Chapter 8: Trigonometry and its application
Exercise: 8.1

Question 1:
If cos A =$\frac{4}{5}$, then the value of tan A is
(a) $\frac{3}{5}$
(b) $\frac{3}{4}$
(c) $\frac{4}{3}$
(d) $\frac{5}{3}$
Solution: (b)
Given, cos A = 4/5
Therefore, sin A = $\sqrt{1-cos^2A}$

$$= \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

Now, tan A = $\frac{\sin A}{\cos A} = \frac{5}{\frac{4}{5}} = \frac{3}{4}$

Hence, required value of tan A is $\frac{3}{4}$

Question 2:
If sin A =$\frac{1}{2}$, then the value of cot A is
(a) $\sqrt{3}$
(b) $\frac{1}{\sqrt{3}}$
(c) $\frac{\sqrt{3}}{1}$
(d) 1
Solution: (a) Given, sin A = $\frac{1}{2}$

Hence, cos A = $\sqrt{1-sin^2A}$

$$= \sqrt{1-\left(\frac{1}{2}\right)^2} = \sqrt{1-\frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

Now, cot A = $\frac{\cos A}{\sin A} = \frac{2}{\frac{1}{2}} = \sqrt{3}$

Question 3:
The value of the expression cosec (75° + 0) – sec (15° – 0) – tan (55° + 0) + cot (35° – 0) is
(a) -1 (b) 0 (c)1 (d) $\frac{3}{2}$
Solution:
(b) Given, expression = cosec (75° + 0) – sec (15° – 0) – tan (55° + 0) + cot (35° – 0)
=cosec [90° – (15° – 0)] – sec (15° – 0)- tan (55° + 0) + cot (90° – (55° + 0)}
= sec (15° – 0) – sec (15° – 0) – tan (55° + 0) + tan (55° + 0)
[∴ cosec (90° – 0) = sec 0 and cot (90° – 0) = tan 0]
= 0
Hence, the required value of the given expression is 0.

Question 4:
If sinθ = $\frac{3}{5}$, then cosθ is equal to

(a) $\frac{b}{\sqrt{b^2-a^2}}$ (b) $\frac{b}{a}$ (c) $\frac{\sqrt{b^2-a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2-a^2}}$

Solution: (c) Given, sin$\theta = \frac{a}{b}$
hence, cos $\theta = \sqrt{1 - sin^2\theta}$

$$= \sqrt{1 - \left(\frac{a}{b}\right)^2} = \sqrt{1 - \frac{a^2}{b^2}} = \frac{\sqrt{b^2-a^2}}{b}$$

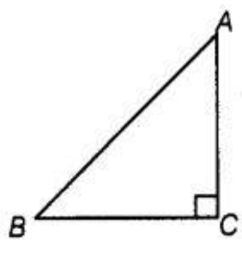
Question 5:
If cos(α+ β) = 0, then sin (α – β) can be reduced to
(a) cos β (b) cos 2β (c) sin α (d) sin 2α
Solution:
(b) Given, $\cos(\alpha + \beta) = 0 = \cos 90°$ [∵cos 90° = 0]
$\Rightarrow \alpha + \beta = 90°$
$\Rightarrow \alpha = 90° - \beta$ ...(i)
Now, $\sin(\alpha - \beta) = \sin(90° - \beta - \beta)$ [put the value from Eq. (i)]
$= \sin(90° - 2\beta)$
$= \cos 2\beta$ [∵ sin (90° – θ) = cos θ]
Hence, sin (α – β) can be reduced to cos2β.

Question 6:
The value of (tanl° tan2° tan3°… tan89°) is
(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

Solution:
(b) tan1°-tan2°-tan3°… tan 89°
= tan1°-tan2°-tan3°… tan44° . tan 45° . tan 46°… tan 87°-tan 88°tan 89°
= tan 1°- tan2 °- tan 3°… tan 44° . (1)- tan (90° – 44°)… tan (90° – 3°)
tan (90° -2°)- tan (90° -1°) (∴ tan 45° = 1)
= tan1°-tan2°-tan3°…. tan44° (1) . cot 44°……. cot3°-cot2°-cot1°
= 1

Question 7:
If cos 9α = sin α and 9α < 90° ,then the value of tan 5α is
(a) $\frac{1}{\sqrt{3}}$
(b) $\frac{\sqrt{3}}{1}$
(c) 1
(d) 0
Solution:
(c) Given, $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90°$ *i.e.*, acute angle.
$\sin(90° - 9\alpha) = \sin\alpha$ $\quad [\because \cos A = \sin(90° - A)]$
$\Rightarrow \quad 90° - 9\alpha = \alpha$
$\Rightarrow \quad 10\alpha = 90°$
$\Rightarrow \quad \alpha = 9°$
$\therefore \quad \tan 5\alpha = \tan(5 \times 9°) = \tan 45° = 1$ $\quad [\because \tan 45° = 1]$

Question 8:
If ΔABC is right angled at C, then the value of cos (A + B) is
(a) 0
(b) 1
(c) $\frac{1}{2}$
(d) $\frac{\sqrt{3}}{2}$
Solution:
(a) We know that, in ΔABC, sum of three angles = 180°
i.e., $\angle A + \angle B + \angle C = 180°$
But right angled at C *i.e.*, $\angle C = 90°$ $\quad$ [given]
$\angle A + \angle B + 90° = 180°$
$\Rightarrow \quad A + B = 90°$ $\quad [\because \angle A = A]$
$\therefore \quad \cos(A + B) = \cos 90° = 0$



Question 9:
If sin A + $\sin^2$ A = 1, then the value of ($\cos^2$ A + $\cos^4$ A) is
(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

Solution:

(a) Given, $\sin A + \sin^2 A = 1$

$$\Rightarrow \quad \sin A = 1 - \sin^2 A = \cos^2 A \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

On squaring both sides, we get

$$\sin^2 A = \cos^4 A$$

$$\Rightarrow \quad 1 - \cos^2 A = \cos^4 A$$

$$\Rightarrow \quad \cos^2 A + \cos^4 A = 1$$

Question 10:

If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$ then the value of $(\alpha + \beta)$ is

(a) 0° (b) 30° (c) 60° (d) 90°

Solution:

(d) Given, $\sin\alpha = \frac{1}{2} = \sin 30° \qquad \left[\because \sin 30° = \frac{1}{2}\right]$

$$\Rightarrow \quad \alpha = 30°$$

and $\cos\beta = \frac{1}{2} = \cos 60° \qquad \left[\because \cos 60° = \frac{1}{2}\right]$

$$\Rightarrow \quad \beta = 60°$$

$$\therefore \quad \alpha + \beta = 30° + 60° = 90°$$

Question 11:

The value of the expression

$$\left(\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°\right) \text{ is}$$

(a) 3 (b) 2 (c) 1 (d) 0

Solution:

(b) Given expression, $\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°$

$$= \frac{\sin^2 22° + \sin^2(90° - 22°)}{\cos^2(90° - 68°) + \cos^2 68°} + \sin^2 63° + \cos 63° \sin(90° - 63°)$$

$$= \frac{\sin^2 22° + \cos^2 22°}{\sin^2 68° + \cos^2 68°} + \sin^2 63° + \cos 63° \cdot \cos 63° \quad \left[\begin{array}{l}\because \sin(90° - \theta) = \cos\theta \\ \text{and } \cos(90° - \theta) = \sin\theta\end{array}\right]$$

$$= \frac{1}{1} + (\sin^2 63° + \cos^2 63°) \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= 1 + 1 = 2$$

Question 12:

If $4\tan\theta = 3$, then $\left(\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to

(a) $\frac{2}{3}$
(b) $\frac{1}{3}$
(c) $\frac{1}{2}$
(d) $\frac{3}{4}$
Solution:
(c) Given, $4\tan\theta = 3$

$\Rightarrow \quad \tan\theta = \frac{3}{4}$ ...(i)

$\therefore \quad \frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \frac{4\frac{\sin\theta}{\cos\theta} - 1}{4\frac{\sin\theta}{\cos\theta} + 1}$

[divide by $\cos\theta$ in both numerator and denominator]

$= \frac{4\tan\theta - 1}{4\tan\theta + 1}$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$= \frac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$ [put the value from Eq. (i)]

