Chapter 12- Areas Related to Circle
Exercise – 12.1

Question 1: The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has a circumference equal to the sum of the circumferences of the two circles.

Answer: The radius of the 1st circle = 19 cm [Given]
hence, the circumference of the 1st circle = $2\pi\times19 = 38\pi$ cm

The radius of the 2nd circle = 9 cm [Given]
Hence, the circumference of the 2nd circle = $2\pi\times9 = 18\pi$ cm

Therefore, the sum of the circumference of two circles = $38\pi+18\pi = 56\pi$ cm

Now, let the radius of the 3rd circle = R
And the circumference of the 3rd circle = $2\pi R$

The sum of the circumference of two circles = circumference of the 3rd circle [Given]

Hence, $56\pi = 2\pi R$
or, R = 28 cm.

Question 2: The radii of two circles are 8 cm and 6 cm, respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

Answer: Radius of 1st circle = 8 cm [Given]
Therefore, the area of 1st circle = $\pi(8)^2 = 64\pi$

Radius of 2nd circle = 6 cm [Given]
Therefore, the area of 2nd circle = $\pi(6)^2 = 36\pi$

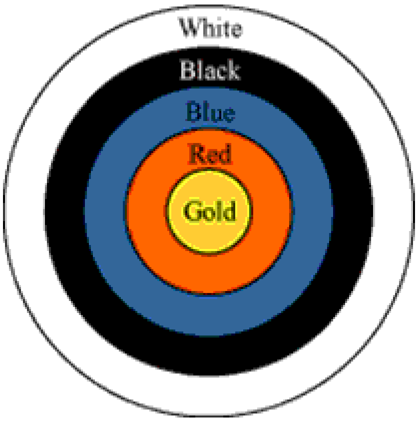
So, the sum of 1st and 2nd circle will be = $64\pi + 36\pi = 100\pi$

Now, assume that the radius of 3rd circle = R
Hence, the area of the circle 3rd circle = πR^2

The area of the circle 3rd circle = Area of 1st circle + Area of 2nd circle [Given]
or, $\pi R^2 = 100\pi cm^2$
or, $R^2 = 100cm^2$

So, R = 10cm

Question 3: The given figure depicts an archery target marked with its five scoring regions from the center outwards as Gold, Red, Blue, Black, and White.



The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

Answer: Diameter of the region representing gold region = 21 cm

Hence, the radius of the gold region = $\frac{21}{2}$ cm

So, the area of the gold region = $\pi r^2 = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{11 \times 3 \times 21}{2}$

Hence, the area of the gold region is 346.5 cm^2

Width of the red region = 10.5 cm = $\frac{21}{2}$ cm

Radius of the red region including gold region

= radius of the gold region + width of the red region

= $\left(\frac{21}{2} + \frac{21}{2}\right)$cm = 21cm

Hence, the area of the red region including gold region

= $\pi r^2 = \frac{22}{7} \times 21 \times 21 = 22 \times 3 \times 21 = 1386$ cm^2

Area of the red region only = Area of the red region including gold region – Area of the gold region

= (1386 – 346.5) cm^2

Area of the red region only = 1039.5 cm^2

Now, width of the blue circle = 10.5 cm = $\frac{21}{2}$ cm

Therefore, the radius upto the blue region

= Radius of the gold region + width of the red region + width of the blue region

$= \left(\frac{21}{2} + \frac{21}{2} + \frac{21}{2}\right)$ cm
$= \frac{63}{2}$ cm

Hence, area of the blue region including red and gold region = $\pi r^2 = \frac{22}{7} \times \frac{63}{2} \times \frac{63}{2} =$ 3118.5 cm^2

Area of the blue region only
= Area of the (black + blue + red + gold) region - Area of the (red + gold) region
= (3118.5 – 1386) cm^2
<u>Area of the blue region only = 1732.5 cm^2</u>

Radius upto the black region = $\left(\frac{21}{2} + \frac{21}{2} + \frac{21}{2} + \frac{21}{2}\right)$ cm = $\frac{84}{2}$ cm = 42 cm

Area including (black + blue + red + gold) region = $\frac{22}{7} \times 42 \times 42$ = 5544cm^2

Area of the (blue + red + gold) region = 3118.5 cm^2
Area of the black region only
= Area of the (black + blur + red + gold) region – Area of the (blue + red + gold) region
= 5544 cm^2 – 3118.5 cm^2
= 2425.5 cm^2

Radius upto the white boundary
= Radius upto the black region + width of the white region
= 42 cm + $\frac{21}{2}$ cm
$= \frac{105}{2}$ cm

Area of the (white + black + blue + red + gold) region
$= \frac{22}{7} \times \frac{105}{2} \times \frac{105}{2}$ cm^2
= 8662.5 cm^2

