Chapter 13: Surface area volume
Exercise 13.1

Question 1:
If the radius of a sphere is 2r, then its volume will be
(a) $\frac{4}{3}\pi r^3$ (b) $4\pi r^3$ (c) $\frac{8}{3}\pi r^3$ (d) $\frac{32}{3}\pi r^3$

Solution:
(d) Given, radius of a sphere = 2r
Volume of a sphere =4/3 π$(\text{Radius})^3$
= 4/3 π$(2r)^3$ = 4/3 π $8r^3$
= (32 πr^3)/3 cu units
Hence the volume of a sphere is (32 πr^3)/3 cu units.

Question 2:
The total surface area of a cube is 96 cm^2 . The volume of the cube is
(a) 8 cm^3
(b) 512 cm^3
(c) 64 cm^3
(d) 27 cm^3
Solution:
(c) Surface area of a cube = 96 cm^2
Surface area of a cube = 6 $(\text{Side})^2$ = 96 => $(\text{Side})^2$ = 16
=> (Side) = 4 cm
[taking positive square root because side is always a positive quantity]
Volume of cube = $(\text{Side})^3$ = $(4)^3$ = 64 cm^3
Hence, the volume of the cube is 64 cm^3.

Question 3:
A cone is 8.4 cm high and the radius of its base is 2.1 cm. It is melted and recast into a sphere. The radius of the sphere is
(a) 4.2 cm
(b) 2.1 cm
(c) 2.4 cm
(d) 1.6 cm
Thinking Process

1. Firstly, determine the volume of a cone and volume of sphere using formula, volume of sphere = 4/3 πr^3 and volume of cone = 1/3 πr^2h
2. Further, equate volume of a cone and volume of sphere; so that we get the radius of sphere.

Solution:

(b) Given, height of a cone = 8.4 cm

Radius of the base of cone = 2.1 cm

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times (2.1)^2 \times 8.4$$

$$= \frac{1}{3} \times \pi \times 2.1 \times 2.1 \times 8.4 = \pi \times 4.41 \times 2.8 \text{ cm}^3$$

Since, cone is melted and recast into a sphere.

Let the radius of a sphere be R.

Then, volume of a sphere = Volume of a cone

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \pi \times 4.41 \times 2.8 \quad \Rightarrow \quad r^3 = \frac{4.41 \times 2.8 \times 3}{4}$$

$$\Rightarrow \quad r^3 = 4.41 \times 0.7 \times 3$$

$$\Rightarrow \quad r^3 = 4.41 \times 2.1$$

$$\therefore \quad r = 2.1$$

Hence, the radius of the sphere is 2.1 cm.

Question 4:
In a cylinder, radius is doubled and height is halved, then curved surface area will be
(a) halved
(b) doubled
(c) same
(d) four times
Solution:
(e) Let the radius be r and height be h of a cylinder.,
∴ Curved surface area of cylinder = 2πrh
We have, radius = 2r, height = h/2
New curved surface area = 2π (2r) x (h/2) = 2 πrh
Hence, the curved surface area will be same.

Question 5:
The total surface area of a cone whose radius is r/2 and slant height 2l is

(a) $2\pi r(l + r)$ (b) $\pi r\left(l + \frac{r}{4}\right)$ (c) $\pi r(l + r)$ (d) $2\pi rl$

Solution:

Question 6:
The radii of two cylinders are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 3. The ratio of their volumes is
(a) 10:17
(b) 20:27
(c) 17:27
(d) 20:37

Solution:

(b) Let the radii of two cylinders be r_1 and r_2 and height of the cylinder be h_1 and h_2.

Given, $\frac{r_1}{r_2} = \frac{2}{3}$ and $\frac{h_1}{h_2} = \frac{5}{3}$

$$\therefore \text{ Ratio of volumes} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{h_1}{h_2}\right)$$

$$= \left(\frac{2}{3}\right)^2 \left(\frac{5}{3}\right) = \frac{4}{9} \times \frac{5}{3} = 20:27$$

Hence, the ratio of their volumes is 20 : 27.

