$$(-4k)^2 - 4(3k)(4) = 0$$

$$16k^2 - 48k = 0$$

$$16k\,(k - 3) = 0$$

Either $16k = 0$ or $k - 3 = 0$

$k = 0$ $\quad k = 3$

But $k = 0$ is not possible because if $k = 0$, then our equation is not satisfied.

So, $k = 3$

Question 106.

Find the value of ip' for which the roots of the equation px(x – 2) + 6 = 0, are equal.

Solution :

Given that: $px(x - 2) + 6 = 0$

$$\Rightarrow \quad px^2 - 2px + 6 = 0$$

Now Discriminant, $D = b^2 - 4ac = (-2p)^2 - 4 \times p \times 6 = 4p^2 - 24p$

For equal roots, $D = 0 \Rightarrow 4p^2 - 24p = 0 \Rightarrow 4p(p - 6) = 0$

$$\Rightarrow \quad p = 0 \text{ or } p = 6$$

$p = 0$ will be rejected as at $p = 0$ there will be no quadratic equation.

so, $p = 6$

[$\because$ For $p = 0$, quadratic equation becomes zero, so rejected $p = 0$]

Question 107.

Find the value ofp for which the roots of the quadratic equation $(p + 3)\,x^2 + 2(p+3)x + 4 = 0$ are equal.

Solution :

Given: $(p+3)x^2 + 2(p+3)x + 4 = 0$. Then,

Discriminant, $D = b^2 - 4ac$

$$D = [2(p+3)]^2 - 4 \times (p+3) \times 4 = 4(p+3)^2 - 16(p+3)$$

Since roots are equal, so

$$\therefore \quad D = 0$$

$$4(p+3)^2 - 16(p+3) = 0 \Rightarrow (p+3)^2 - 4(p+3) = 0$$

$$\Rightarrow \quad (p+3)[p+3-4) = 0 \Rightarrow (p+3)(p-1) = 0$$

But $p = -3$ or $p = 1$

For $p = -3$, given equation is not quadratic. Therefore, $p = 1$ is the required value.

Question 108.

Find the value of* for which the roots of the quadratic equation $(k-4)x^2 + 2(k-4)x + 2 = 0$ are equal.

Solution :

Given quadratic equation is: $(k-4)x^2 + 2(k-4)x + 2 = 0$

Here, $a = (k-4), b = 2(k-4), c = 2$. So,

$$\text{Discriminant, } D = b^2 - 4ac$$

$$= [2(k-4)]^2 - 4 \times (k-4) \times 2$$

$$= 4(k-4)^2 - 8(k-4)$$

For equal roots, $D = 0$

$$\Rightarrow \quad 4(k-4)^2 - 8(k-4) = 0 \Rightarrow 4(k-4)[(k-4)-2] = 0 \Rightarrow (k-4)(k-6) = 0$$

$$\Rightarrow \quad k = 4 \text{ or } k = 6$$

But $k \neq 4$, because at $k = 4$, quadratic equation becomes zero.

$$\therefore \quad k = 6$$

Question 109.

Find the value of 'a' for which the roots of the quadratic equation $2(a+5)x^2 - (a+5)x + 1 = 0$ are equal.

Solution :

Given quadratic equation is: $2(a+5)x^2 - (a+5)x + 1 = 0$

Discriminant, $D = \{-(a+5)^2\} - 4 \times 2(a+5)$, where $D = b^2 - 4ac$

$$= a^2 + 10a + 25 - 8a - 40$$

$$= a^2 + 2a - 15$$

For equal roots, $D = 0 \Rightarrow a^2 + 2a - 15 = 0 \Rightarrow a^2 + 5a - 3a - 15 = 0$

$$\Rightarrow \quad (a+5)(a-3) = 0 \Rightarrow a = 3, -5$$

$$\therefore \quad a = 3$$

But $a = -5$ is rejected since quadratic equation becomes zero, at $a = -5$

Short Answer Type Questions II [3 Marks]

Question 110.

Solve for x: $4x^2 - 4ax + (a^2 - b^2) = 0$

Solution :

Given equation, $4x^2 - 4ax + a^2 - b^2 = 0$

Compare the equation with $Ax^2 + Bx + C = 0$,

we have $A = 4, B = -4a, C = a^2 - b^2$.

$$\text{Roots are } = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-(-4a) \pm \sqrt{(-4a)^2 - 4 \times 4(a^2 - b^2)}}{2 \times 4}$$

$$= \frac{4a \pm \sqrt{16a^2 - 16a^2 + 16b^2}}{8} = \frac{4a \pm 4b}{8} = \frac{a \pm b}{2}$$

$\therefore$ Values of x are $\frac{a+b}{2}$ and $\frac{a-b}{2}$.

Question 111.

Solve for x: $3x^2 - 2\text{-}\sqrt{6}x + 2 = 0$

Solution :

Given equation, $3x^2 - 2\sqrt{6}x + 2 = 0$

$\Rightarrow$ $3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\Rightarrow$ $\sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$

$\Rightarrow$ $(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$

$\Rightarrow$ $(\sqrt{3}x - \sqrt{2}) = 0$ or $[\sqrt{3}x - \sqrt{2}] = 0$

$\Rightarrow$ $x = \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$ is solution

$$x = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

Question 112.