Area of the white region only
= Area of the (white + black + blue + red + gold) region – area of the (black + blue + red + gold) region
= 8662.5 cm^2 – 5544 cm^2
= 3118.5 cm^2

Question 4: The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

Answer: Radius of one wheel of the car = 40 cm = 0.4 m
Distance covered by the wheel to complete one revolution = 2πr $\times 0.4 = 0.8$ m

Let the wheel of the car complete revolutions in 10 mins at a speed of 66km/hr
Then the distance covered by the wheel in making “r” complete revolutions in 10 mins
= $(0.8\pi \times n)$ m

Also, distance travelled by car in 60 mins = (66×1000) m

Therefore, distance travelled by car in 1min = $\frac{66\times1000}{60}$ m

Distance travelled by car in 10mins = $\frac{66\times1000\times10}{60}$ m = 11000m

Hence, according to the problem,
$0.8\pi \times n = 11000$

or, $n = \frac{11000}{0.8\pi}$
or, $n = \frac{11000\times7}{0.8\times22}$
or, $n = 625 \times 7 = 4375$

Hence, the wheel makes 4375 complete revolutions in 10 mins.

Question 5: Tick the correct answer in the following and justify your choice: If the perimeter and the area of a circle are numerically equal, then the radius of the circle is
(a) 2 units
(b) n units
(c) 4 units
(d) 7 units

Answer: Let the radius of the circle = r units
Perimeter of the circle = $2\pi r$
Area of the circle = πr^2

Now, according to the question,
Perimeter of the circle = Area of the circle
or, $2\pi r = \pi r^2$
or, r = 2 units

Hence, option (a) is correct.

Exercise 2.2

Question 1: Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Answer: Radius of the sector (r) = 6 cm
Angle of sector (θ) = 60°

Hence, the area of the sector = $\pi r^2 \times \frac{\theta}{360°}$ = $\left[\frac{22}{7} \times 6 \times 6 \times \frac{60°}{360°}\right]$ = $\frac{22\times6\times6}{7\times6}$ = 18.86 cm^2

Question 2: Find the area of a quadrant of a circle whose circumference is 22 cm.

Answer: Let radius if the circle be "r"
Hence, circumference of the circle = $2\pi r$

According to the question,
$2\pi r = 22$ cm
or, $2 \times \frac{22}{7} \times r = 22$

or, $r = \frac{22\times7}{2\times22}$

or, $r = \frac{7}{2}$ cm

Area of the quadrant of the circle = $\frac{\pi r^2\theta}{360°} = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{90°}{360°} = \frac{77}{8}$ cm^2

Question 3: The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Answer: Angle described by the minute hand in 60 minutes = 360^0
Angle described by the minute hand in 5 minutes = $\frac{360°\times5}{60°}$ = 30°
Now, we have $\theta = 30°$ and r = 14 cm

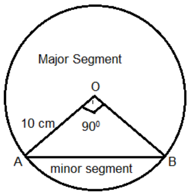
Hence, required area swept by the minute hand in 5 minutes = Area of the sector with r = 14cm
and $\theta = 30°$
$= \frac{\pi r^2\theta}{360°}$ cm^2

$= \left[\frac{22}{7} \times 14 \times 14 \times \frac{30°}{360°}\right]$ cm^2

= 51.33 cm^2

Question 4: A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:
(i) minor segment
(ii) major segment (Use π = 3.14)

Answer:



Radius of the circle = 10 cm.
Angle subtended by chord at centre = 90^0

(i) Area of the minor segment
= Area of the sector OAPB – area of ΔAOB
= $\left[\frac{\pi r^2\theta}{360°} - \frac{1}{2}r^2\sin\theta\right]$
= $3.14 \times \frac{10\times10\times90°}{360°} - \frac{1}{2} \times 10 \times 10 \times \sin 90°$

= 3.14 × 25 – 50
= 78.5 – 50
= 28.5 cm^2

(ii) Area of the major segment = Area of the circle – area of the minor segment
= πr^2 – 28.5
= $3.14 \times 10 \times 10 - 28.5$
= 314 – 28.5
= 285.5 cm^2

Question 5: In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
(i) length of the arc.
(ii) area of the sector formed by the arc.
(iii) area of the segment formed by the corresponding chord.

Answer: Radius of the circle (r) = 21 cm
Angle of the sector, θ = 60°

(i) Length of the arc = $\frac{\theta}{360°} \times 2\pi r$
= $\frac{60°}{360°} \times 2 \times \frac{22}{7} \times 21$

= $\frac{1}{6} \times 2 \times 22 \times 3$
= 22 cm

(ii) Area of the sector formed by the arc,
= $\pi r^2 \times \frac{\theta}{360°}$

= $\frac{22}{7} \times 21 \times 21 \times \frac{60°}{360°}$

= 11 × 21
= 231 cm^2

(iii) From the figure, OA = OB [Radii of the same circle]
$\angle A = \angle B = \frac{1}{2}(180° - 60°)$; $\angle OAB$ is an equilateral triangle.