Question 13:
If sinθ – cosθ= 0, then the value of ($\sin^4\theta + \cos^4\theta$) is
(a) 1
(b) $\frac{3}{4}$
(c) $\frac{1}{2}$
(d) $\frac{1}{4}$
Solution:
(c) Given, $\sin\theta - \cos\theta = 0$

$\Rightarrow \quad \sin\theta = \cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \quad \tan\theta = 1$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \tan 45^\circ = 1\right]$

$\Rightarrow \quad \tan\theta = \tan 45^\circ$

$\therefore \quad \theta = 45^\circ$

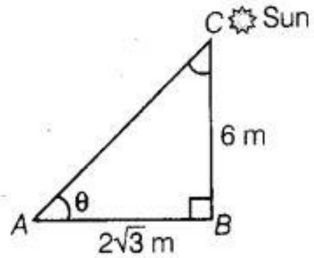
Now, $\sin^4\theta + \cos^4\theta = \sin^4 45^\circ + \cos^4 45^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$ $\left[\because \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}\right]$

$= \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Question 14:
sin (45° + θ) – cos (45° – θ) is equal to
(a) 2 cosθ (b) 0 (c) 2 sinθ (d) 1
Solution:
(b) sin(45° + θ) – cos(45° – θ) = cos[90°- (45° + θ)] – cos(45°- 6) [∴ cos(90° – θ) = sin0]
= cos (45° – 0) – cos (45° – 0)
= 0

Question 15:
If a pole 6 m high casts a shadow 2√3 m long on the ground, then the Sun's elevation is
(a) 60° (b) 45° (c) 30° (d) 90°
Solution:
(a) Let $BC = 6$ m be the height of the pole and $AB = 2\sqrt{3}$ m be the length of the shadow on the ground. let the Sun's makes an angle θ on the ground.



Now, in ΔBAC, $\tan\theta = \dfrac{BC}{AB}$

$\Rightarrow$ $\tan\theta = \dfrac{6}{2\sqrt{3}} = \dfrac{3}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow$ $\tan\theta = \dfrac{3\sqrt{3}}{3} = \sqrt{3} = \tan 60°$ [$\because \tan 60° = \sqrt{3}$]

$\therefore$ $\theta = 60°$

Hence, the Sun's elevation is 60°.

Exercise 8.2 Very Short Answer Type Questions

Write whether **True** or **False** and justify your answer.

Question 1:
$$\frac{\tan 47°}{\cot 43°} = 1$$
Solution:
True
$$\frac{\tan 47°}{\cot 43°} = \frac{\tan(90° - 43°)}{\cot 43°} = \frac{\cot 43°}{\cot 43°} = 1 \qquad [\because \tan(90° - \theta) = \cot\theta]$$

Question 2:
The value of the expression ($\cos^2 23° - \sin^2 67°$) is positive.
Solution:
False
$\cos^2 23° - \sin^2 67° = (\cos 23° - \sin 67°)(\cos 23° + \sin 67°)$ [$\therefore (a^2 - b^2) = (a - b)(a + b)$]
= [cos23° – sin (90° – 23°)] (cos23° + sin67°)

= (cos23° – cos23°) (cos23° + sin67°) [∴sin (90° – 0) = cos 0]
= 0.(cos23° + sin67°) = 0
which may be either positive or negative.

Question 3:
The value of the expression (sin 80° – cos 80°) is negative.
Solution:
False
We know that, sine is increasing when, O°<θ<9O° and cosθ is decreasing when, O°<θ<9O°.
∴ sin80°-cos80°> 0 [positive]

Question 4:

$$\sqrt{(1-\cos^2\theta)\sec^2\theta} = \tan\theta$$

Solution:
True

$$\sqrt{(1-\cos^2\theta)\sec^2\theta} = \sqrt{\sin^2\theta\cdot\sec^2\theta} \qquad [\because \sin^2\theta+\cos^2\theta=1]$$

$$= \sqrt{\sin^2\theta\cdot\frac{1}{\cos^2\theta}} = \sqrt{\tan^2\theta} = \tan\theta \quad \left[\because\ \sec\theta=\frac{1}{\cos\theta},\ \tan\theta=\frac{\sin\theta}{\cos\theta}\right]$$

Question 5:
If cosA + $\cos^2$ A = 1, then $\sin^2$ A + $\sin^4$ A = 1
Solution:
True

$$\because \quad \cos A+\cos^2 A=1$$

$$\Rightarrow \quad \cos A = 1-\cos^2 A=\sin^2 A \qquad [\because \sin^2 A+\cos^2 A=1]$$

$$\Rightarrow \quad \cos^2 A=\sin^4 A$$

$$\Rightarrow \quad 1-\sin^2 A=\sin^4 A$$

$$\Rightarrow \quad \sin^2 A+\sin^4 A=1 \qquad [\because \cos^2 A=1-\sin^2 A]$$

Question 6:
(tanθ + 2) (2 tanθ + 1) = 5 tanθ + $\sec^2$ θ
Solution:
False

$$\text{LHS} = (\tan\theta+2)(2\tan\theta+1)$$

$$= 2\tan^2\theta+4\tan\theta+\tan\theta+2$$

$$= 2(\sec^2\theta-1)+5\tan\theta+2 \qquad [\because \sec^2\theta-\tan^2\theta=1]$$

$$= 2\sec^2\theta+5\tan\theta = \text{RHS}$$

Question 7:
If the length of the shadow of a tower is increasing, then the angle of elevation of the

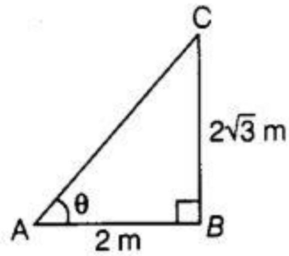
Sun is also increasing.

Solution:

False

To understand the fact of this question, consider the following example

I. A tower $2\sqrt{3}$ m high casts a shadow 2 m long on the ground, then the Sun's elevation is 60°.

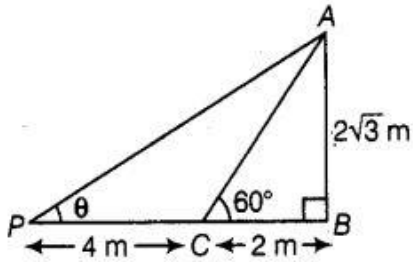


In ΔACB, $\tan\theta = \frac{AB}{BC} = \frac{2\sqrt{3}}{2}$

$\Rightarrow \tan\theta = \sqrt{3} = \tan 60°$

$\therefore \theta = 60°$

II. A same hight of tower casts a shadow 4m more from preceding point, then the Sun's elevation is 30°.



In ΔAPB, $\tan\theta = \frac{AB}{PB} = \frac{AB}{PC + CB}$

$\Rightarrow \tan\theta = \frac{2\sqrt{3}}{4+2} = \frac{2\sqrt{3}}{6}$

$\Rightarrow \tan\theta = \frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{3}{3\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} = \tan 30°$

$\therefore \theta = 30$

Hence, we conclude from above two examples that if the length of the shadow of a tower is increasing, then the angle of elevation of the Sun is decreasing.

Alternate Method

False, we know that, if the elevation moves towards the tower, it increases and if its elevation moves away the tower, it decreases. Hence, if the shadow of a tower is increasing, then the angle of elevation of a Sun is not increasing.

Question 8:

If a man standing on a plat form 3 m above the surface of a lake observes a cloud and its reflection in the lake, then the angle of elevation of the cloud is equal to the angle of depression of its reflection.

Solution:

False

From figure, we observe that, a man standing on a platform at point P, 3 m above the surface of a lake observes a cloud at point C. Let the height of the cloud from the surface of the platform is h and angle of elevation of the cloud is θ_1.

Now at same point P a man observes a cloud reflection in the lake at this time the height of reflection of cloud in lake is

(h + 3) because in lake platform height is also added to reflection of cloud.