Question 7:
The lateral surface area of a cube is 256 m^2. The volume of the cube is
(a) 512 m^3
(b) 64 m^3
(c) 216 m^3
(d) 256 m^3
Solution:
(a) Given, lateral surface area of a cube = 256 m^2
We know that, lateral surface area of a cube = 4 x $(\text{Side})^2$
=> 256 = 4 x $(\text{Side})^2$
=> $(\text{Side})^2$ = 256/4 = 64
=> Side = √64 = 8 m
[taking positive square root because side is always a positive quantity]
Now, volume of a cube = $(\text{Side})^3$ = $(8)^3$ = 8 x 8 x 8 = 512 m^3
Hence, the volume of the cube is 512 m^3.

Question 8:
The number of planks of dimensions (4 m x 50cm x 20cm) that can be stored in a pit which is 16 m long, 12 m wide and 40 m deep is
(a) 1900
(b) 1920
(c) 1800
(d) 1840
Thinking Process

1. Firstly, determine the volume of plank and volume of pit by using the formula = l x b x h
2. Further, find the ratio of the volume of pit to the volume of the plank, to get the number of planks.

Solution:

(b) Given, dimensions of the plank are

$$l = 4\text{m},\ b = 50\text{cm} = \frac{50}{100}\text{m} = 0.5\text{ m}$$

and $$h = 20\text{cm} = \frac{20}{100}\text{ m} = 0.2\text{m}$$

$\therefore$ Volume of the plank $= l \times b \times h = 4 \times 0.5 \times 02 = 4\text{m}^3$

Also, given dimensions of the pit are

$$l = 16\text{m},\ b = 12\text{ m and } h = 40\text{ m}$$

$\therefore$ Volume of a pit $= l \times b \times h = (16 \times 12 \times 40)\text{m}^3$

$\therefore$ Number of planks $= \dfrac{\text{Volume of the pit}}{\text{Volume of the plank}} = \dfrac{16 \times 12 \times 40}{4}$

$$= 16 \times 12 \times 10 = 1920$$

Hence, the number of planks are 1920.

Question 9:
The length of the longest pole that can be put in a room of dimensions (10m x 10m x 5m)is
(a) 15 m
(b) 16 m
(c) 10 m
(d) 12 m
Solution:

(a) Given, dimensions of a room, $l = 10\text{m}$, $b = 10\text{m}$, $h = 5\text{m}$

$\therefore$ Length of the longest pole = Diagonal of a cuboid (room)

$$= \sqrt{l^2 + b^2 + h^2}$$

$$= \sqrt{(10)^2 + (10)^2 + (5)^2}$$

$$= \sqrt{100 + 100 + 25}$$

$$= \sqrt{225} = 15\text{m}$$

Hence, the length of the longest pole is 15 m.

Question 10:
The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. The ratios of the surface areas of the balloon in the two cases is
(a) 1 : 4
(b) 1 : 3
(c) 2 : 3
(d) 2 : 1

Solution:

(a) Given, radius of a hemispherical balloon, $r_1 = 6$ cm

Since, air is pumped into balloon. Then, radius of a hemispherical balloon, $r_2 = 12$ cm

$\therefore$ Ratio of the surface areas of the balloon in two cases $= \dfrac{3\pi r_1^2}{3\pi r_2^2}$

$$\Rightarrow \quad \frac{r_1^2}{r_2^2} = \frac{(6)^2}{(12)^2} = \frac{36}{144} = \frac{1}{4} = 1:4$$

Hence, ratio of the surface areas of the balloon in the two cases in 1 : 4.

Exercise 13.2: Very Short Answer Type Questions

Write whether True or False and justify your answer

Question 1:

The volume of a sphere is equal to two-third of the volume of a cylinder whose height and diameter are equal to the diameter of the sphere.

Solution:

True

Let the radius of the sphere and cylinder be r.

Given, height of the cylinder = diameter of the base => h = 2r

According to the given condition,

Volume of sphere = (2/3) x Volume of cylinder

$4/3\ \pi r^3 = (4/3) \times \pi r^2 \times 2r$

$4/3\ \pi r^3 = 4/3\ \pi r^3$

Hence, the volume of a sphere is equal to two-third of the volume of a cylinder.

Question 2:

If the radius of a right circular cone is halved and height is doubled, then volume will remain unchanged,

Solution:

False

Let the radius of a right circular cone be r and height of a right circular cone be h.