If the sum of two natural numbers is 8 and their product is 15, find the numbers.

Solution :

Let 1st number $= x$

Sum of numbers $= 8$

2nd number $= 8 - x$

Product of numbers $= x(8 - x)$. As per condition,

$15 = x(8 - x)$

$15 = 8x - x^2$

$\Rightarrow$ $x^2 - 8x + 15 = 0 \Rightarrow x^2 - 5x - 3x + 15 = 0$

$\Rightarrow$ $(x - 5)(x - 3) \Rightarrow x = 5, 3$

Two numbers are 5 and 3.

Question 113.

Solve for x: $x^2 - 4ax - b + 4a^2 = 0$.

Solution :

Given: $x^2 - 4ax - b^2 + 4a^2 = 0$

Discriminant, $D = (-4a)^2 - 4 \times 1 \times (4a^2 - b^2) = 16a^2 - 16a^2 + 4b^2 = 4b^2$ $[\because D = b^2 - 4ac]$

Now, Solution is: $x = \dfrac{4a \pm \sqrt{4b^2}}{2}$, where $x = \dfrac{-b \pm \sqrt{D}}{2a}$

$$x = \frac{4a \pm 2b}{2}$$

$$x = 2a \pm b$$

$\therefore$ Roots are $(2a + b), (2a - b)$.

Question 114.

Solve for x: $x^2 - 5\sqrt{5}x + 30 = 0$.

Solution :

Given equation: $x^2 - 5\sqrt{5}\cdot x + 30 = 0 \Rightarrow x^2 - 2\sqrt{5}x - 3\sqrt{5}x + 30 = 0$

$\Rightarrow$ $x(x - 2\sqrt{5}) - 3\sqrt{5}(x - 2\sqrt{5}) = 0$

$\Rightarrow$ $(x - 3\sqrt{5})(x - 2\sqrt{5}) = 0 \Rightarrow$ either $x - 3\sqrt{5} = 0$ or $x - 2\sqrt{5} = 0$

$\Rightarrow$ $x = 3\sqrt{5}$ or $x = 2\sqrt{5}$

Question 115.

Solve for x: $x^2 + \sqrt{5}x - 60 = 0$

Solution :

Given equation: $x^2 + \sqrt{5}x - 60 = 0$

Discriminant, $D = b^2 - 4ac = (\sqrt{5})^2 - 4 \times 1 \times (-60) = 5 + 240 = 245$

Now, Solution is $x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{-\sqrt{5} \pm \sqrt{245}}{2 \times 1} = \dfrac{\sqrt{5} \pm 7\sqrt{5}}{2}$

$\therefore$ $x = \dfrac{-\sqrt{5} + 7\sqrt{5}}{2}$ or $\dfrac{-\sqrt{5} - 7\sqrt{5}}{2}$

$x = 3\sqrt{5}$ or $-4\sqrt{5}$

Question 116.

Solve for x: $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Solution :

Given equation, $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Discriminant, $D = b^2 - 4ac = 5^2 - 4 \times 4\sqrt{3} \times (-2\sqrt{3}) = 25 + 96 = 121$

Now, Solution is: $x = \frac{-b \pm \sqrt{D}}{2a}$

$\Rightarrow$ $x = \frac{-5 \pm \sqrt{121}}{2 \times 4\sqrt{3}} = \frac{-5 \pm 11}{8\sqrt{3}}$

$\Rightarrow$ $x = \frac{-5+11}{8\sqrt{3}}$ or $\frac{-5-11}{8\sqrt{3}}$

$\Rightarrow$ $x = \frac{6}{8\sqrt{3}}$ or $\frac{-16}{8\sqrt{3}}$

$\Rightarrow$ $x = \frac{\sqrt{3}}{4}$ or $\frac{-2\sqrt{3}}{3}$

long Answer Type Questions [4 Marks]

Question 117.

A shopkeeper buys some books for rs 80. If he had bought 4 more books for the same amount, each book would have cost rs 1 less. Find the number of books he bought.

Solution :

Let number of books $= x$

Cost of each book $= y$

Total cost $= ₹\ 80$

$\therefore \quad x \times y = 80 \quad ...(i)$

New number of books $= x + 4$

Cost of each book $= ₹\ (y - 1)$

Total cost $= ₹\ 80$

$\therefore \quad (x + 4)(y - 1) = 80$

$xy - x + 4y - 4 = 80$

$80 - \frac{80}{y} + 4y - 4 = 80$ [From (i)]

$-\frac{80}{y} + 4y - 4 = 0$

$-80 + 4y^2 - 4y = 0$

$y^2 - y - 20 = 0$

$y^2 - 5y + 4y - 20 = 0$

$y(y - 5) + 4(y - 5) = 0$

$(y + 4)(y - 5) = 0$

$y = -4, \quad y = 5$

But, $y = -4$ is rejected because cost of books cannot be negative.