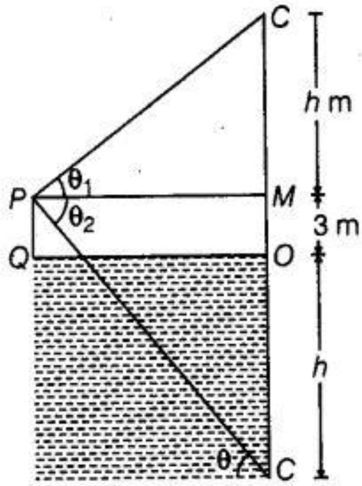
So,angle of depression is different in the lake from the angle of elevation of the cloud above the surface of a lake

In ΔMPC, $\tan\theta_1 = \frac{CM}{PM} = \frac{h}{PM}$

$\Rightarrow$ $\frac{\tan\theta_1}{h} = \frac{1}{PM}$ ...(i)

In ΔCPM, $\tan\theta_2 = \frac{CM}{PM} = \frac{OC + OM}{PM} = \frac{h+3}{PM}$

$\Rightarrow$ $\frac{\tan\theta_2}{h+3} = \frac{1}{PM}$ ...(ii)



From Eqs. (i) and (ii),

$$\frac{\tan\theta_1}{h} = \frac{\tan\theta_2}{h+3}$$

$\Rightarrow$ $\tan\theta_2 = \left(\frac{h+3}{h}\right)\tan\theta_1$

Hence, $\theta_1 \neq \theta_2$

Question 9:

The value of 2 sinθ can be a + $\frac{1}{a}$ where a is a positive number and a ≠ 1.

Solution:

False

Given, a is a positive number and a ≠ 1, then AM > GM

$\Rightarrow$ $\frac{a+\frac{1}{a}}{2} > \sqrt{a\cdot\frac{1}{a}} \Rightarrow \left(a+\frac{1}{a}\right) > 2$

[since, AM and GM of two number's a and b are $\frac{(a+b)}{2}$ and $\sqrt{ab}$, respectively]

$\Rightarrow$ $2\sin\theta > 2$ $\left[\because 2\sin\theta = a + \frac{1}{a}\right]$

$\Rightarrow$ $\sin\theta > 1$ $[\because -1 \leq \sin\theta \leq 1]$

Which is not possible.

Hence,the value of 2 sinθ can not be a + $\frac{1}{a}$

Question 10:

cosθ = $\frac{a^2+b^2}{2ab}$ where a and b are two distinct numbers such that ab > 0.

Solution:

False

Given, a and b are two distinct numbers such that ab > 0.

Using, AM > GM

[since, AM and GM of two number a and b are $\frac{a+b}{2}$ and $\sqrt{ab}$, respectively]

$$\Rightarrow \quad \frac{a^2+b^2}{2} > \sqrt{a^2 \cdot b^2}$$

$$\Rightarrow \quad a^2+b^2 > 2ab$$

$$\Rightarrow \quad \frac{a^2+b^2}{2ab} > 1 \qquad \left[\because \cos\theta = \frac{a^2+b^2}{2ab}\right]$$

$$\Rightarrow \quad \cos\theta > 1 \qquad [\because -1 \le \cos\theta \le 1]$$

which is not possible.

Hence, $\cos\theta \ne \frac{a^2+b^2}{2ab}$

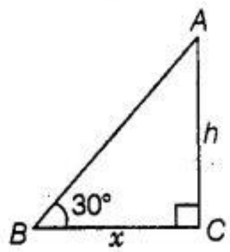
Question 11:
The angle of elevation of the top of a tower is 30°. If the height of the tower is doubled, then the angle of elevation of its top will also be doubled.
Solution:
False

Case I Let the height of the tower is h and $BC = x$ m

In ΔABC,

$$\tan 30^\circ = \frac{AC}{BC} = \frac{h}{x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x} \qquad \text{....(i)}$$



Case II By condition, the height of the tower is doubled. *i.e.*, $PR = 2h$.

In ΔPQR, $\tan\theta = \frac{PR}{QR} = \frac{2h}{x}$

$$\Rightarrow \quad \tan\theta = \frac{2}{x} \times \frac{x}{\sqrt{3}} \qquad \left[\because h = \frac{x}{\sqrt{3}}, \text{ from Eq. (i)}\right]$$

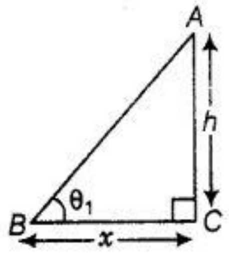
$$\Rightarrow \quad \tan\theta = \frac{2}{\sqrt{3}} = 1.15$$

$$\therefore \quad \theta = \tan^{-1}(1.15) < 60^\circ$$

Hence, the required angle is not doubled.

Question 12:
If the height of a tower and the distance of the point of observation from its foot, both.are increased by 10%, then the angle of elevation of its top remains unchanged.
Solution:
True

Case I Let the height of a tower be h and the distance of the point of observation from its foot is x.



In ΔABC,

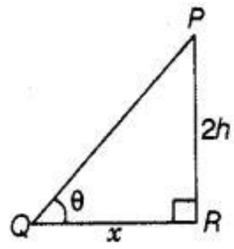
$$\tan\theta_1 = \frac{AC}{BC} = \frac{h}{x}$$

$$\Rightarrow \qquad \theta_1 = \tan^{-1}\left(\frac{h}{x}\right) \qquad \text{...(i)}$$

Case II Now, the height of a tower increased by $10\% = h + 10\%$ of $h = h + h \times \frac{10}{100} = \frac{11h}{10}$

and the distance of the point of observation from its foot $= x + 10\%$ of x

$$= x + x \times \frac{10}{100} = \frac{11x}{10}$$



In ΔPQR, $\qquad \tan\theta_2 = \frac{PR}{QR} = \frac{\left(\frac{11h}{10}\right)}{\left(\frac{11x}{10}\right)}$

$$\Rightarrow \qquad \tan\theta_2 = \frac{h}{x}$$

$$\Rightarrow \qquad \theta_2 = \tan^{-1}\left(\frac{h}{x}\right) \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\theta_1 = \theta_2$$

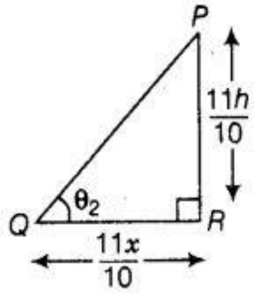
Hence,the required angle of elevation of its top remains unchanged

Exercise 8.3 Short Answer Type Questions

Question 1:

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\ \text{cosec}\theta$$

Solution:



$$\text{LHS} = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta\,(1+\cos\theta)}$$

$$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{\sin\theta\,(1+\cos\theta)} \qquad [\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$= \frac{1+1+2\cos\theta}{\sin\theta\,(1+\cos\theta)} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2\,(1+\cos\theta)}{\sin\theta\,(1+\cos\theta)} = \frac{2}{\sin\theta}$$

$$= 2\operatorname{cosec}\theta = \text{RHS} \qquad \left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$$

Question 2:

$$\frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A} = 2\operatorname{cosec} A$$

Solution:

$$\text{LHS} = \frac{\tan A}{1+\sec A} - \frac{\tan A}{1-\sec A} = \frac{\tan A\,(1-\sec A-1-\sec A)}{(1+\sec A)(1-\sec A)}$$

$$= \frac{\tan A\,(-2\sec A)}{(1-\sec^2 A)} = \frac{2\tan A\cdot\sec A}{(\sec^2 A-1)} \qquad [\because (a+b)(a-b) = a^2 - b^2]$$

$$= \frac{2\tan A\cdot\sec A}{\tan^2 A} \qquad [\because \sec^2 A - \tan^2 A = 1] \left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= \frac{2\sec A}{\tan A} = \frac{2}{\sin A} = 2\operatorname{cosec} A = \text{RHS} \qquad \left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$$

Question 3:

If tan A = $\frac{3}{4}$,then sin A cos A = $\frac{12}{25}$

Solution:

Given, $\tan A = \frac{3}{4} = \frac{P}{B} = \frac{\text{Perpendicular}}{\text{Base}}$

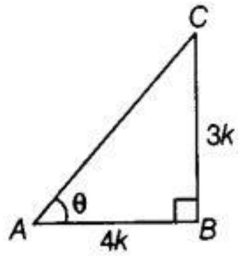
Let $P = 3k$ and $B = 4k$

By Pythagoras theorem,

$$H^2 = P^2 + B^2 = (3k)^2 + (4k)^2$$
$$= 9k^2 + 16k^2 = 25k^2$$

$\Rightarrow$ $H = 5k$ [since, side cannot be negative]

$\therefore$ $\sin A = \frac{P}{H} = \frac{3k}{5k} = \frac{3}{5}$ and $\cos A = \frac{B}{H} = \frac{4k}{5k} = \frac{4}{5}$



Now, $\sin A \cos A = \frac{3}{5} \cdot \frac{4}{5} = \frac{12}{25}$

Hence proved.