Then, the volume of a right circular cone $= \dfrac{1}{3}\pi r^2 h$

Now, $\quad r = \dfrac{r}{2},\ h = 2h$

New volume of a right circular cone $= \dfrac{1}{3}\pi\left(\dfrac{r}{2}\right)^2 \times (2h) = \dfrac{1}{3}\pi \times \dfrac{r^2}{4} \times 2h$

$$= \frac{1}{3}\pi \times \frac{r^2}{2}h = \frac{1}{2}\left(\frac{1}{3}\pi r^2 h\right)$$

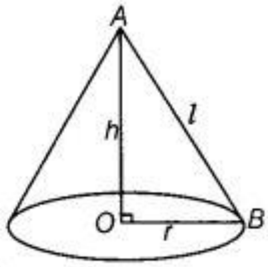
Hence, the new volume of a right circular cone becomes half of original volume of a right circular cone.

Question 3:

In a right circular cone, height, radius and slant height do not always be sides of a right triangle,

Solution:

True
On rotating a right-angled triangular lamina AOB about OA, it generates a cone. The point A is the vertex of a cone. Its base is a circle with centre O and radius OB. The length OA is the height of the cone and the length AB is called its slant height.



Clearly, $\angle AOB = 90^\circ$
Let the radius of the base = r units, height = h units
and slant height = l units, then

$$l^2 = h^2 + r^2 \Rightarrow l = \sqrt{h^2 + r^2}$$

Question 4:
If the radius of a cylinder is doubled and its curved surface area is not changed, the height must be halved.
Solution:
True
Let radius of hemisphere is r.

Volume of a cone, $V_1 = \frac{1}{3}\pi r^2 h$

$V_1 = \frac{1}{3}\pi r^2 (r)$ $[\because h = r]$

$\Rightarrow = \frac{1}{3}\pi r^3$

Now, Volume of a hemisphere, $V_2 = \frac{2}{3}\pi r^3$

and volume of cylinder, $V_3 = \pi r^2 h = \pi r^2 \times r = \pi r^3$ $[\because h = r]$

$\therefore V_1 : V_2 : V_3 = \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 = 1 : 2 : 3$

Hence, the ratio of their volumes is 1 : 2 : 3.

Question 5:
The volume of the largest right circular cone that can be fitted in a cube whose edge is 2r equals to the volume of a hemisphere radius r.
Solution:
True
Given, edge of cube = 2r, then height of cube becomes h = 2r.
Volume of a cone = $1/3\ \pi r^2 h = 1/3\ \pi r^2 (2r) = 2/3\ \pi r^3$
Volume of a hemisphere = $2/3\ \pi r^3$
Hence, the volume of a cone is equal to the volume of a hemisphere.

Question 6:
A cylinder and a right circular cone are having the same base and same height. The volume of the cylinder is three times the volume of the cone.

Solution:
True
Let the radius of the base of a cylinder and a right circular cone be r and height be h.
Then, Volume of a cylinder = $\pi r^2 h$
Volume of a cone = 1/3 $\pi r^2 h$
Volume of a cylinder = 3 x Volume of a cone
Hence, the volume of a cylinder is three times the volume of the right circular cone.

Question 7:
A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is 1 : 2 : 3.
Solution:
True
Let radius of hemisphere is r.
Volume of a cone, V_1= 1/3 $\pi r^2 h$
V_1 = 1/3 $\pi r^2(r)$ [$\therefore h = r$]
= 1/3 πr^3
Volume of a hemisphere, V_2=2/3 πr^3
volume of cylinder, $V_3 = \pi r^2 h = \pi r^2 \times r = \pi r^3$ [$\therefore h=r$]
$V_1 : V_2 : V_3$ =1/2 πr^3 : 2/3 πr^3 : πr^3 = 1 : 2 : 3
Hence, the ratio of their volumes is 1 : 2 : 3.

Question 8:
If the length of the diagonal of a cube is $6\sqrt{3}$ cm, then the length of the edge of the cube is 3 cm.
Solution:
False
Given, the length of the diagonal of a cube = $6\sqrt{3}$ cm
Let the edge (side) of a cube be a cm.
Then, diagonal of a cube =$a\sqrt{3}$
=> $6\sqrt{3} = a\sqrt{3}$
=> a = 6 cm
Hence, the edge of a cube is 6 cm.

Question 9:
If a sphere is inscribed in a cube, then the ratio of the volume of the cube to the volume of the sphere will be 6 : π.

