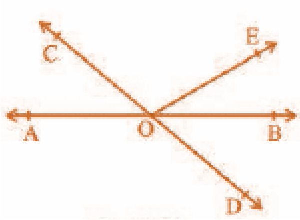
Chapter 6: lines and angles

Q.1: In the figure, lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.



Solution:

From the given figure, we can see;

∠AOC, ∠BOE, ∠COE and ∠COE, ∠BOD, ∠BOE form a straight line each.

So, ∠AOC + ∠BOE +∠COE = ∠COE +∠BOD + ∠BOE = 180°

Now, by substituting the values of ∠AOC + ∠BOE = 70° and ∠BOD = 40° we get:

70° +∠COE = 180°

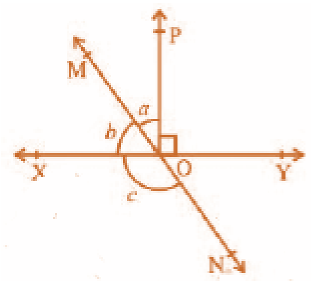
∠COE = 110°

Similarly,

110° + 40° + ∠BOE = 180°

∠BOE = 30°

Q.2: In the Figure, lines XY and MN intersect at O. If ∠POY = 90° and a: b = 2 : 3, find c.



Solution:

As we know, the sum of the linear pair is always equal to 180°

So,

∠POY + a + b = 180°

Substituting the value of ∠POY = 90° (as given in the question) we get,

a + b = 90°

Now, it is given that a: b = 2 : 3 so,

Let a be 2x and b be 3x.

∴ 2x + 3x = 90°

Solving this we get

5x = 90°

So, x = 18°

∴ a = 2 × 18° = 36°

Similarly, b can be calculated and the value will be

b = 3 × 18° = 54°

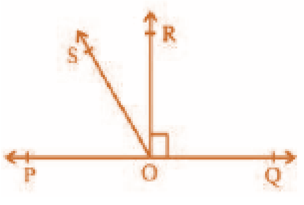
From the diagram, b + c also forms a straight angle so,

b + c = 180°

=> c + 54° = 180°

∴ c = 126°

Q.3: In Figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that ∠ROS = 1/2(∠QOS – ∠POS).



Solution:

In the question, it is given that (OR ⊥ PQ) and ∠POQ = 180°

So, ∠POS + ∠ROS + ∠ROQ = 180° (Linear pair of angles)

Now, ∠POS + ∠ROS = 180° – 90° (Since ∠POR = ∠ROQ = 90°)

∴ ∠POS + ∠ROS = 90°

Now, ∠QOS = ∠ROQ + ∠ROS

It is given that ∠ROQ = 90°,

∴ ∠QOS = 90° + ∠ROS

Or, ∠QOS – ∠ROS = 90°

As ∠POS + ∠ROS = 90° and ∠QOS – ∠ROS = 90°, we get

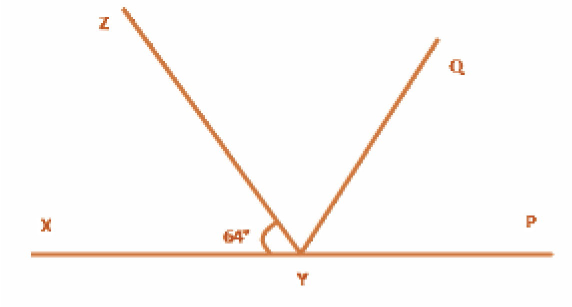
∠POS + ∠ROS = ∠QOS – ∠ROS

=>2 ∠ROS + ∠POS = ∠QOS

Or, ∠ROS = ½ (∠QOS – ∠POS) (Hence proved).

Q.4: It is given that ∠XYZ = 64° and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠ZYP, find ∠XYQ and reflex ∠QYP.