Question 4:
(sinα + cosα) (tanα + cotβ) = secα + cosecβ
Solution:

$$\text{LHS} = (\sin\alpha + \cos\alpha)(\tan\alpha + \cot\alpha)$$
$$= (\sin\alpha + \cos\alpha)\left(\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}\right) \quad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$
$$= (\sin\alpha + \cos\alpha)\left(\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha \cdot \cos\alpha}\right)$$
$$= (\sin\alpha + \cos\alpha) \cdot \frac{1}{(\sin\alpha \cdot \cos\alpha)} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$
$$= \frac{1}{\cos\alpha} + \frac{1}{\sin\alpha} \quad \left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$$
$$= \sec\alpha + \cos\alpha = \text{RHS}$$

Question 5:
(√3 + 1) (3 – cot 30°) = $\tan^3$ 60° – 2sin60°
Solution:

$$\text{RHS} = \tan^3 60° - 2\sin 60° = (\sqrt{3})^3 - 2\frac{\sqrt{3}}{2} = 3\sqrt{3} - \sqrt{3} = 2\sqrt{3}$$
$$\text{LHS} = (\sqrt{3} + 1)(3 - \cot 30°) = (\sqrt{3} + 1)(3 - \sqrt{3})$$
$$[\because \tan 60° = \sqrt{3} \ \sin 60° = \frac{\sqrt{3}}{2} \text{ and } = (\sqrt{3} + 1)\sqrt{3}(\sqrt{3} - 1) \cot 30° = \sqrt{3}]$$
$$= \sqrt{3}((\sqrt{3})^2 - 1) = \sqrt{3}(3 - 1) = 2\sqrt{3}$$

$\therefore$ LHS = RHS **Hence proved.**

Question 6:

$$1 + \frac{\cot^2\alpha}{1 + \operatorname{cosec}\alpha} = \operatorname{cosec}\alpha$$

Solution:

$$\text{LHS} = 1 + \frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha} = 1 + \frac{\cos^2\alpha/\sin^2\alpha}{1+1/\sin\alpha} \qquad \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta} \text{ and } \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$$

$$= 1 + \frac{\cos^2\alpha}{\sin\alpha\,(1+\sin\alpha)} = \frac{\sin\alpha\,(1+\sin\alpha)+\cos^2\alpha}{\sin\alpha\,(1+\sin\alpha)}$$

$$= \frac{\sin\alpha + (\sin^2\alpha+\cos^2\alpha)}{\sin\alpha\,(1+\sin\alpha)} \qquad [\because \sin^2\theta+\cos^2\theta = 1]$$

$$= \frac{(\sin\alpha+1)}{\sin\alpha\,(\sin\alpha+1)} = \frac{1}{\sin\alpha} \qquad \left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$$

$$= \operatorname{cosec}\alpha = \text{RHS}$$

Question 7:
$\tan\theta + \tan(90° - \theta) = \sec\theta\,\sec(90° - \theta)$
Solution:

$$\text{LHS} = \tan\theta + \tan(90° - \theta) \qquad [\because \tan(90° - \theta) = \cot\theta]$$

$$= \tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

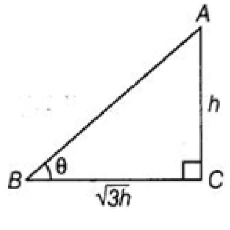
$$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{1}{\sin\theta\cos\theta} \qquad [\because \sin^2\theta+\cos^2\theta = 1]$$

$$= \sec\theta\operatorname{cosec}\theta \qquad \left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \cos\theta = \frac{1}{\sin\theta}\right]$$

$$= \sec\theta\,\sec(90° - \theta) = \text{RHS} \qquad [\because \sec(90° - \theta) = \operatorname{cosec}\theta]$$

Question 8:
Find the angle of elevation of the sun when the shadow of a pole h m high is √3 h m long.
Solution:



Let the angle of elevation of the sun is θ.
Given, height of pole = h
Now, in triangle ABC, $\tan\theta = \frac{AB}{BC} = \frac{h}{\sqrt{3}h}$
$\tan\theta = \frac{1}{\sqrt{3}} = \tan 30^0$

$\theta = 30°$
Hence, the angle of elevation of the sun is 30^0

Question 9:
If $\sqrt{3}\tan\theta = 1$, then find the value of $\sin^2\theta - \cos^2\theta$
Solution:

Given that, $\sqrt{3}\tan\theta = 1$

$\Rightarrow$ $\tan\theta = \frac{1}{\sqrt{3}} = \tan 30°$

$\Rightarrow$ $\theta = 30°$

Now, $\sin^2\theta - \cos^2\theta = \sin^2 30° - \cos^2 30°$

$$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} - \frac{3}{4} = \frac{1-3}{4} = -\frac{2}{4} = -\frac{1}{2}$$

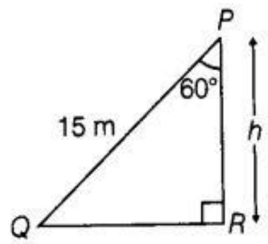
Question 10:
A ladder 15 m long just reaches the top of a vertical wall.If the ladders makes an angle of 60° with the wall,then find the height of the wall.
Solution:
Given that,the height of the ladder = 15 m
Let the height of the vertical wall = h
and the ladder makes an angle of elevation 60° with the wall i.e $\theta = 60°$

In ΔQPR, $\cos 60° = \frac{PR}{PQ} = \frac{h}{15}$

$\Rightarrow$ $\frac{1}{2} = \frac{h}{15}$

$\Rightarrow$ $h = \frac{15}{2}$ m.

Hence, the required height of the wall $\frac{15}{2}$ m.



Question 11:
Simplify $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$
Solution:

$(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) = (1 + \tan^2\theta)(1 - \sin^2\theta)$ $[\because (a-b)(a+b) = a^2 - b^2]$

$= \sec^2\theta \cdot \cos^2\theta$

$[\because 1 + \tan^2\theta = \sec^2\theta$ and $\cos^2\theta + \sin^2\theta = 1]$

$= \frac{1}{\cos^2\theta} \cdot \cos^2\theta = 1$ $\left[\because \sec\theta = \frac{1}{\cos\theta}\right]$

Question 12:
If 2 $\sin^2\theta - \cos^2\theta = 2$, then find the value of θ.
Solution:

Given, $2\sin^2\theta - \cos^2\theta = 2$

$\Rightarrow$ $2\sin^2\theta - (1 - \sin^2\theta) = 2$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow$ $2\sin^2\theta + \sin^2\theta - 1 = 2$

$\Rightarrow$ $3\sin^2\theta = 3$

$\Rightarrow$ $\sin^2\theta = 1$ [$\because \sin 90° = 1$]

$\Rightarrow$ $\sin\theta = 1 = \sin 90°$

$\therefore$ $\theta = 90°$

Question 13:

Show that $\dfrac{\cos^2(45° + \theta) + \cos^2(45° - \theta)}{\tan(60° + \theta)\tan(30° - \theta)} = 1$

Solution:

$$\text{LHS} = \frac{\cos^2(45° + \theta) + \cos^2(45° - \theta)}{\tan(60° + \theta)\cdot\tan(30° - \theta)}$$

$$= \frac{\cos^2(45° + \theta) + [\sin\{90° - (45° - \theta)\}]^2}{\tan(60° + \theta)\cdot\cot\{90° - (30° - \theta)\}}$$

[$\because \sin(90° - \theta) = \cos\theta$ and $\cot(90° - \theta) = \tan\theta$]

$$= \frac{\cos^2(45° + \theta) + \sin^2(45° + \theta)}{\tan(60° + \theta)\cdot\cot(60° + \theta)}$$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$$= \frac{1}{\tan(60° + \theta)\cdot\dfrac{1}{\tan(60° + \theta)}} = 1 = \text{RHS}$$ [$\because \cot\theta = 1/\tan\theta$]