Solution:

True

$$\text{Volume of cube } (V_1) = (\text{Side})^3$$

$$\text{Radius of sphere}(r) = \frac{\text{Side}}{2}$$

$$\text{Volume of sphere } (V_1) = \frac{4}{3}\pi(r^3)$$

$$= \frac{4}{3}\pi\left(\frac{\text{Side}}{2}\right)^3 = \frac{4}{3}\pi \times \frac{(\text{Side})^3}{8}$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{(\text{side})^3}{\frac{4}{3}\pi\frac{(\text{side})^3}{8}} = \frac{6}{\pi} \text{ or } 6 : \pi$$

Hence, the ratio of the volume of the cube to the volume of the sphere is $6 : \pi$.

Question 10:

If the radius of a cylinder is doubled and height is halved, the volume will be doubled.

Solution:

True

Let the radius of a cylinder be r and height be h.

Then, volume of a cylinder, $V_1 = \pi r^2 h$

If radius of a cylinder, $R = 2r$

and height of a cylinder, $H = \frac{h}{2}$

$\therefore$ Volume of a cylinder, $V_2 = \pi R^2 H = \pi(2r)^2 \frac{h}{2}$

$$= 2\pi r^2 h = 2 \times V_1$$

Hence, if the radius of a cylinder is doubled and height of a cylinder is halved, then volume of a cylinder is doubled.

Exercise 13.3: Short Answer Type Questions

Question 1:

Metal spheres, each of radius 2 cm, are packed into a rectangular box of internal dimensions 16 cm x 8 cm x 8 cm. When 16 spheres are packed the box is filled with preservative liquid. Find the volume of this liquid. Give your answer to the nearest integer, [use π = 3.14]

Thinking Process

1. Firstly, determine the volume of metallic sphere by using formula 4/3 πr^3 and then multiply by 16.
2. Secondly, determine the volume of internal rectangular box by using the formula = l x b x h
3. Finally, substract the volume of 16 metallic sphere from volume of internal rectangular box, to get volume of preservative liquid.

Solution:

Given, radius of each metal sphere = 2 cm

Internal dimensions of a rectangular box,

$$l = 16\text{cm},\ b = 8\text{cm and } h = 8\text{cm}$$

$\therefore$ Volume of a metallic sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times (2)^3$

$= \frac{4}{3} \times 3.14 \times 8$

Now, volume of 16 metallic spheres $= \frac{4 \times 3.14 \times 8 \times 16}{3} = \frac{100.48 \times 16}{3}$

$= \frac{1607.68}{3} = 535.89\text{cm}^3$

and internal volume of a rectangular box

$= l \times b \times h = 16 \times 8 \times 8$

$= 1024\ \text{cm}^3$

$\therefore$ Volume of preservative liquid $= 1024 - 535.89$

$= 488.11\ \text{cm}^3 \approx 488\ \text{cm}^3$

Hence, the volume of preservative liquid when 16 metallic spheres are packed in rectangular box is 488 cm^3.

Question 2:

A storage tank is in the form of a cube. When it is full of water, the volume of water is 15.625 m^3. If the present depth of water is 1.3 m, then find the volume of water already used from the tank.

Solution:

Let side of a cube be = x m

∴ Volume of cubical tank = 15.625 m^3 [given]

=> x^3 = 15.625 m^3

=> x = 2.5 m

and present depth of water in cubical tank = 1.3 m

∴ Height of water used =2.5 – 1.3 m = 1.2 m

Now, volume of water used = 1.2 x 2.5 x 2.5= 7.5 m^3

= 7. 5 x 1000 = 7500 L [∴ 1 m^3 = 1000 L]

Hence, the volume of water already used from the tank is 7500 L.

Question 3:

Find the amount of water displaced by a solid spherical ball of diameter 4.2 cm, when it is completely immersed in water.

Solution:

Given, diameter of a solid spherical ball = 42 cm

∴ Radius of a solid spherical ball, $r = \frac{42}{2} = 2.1\,\text{cm}$

Now, volume of water displaced by a solid spherical ball when it is completely immersed in water = Volume of a solid spherical ball

$$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1$$

$$= \frac{814.968}{21} = 38.808 = 38.81\,\text{cm}^3$$

Hence, the volume of water displaced by a solid spherical ball, when it is completely immersed in water is $38.81\ \text{cm}^3$.

Question 4:

How many square metres of canvas is required for a conical tent whose height is 3.5 m and the radius of the base is 12 m?