Therefore, the area of the equilateral triangle OAB = $\frac{\sqrt{3}}{4}$ (side)^2 = $\frac{\sqrt{3}}{4}$ (21)^2 = $\frac{441\sqrt{3}}{4}$ cm^2

Hence, the area of the segment formed by the chord
= Area of the sector – Area of the equilateral triangle
= $\left(231 - \frac{441\sqrt{3}}{4}\right)$ cm^2

Question 6: A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and √3 = 1.73)

Answer: Radius of the circle = 15 cm
Angle of the subtended by chord at centre = 60°

Area of the sector = $\frac{\pi r^2 \theta}{360°}$ = 3.14 × $\frac{15 \times 15 \times 60°}{360°}$ = 117.75 cm^2

Area of the triangle formed by radii and chord
= $\frac{1}{2} r^2 \sin\theta$

= $\frac{1}{2}(15)^2 \sin 60°$

= $\frac{1}{2} \times 15 \times 15 \times \frac{\sqrt{3}}{2}$

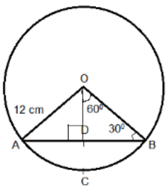
= 97.31 cm^2

Area of the minor segment
= Area of the sector – Area of the triangle formed by radii and chord
= (117.75 – 97.31) cm^2
= 20.44 cm^2

Area of the circle = πr^2 = $(3.14 \times 15 \times 15)$cm^2 = 706.5 cm^2
Area of the major segment = Area of the circle – Area of the minor segment
= (706.5 – 20.44)cm^2 = 686.06 cm^2

Question 7: A chord of a circle of the radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and √3 = 1.73).

Answer:



Let AB be a chord which subtends an angle 120^0 at the centre O of the circle.
Area of the segment ACB = Area of sector OACB – Area of triangle OAB………………………..…(1)

Area of the sector OACB $= \frac{120°}{360°} \times 3.14 \times 12 \times 12$

$= \frac{1}{3} \times 3.14 \times 12 \times 12$
= 150.72 cm^2
…………………………………………………………………..(2)

We draw OD⊥AB.
Therefore, $\angle OBD$ = 180^0 – $(90^0 + 60^0)$ = 30^0

Now from triangle ODB ,
sin 30^0 = $\frac{OD}{OB}$
or, $\frac{1}{2} = \frac{OD}{12}$
or, OD = 6 cm

Also, cos 30^0 = $\frac{BD}{OB}$
or, $\frac{\sqrt{3}}{2} = \frac{BD}{12}$
or, BD = $12\sqrt{3}$ cm

Area of triangle OAB = $\frac{1}{2} \times AB \times OD = \frac{1}{2} \times 12\sqrt{3} \times 6 = 36\sqrt{3}$ cm^2
………………………………..(3)

From equations, (1), (2) and (3), we get,
Area of the segment ACB = $(150.72 - 36\sqrt{3})$ cm^2
= 88.37 cm^2

Hence, the are of the segment of the circle = 99.37 cm^2

Question 8: . A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 12.11). Find

(i) the area of that part of the field in which the horse can graze.
(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

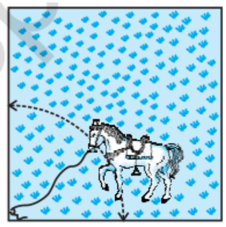


Fig. 12.11

Answer: As per the question, the horse is tied at one end of a square field, it will graze only a quarter (i.e. θ = 90°) of the field with radius 5 m.
The length of rope = the radius of the circle (r) = 5 m

It is also known that the side of square field = 15 m

(i) Area of circle = $\pi r^2 = \frac{22}{7} \times 5^2 = 78.5$ m^2
Now, the area of the part of the field where the horse can graze
= ¼ (the area of the circle)
= $\frac{78.5}{4}$
= 19.625 m^2

(ii) If the length of the rope, increased to 10 m,

Area of circle will be
= πr^2
= $\frac{22}{7} \times 10^2$
= 314 m^2

Now, the area of the part of the field where the horse can graze =
¼ (the area of the circle)

= $\frac{314}{4}$ m^2
= 78.5 m^2

Hence, increase in the grazing area = 78.5 m^2 – 19.625 m^2 = 58.875 m^2

Question 9: **A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. 12.12. Find:**

(i) the total length of the silver wire required.
(ii) the area of each sector of the brooch.