Question 14:
An observer 1.5 m tall is 20.5 m away from a tower 22 m high. Determine the angle of elevation of the top of the tower from the eye of the observer.
Solution:

Let the angle of elevation of the top of the tower from the eye of the observe is θ

Given that, $AB = 22$ m, $PQ = 1.5$ m $= MB$

and $QB = PM = 20.5$ m

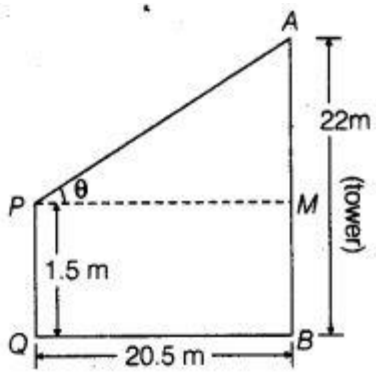
$\Rightarrow$ $AM = AB - MB$

$= 22 - 1.5 = 20.5$ m

Now, in ΔAPM, $\tan\theta = \dfrac{AM}{PM} = \dfrac{20.5}{20.5} = 1$

$\Rightarrow$ $\tan\theta = \tan 45°$

$\therefore$ $\theta = 45°$



which may be either positive or negative.Hence, required angle of elevation of the top of the tower from the eye of the observer is 45°.

Question 15:
Show that $\tan^4 \theta + \tan^2 \theta = \sec^4 \theta - \sec^2 \theta$.
Solution:
LHS = $\tan^4 \theta + \tan^2 \theta = \tan^2 \theta (\tan^2 \theta+1)$
$= \tan^2 \theta.\sec^2 \theta$ [$\therefore \sec^2\theta = \tan^2 \theta+1$]
$= (\sec^2 \theta-1) . \sec^2 \theta$ [$\therefore \tan^2\theta = \sec^2 \theta - 1$]
$= \sec^4 \theta - \sec^2 \theta$ = RHS

Exercise 8.4 Long Answer Type Questions

Question 1:
If cosec θ+ cot θ = p, then prove that $\cos\theta = \frac{p^2-1}{p^2+1}$
Solution:
Given, $\operatorname{cosec} \theta + \cot \theta = p$

$\Rightarrow \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} = p$ $\left[\because \operatorname{cosec} \theta = \frac{1}{\sin \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta}\right]$

$\Rightarrow \frac{1+\cos \theta}{\sin \theta} = \frac{p}{1}$

$\Rightarrow \frac{(1+\cos \theta)^2}{\sin^2 \theta} = \frac{p^2}{1}$ [take square on both sides]

$\Rightarrow \frac{1+\cos^2 \theta + 2\cos \theta}{\sin^2 \theta} = \frac{p^2}{1}$

Using componendo and dividendo rule, we get

$$\frac{(1+\cos^2 \theta + 2\cos \theta) - \sin^2 \theta}{(1+\cos^2 \theta + 2\cos \theta) + \sin^2 \theta} = \frac{p^2-1}{p^2+1}$$

$\Rightarrow \frac{1+\cos^2 \theta + 2\cos \theta - (1-\cos^2 \theta)}{1 + 2\cos \theta + (\cos^2 \theta + \sin^2 \theta)} = \frac{p^2-1}{p^2+1}$ [$\because \sin^2 \theta + \cos^2 \theta = 1$]

$\Rightarrow \frac{2\cos^2 \theta + 2\cos \theta}{2 + 2\cos \theta} = \frac{p^2-1}{p^2+1}$

$\Rightarrow \frac{2\cos \theta (\cos \theta + 1)}{2(\cos \theta + 1)} = \frac{p^2-1}{p^2+1}$

$\therefore \cos \theta = \frac{p^2-1}{p^2+1}$ Hence proved.

Question 2:
Prove that $\sqrt{sec^2\theta + cosec^2\theta} = \tan \theta + \cot \theta$.

Solution:

$$\text{LHS} = \sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}$$

$$= \sqrt{\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}} \qquad \left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$$

$$= \sqrt{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta \cdot \cos^2\theta}} = \sqrt{\frac{1}{\sin^2\theta \cdot \cos^2\theta}} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{\sin\theta \cdot \cos\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cdot \cos\theta} \qquad [\because 1 = \sin^2\theta + \cos^2\theta]$$

$$= \frac{\sin^2\theta}{\sin\theta \cdot \cos\theta} + \frac{\cos^2\theta}{\sin\theta \cdot \cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \tan\theta + \cot\theta = \text{RHS}$$

Question 3:

The angle of elevation of the top of a tower from certain point is 30°. If the observer moves 20 m towards the tower, the angle of elevation of the top increases by 15°. Find the height of the tower.

Solution:

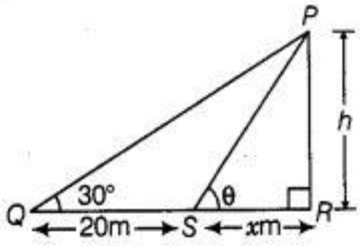
Let the height of the tower be h.

also, $SR = x$ m, $\angle PSR = \theta$

Given that, $QS = 20$ m

and $\angle PQR = 30°$

Now, in ΔPSR,



$$\tan\theta = \frac{PR}{SR} = \frac{h}{x}$$

$$\Rightarrow \quad \tan\theta = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\tan\theta} \quad \text{...(i)}$$

Now, in ΔPQR,

$$\tan 30° = \frac{PR}{QR} = \frac{PR}{QS + SR}$$

$$\Rightarrow \quad \tan 30° = \frac{h}{20 + x}$$

$$\Rightarrow \quad 20 + x = \frac{h}{\tan 30°} = \frac{h}{1/\sqrt{3}}$$

$$\Rightarrow \quad 20 + x = h\sqrt{3}$$

$$\Rightarrow \quad 20 + \frac{h}{\tan\theta} = h\sqrt{3} \quad \text{...(ii) [from Eq. (i)]}$$

Since, after moving 20 m towards the tower the angle of elevation of the top increases by 15°.

i.e., $\angle PSR = \theta = \angle PQR + 15°$

$$\Rightarrow \quad \theta = 30° + 15 = 45°$$

$$\therefore \quad 20 + \frac{h}{\tan 45°} = h\sqrt{3} \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad 20 + \frac{h}{1} = h\sqrt{3}$$

$$\Rightarrow \quad 20 = h\sqrt{3} - h$$

$$\Rightarrow \quad h(\sqrt{3} - 1) = 20$$

$$\therefore \quad h = \frac{20}{\sqrt{3} - 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \quad \text{[by rationalisation]}$$

$$\Rightarrow \quad = \frac{20(\sqrt{3} + 1)}{3 - 1} = \frac{20(\sqrt{3} + 1)}{2}$$

$$\Rightarrow \quad = 10(\sqrt{3} + 1)\,\text{m}$$

Hence, the required height of tower is $10(\sqrt{3} + 1)$ m.

Question 4:

If 1 + $\sin^2\theta$ = 3 sin0 cos0, then prove that tan0 = 1 or $\frac{1}{2}$

Solution:

Given, $1 + \sin^2\theta = 3\sin\theta \cdot \cos\theta$

On dividing by $\sin^2\theta$ on both sides, we get

$$\frac{1}{\sin^2\theta} + 1 = 3 \cdot \cot\theta \qquad \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$\Rightarrow \quad \operatorname{cosec}^2\theta + 1 = 3 \cdot \cot\theta \qquad \left[\operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$$

$$\Rightarrow \quad 1 + \cot^2\theta + 1 = 3 \cdot \cot\theta \qquad [\because \operatorname{cosec}^2\theta - \cot^2\theta = 1]$$

$$\Rightarrow \quad \cot^2\theta - 3\cot\theta + 2 = 0$$

$$\Rightarrow \quad \cot^2\theta - 2\cot\theta - \cot\theta + 2 = 0 \qquad \text{[by splitting the middle term]}$$

$$\Rightarrow \quad \cot\theta(\cot\theta - 2) - 1(\cot\theta - 2) = 0$$

$$\Rightarrow \quad (\cot\theta - 2)(\cot\theta - 1) = 0 \Rightarrow \cot\theta = 1 \text{ or } 2$$

$$\Rightarrow \quad \tan\theta = 1 \text{ or } \frac{1}{2} \qquad \left[\because \tan\theta = \frac{1}{\cot\theta}\right]$$

Hence proved.