So, $y = 5$

$x = \frac{80}{5} = 16 \qquad \therefore x = 16$ [From (i)]

Hence, numer of books he bought $= 16$

Question 118.

The sum of two numbers is 9 and the sum of their reciprocals is 1/2. Find the numbers.

Solution :

Let first number $= x$

So, second number $= 9 - x$

According to question,

$\frac{1}{x} + \frac{1}{9-x} = \frac{1}{2}$

$\frac{9-x+x}{9x-x^2} = \frac{1}{2}$

$18 = 9x - x^2$

$x^2 - 9x + 18 = 0$

$x^2 - 6x - 3x + 18 = 0$

$x(x - 6) - 3(x - 6) = 0$

$(x - 3)(x - 6) = 0$

$x = 3 \quad \text{and} \quad x = 6.$

Hence, the first number $= 3$ and the second number $= 9 - 3 = 6$.

Question 119.
In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and time increased by 30 minutes. Find the original duration of the flight.
Solution :
Let original time taken by the flight to reach destination be x hours.

$$\text{Distance} = 2800 \text{ km}$$

$$\therefore \quad \text{Usual speed} = \frac{2800}{x} \text{ km/h}$$

When time is increased, then time taken $= \left(x + \frac{1}{2}\right)$ hrs

And $$\text{new speed} = \frac{2800}{x + \frac{1}{2}} = \frac{5600}{2x+1} \text{ km/h}$$

A.T.Q.,

$$\frac{2800}{x} - \frac{5600}{2x+1} = 100 \Rightarrow \frac{2800(2x+1) - 5600x}{(2x+1)x} = 100$$

$$\Rightarrow \quad 2800 = 100\,(2x^2 + x)$$

$$\Rightarrow \quad 2x^2 + x - 28 = 0 \Rightarrow 2x^2 + 8x - 7x - 28 = 0$$

$$\Rightarrow \quad 2x(x+4) - 7(x+4) = 0 \Rightarrow (x+4)\,(2x-7) = 0$$

$$\Rightarrow \quad \therefore x = -4 \text{ or } x = \frac{7}{2} = 3\frac{1}{2}$$

$\therefore$ Original duration of the flight $= 3\frac{1}{2}$ hours, since time cannot be negative.

Question 120.
The numerator of a fraction is 3 less than its denominator. If 1 is added to the denominator, the fraction is decreased by 1/15. Find the fraction.
Solution :
Let denominator be x and numerator be $x - 3$.

$\therefore$ Fraction is $\frac{x-3}{x} = \frac{\text{numerator}}{\text{denominator}}$.

A.T.Q.,

$$\frac{x-3}{x+1} + \frac{1}{15} = \frac{x-3}{x} \Rightarrow \frac{x-3}{x} - \frac{x-3}{x+1} = \frac{1}{15}$$

$$\Rightarrow \quad (x-3)\left[\frac{x+1-x}{x(x+1)}\right] = \frac{1}{15} \Rightarrow 15x - 45 = x^2 + x$$

$$\Rightarrow \quad x^2 - 14x + 45 = 0 \Rightarrow (x-9)\,(x-5) = 0$$

$$\therefore \quad x = 5, 9$$

$x = 9$ will be rejected as it does not satisfy the conditions. So, required fraction is $\frac{2}{5}$.

Question 121.
A two-digit number is such that the product of its digits is 14. When 45 is added to

the number, the digits interchange their places. Find the number.

Solution :

Let the units place digit is x

As we know, product of digits is 14, so the tens place digit will be $\frac{14}{x}$

The number is $\left(10\times\frac{14}{x}+x\right)$

According to question,

$$\frac{140}{x}+x+45 = 10x+\frac{14}{x}$$

$$\frac{140+x^2+45x}{x} = \frac{10x^2+14}{x}$$

$$140+x^2+45x = 10x^2+14$$

$$10x^2+14-140-x^2-45x = 0$$

$$10x^2-x^2-45x+14-140 = 0$$

$$9x^2-45x-126 = 0$$

$$9(x^2-5x-14) = 0$$

$$x^2-5x-14 = 0$$

$$x^2-7x+2x-14 = 0$$

$$x(x-7)+2(x-7) = 0$$

$$(x-7)(x+2) = 0$$

$$x-7 = 0 \Rightarrow x = 7$$

$$x+2 = 0 \Rightarrow x = -2$$

$x = -2$ is not possible

$\therefore$ The units place digit is 7 and tens place digit is 20.

Hence, required number is 27.

Question 122.

Find two consecutive natural numbers, the sum of whose squares is 145

Solution :

Let the smaller of the two consecutive natural number be x.

Then, second natural number will be $x + 1$.