Thinking Process

1. Firstly, determine the slant height of conical test by using the formula, $l = \sqrt{h^2 + r^2}$
2. Further, using the formula for curved surface area =πrl which we get the required canvas.

Solution:

Given, height of a conical tent, $h = 3.5$ m and radius of the base of a conical tent, $r = 12$ m

Slant height, $l = \sqrt{h^2 + r^2} = \sqrt{(3.5)^2 + (12)^2}$

$$= \sqrt{12.25 + 144}$$

$$= \sqrt{156.25} = 12.5\ \text{m}$$

∴ Canvas required = Curved surface area of the cone (conical tent)

$$= \pi r l = \frac{22}{7} \times 12 \times 12.5$$

$$= 471.42\ \text{m}^2$$

Hence, the canvas required to make a conical tent is $471.42\ \text{m}^2$.

Question 5:

Two solid spheres made of the same metal have weights 5920 g and 740 g, respectively. Determine the radius of the larger sphere, if the diameter of the smaller one is 5 cm.

Given, Weight of one solid sphere, $m_1 = 5920$ g
and weight of another solid sphere, $m_2 = 740$ g
Diameter of the smaller sphere = 5 cm

$\therefore$ Radius of the smaller sphere, $r_2 = \frac{5}{2}$, $m_2 = 740$ g

We know that, $$\text{Density} = \frac{\text{Mass } (M)}{\text{Volume } (D)}$$

$$\Rightarrow \quad \text{Volume, } V = \frac{M}{D}$$

$$\Rightarrow \quad V_1 = \frac{5920}{D} \text{ cm}^3 \quad \text{...(i)}$$

and $$V_2 = \frac{740}{D} \text{ cm}^3 \quad \text{..(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{V_1}{V_2} = \frac{\frac{5920}{D}}{\frac{740}{D}}$$

$\because$ $$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{5920}{740} \Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \frac{592}{74}$$

$$\Rightarrow \quad \left(\frac{r_1}{5/2}\right)^3 = \frac{592}{74} \quad \left[\because r_2 = \frac{5}{2} \text{ cm}\right]$$

$$\Rightarrow \quad \frac{r_1^3}{\frac{125}{8}} = \frac{592}{74} \Rightarrow \frac{8r_1^3}{125} = \frac{592}{74}$$

$$\Rightarrow \quad r_1^3 = \frac{592}{74} \times \frac{125}{8} = \frac{74000}{592} = 125$$

$$\therefore \quad r_1 = 5 \text{ cm} \quad \text{[taking positive value of the cube root]}$$

Hence, the radius of larger sphere is 5 cm.

Question 6:
A school provides milk to the students daily in a cylindrical glasses of diameter 7 cm. If the glass is filled with milk upto an height, of 12 cm, find how many litres of milk is needed to serve 1600 students.
Solution:

Given, diameter of glass = 7 cm

Radius of glass, $r = \frac{7}{2}$ cm

$\therefore$ Milk contained in the cylindrical glass = Volume of cylindrical glass

$$= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 = 462 \text{ cm}^3$$

Now, milk required for 1600 students = $462 \times 1600 = 739200$ cm^3

$$= \frac{739200}{1000} = 7392 \text{ L} \quad \left[\because 1 \text{ cm}^3 = \frac{1}{1000} \text{ L}\right]$$

Hence, 739.2 L milk is needed to serve 1600 students.

Question 7:
A cylindrical roller 2.5 m in length, 1.75 m in radius when rolled on a road was found to cover the area of 5500 m^2. How many revolutions did it make?
Thinking Process

1. Firstly, determine the covered surface area of cylinder by using the formula 2πrh and surface area of cylinder covered in one revolution.
2. Divide total area covered by area covered by area covered by cylindrical roller in one revolution to get number of revolutions.

Solution:

Given, length of a cylindrical roller, $h = 2.5$ m
Radius of a cylindrical roller, $r = 1.75$ m
and total area rolled on a road by cylindrical roller = 5500 m^2

∴ Area rolled to cover in one revolution = Curved surface area of a cylindrical roller

$$= 2\pi rh = 2 \times \frac{22}{7} \times 1.75 \times 2.5$$

$$= \frac{44 \times 4.375}{7} = \frac{192.5}{7} = 27.5 \text{ m}^2$$

∴ Number of revolutions rolled by a cylindrical roller

$$= \frac{\text{Total area rolled by a cylindrical roller}}{\text{Area rolled to cover by a cylindrical roller in one revolution}} = \frac{5500}{27.5} = 200$$

Hence, the number of revolutions rolled by a cylindrical roller on a road is 200.