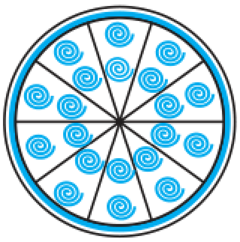


Fig. 12.12

Answer : Diameter of the brooch (D) = 35 mm [Given]
Total number of diameters to be considered= 5

Now, the total length of 5 diameters that would be required = 35 × 5 = 175

Circumference of the circle = $2\pi r$
or, $C = \pi D = \frac{22}{7} \times 35 = 110$

Area of the circle = πr^2
or, $A = \frac{22}{7} \times \left[\frac{35}{2}\right]^2 = \frac{1925}{2}$ mm^2

(i) Total length of silver wire required
= Circumference of the circle + Length of 5 diameter
= 110 + 175
= 185 mm

(ii) Total Number of sectors in the brooch = 10
So, the area of each sector = total area of the circle ÷ number of sectors
Hence, the area of each sector = $\frac{1925}{2} \times \frac{1}{10} = \frac{385}{4}$ mm^2

Question 10: **An umbrella has 8 ribs which are equally spaced (see Fig. 12.13). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.**

Fig. 12.13

Answer: Angle between two consecutive ribs = $\frac{central\ angle\ of\ the\ circle}{number\ of\ the\ sectors} = \frac{360°}{8}$ = 45°

Area between two consecutive ribs = Area of one sector of the circle.

$$= \frac{\pi r^2 \theta}{360°}$$
$$= \frac{22}{7} \times \frac{45 \times 45 \times 45°}{360°}$$
$$= \frac{22275}{28} \text{ cm}^2$$

Question 11: A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Answer: The radius (r) = 25 cm and sector angle (θ) = 115° [Given]

Since there are 2 blades, the total area of the sector made by wiper

$= 2 \times \frac{\theta}{360°} \times \pi r^2$

$= 2 \times \frac{115}{360} \times \frac{22}{7} \times 25^2$

$= 1254.96 \text{ cm}^2$

Question 12: To warn ships for underwater rocks, a lighthouse spreads a red colored light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned.

(Use π = 3.14)

Answer: Here the radius will be the distance over which light spreads.
Radius (r) = 16.5 km [Given]

Sector angle (θ) = 80°

Now, the total area of the sea over which the ships are warned = Area made by the sector

or, Area of sector = $\frac{\theta}{360°} \times \pi r^2$

$= \frac{80°}{360°} \times \pi r^2$ km^2

$= 189.97$ km^2

Question 13: 13. A round table cover has six equal designs as shown in Fig. 12.14. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2 . (Use $\sqrt{3}$ = 1.7)

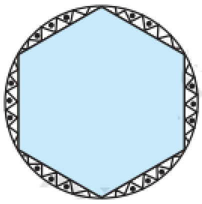
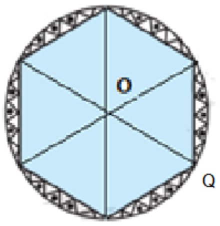


Fig. 12.14

Answer:



Here, the radius of the cover (r) = 28 cm and the central angle (θ) = $\frac{360°}{6}$ = 60^0
We need to join OP and OQ.

Then, triangle OPQ, is an equilateral triangle with side 28cm.

Therefore, area of one designed segment,
= Area of sector OPQ – Area of the triangle OPQ

$= \frac{60°}{360°} \times \pi \times 28^2 - \frac{\sqrt{3}}{4} \times 28^2$

$= 28^2 \times \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

$= 28^2 \times \left(\frac{22}{7\times6} - \frac{1.7}{4}\right)$

$= 28^2 \times \left(\frac{44-35.7}{84}\right)$

= 77.47 cm^2

Total area of the 6 designed segments = 6 × 77.47 cm^2 = 464.82 cm^2

Hence, cost of making the designs = Rs. 464.82 × 0.35 = Rs. 162.69

Question 14: Tick the correct solution in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is

(A) $\frac{p}{180} \times 2\pi R$
(B) $\frac{p}{180} \times \pi R^2$

(C) $\frac{p}{360}$ **× 2πR**
(D) $\frac{p}{720}$ **× 2πR²**

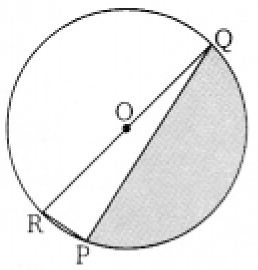
Answer: Angle of the sector is p degrees.
Radius of the circle = R

Area of the sector = $\frac{\pi R^2 p}{360°} = \frac{(\pi R^2 p)2}{720°} = \frac{p}{720°} \times 2\pi R^2$

Hence, the correct option is (D).

Exercise 12.3

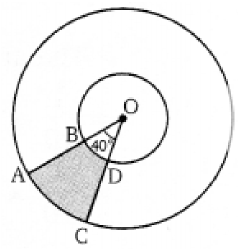
Question 1: Find the area of the shaded region in the given figure, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.