According to question,

$$\begin{aligned} x^2 + (x+1)^2 &= 145 \\ x^2 + x^2 + 2x + 1 &= 145 \\ 2x^2 + 2x + 1 &= 145 \\ 2x^2 + 2x + 1 - 145 &= 0 \\ 2x^2 + 2x - 144 &= 0 \\ 2(x^2 + x - 72) &= 0 \\ x^2 + x - 72 &= 0 \\ x^2 - 8x + 9x - 72 &= 0 \\ x(x-8) + 9(x-8) &= 0 \\ (x-8)(x+9) &= 0 \\ x - 8 = 0 &\Rightarrow x = 8 \\ x + 9 = 0 &\Rightarrow x = -9 \end{aligned}$$

But x is given to be a natural number.

Therefore, $x \neq -9, x = 8$

Thus, the two consecutive natural numbers are 8 and 9.

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Short Answer Type Questions I [2 Marks]

Question 123.

Find the value of p so that the quadratic equation px(x – 3) + 9 = 0 has two equal roots.

Solution :

Given quadratic equation, $px(x-3) + 9 = 0 \Rightarrow px^2 - 3px + 9 = 0$

Here, $a = p, b = -3p, c = 9$

When roots are equal, $D = 0$, where D is discriminant.

$$D = b^2 - 4ac$$

$$\Rightarrow (-3p)^2 - 4 \times p \times 9 = 0$$

$$9p^2 - 36p = 0 \Rightarrow 9p(p-4) = 0$$

$$\Rightarrow 9p = 0, p - 4 = 0 \Rightarrow p = 0, p = 4$$

But $p \neq 0$ [$\because$ In quadratic equation, $a \neq 0$]

$\therefore p = 4$

Question 124.

Find the value of & so that the quadratic equation kx(3x -10) + 25 = 0, has two equal roots.

Solution :

Given quadratic equation, $kx(3x - 10) + 25 = 0 \Rightarrow 3kx^2 - 10kx + 25 = 0$

$\Rightarrow$ Discriminant $D = (-10k)^2 - 4 \times 3k \times 25 = 100k^2 - 300k$, where $D = b^2 - 4ac$

For equal roots,

$D = 0$

$\Rightarrow \quad 100k^2 - 300k = 0 \Rightarrow 100k(k - 3) = 0$

$\Rightarrow \quad k = 0$ or $k = 3 \Rightarrow k = 0$ (Rejected) [$\because$ In quadratic equation, $a \neq 0$]

Hence, $k = 3$.

Question 125.

Find the value of m so that the quadratic equation mx(5x – 6) + 9 = 0 has two equal roots.

Solution :

Given quadratic equation, $mx(5x - 6) + 9 = 0 \Rightarrow 5mx^2 - 6mx + 9 = 0$

$\Rightarrow$ Discriminant, $D = (-6m)^2 - 4 \times 5m \times 9 = 36m^2 - 180m$ [$\therefore D = b^2 - 4ac$]

For equal roots, $D = 0$

$\Rightarrow \quad 36m^2 - 180m = 0 \Rightarrow 36m(m - 5) = 0$

$\Rightarrow \quad m = 5$ or $m = 0$ (Rejected) [$\because$ In quadratic equation, $a \neq 0$]

Hence, $m = 5$

Question 126.

Find the value of m so that the quadratic equation mx(x – 7) + 49 = 0 has two equal roots.

Solution :

Simplifying given quadratic equation,

$mx(x - 7) + 49 = 0$, We get

$\Rightarrow \quad mx^2 - 7mx + 49 = 0$

In above equation, $a = m, b = -7m, c = 49$

For equal roots, $D = 0$, where D is discriminant

$D = b^2 - 4ac$

$0 = (-7m)^2 - 4 \times m \times 49$

$0 = 49m^2 - 196m$

$\Rightarrow \quad 49m^2 - 196m = 0 \Rightarrow 7m(7m - 28) = 0$

$\Rightarrow \quad 7m = 0$ or $7m - 28 = 0 \Rightarrow m = 0$ or $7m = 28$

$\Rightarrow \quad m = 0$ or $m = \frac{28}{7} = 4$

But $m \neq 0$ [$\because$ In quadratic equation, $a \neq 0$]

$\therefore \quad m = 4$

Question 127.

For what value of k does the quadratic equation $(k - 5)x^2 + 2(k - 5)x + 2 = 0$ have equal roots?

Solution :

Given quadratic equation, $(k-5)x^2 + 2(k-5)x + 2 = 0$

Here, $a = k-5, b = 2(k-5), c = 2$

When roots are equal, $D = 0 \Rightarrow b^2 - 4ac = 0$, where D is discriminant

$[2(k-5)]^2 - 4 \times (k-5) \times 2 = 0 \Rightarrow 4k^2 + 100 - 40k - 8k + 40 = 0$

$\Rightarrow 4k^2 - 48k + 140 = 0 \Rightarrow k^2 - 12k + 35 = 0$

$\Rightarrow k^2 - 7k - 5k + 35 = 0 \Rightarrow k(k-7) - 5(k-7) = 0$

$\Rightarrow (k-5)(k-7) = 0 \Rightarrow k = 5, 7$

But $k \neq 5$ [$\because$ In quadratic equation, $a \neq 0$]

$\therefore k = 7$

Short Answer Type Questions II [3 Marks]

Question 128.