Question 5:

If sinθ + 2 cosθ = 1, then prove that 2 sinθ – cosθ = 2.

Solution:

Given, $\sin\theta + 2\cos\theta = 1$

On squaring both sides, we get

$$(\sin\theta + 2\cos\theta)^2 = 1$$

$$\Rightarrow \quad \sin^2\theta + 4\cos^2\theta + 4\sin\theta \cdot \cos\theta = 1$$

$$\Rightarrow \quad (1 - \cos^2\theta) + 4(1 - \sin^2\theta) + 4\sin\theta \cdot \cos\theta = 1 \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$\Rightarrow \quad -\cos^2\theta - 4\sin^2\theta + 4\sin\theta \cdot \cos\theta = -4$$

$$\Rightarrow \quad 4\sin^2\theta + \cos^2\theta - 4\sin\theta \cdot \cos\theta = 4$$

$$\Rightarrow \quad (2\sin\theta - \cos\theta)^2 = 4 \qquad [\because a^2 + b^2 - 2ab = (a - b)^2]$$

$$\Rightarrow \quad 2\sin\theta - \cos\theta = 2$$

Hence proved.

Question 6:

The angle of elevation of the top of a tower from two points distant s and t from its foot are complementary. Prove that the height of the tower is $\sqrt{st}$.

Solution:

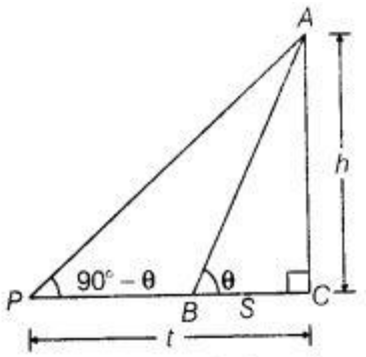
Let the height of the tower is h.

and $\angle ABC = \theta$

Given that, $BC = s$, $PC = t$

and angle of elevation on both positions are complementary.

i.e , $\angle APC = 90° - \theta$



[if two angles are complementary to each other, then the sum of both angles is equal to 90°.]

Now in ΔABC, $$\tan\theta = \frac{AC}{BC} = \frac{h}{s} \quad \dots(i)$$

and in ΔAPC

$$\tan(90° - \theta) = \frac{AC}{PC} \quad [\because \tan(90° - \theta) = \cot\theta]$$

$$\Rightarrow \cot\theta = \frac{h}{t}$$

$$\Rightarrow \frac{1}{\tan\theta} = \frac{h}{t} \quad \left[\because \cot\theta = \frac{1}{\tan\theta}\right] \dots(ii)$$

On, multiplying Eqs. (i) and (ii), we get

$$\tan\theta \cdot \frac{1}{\tan\theta} = \frac{h}{s} \cdot \frac{h}{t}$$

$$\Rightarrow \frac{h^2}{st} = 1$$

$$\Rightarrow h^2 = st$$

$$\Rightarrow h = \sqrt{st}$$

So, the required height of the tower is √st.
Hence proved.

Question 7:
The shadow of a tower standing on a level plane is found to be 50 m longer when Sun's elevation is 30° than when it is 60°. Find the height of the tower.
Solution:
Let the height of the tower be h and RQ = x m

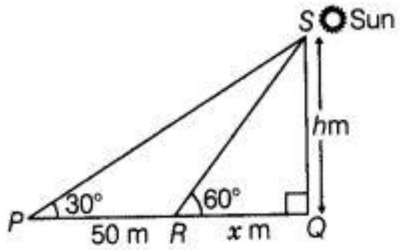
Given that, $PR = 50$ m

and $\angle SPQ = 30°$, $\angle SRQ = 60°$

Now, in ΔSRQ, $$\tan 60° = \frac{SQ}{RQ}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}} \quad \dots(i)$$

and in ΔSPQ, $\tan 30° = \frac{SQ}{PQ} = \frac{SQ}{PR + RQ} = \frac{h}{50 + x}$



$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{50 + x}$

$\Rightarrow \quad \sqrt{3} \cdot h = 50 + x$

$\Rightarrow \quad \sqrt{3} \cdot h = 50 + \frac{h}{\sqrt{3}}$ [from Eq. (i)]

$\Rightarrow \quad (\sqrt{3} - \frac{1}{\sqrt{3}}) h = 50$

$\Rightarrow \quad \frac{(3 - 1)}{\sqrt{3}} h = 50$

$\therefore \quad h = \frac{50\sqrt{3}}{2}$

$h = 25\sqrt{3}$ m

Hence, the required height of tower is 25√3 m.

Question 8:

A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h. At a point on the plane, the angles of elevation of the bottom and the top of the flag staff are α and β respectively. Prove that the height of the tower is $\left(\frac{h \quad tan\alpha}{tan\beta - tan\alpha}\right)$

Solution:

Let the height of the tower be H and OR = x

Given that, height of flag staff = h = FP and ∠PRO = α, ∠FRO = β

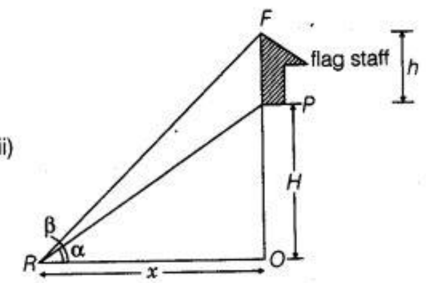
Now, in ΔPRO, $\tan \alpha = \frac{PO}{RO} = \frac{H}{x}$

$\Rightarrow \quad x = \frac{H}{\tan \alpha}$ ...(i)

and in ΔFRO, $\tan \beta = \frac{FO}{RO} = \frac{FP + PO}{RO}$

$\tan \beta = \frac{h + H}{x}$

$\Rightarrow \quad x = \frac{h + H}{\tan \beta}$ ...(ii)



From Eqs. (i) and (ii),

$\frac{H}{\tan\alpha} = \frac{h + H}{\tan\beta}$

$\Rightarrow \quad H \tan \beta = h \tan \alpha + H \tan \alpha$

$\Rightarrow \quad H \tan \beta - H \tan \alpha = h \tan \alpha$

$\Rightarrow \quad H (\tan \beta - \tan \alpha) = h \tan \alpha \quad \Rightarrow \quad H = \frac{h \tan \alpha}{\tan \beta - \tan \alpha}$

Hence the required height of tower is $\frac{h \tan \alpha}{\tan \beta - \tan \alpha}$. **Hence proved.**

Question 9:

If tan θ + sec θ = l, then prove that secθ = $\frac{l^2+1}{2l}$.

Solution:

Given, $\tan\theta + \sec\theta = l$ ...(i)

[multiply by $(\sec\theta - \tan\theta)$ on numerator and denominator LHS]

$\Rightarrow \dfrac{(\tan\theta + \sec\theta)(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)} = l \Rightarrow \dfrac{(\sec^2\theta - \tan^2\theta)}{(\sec\theta - \tan\theta)} = l$

$\Rightarrow \dfrac{1}{\sec\theta - \tan\theta} = l$ $[\because \sec^2\theta - \tan^2\theta = 1]$

$\Rightarrow \sec\theta - \tan\theta = \dfrac{1}{l}$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2\sec\theta = l + \dfrac{1}{l}$

$\Rightarrow \sec\theta = \dfrac{l^2+1}{2l}$ **Hence proved.**

Question 10:

If sinθ + cosθ = p and sec θ + cosec θ = q, then prove that $q(p^2-1) = 2p$.