Answer: In ΔPQR = 90° [Angle in semicircle]
$QR^2 = PQ^2 + RP^2$
or, $QR^2 = (24^2 + 7^2)\ cm^2$
or, $QR^2 = (576 + 49)\ cm^2$
or, $QR^2 = 625\ cm^2$
or, QR = 25 cm

Hence the area of the shaded region
= area of the semicircle – area of the triangle
$= \frac{1}{2}\pi r^2$ – area of the triangle PQR
$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{25}{2}\right)^2 - \frac{1}{2} \times PR \times QP$
$= \frac{22 \times 625}{28 \times 2} - 84$
$= \frac{13750 - 4704}{56}$
$= 161.54\ cm^2$

Question 2: Find the area of the shaded region in the given figure, ¡f radii of the two concentric circles with centre O are 7 cm and 14 cm respectively and ∠AOC = 40°.



Answer: The radius of the smaller circle = 7cm
Angle of the sector (θ) = 40^0

Area of the smaller sector BOD = $\frac{\theta}{360°} \times \pi r^2$
= $\frac{40°}{360°} \times \frac{22}{7} \times 7 \times 7$

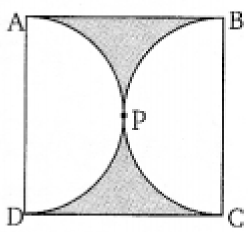
= $\frac{1}{9} \times 22 \times 7$
= $\frac{154}{9}$ cm^2
Area of the bigger sector AOC
= $\frac{40°}{360°} \times \frac{22}{7} \times 14 \times 14$

= $\frac{1}{9} \times 22 \times 28$

= $\frac{616}{9}$ cm^2

Area of the shaded region
= area of bigger sector AOC – area of smaller sector BOD
= $\frac{616}{9} - \frac{154}{9} = \frac{462}{9}$ cm^2 = $\frac{154}{3}$ cm^2

Question 3: Find the area of the shaded region in the given figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

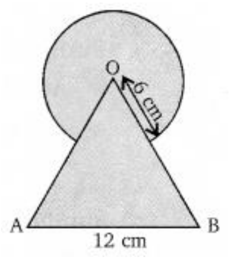


Answer: ABCD is a square, having side 14cm.
Area of the square = $(\text{side})^2$ = 14^2 = 196 cm^2
Radius of the semicircle APD = $\frac{1}{2}$(side of square) = $\frac{1}{2} \times 14$ = 7cm

Area of the semicircle APD = $\frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 11 \times 7 = 77$ cm^2

Similarly, area of the semicircle BPC = 77cm^2
Total area of the both the semicircles = 77 + 77 = 154 cm^2
Area of the shaded region = area of square – area of both semicircles
= 196 – 154 cm^2
= 42 cm^2

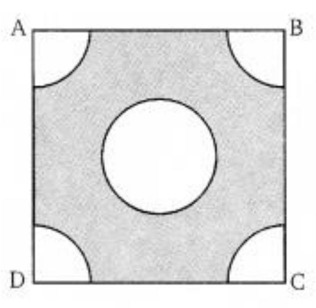
Question 4: Find the area of the shaded region in the figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



Answer: Area of the equilateral triangle OAB = $\frac{\sqrt{3}}{4}$ (side)^2 = $\frac{\sqrt{3}}{4} \times 12 \times 12 = 36\sqrt{3}$ cm^2
Angle of sector, θ = 60^0
Area of the major sector of the circle = πr^2 – Area of Minor Sector
= $\pi \times 6 \times 6(1 - \frac{60^o}{360^o})$
= $36\pi - 6\pi = 30\pi = 30 \times \frac{22}{7} = \frac{660}{7}$ cm^2

Area of the shaded region = Area of the major sector of the circle + Area of Triangle
=$\left(\frac{660}{7} + 36\sqrt{3}\right)$ cm^2

Question 5: From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in the figure. Find the area of the remaining portion of the square.



Answer: Side of the square = 4 cm
Area of the square ABCD = 4 x 4 = 16 cm^2
Radius of the quadrant in the corner = 1 cm
Hence, Area of each quadrant = $\frac{\pi r^2\theta}{360^o} = \frac{22}{7} \times \frac{1\times1\times90^o}{360^o} = \frac{22}{28}$ cm^2

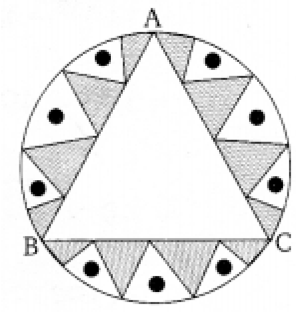
Area covered by four quadrants = 4 x $\frac{22}{28}$ =$\frac{22}{7}$ cm^2

Area of the circle at middle = $\pi r^2 = \frac{22}{7} \times 1 \times 1 = \frac{22}{7}$ cm^2

Therefore, total area to be cut from the square = $\frac{22}{7} + \frac{22}{7} = \frac{44}{7}$ cm^2

Area of the remaining portion = Area of square – Area cut from square
$= 16 - \frac{44}{7} = \frac{112-44}{7} = \frac{68}{7}$ cm^2

Question 6: In a circular table cover of (he radius 32 cm, a design Is formed leaving an equilateral triangle ABC in the middle as shown in the figure. Find the area of the design (shaded region).