Find the roots of the following quadratic equation. $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

Solution :

Given quadratic equation, $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$

In the above equation, $a = 2\sqrt{3}, b = -5, c = \sqrt{3}$. Then,

discriminant: $D = b^2 - 4ac$

$= (-5)^2 - 4 \times 2\sqrt{3} \times \sqrt{3}$

$= 25 - 24 = 1$

$\therefore$ Solution is: $x = \dfrac{-b \pm \sqrt{D}}{2a}$

$\Rightarrow x = \dfrac{5+1}{2 \times 2\sqrt{3}} = \dfrac{6}{4\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{6\sqrt{3}}{4 \times 3} = \dfrac{\sqrt{3}}{2}$

or $x = \dfrac{5-1}{4\sqrt{3}} = \dfrac{4}{4\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\therefore$ Roots are $\dfrac{\sqrt{3}}{2}, \dfrac{1}{\sqrt{3}}$.

Question 129.

Find the roots of the following quadratic equation: $x^2 - 3\sqrt{5}x + 10 = 0$

Solution :

Given quadratic equation, $x^2 - 3\sqrt{5}x + 10 = 0$

Discriminant, $D = b^2 - 4ac$

$= (-3\sqrt{5})^2 - 4 \times 1 \times 10$

$= 45 - 40 = 5$

$\therefore$ Solution is: $x = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{3\sqrt{5} \pm \sqrt{5}}{2 \times 1} \Rightarrow x = \frac{3\sqrt{5} + \sqrt{5}}{2}$ or $\frac{3\sqrt{5} - \sqrt{5}}{2}$

$\Rightarrow x = \frac{4\sqrt{5}}{2}$ or $\frac{2\sqrt{5}}{2} \Rightarrow x = 2\sqrt{5}$ or $x = \sqrt{5}$.

$\therefore$ Roots are $2\sqrt{5}, \sqrt{5}$

Question 130.

Find the roots of the following quadratic equation:$\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

Solution :

Given quadratic equation, $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$. Then, discriminant

$D = b^2 - 4ac$

$= (-2\sqrt{2})^2 - 4 \times \sqrt{3} \times (-2\sqrt{3}) = 8 + 24 = 32$

Now, Solution is: $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{2\sqrt{2} \pm \sqrt{32}}{2\sqrt{3}}$

$\Rightarrow x = \frac{2\sqrt{2} \pm 4\sqrt{2}}{2\sqrt{3}}$

$\Rightarrow x = \frac{2\sqrt{2} + 4\sqrt{2}}{2\sqrt{3}}$ or $x = \frac{2\sqrt{2} - 4\sqrt{2}}{2\sqrt{3}}$

$\Rightarrow x = \frac{6\sqrt{2}}{2\sqrt{3}}$ or $x = \frac{-2\sqrt{2}}{2\sqrt{3}}$

$\Rightarrow x = \frac{3\sqrt{2}}{\sqrt{3}}$ or $x = \frac{-\sqrt{2}}{\sqrt{3}}$

$\therefore$ Roots are $\frac{3\sqrt{2}}{\sqrt{3}}, \frac{-\sqrt{2}}{\sqrt{3}}$.

Question 131.

Find the roots of the following quadratic equation: $3x^2 + 2\sqrt{5}x - 5 = 0$

Solution :

Given quadratic equation, $3x^2 + 2\sqrt{5}x - 5 = 0$. Then, discriminant

$$D = b^2 - 4ac$$
$$= (2\sqrt{5})^2 - 4 \times 3 \times (-5)$$
$$= 20 + 60 = 80$$

Now, solution is:

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-2\sqrt{5} \pm \sqrt{80}}{2 \times 3} = \frac{-2\sqrt{5} \pm 4\sqrt{5}}{6}$$

$$x = \frac{-2\sqrt{5} + 4\sqrt{5}}{6} \text{ or } x = \frac{-2\sqrt{5} - 4\sqrt{5}}{6}$$

$$x = \frac{2\sqrt{5}}{6} \text{ or } x = \frac{-6\sqrt{5}}{6}$$

$\therefore$

$$x = \frac{\sqrt{5}}{3} \text{ or } x = -\sqrt{5}.$$

$\therefore$ Roots are $\frac{\sqrt{5}}{3}, -\sqrt{5}$.

Long Answer Type Questions [4 Marks]

Question 132.