Solution:

Given that, $\sin\theta + \cos\theta = p$ ...(i)

and $\sec\theta + \operatorname{cosec}\theta = q$

$\Rightarrow \dfrac{1}{\cos\theta} + \dfrac{1}{\sin\theta} = q$ $\left[\because \sec\theta = \dfrac{1}{\cos\theta} \text{ and } \operatorname{cosec}\theta = \dfrac{1}{\sin\theta}\right]$

$\Rightarrow \dfrac{\sin\theta + \cos\theta}{\sin\theta . \cos\theta} = q$

$\Rightarrow \dfrac{p}{\sin\theta \cdot \cos\theta} = q$ [from Eq. (i)]

$\Rightarrow \sin\theta \cdot \cos\theta = \dfrac{p}{q}$ [from Eq. (i)]...(ii)

$\sin\theta + \cos\theta = p$

On squaring both sides, we get

$(\sin\theta + \cos\theta)^2 = p^2$

$\Rightarrow (\sin^2\theta + \cos^2\theta) + 2\sin\theta.\cos\theta = p^2$ $[\because (a+b)^2 = a^2 + 2ab + b^2]$

$\Rightarrow 1 + 2\sin\theta \cdot \cos\theta = p^2$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow 1 + 2 \cdot \dfrac{p}{q} = p^2$ [from Eq. (ii)]

$\Rightarrow q + 2p = p^2 q \Rightarrow 2p = p^2 q - q$

$\Rightarrow q(p^2 - 1) = 2p$ **Hence proved.**

Question 11:

If a sinθ+ b cosθ = c, then prove that a cosθ – b sinθ = $\sqrt{a^2+b^2-c^2}$.

Solution:

Given that, $a\sin\theta + b\cos\theta = c$

On squaring both sides,

$$(a\cdot\sin\theta + \cos\theta\cdot b)^2 = c^2$$

$\Rightarrow$ $a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cdot\cos\theta = c^2$ $[\because (x+y)^2 = x^2 + 2xy + y^2]$

$\Rightarrow$ $a^2(1-\cos^2\theta) + b^2(1-\sin^2\theta) + 2ab\sin\theta\cdot\cos\theta = c^2$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow$ $a^2 - a^2\cos^2\theta + b^2 - b^2\sin^2\theta + 2ab\sin\theta\cdot\cos\theta = c^2$

$\Rightarrow$ $a^2 + b^2 - c^2 = a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cdot\cos\theta$

$\Rightarrow$ $(a^2 + b^2 - c^2) = (a\cos\theta - b\sin\theta)^2$ $[\because a^2 + b^2 - 2ab = (a-b)^2]$

$\Rightarrow$ $(a\cos\theta - b\sin\theta)^2 = a^2 + b^2 - c^2$

$\Rightarrow$ $a\cos\theta - b\sin\theta = \sqrt{a^2 + b^2 - c^2}$ **Hence proved.**

Question 12:

Prove that $\dfrac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \dfrac{1-\sin\theta}{\cos\theta}$

Solution:

$$\text{LHS} = \frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$$

$$= \frac{1+1/\cos\theta-\sin\theta/\cos\theta}{1+1/\cos\theta+\sin\theta/\cos\theta} \quad \left[\because \sec\theta = \frac{1}{\cos\theta} \text{ and } \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= \frac{\cos\theta+1-\sin\theta}{\cos\theta+1+\sin\theta} = \frac{(\cos\theta+1)-\sin\theta}{(\cos\theta+1)+\sin\theta} = \frac{2\cos^2\frac{\theta}{2} - 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}$$

$$\left[\because 1+\cos\theta = 2\cos^2\frac{\theta}{2} \text{ and } \sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]$$

$$= \frac{2\cos^2\frac{\theta}{2} - 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}} = \frac{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)}$$

$$= \frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}} \times \frac{\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)} \quad [\text{ by rationalisation}]$$

$$= \frac{\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)^2}{\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)} \quad [\because (a-b)^2 = a^2 + b^2 - 2ab \text{ and } (a-b)(a+b) = (a^2 - b^2)]$$

$$= \frac{\left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right) - \left(2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}\right)}{\cos\theta} \quad \left[\because \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = \cos\theta\right]$$

$$= \frac{1-\sin\theta}{\cos\theta} \quad \left[\because \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} = 1\right]$$

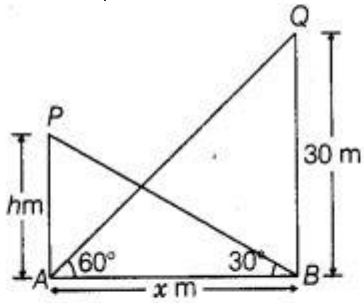
$= \text{RHS}$ **Hence proved.**

Question 13:
The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. Find the distance between the two towers and also the height of the tower.
Solution:
Let distance between the two towers = AB = x m
and height of the other tower = PA = h m
Given that, height of the tower = QB = 30 m and ∠QAB = 60°, ∠PBA = 30°

Now, in ΔQAB, $\tan 60° = \frac{QB}{AB} = \frac{30}{x}$

$\Rightarrow \sqrt{3} = \frac{30}{x}$

$\therefore x = \frac{30}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}$ m



and in ΔPBA,

$\tan 30° = \frac{PA}{AB} = \frac{h}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{10\sqrt{3}}$ $\quad [\because x = 10\sqrt{3}$ m$]$

$\Rightarrow h = 10$ m

Hence, the required distance and height are 10√3 m and 10 m, respectively.

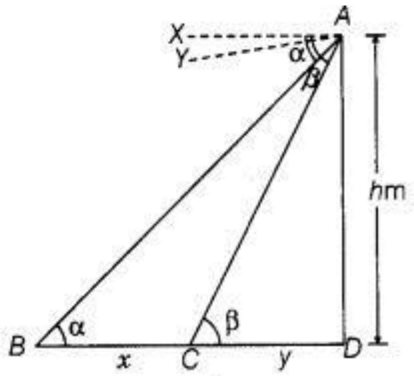
Question 14:
From the top of a tower h m high, angles of depression of two objects, which are in line with the foot of the tower are a and β (β > a). Find the distance between the two objects.
Solution:
Let the distance between two objects is x m,
and CD = y m.
Given that, ∠BAX = α = ∠ABD, [alternate angle]

∠CAY = p = ∠ACD [alternate angle]

Now, in ΔACD,

$$\tan\beta = \frac{AD}{CD} = \frac{h}{y}$$

$$\Rightarrow \quad y = \frac{h}{\tan\beta} \quad \ldots(i)$$



and in ΔABD,

$$\tan\alpha = \frac{AD}{BD} \quad \Rightarrow \quad = \frac{AD}{BC + CD}$$

$$\Rightarrow \quad \tan\alpha = \frac{h}{x+y} \Rightarrow x + y = \frac{h}{\tan\alpha}$$

$$\Rightarrow \quad y = \frac{h}{\tan\alpha} - x \quad \ldots(ii)$$

From Eqs. (i) and (ii),

$$\frac{h}{\tan\beta} = \frac{h}{\tan\alpha} - x$$

$$\therefore \quad x = \frac{h}{\tan\alpha} - \frac{h}{\tan\beta}$$

$$= h\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) = h(\cot\alpha - \cot\beta) \quad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

which is the required distance between the two objects. **Hence proved.**

Question 15:

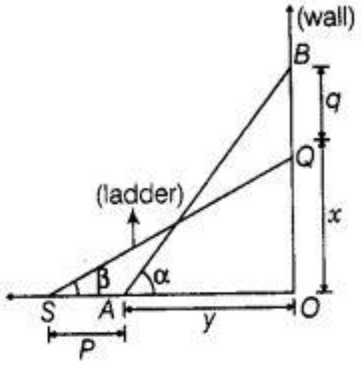
A ladder against a vertical wall at an inclination a to the horizontal. Sts foot is pulled away from the wall through a distance p, so that its upper end slides a distance q down the wall and then the ladder makes an angle B to the horizontal. Show that $\frac{p}{q} = \frac{cos\beta - cos\alpha}{sin\alpha - sin\beta}$

Solution:

Let $\quad OQ = x$ and $OA = y$

Given that, $\quad BQ = q, SA = P$ and $AB = SQ =$ Length of ladder

Also, $\quad \angle BAO = \alpha$ and $\angle QSO = \beta$



Now, in ΔBAO,

$$\cos\alpha = \frac{OA}{AB}$$

$$\Rightarrow \quad \cos\alpha = \frac{y}{AB}$$

$$\Rightarrow \quad y = AB\cos\alpha = OA \quad \ldots(i)$$

and $$\sin\alpha = \frac{OB}{AB}$$

$$\Rightarrow \quad OB = BA\sin\alpha \quad \ldots(ii)$$

Now, in ΔQSO

$$\cos\beta = \frac{OS}{SQ}$$

$$\Rightarrow \quad OS = SQ\cos\beta = AB\cos\beta \quad [\because AB = SQ]\ldots(iii)$$

and $$\sin\beta = \frac{OQ}{SQ}$$

$$\Rightarrow \quad OQ = SQ\sin\beta = AB\sin\beta \quad [\because AB = SQ]\ldots(iv)$$

Now, $$SA = OS - AO$$

$$P = AB\cos\beta - AB\cos\alpha$$

$$\Rightarrow \quad P = AB(\cos\beta - \cos\alpha) \quad \ldots(v)$$

and $$BQ = BO - QO$$

$$\Rightarrow \quad q = BA\sin\alpha - AB\sin\beta$$

$$\Rightarrow \quad q = AB(\sin\alpha - \sin\beta) \quad \ldots(vi)$$

Eq. (v) divided by Eq. (vi), we get

$$\frac{p}{q} = \frac{AB(\cos\beta - \cos\alpha)}{AB(\sin\alpha - \sin\beta)} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$$

$$\Rightarrow \quad \frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$$

Hence proved.