Question 8:
A small village, having a population of 5000, requires 75 L of water per head per day. The village has got an overhead tank of measurement 40 m x 25 m x 15 m. For how many days will the water of this tank last?
Solution:

Given, total population of a small village = 5000
Water required per head per day = 75 L
Volume of water required for a small village per day = 5000 × 75 = 375000 L

$$= \frac{375000}{1000} \text{ m}^3 = 375 \text{ m}^3 \qquad [\because 1 \text{ m}^3 = 1000 \text{ L}]$$

Total capacity of water in overhead tank = Volume of overhead tank

$$= 40 \times 25 \times 15 = 15000 \text{ m}^3$$

$$\therefore \quad \text{Number of days} = \frac{\text{Total capacity of water in over spead tank}}{\text{Volume of water required for a small village per day}}$$

$$= \frac{15000}{375} = 40 \text{ days}$$

Hence, water of this tank will be last in 40 days.

Question 9:
A shopkeeper has one spherical laddoo of radius 5 cm. With the same amount of material, how many laddoos of radius 2.5 cm can be made?
Thinking Process

1. Firstly, determine the volumes of laddoo and small laddoo by using the formula 4/3 πr^3
2. Further, find the quotient of volume of laddoo to the volume of small laddoo, to get the number of laddoos.

Solution:

Given, radius of a spherical laddoo, $r = 5$ cm

$\therefore$ Volume of a spherical laddoo $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (5)^3$

$= \frac{4}{3} \times 125\pi = \frac{500}{3}\pi \text{ cm}^3$

Now, radius of small laddoo = 2.5 cm

So, volume of small laddoo $= \frac{4}{3}\pi \times (2.5)^3 = \frac{62.5}{3}\pi \text{ cm}^3$

$$\therefore \text{Number of laddoos} = \frac{\text{Volume of laddoo}}{\text{Volume of small laddoo}} = \frac{\frac{500\pi}{3}}{\frac{62.5\pi}{3}} = \frac{500}{62.5} = 8$$

Hence, the number of laddoos are 8.

Question 10:
A right triangle with sides 6 cm, 8 cm and 10 cm is revolved about the side 8 cm. Find the volume and the curved surface of the solid so formed.
Solution:
When a right triangle with sides 6 cm, 8 cm and 10 cm is revolved about the side 8 cm, then solid formed is a cone whose height of a cone, h = 8 cm and radius of a cone, r = 6 cm. Slant height of a cone, l = 10 cm
Volume of a cone = 1/3 $\pi r^2 h$ = (1/3) x (22/7) x 6 x 6 x 8
=> 6336/21 = 301.7 cm^3
and curved surface of the area of cone = $\pi r l$
=> (22/7) x 6 x 10 = 1320/7 = 188.5 cm^2
Hence, the volume and surface area of a cone are 301.7 cm^3 and 188.5 cm^2, respectively.

Exercise 13.4: Long Answer Type Questions

Question 1:
A cylindrical tube opened at both the ends is made of iron sheet which is 2 cm thick. If the outer diameter is 16 cm and its length is 100 cm, find how many cubic centimetres of iron has been used in making the tube?

Solution:

Given, outer diameter of a cylindrical tube, $d = 16$ cm

$\Rightarrow$ r_1 = outer radius of a cylindrical tube = 8 cm [$\because d = 16$ cm]

r_2 = inner radius of a cylindrical tube = (r_1 − thickness of the iron sheet)

$= 8 - 2 = 6$ cm

and height of a cylindrical tube, $h = 100$ cm

Volume of metal used in making cylindrical tube

= Outer volume of a cylindrical tube − Inner volume

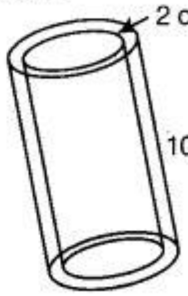
$= \pi r_1^2 h - \pi r_2^2 h = \pi h (r_1^2 - r_2^2)$

$= \frac{22}{7} \times 100(8^2 - 6^2)$

$= \frac{22}{7} \times 100\,(8 + 6)(8 - 6)$

$= \frac{22}{7} \times 100 \times 14 \times 2$

$= 2200 \times 4 = 8800\ \text{cm}^3$



Hence, $8800\ \text{cm}^3$ of iron has been used in making a cylindrical tube.