A motor boat whose speed is 20 km/h in still water, takes 1 hour more to go 48 km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution :

Let the speed of the stream = km/h

Speed of the boat in still water = 20 km/h

Now, speed of boat during downstream = (20 + x) km/h

Distance covered during downstream = 48km.

$$\text{Time taken} = \left(\frac{48}{20+x}\right) \text{hours.}$$

Speed of boat during upstream = $(20-x)$ km/h

Distance covered during upstream = 48 km.

$$\text{Time taken} = \left(\frac{48}{20-x}\right) \text{hours.}$$

According to question, $\frac{48}{20-x} = 1 + \frac{48}{20+x}$

$$\frac{48}{20-x} - \frac{48}{20+x} = 1$$

$$\Rightarrow \quad \frac{960+48x-960+48x}{400-x^2} = 1$$

$$\Rightarrow \quad 96x = 400-x^2$$

$$\Rightarrow \quad x^2 + 96x - 400 = 0. \text{ As we know, } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-96 \pm \sqrt{(96)^2+4\times1\times400}}{2\times1} \Rightarrow x = \frac{-96 \pm \sqrt{10816}}{2}$$

$$\Rightarrow \quad x = \frac{-96 \pm 104}{2}$$

$$\Rightarrow \quad x = \frac{-96+104}{2} \text{ or } x = \frac{-96-104}{2}$$

$$\Rightarrow \quad x = \frac{8}{2} = 4 \text{ or } x = \frac{-200}{2} = -100 \text{ (Not possible)}$$

Thus, speed of stream = 4km/h

Question 133.

Find the roots of the equation:

$$\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, x \neq -4, 7$$

Solution :

Given that: $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}; x \neq -4, 7 \Rightarrow \frac{(x-7)-(x+4)}{(x+4)(x-7)}=\frac{11}{30}$

$\Rightarrow \frac{x-7-x-4}{x^2-3x-28}=\frac{11}{30} \Rightarrow \frac{-11}{x^2-3x-28}=\frac{11}{30}$

$\Rightarrow \frac{-1}{x^2-3x-28}=\frac{1}{30} \Rightarrow x^2-3x-28=-30$

$\Rightarrow x^2-3x+2=0 \Rightarrow (x-1)(x-2)=0$

$\Rightarrow x=1, 2$

Hence, roots of given quadratic equation are 1 and 2.

Question 134.

Find the roots of the equation:

$$\frac{1}{2x-3}+\frac{1}{x-5}=1, x \neq \frac{3}{2}, 5$$

Solution :

Given that: $\frac{1}{2x-3}+\frac{1}{x-5}=1; x \neq \frac{3}{2}, 5$

$\Rightarrow \frac{x-5+2x-3}{(x-5)(2x-3)}=1$

$\Rightarrow 3x-8=2x^2-13x+15$

$\Rightarrow 2x^2-16x+23=0$

$\Rightarrow$ Solution is: $x=\frac{16 \pm \sqrt{(-16)^2-4\times2\times23}}{2\times2}$ $\left[\because x=\frac{-b \pm \sqrt{b^2-4ac}}{2a}\right]$

$\Rightarrow x=\frac{16 \pm \sqrt{256-184}}{4} \Rightarrow x=\frac{16 \pm \sqrt{72}}{4}$

$\Rightarrow x=\frac{16 \pm 6\sqrt{2}}{4} \Rightarrow x=\frac{8 \pm 3\sqrt{2}}{2}$

$\Rightarrow x=\frac{8+3\sqrt{2}}{2}$ and $x=\frac{8-3\sqrt{2}}{2}$

Hence, $\frac{8+3\sqrt{2}}{2}$ and $\frac{8-3\sqrt{2}}{2}$ are the roots of given equation.

Question 135.

A train travels 180 km at a uniform speed. If the speed had been 9 km/hour more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Solution :

Let the speed of the train $= x$ km/h

Distance travelled = 180 km

Time taken $= \frac{180}{x}$ hours

When speed of the train is increased, then new speed of the train $= (x + 9)$km/h

Distance travelled = 180 km

Time taken $= \frac{180}{x+9}$ hours

According to question, $\frac{180}{x+9} = \frac{180}{x} - 1$

$$\frac{180}{x} - \frac{180}{x+9} = 1$$

$$\Rightarrow \quad \frac{180x + 1620 - 180x}{x(x+9)} = 1$$

$$\Rightarrow \quad x(x + 9) = 1620$$

$$\Rightarrow \quad x^2 + 9x - 1620 = 0 \Rightarrow x^2 + 45x - 36x - 1620 = 0$$

$$\Rightarrow \quad (x - 36)(x + 45) = 0$$

$$\Rightarrow \quad x = 36 \text{ or } x = -45 \text{ (Rejected)}$$

Hence, speed of the train = 36 km/h.

Question 136.

Two water taps together can fill a tank in 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution :

Let x hours be the time taken by smaller one water tap. $\therefore$ $(x - 9)$ hours be the time taken by larger one water tap.

According to question,

$$\frac{1}{x} + \frac{1}{x-9} = \frac{1}{6} \Rightarrow \frac{x-9+x}{x(x-9)} = \frac{1}{6}$$

$$\Rightarrow \quad 6(2x - 9) = x(x - 9)$$

$$\Rightarrow \quad 12x - 54 = x^2 - 9x$$

$$\Rightarrow \quad x^2 - 21x + 54 = 0$$

$$\Rightarrow \quad x^2 - 18x - 3x + 54 = 0 \Rightarrow x(x - 18) - 3(x - 18) = 0$$

$$\Rightarrow \quad (x - 3)(x - 18) = 0$$

$$\Rightarrow \quad x = 3 \text{ or } x = 18$$

$\therefore$ Tap of smaller diameter takes 18 hours and that of larger diameter takes $(18 - 9) = 9$ hrs.