Question 16:

The angle of elevation of the top of a vertical tower from a point on the ground is 60° From another point 10 m vertically above the first, its angle of elevation is 45°. Find the height of the tower.

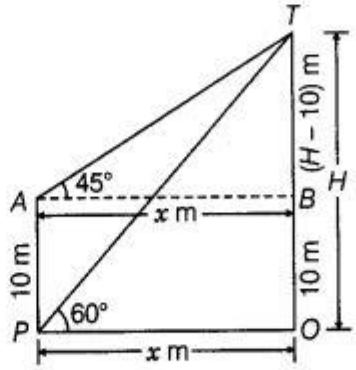
Solution:

Let the height the vertical tower, OT = H and $\quad OP = AB = x$ m

Given that, $\quad AP = 10$ m

and $\quad \angle TPO = 60°, \angle TAB = 45°$

Now, in ΔTPO, $\quad \tan 60° = \frac{OT}{OP} = \frac{H}{x}$



$\Rightarrow \quad \sqrt{3} = \frac{H}{x}$

$\Rightarrow \quad x = \frac{H}{\sqrt{3}} \quad$ ...(i)

and in ΔTAB, $\quad \tan 45° = \frac{TB}{AB} = \frac{H-10}{x}$

$\Rightarrow \quad 1 = \frac{H-10}{x} \Rightarrow x = H - 10$

$\Rightarrow \quad \frac{H}{\sqrt{3}} = H - 10 \quad$ [from Eq. (i)]

$\Rightarrow \quad H - \frac{H}{\sqrt{3}} = 10 \Rightarrow H\left(1 - \frac{1}{\sqrt{3}}\right) = 10$

$\Rightarrow \quad H\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 10$

$\therefore \quad H = \frac{10\sqrt{3}}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1} \quad$ [by rationalisation]

$= \frac{10\sqrt{3}(\sqrt{3}+1)}{3-1} = \frac{10\sqrt{3}(\sqrt{3}+1)}{2}$

$\Rightarrow \quad = 5\sqrt{3}(\sqrt{3}+1) = 5(\sqrt{3}+3)$ m.

Hence, the required height of the tower is 5 (√3 + 3) m,

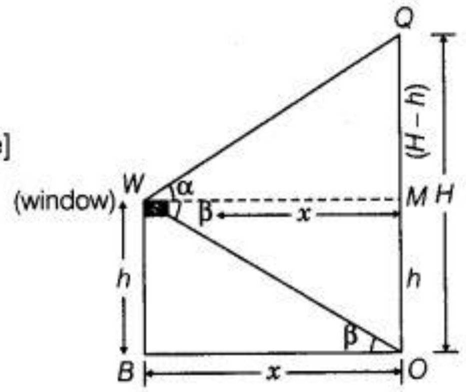
Question 17:
A window of a house is h m above the ground. Form the window, the angles of elevation and depression of the top and the bottom of another house situated on the opposite side of the lane are found to be a and p, respectively. Prove that the height of the other house is h(1+tanα cot β)m.
Solution:

Let the height of the other house = $OQ = H$

and $OB = MW = x$ m

Given that, height of the first house = $WB = h = MO$

and $\angle QWM = \alpha$, $\angle OWM = \beta = \angle WOB$ [alternate angle]



Now, in ΔWOB, $\tan\beta = \frac{WB}{OB} = \frac{h}{x}$

$\Rightarrow$ $x = \frac{h}{\tan\beta}$ ...(i)

And in ΔQWM, $\tan\alpha = \frac{QM}{WM} = \frac{OQ - MO}{WM}$

$\Rightarrow$ $\tan\alpha = \frac{H-h}{x}$

$\Rightarrow$ $x = \frac{H-h}{\tan\alpha}$ ...(ii)

From Eqs. (i) and (ii),

$$\frac{h}{\tan\beta} = \frac{H-h}{\tan\alpha}$$

$\Rightarrow$ $h\tan\alpha = (H-h)\tan\beta$

$\Rightarrow$ $h\tan\alpha = H\tan\beta - h\tan\beta$

$\Rightarrow$ $H\tan\beta = h(\tan\alpha + \tan\beta)$

$\therefore$ $H = h\left(\frac{\tan\alpha + \tan\beta}{\tan\beta}\right)$

$= h\left(1 + \tan\alpha \cdot \frac{1}{\tan\beta}\right) = h(1 + \tan\alpha \cdot \cot\beta)$ $\left[\because \cot\theta = \frac{1}{\tan\theta}\right]$

Hence, the required height of the other house is $h(1 + \tan\alpha \cdot \cot\beta)$ **Hence proved.**

Question 18:

The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain instant the angles of elevation of a balloon from these windows are observed to be 60° and 30°, respectively. Find the height of the balloon above the ground.

Solution:

Let the height of the balloon from above the ground is H.

A and $OP=w_2R=w_1Q=x$

Given that, height of lower window from above the ground = w_2P = 2 m = OR

Height of upper window from above the lower window = w_1w_2 = 4 m = QR

$\therefore$ $$BQ = OB - (QR + RO)$$
$$= H - (4 + 2)$$
$$= H - 6$$

and $$\angle Bw_1 Q = 30^\circ$$

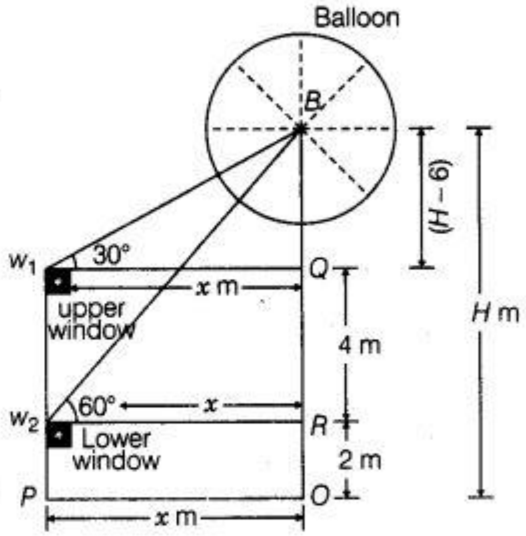
$\Rightarrow$ $$\angle Bw_2 R = 60^\circ$$

Now, in ΔBw_2R,

$$\tan 60^\circ = \frac{BR}{w_2R} = \frac{BQ + QR}{x}$$

$\Rightarrow$ $$\sqrt{3} = \frac{(H-6)+4}{x}$$

$\Rightarrow$ $$x = \frac{H-2}{\sqrt{3}} \quad \text{...(i)}$$



and in ΔBw_1Q,

$$\tan 30^\circ = \frac{BQ}{w_1Q}$$

$$\tan 30^\circ = \frac{H-6}{x} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $$x = \sqrt{3}(H-6) \quad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$\sqrt{3}(H-6) = \frac{(H-2)}{\sqrt{3}}$$

$$3(H-6) = H - 2 = 3H - 18 = H - 2$$

$\Rightarrow$ $$2H = 16 \Rightarrow H = 8$$

So, the required height is 8 m.

Hence, the required height of the balloon from above the ground is 8 m.