Question 2:
A semi-circular sheet of metal of diameter 28 cm is bent to form an open conical cup. Find the capacity of the cup.
Thinking Process

1. Firstly, determine the circumference of cone by using formula $2\pi R$ and circumference of semi-circle by using formula πr and equating them to get the radius of cone.
2. Secondly, determine the height of cone by using the formula $h = \sqrt{l^2 - R^2}$.
3. Further, determine the capacity of conical cup by using the formula, Capacity of cup = Volume of cup $= 1/3\ \pi R^2 h$

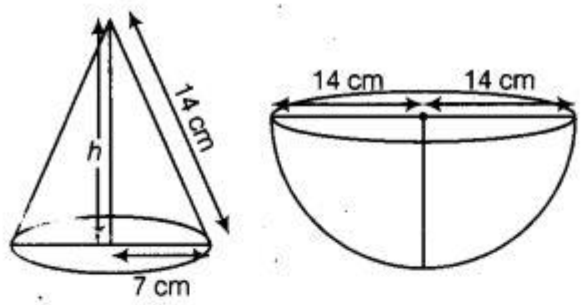
Solution:

Given, diameter of a semi-circular sheet = 28 cm

$\therefore$ Radius of a semi-circular sheet, $r = \frac{28}{2} = 14$ cm

Since, a semi-circular sheet of metal is bent to form an open conical cup.

Let the radius of a conical cup be R.



$\therefore$ Circumference of base of cone = Circumference of semi-circle

$$2\pi R = \pi r$$

$\Rightarrow$ $$2\pi R = \pi \times 14 \Rightarrow R = 7\text{cm}$$

Now, $$h = \sqrt{l^2 - R^2} = \sqrt{14^2 - 7^2} \qquad [\because l^2 = h^2 + R^2]$$

$$= \sqrt{196 - 49} = \sqrt{147} = 12.1243\text{ cm}$$

Volume (capacity) of conical cup $= \frac{1}{3}\pi R^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12.1243 = 622.38\text{ cm}^3$$

Hence, the capacity of an open conical cup is 622.38 cm^3.

Question 3:

A cloth having an area of 165 m^2 is shaped into the form of a conical tent of radius 5 m.

(i) How many students can sit in the tent, if a student on an average occupies 5/7 m^2 on the ground?

(ii) Find the volume of the cone.

Solution:

(i) Given, radius of the base of a conical tent = 5 m

and area needs to sit a student on the ground $= \frac{5}{7}\ m^2$

$\therefore$ Area of the base of a conical tent $= \pi r^2$

$$= \frac{22}{7} \times 5 \times 5\ m^2$$

Now, number of students $= \dfrac{\text{Area of the base of a conical tent}}{\text{Area needs to sit a student on the ground}}$

$$= \frac{\frac{22 \times 5 \times 5}{7}}{5/7} = \frac{22}{7} \times 5 \times 5 \times \frac{7}{5} = 110$$

Hence, 110 students can sit in the conical tent.

(ii) Given, area of the cloth to form a conical tent = 165 m^2

Radius of the base of a conical tent, $r = 5$ m

Curved surface area of a conical tent = Area of cloth to form a conical tent

$\Rightarrow$ $\pi r l = 165$

$\Rightarrow$ $\frac{22}{7} \times 5 \times l = 165$

$\therefore$ $l = \frac{165 \times 7}{22 \times 5} = \frac{33 \times 7}{22} = 10.5\ m$

Now, height of a conical tent $= \sqrt{l^2 - r^2} = \sqrt{(10.5)^2 - (5)^2}$

$$= \sqrt{110.25 - 25} = \sqrt{85.25} = 9.23\ m$$

Volume of a cone (conical tent) $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 9.23$

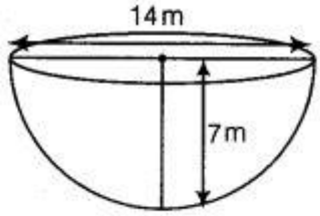
$$= \frac{1}{3} \times \frac{1550 \times 9.23}{7} = \frac{5076.5}{7 \times 3} = 241.7\ m^3$$

Hence, the volume of the cone (conical tent) is 241.7 m^3.