Question 137.

Solve the following equation for x:

$$\frac{1}{x+1}+\frac{2}{x+2}=\frac{5}{x+4}, x \neq -1, -2, -4$$

Solution :

Given that: $\frac{1}{x+1}+\frac{2}{x+2}=\frac{5}{x+4}; x \neq -1, -2, -4 \Rightarrow \frac{x+2+2(x+1)}{(x+1)(x+2)}=\frac{5}{x+4}$

$\Rightarrow \frac{x+2+2x+2}{x^2+3x+2}=\frac{5}{x+4} \Rightarrow (3x+4)(x+4)=5(x^2+3x+2)$

$\Rightarrow 3x^2+12x+4x+16=5x^2+15x+10$

$\Rightarrow 3x^2+16x+16=5x^2+15x+10$

$\Rightarrow 2x^2-x-6=0 \Rightarrow 2x^2-4x+3x-6=0$

$\Rightarrow 2x(x-2)+3(x-2)=0 \Rightarrow (2x+3)(x-2)=0$

$\Rightarrow x=2 \text{ or } x=\frac{-3}{2}$

$\therefore$ Roots are $2, \frac{-3}{2}$.

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Very Short Answer Type Questions [1 Mark].

Question 138.

. If α, β are the zeroes of the polynomial $2y^2 + 7y + 5$, write the value of $\alpha + \beta + \alpha\beta$.

Solution :

Given polynomial is $2y^2 + 7y + 5$, α & β are the zeroes of polynomial.

$$\text{Sum of zeroes} = \alpha+\beta=\frac{-7}{2}=\frac{-b}{a}$$

$$\text{Product of zeroes} = \alpha\beta=\frac{5}{2}=\frac{c}{a}$$

$$\therefore \quad \alpha+\beta+\alpha\beta=\frac{-7}{2}+\frac{5}{2}=-1$$

Question 139.

If one zero of the polynomial $x^2 - 4x + 1$ is $2 + \sqrt{3}$, write the other zero.

Solution :

Given polynomial is $x^2 - 4x - 1$. Given zero is $2 + \sqrt{3}$. We know that irrational roots always occur in pair which are conjugate of each other. So, other zero is $2-\sqrt{3}$.

Short Answer Type Questions I [2 Marks]

Question 140.

For what value of k, is -2 a zero of the polynomial $3x^2 + 4x + 2k$?

Solution :

Let $p(x) = 3x^2 + 4x + 2k$

Given: -2 is a zero of $p(x)$

$$p(-2) = 0$$

$$3(-2)^2 + 4(-2) + 2k = 0 \Rightarrow 3 \times 4 - 8 + 2k = 0$$

$$\Rightarrow \quad 12 - 8 + 2k = 0 \Rightarrow 2k = -4 \Rightarrow k = -2$$

Hence, for $k = -2, -2$ is a zero of $p(x)$.

Question 141.

For what value of k, is 3 a zero of the polynomial $2x^2 + x + k$?

Solution :

Let $p(x) = 2x^2 + x + k$

Given: 3 is a zero of $p(x)$.

$$\therefore \quad p(3) = 0$$

$$2(3)^2 + 3 + k = 0 \Rightarrow 2 \times 9 + 3 + k = 0$$

$$\Rightarrow \quad 18 + 3 + k = 0 \Rightarrow 21 + k = 0 \Rightarrow k = -21$$

Hence, for $k = -21$, 3 is a zero of $p(x)$

Question 142.

For what value of It, is 3 a zero of the polynomial $x^2 + 11x + k$?

Solution :

Let $p(x) = x^2 + 11x + k$

Given: -3 is a zero of $p(x)$.

$$\therefore \quad p(-3) = 0$$

$$(-3)^2 + 11(-3) + k = 0 \Rightarrow 9 - 33 + k = 0 \Rightarrow -24 + k = 0 \Rightarrow k = 24$$

$\therefore$ Hence for $k = 24$, 3 is zero of $p(x)$.

Long Answer Type Questions [4 Marks]

Question 143.

The difference of squares of two numbers is 88. If the larger number is 5 less than twice the smaller number, then find the two numbers.

Solution :

Let smaller number be x and other larger number be $2x - 5$.

A.T.Q. $(2x-5)^2 - x^2 = 88 \Rightarrow 4x^2 - 20x + 25 - x^2 = 88$

$\Rightarrow \quad 3x^2 - 20x - 63 = 0$

$\therefore \quad$ Discriminant, $D = (-20)^2 - 4 \times 3 \times (-63) = 400 + 756 = 1156 \quad [\because D = b^2 - 4ac]$

Now, solution is: $x = \dfrac{20 \pm \sqrt{1156}}{2 \times 3} \quad \left[\because x = \dfrac{-b \pm \sqrt{D}}{2a}\right]$

$\Rightarrow \quad x = \dfrac{20 \pm 34}{6}; \Rightarrow x = \dfrac{20+34}{6}$ or $x = \dfrac{20-34}{6}$

$\Rightarrow \quad x = \dfrac{54}{6};$ or $x = \dfrac{-14}{6}$

$\Rightarrow \quad x = 9;$ or $x = \dfrac{-7}{3}$ (Rejected)

$\therefore$ The required numbers are 9 and 13.