Answer: Let O be the centre of the circular table and ABC be the equilateral triangle. We draw OD⊥BC.

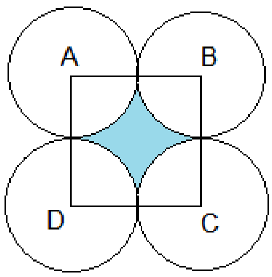
In ΔOBD, we have Cos $60^0 = \frac{OD}{OB}$

or, $\frac{1}{2} = \frac{OD}{32}$ [because OB = radius = 32 cm]
or, OD = 16 cm

Sin 60^0= $\frac{BD}{OB}$
or, $\frac{\sqrt{3}}{2} = \frac{BD}{32}$
or, BD = $16\sqrt{3}$ cm
Therefore, BC = 2BD = $2(16\sqrt{3}) = 32\sqrt{3}$ cm
Area of the shaded region = Area of circle – Area of equilateral triangle
= $\left[\pi \times (32)^2 - \frac{\sqrt{3}}{4} \times (32\sqrt{3})^2\right]$ cm^2
= $\left(\frac{22}{7} \times 32 \times 32 - \frac{\sqrt{3}}{4} \times 32 \times 32 \times 3\right)$ cm^2

$= 1024\left(\frac{22}{7} - \frac{3\sqrt{3}}{4}\right)$ cm^2 = $\left(\frac{22528}{7} - 768\sqrt{3}\right)$ cm^2

Question 7 : ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.



Answer: Side of square = 14 cm

Four quadrants are included in the four sides of the square.

∴ Radius of the circles = $\frac{14}{2}$ cm = 7 cm

Area of the square ABCD = 14^2 = 196 cm^2

Area of the quadrant at each vertex = $\frac{\pi r^2}{4}$ cm^2 = $\frac{22}{7} \times \frac{7^2}{4}$ cm^2

= $\frac{77}{2}$ cm^2

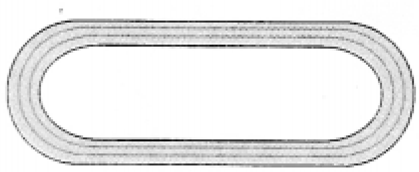
Total area of the quadrant = $4\times\frac{77}{2}$ cm^2 = 154cm^2

Area of the shaded region = Area of the square ABCD – Area of the quadrant

= 196 cm^2 – 154 cm^2

= 42 cm^2

Question 8 : The given figure depicts a racing track whose left and right ends are semicircular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:



(i) the distance around the track along its inner edge.
(ii) the area of the track.

Answer: Width of the track = 10 m

Distance between two parallel lines = 60 m

Length of parallel tracks = 106 m

DE = CF = 60 m

Radius of inner semicircle, r = OD = O'C

= $\frac{60}{2}$ m = 30 m

Radius of outer semicircle, R = OA = O'B

= 30+10 m = 40 m

Also, AB = CD = EF = GH = 106 m

Distance around the track along its inner edge = CD+EF+2×(Circumference of inner semicircle)

= 106+106+(2×πr) m = 212+(2×$\frac{22}{7}$×30) m

= 212+$\frac{1320}{7}$ m = 2804/7 m

Area of the track = Area of ABCD + Area EFGH + 2 × (area of outer semicircle) – 2 × (area of inner semicircle)

= (AB×CD)+(EF×GH)+2×($\frac{\pi R^2}{2}$) -2×($\frac{\pi r^2}{2}$) m^2

= (106×10)+(106×10)+2×$\frac{\pi(r^2-R^2)}{2}$ m^2

= 2120+$\frac{22}{7}$×70×10 m^2

= 4320 m^2

Question 9: In Fig. 12.27, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

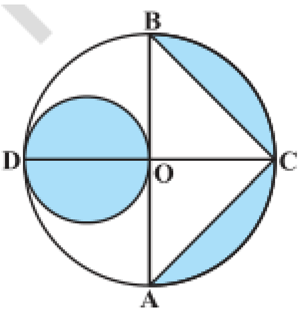


Fig. 12.27

Answer: Radius of larger circle, R = 7 cm

Radius of smaller circle, r = $\frac{7}{2}$ cm

Height of ΔBCA = OC = 7 cm

Base of ΔBCA = AB = 14 cm

Area of ΔBCA = $\frac{1}{2}$ × AB × OC = $(\frac{1}{2})$×7×14 = 49 cm^2

Area of larger circle = πR^2 = $(\frac{22}{7})$×7^2 = 154 cm^2

Area of larger semicircle = $\frac{154}{2}$ cm^2 = 77 cm^2

Area of smaller circle = πr^2 = $(\frac{22}{7})$×$(\frac{7}{2})$×$(\frac{7}{2})$ = $\frac{77}{2}$$cm^2$