Solution:



Here, XP is a straight line

So, ∠XYZ +∠ZYP = 180°

substituting the value of ∠XYZ = 64° we get,

64° +∠ZYP = 180°

∴ ∠ZYP = 116°

From the diagram, we also know that ∠ZYP = ∠ZYQ + ∠QYP

Now, as YQ bisects ∠ZYP,

∠ZYQ = ∠QYP

Or, ∠ZYP = 2∠ZYQ

∴ ∠ZYQ = ∠QYP = 58°

Again, ∠XYQ = ∠XYZ + ∠ZYQ

By substituting the value of ∠XYZ = 64° and ∠ZYQ = 58° we get.

∠XYQ = 64° + 58°

Or, ∠XYQ = 122°

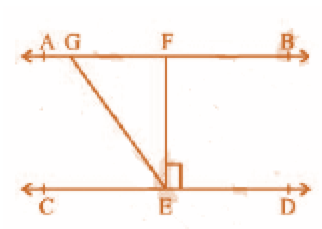
Now, reflex ∠QYP = 180° + ∠XYQ

We computed that the value of ∠XYQ = 122°. So,

∠QYP = 180° + 122°

∴ ∠QYP = 302°

Q.5: In the Figure, if AB || CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.



Solution:

Since AB || CD GE is a transversal.

It is given that ∠GED = 126°

So, ∠GED = ∠AGE = 126° (alternate interior angles)

Also,

∠GED = ∠GEF + ∠FED

As

EF ⊥ CD, ∠FED = 90°

∴ ∠GED = ∠GEF + 90°

Or, ∠GEF = 126° – 90° = 36°

Again, ∠FGE + ∠GED = 180° (Transversal)

Substituting the value of ∠GED = 126° we get,

∠FGE = 54°

So,

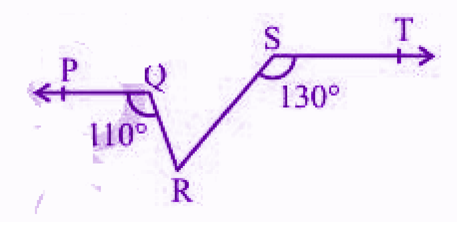
∠AGE = 126°

∠GEF = 36° and

∠FGE = 54°

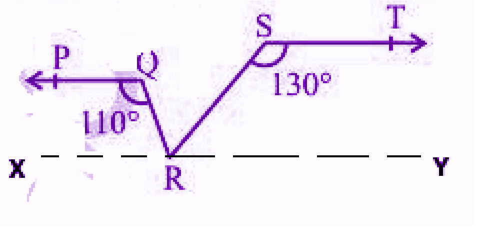
Q.6: In Figure, if PQ || ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.

[Hint: Draw a line parallel to ST through point R.]



Solution:

First, construct a line XY parallel to PQ.



As we know, the angles on the same side of the transversal are equal to 180°.

So, ∠PQR + ∠QRX = 180°

Or,∠QRX = 180° – 110°

∴ ∠QRX = 70°

Similarly,

∠RST + ∠SRY = 180°

Or, ∠SRY = 180° – 130°

∴ ∠SRY = 50°

Now, for the linear pairs on the line XY-

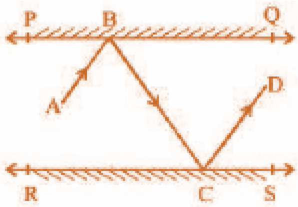
∠QRX + ∠QRS + ∠SRY = 180°

Substituting their respective values we get,

∠QRS = 180° – 70° – 50°

Or, ∠QRS = 60°

Q.7: In Fig. 6.33, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects along CD. Prove that AB || CD.

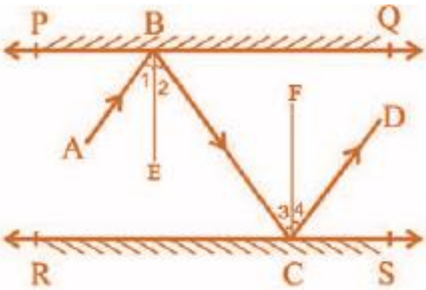


Solution:

First, draw two lines BE and CF such that BE ⊥ PQ and CF ⊥ RS.