Question 144.

Three consecutive positive integers are such that the sum of the square of the first and the product of the other two is 46. Find the integers.

Solution :

Let the three consecutive positive integers be $n, n + 1, n + 2$.

According to question,

$n^2 + (n+1)(n+2) = 46 \Rightarrow 2n^2 + 3n - 44 = 0$

$\Rightarrow \quad 2n^2 + 11n - 8n - 44 = 0 \Rightarrow n(2n + 11) - 4(2n + 11) = 0$

$\Rightarrow \quad (n-4)(2n+11) = 0 \Rightarrow n - 4 = 0$ or $2n + 11 = 0$

$\Rightarrow \quad n = 4$ or $n = -\dfrac{11}{2}$ (is not possible)

$\therefore$ Numbers are 4, 5, 6.

Question 145.

A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

Solution :

Let sister's present age be x years.

And girl's present age be $2x$ years.

Given: relation between the ages of both four years hence

$(2x+4)(x+4) = 160 \Rightarrow 2x^2 + 12x + 16 = 160$

$\Rightarrow \quad 2x^2 + 12x - 144 = 0 \Rightarrow x^2 + 6x - 72 = 0 \Rightarrow x^2 + 12x - 6x - 72 = 0$

$\Rightarrow \quad (x+12)(x-6) = 0 \Rightarrow x + 12 = 0$ or $x - 6 = 0$

$\Rightarrow \quad x = -12$ (rejected) or $x = 6$

$\therefore \quad$ Sister's present age = 6 years, Girl's present age = 12 years.

Question 146.

Solve the following equation for x.

$$\frac{3x-4}{7} + \frac{7}{3x-4} = \frac{5}{2}, x \neq \frac{4}{3}$$

Solution :

Given that: $\frac{3x-4}{7}+\frac{7}{3x-4} = \frac{5}{2}; x \neq \frac{4}{3}$

Let us consider: $\frac{3x-4}{7} = y$

$\Rightarrow$ The given equation becomes $y+\frac{1}{y} = \frac{5}{2}$

$\Rightarrow$ $2y^2 - 5y + 2 = 0 \Rightarrow 2y^2 - 4y - y + 2 = 0$

$\Rightarrow$ $2y(y-2) - 1(y-2) = 0 \Rightarrow (2y-1)(y-2) = 0$

$\Rightarrow$ $2y - 1 = 0$ or $y - 2 = 0 \Rightarrow y = \frac{1}{2}$ or 2

$\Rightarrow$ from (i); $\frac{3x-4}{7} = \frac{1}{2}$ = or $\frac{3x-4}{7} = 2$

$\Rightarrow$ $6x - 8 = 7$ or $3x - 4 = 14 \Rightarrow 6x = 15$ or $3x = 18 \Rightarrow x = \frac{5}{2}$ or $x = 6$

$\therefore$ Solution is $\frac{5}{2}$ or 6.

Question 147.

Some students planned a picnic. The total budget for food was rs 2,000. But 5 students failed to attend the picnic and thus the cost of food for each member increased by rs 20. How many students attended the picnic and how much did each student pay for the food.

Solution :

Case (*i*) Let no. of students = x

Cost of food for each member = ₹ y

Total cost = ₹ 2000

$$x \times y = 2{,}000 \quad \ldots(1)$$

Case (*ii*) New no. of student = $x - 5$

New cost of food for each member = ₹ $(y + 20)$

Total cost = ₹ 2,000

$$(x - 5)(y + 20) = 2{,}000$$

$$xy + 20x - 5y - 100 = 2{,}000 \quad \ldots(2)$$

Solving (1) and (2), we get

$$2000 + 20x - 5y - 100 = 2{,}000$$

$$20x - 5y = 100$$

$$4x - y = 20$$

$$4x - \frac{2000}{x} = 20 \quad \text{[From (}i\text{)]}$$

$$x - \frac{500}{x} = 5$$

$$\Rightarrow \quad x^2 - 5x - 500 = 0 \quad \Rightarrow \quad x^2 - 25x + 20x - 500 = 0$$

$$\Rightarrow \quad x(x - 25) + 20(x - 25) = 0 \quad \Rightarrow \quad (x + 20)(x - 25) = 0$$

$$\Rightarrow \quad x + 20 = 0 \quad \text{or} \quad x - 25 = 0$$

$$\therefore \quad x = -20, 25$$

$x = -20$ is rejected because no. of students can't be negative.

So, $x = 25$

$$y = \frac{2000}{x} = \frac{2000}{25} = 80$$

$$\therefore \quad y = 80$$

No. of students = 25

Cost of food for each student = ₹ 80.