Area of the shaded region = Area of larger circle – Area of triangle – Area of larger semicircle + Area of smaller circle

Area of the shaded region = (154-49-77+$\frac{77}{2}$) cm^2

= $\frac{133}{2}$ cm^2 = 66.5 cm^2

Question 10. The area of an equilateral triangle ABC is 17320.5 cm^2. With each vertex of the triangle as centre, a circle is drawn with radius equal to half the length of the side of the triangle (see Fig. 12.28). Find the area of the shaded region. (Use π = 3.14 and √3 = 1.73205)

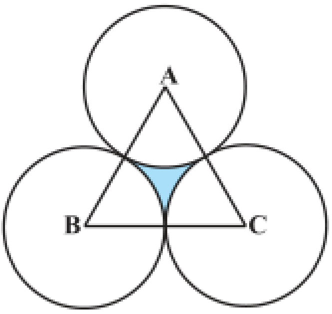


Fig. 12.28

Answer: ABC is an equilateral triangle.

Therefore ∠ A = ∠ B = ∠ C = 60°

There are three sectors each making 60°.

Area of ΔABC = 17320.5 cm^2

or, $\frac{\sqrt{3}}{4}$ ×$(\text{side})^2$ = 17320.5

or, $(\text{side})^2$ =$\frac{17320.5\times4}{1.73205}$

or, $(\text{side})^2$ = 4×10^4

or, side = 200 cm

Radius of the circles = $\frac{200}{2}$ cm = 100 cm

Area of the sector = $(\frac{60^o}{360^o})$×π r^2 cm^2

= $\frac{1}{6}$×3.14×$(100)^2$ cm^2

= $\frac{15700}{3}$ cm^2

Area of 3 sectors = $3\times\frac{15700}{3}$ = 15700 cm^2

Thus, area of the shaded region = Area of equilateral triangle ABC – Area of 3 sectors

= 17320.5-15700 cm^2 = 1620.5 cm^2

Question 11: On a square handkerchief, nine circular designs each of radius 7 cm are made (see Fig. 12.29). Find the area of the remaining portion of the handkerchief.

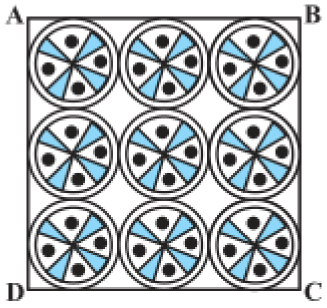


Fig. 12.29

Answer: Number of circular designs = 9

Radius of the circular design = 7 cm

There are three circles in one side of square handkerchief.

Therefore Side of the square = 3×diameter of circle = 3×14 = 42 cm

Area of the square = 42×42 cm^2 = 1764 cm^2

Area of the circle = $\pi r^2 = (\frac{22}{7})\times 7\times 7$ = 154 cm^2

Total area of the design = 9×154 = 1386 cm^2

Area of the remaining portion of the handkerchief = Area of the square – Total area of the design = 1764 – 1386 = 378 cm^2

Question 12: In Fig. 12.30, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the

(i) quadrant OACB,

(ii) shaded region.

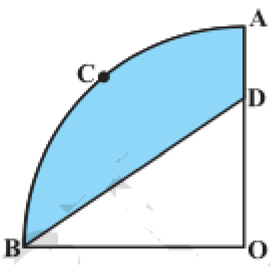


Fig. 12.30

Answer: Radius of the quadrant = 3.5 cm = 7/2 cm

(i) Area of quadrant OACB = $(\pi R^2)/4$ cm^2

= $(\frac{22}{7})\times(\frac{7}{2})\times(\frac{7}{2})/4$ cm^2

= 77/8 cm^2

(ii) Area of triangle BOD = $(\frac{1}{2})\times(\frac{7}{2})\times 2$ cm^2

= $\frac{7}{2}$ cm^2

Area of shaded region = Area of quadrant – Area of triangle BOD

= $(\frac{77}{8})-(\frac{7}{2})$ cm^2 = $\frac{49}{8}$ cm^2

= 6.125 cm^2

Question 13: In Fig. 12.31, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use π = 3.14)