Now, since PQ || RS,

So, BE || CF



BE and CF are normals between the incident ray and reflected ray.

As we know,

The angle of incidence = Angle of reflection (By the law of reflection)

So,

∠1 = ∠2 and

∠3 = ∠4

We also know that alternate interior angles are equal.

Here, BE ⊥ CF and the transversal line BC cuts them at B and C.

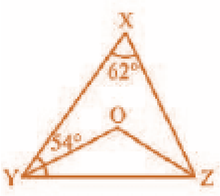
So, ∠2 = ∠3 (As they are alternate interior angles)

Now, ∠1 + ∠2 = ∠3 + ∠4

Or, ∠ABC = ∠DCB

So, AB ∥ CD (alternate interior angles are equal)

Q.8: In Fig. 6.40, ∠X = 62°, ∠XYZ = 54°. If YO and ZO are the bisectors of ∠XYZ and ∠XZY respectively of Δ XYZ, find ∠OZY and ∠YOZ.



Solution:

As we know, the sum of the interior angles of the triangle is 180°.

So, ∠X +∠XYZ + ∠XZY = 180°

substituting the values as given in the question we get,

62° + 54° + ∠XZY = 180°

Or, ∠XZY = 64°

Now, As we know, ZO is the bisector so,

∠OZY = ½ ∠XZY

∴ ∠OZY = 32°

Similarly, YO is a bisector and so,

∠OYZ = ½ ∠XYZ

Or, ∠OYZ = 27° (As ∠XYZ = 54°)

Now, as the sum of the interior angles of the triangle,

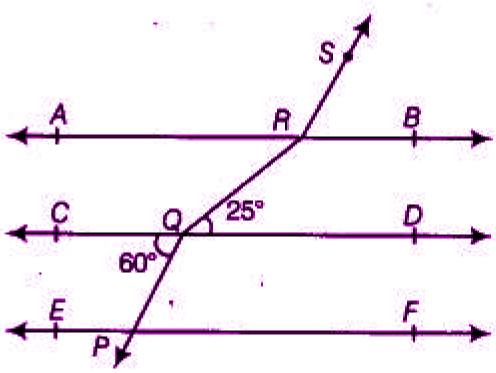
∠OZY +∠OYZ + ∠O = 180°

Substituting their respective values we get,

∠O = 180° – 32° – 27°

Or, ∠O = 121°

Q.9: In the figure, if AB || CD || EF, PQ || RS, ∠RQD = 25° and ∠CQP = 60°, then find ∠QRS.



Solution:

According to the given figure, we have

AB || CD || EF

PQ || RS

∠RQD = 25°

∠CQP = 60°

PQ || RS.

As we know,

If a transversal intersects two parallel lines, then each pair of alternate exterior angles is equal.

Now, since, PQ || RS

⇒ ∠PQC = ∠BRS

We have ∠PQC = 60°

⇒ ∠BRS = 60° … eq.(i)

We also know that,

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

Now again, since, AB || CD

⇒ ∠DQR = ∠QRA

We have ∠DQR = 25°

⇒ ∠QRA = 25° … eq.(ii)

Using linear pair axiom,

We get,

∠ARS + ∠BRS = 180°

⇒ ∠ARS = 180° – ∠BRS

⇒ ∠ARS = 180° – 60° (From (i), ∠BRS = 60°)

⇒ ∠ARS = 120° … eq.(iii)

Now, ∠QRS = ∠QRA + ∠ARS

From equations (ii) and (iii), we have,

∠QRA = 25° and ∠ARS = 120°

Hence, the above equation can be written as:

∠QRS = 25° + 120°

⇒ ∠QRS = 145°

Q. Of the three angles of the triangle, one is twice the smallest and another is three times the smallest. Find the angles.

Let the smallest angle be x^0 and the other two be $2x^0$ and $3x^0$

$x^0 + 2x^0 + 3x^0 = 180^0$

$6x^0 = 180$

$x = 30^0$

Hence, the angles are 30^0, 60^0, 90^0