Question 4:

The water for a factory is stored in a hemispherical tank whose internal diameter is 14 m. The tank contains 50 kL of water. Water is pumped into the tank to fill to its capacity. Calculate the volume of water pumped into the tank.

Solution:
Given, internal diameter of a hemispherical tank = 14m
and internal radius of a hemispherical tank, $r = 7$m



Water in the hemispherical tank = 50 kL = 50×1000 L = 50000 L [$\because$ 1 kL = 1000L]
= $50\,m^3$ [$\because 1\,m^3 = 1000$ L]

$\therefore$ Volume of water in the hemispherical tank $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (7)^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = 718.66\,m^3$$

Now, volume of water pumped in the hemispherical tank = $718.66 - 50 = 668.66\,m^3$
Hence, the volume of water pumped into the tank is $668.66\,m^3$.

Question 5:
The volumes of the two spheres are in the ratio 64 : 27. Find the ratio of their surface areas.
Solution:
Let the radius of two spheres be r_1 and r_2.
Given, the ratio of the volume of two spheres = 64 : 27

$$\frac{V_1}{V_2} = \frac{64}{27} \Rightarrow \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

$\Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{4}{3}\right)^3$ $\left[\because \text{volume of sphere} = \frac{4}{3}\pi r^3\right]$

$\Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$

Let the surface areas of the two spheres be S_1 and S_2.

$\therefore \frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 \Rightarrow S_1 : S_2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

$\Rightarrow S_1 : S_2 = 16 : 9$

Hence, the ratio of the their surface areas is 16 : 9.

Question 6:
A cube of side 4 cm contains a sphere touching its sides. Find the volume of the gap in between.
Solution:
Given, side of a cubp = 4 cm
Side of cube = Diameter of sphere

4 = Diameter of sphere

$\therefore$ Radius of sphere $= \frac{4}{2} = 2$ cm

Volume of the gap = Volume of cube − Volume of sphere

$= (\text{Side})^3 - \frac{4}{3}\pi r^3$

$= (4)^3 - \frac{4}{3}\pi(2)^3$ [$\because$ side of cube = diameter of sphere]

$= \left(64 - \frac{4}{3} \times \frac{22}{7} \times 8\right) = 64 - 33.52 = 30.48 \text{ cm}^3$

Hence, the volume of the gap in between a cube and a sphere is 30.48 cm^3.

Question 7:
A sphere and a right circular cylinder of the same radius have equal volumes. By what percentage does the diameter of the cylinder exceed its height?
Solution:

Let the radius of sphere = r = Radius of a right circular cylinder
According to the question,

Volume of cylinder = Volume of a sphere

$\Rightarrow \pi r^2 h = \frac{4}{3}\pi r^3 \Rightarrow h = \frac{4}{3}r$

$\because$ Diameter of the cylinder = $2r$

$\therefore$ Increased diameter from height of the cylinder $= 2r - \frac{4r}{3} = \frac{2r}{3}$

Now, percentage increase in diameter of the cylinder $= \frac{\frac{2r}{3} \times 100}{\frac{4}{3}r} = 50\%$

Hence, the diameter of the cylinder exceeds its height by 50%.

Question 8:
30 circular plates, each of radius 14 cm and thickness 3 cm are placed one above the another to form a cylindrical solid.
Find

1. the total surface area.
2. volume of the cylinder so formed.

Solution:
Given, radius of a circular plate, r = 14 cm
Thickness of a circular plate = 3 cm
Thickness of 30 circular plates = 30 x 3 = 90 cm
Since, 30 circular plates are placed one above the another to form a cylindrical solid.
Then, Height of the cylindrical solid, h = Thickness of 30 circular plates = 90 cm

1. Total surface area of the cylindrical solid so formed
   = 2πr(h+ r)=2 x (22/7) x 14(90+ 14)

$= 44 \times 2 \times 104 = 9152\ cm^2$
Hence, the total surface area of the cylindrical solid is $9152\ cm^2$,

2. Volume of the cylinder so formed = $\pi r^2 h$
   $= (22/7)\ (14)^2 \times 90 = (22/7) \times 14 \times 14 \times 90$
   $= 22 \times 28 \times 90 = 55440\ cm^3$
   Hence, the volume of the cylinder so formed is $55440\ cm^3$.