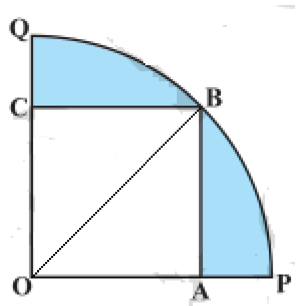


Fig. 12.31

Answer: Side of square = OA = AB = 20 cm

Radius of the quadrant = OB

OAB is right angled triangle

By Pythagoras theorem in ΔOAB,

$OB^2 = AB^2+OA^2$

or, $OB^2 = 20^2 +20^2$

or, $OB^2 = 400+400$

or, $OB^2 = 800$

or, OB= $20\sqrt{2}$ cm

Area of the quadrant = $(\frac{\pi R^2}{4})$ cm^2 = $(\frac{3.14}{4})\times(20\sqrt{2})^2$ cm^2 = $628cm^2$

Area of the square = $20\times20 = 400$ cm^2

Area of the shaded region = Area of the quadrant – Area of the square

= 628-400 cm^2 = $228cm^2$

Question 14: AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O (see Fig. 12.32). If ∠AOB = 30°, find the area of the shaded region.

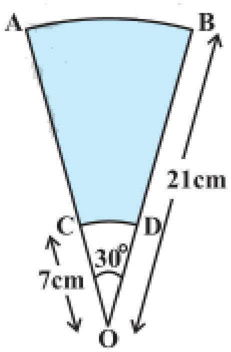


Fig. 12.32

Answer: Radius of the larger circle, R = 21 cm

Radius of the smaller circle, r = 7 cm

Angle made by sectors of both concentric circles = 30°

Area of the larger sector = $(\frac{30^o}{360^o})\times\pi R^2$ cm^2

= $(\frac{1}{12})\times(\frac{22}{7})\times21^2$ cm^2

= $\frac{231}{2}$ cm^2

Area of the smaller circle = $(\frac{30^o}{360^o})\times\pi r^2$ cm^2

= $\frac{1}{12}\times\frac{22}{7}\times7^2$ cm^2

=$\frac{77}{6}$ cm^2

Area of the shaded region = $(\frac{231}{2})$ – $(\frac{77}{6})$ cm^2

= $\frac{616}{6}$ cm^2 = $\frac{308}{3}cm^2$

Question 15: In Fig. 12.33, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.

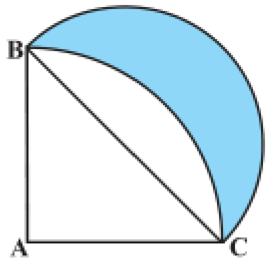


Fig. 12.33

Answer: Radius of the quadrant ABC of circle = 14 cm

AB = AC = 14 cm

BC is diameter of semicircle.

ABC is right angled triangle.

By Pythagoras theorem in ΔABC,

$BC^2 = AB^2 + AC^2$

or, $BC^2 = 14^2 + 14^2$

or, BC = $14\sqrt{2}$ cm

Radius of semicircle = $\frac{14\sqrt{2}}{2}$ cm = $7\sqrt{2}$ cm

Area of ΔABC $=(\frac{1}{2})\times 14\times 14 = 98\ cm^2$

Area of quadrant = $(\frac{1}{4})\times(\frac{22}{7})\times(14\times 14) = 154\ cm^2$

Area of the semicircle = $(\frac{1}{2})\times(\frac{22}{7})\times 7\sqrt{2}\times 7\sqrt{2} = 154\ cm^2$

Area of the shaded region =Area of the semicircle + Area of ΔABC – Area of quadrant

= $154 + 98 - 154\ cm^2 = 98 cm^2$

Question 16: Calculate the area of the designed region in Fig. 12.34 common between the two quadrants of circles of radius 8 cm each.

Solution:

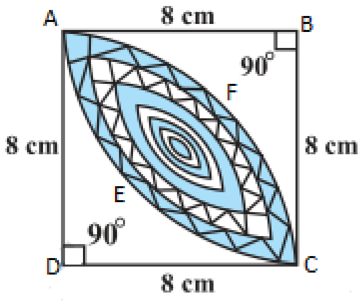


Fig. 12.34

Answer: AB = BC = CD = AD = 8 cm

Area of ΔABC = Area of ΔADC = $(\frac{1}{2})\times8\times8 = 32$ cm^2

Area of quadrant AECB = Area of quadrant AFCD = $(\frac{1}{4})\times22/7\times8^2$

$= \frac{352}{7}$ cm^2

Area of shaded region = (Area of quadrant AECB – Area of ΔABC) = (Area of quadrant AFCD – Area of ΔADC)

$= (\frac{352}{7} - 32) + (\frac{352}{7} - 32)$ cm^2

$= 2\times(\frac{352}{7} - 32)$ cm^2

$= \frac{256}{7}$ cm^2