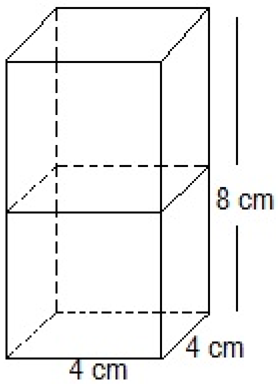
There will be CHAPTER 13 – SURFACE AREAS AND VOLUMES
Exercise – 13.1

Question 1: 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

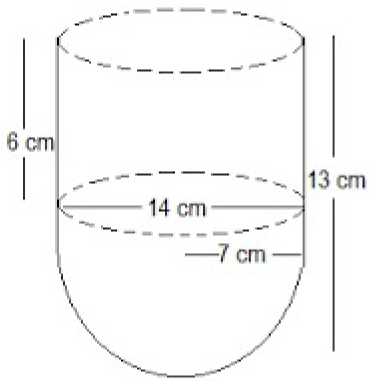
Answer:



Volume of one cube = 64cm^3
Let the edge of one cube = a
Volume of the cube = (edge)^3 = a^3 = 64
or, a = 4
Similarly, the edge of another cube = 4cm
By joining the cubes, a cuboid formed.
Now, length of the cuboid (l) = 8cm
breadth of the cuboid (b) = 4cm
height of the cuboid (h) = 4cm
Surface area of the cuboid so formed
= 2(lb + bh + hl)
= 2($8\times 4 + 4\times 4 + 8\times 4$)
= 2(32 + 16 + 32)
= 2(80) = 160cm^2

Question 2: A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm, and the total height of the vessel is 13 cm. Find the inner surface area of the ship.

Answer:



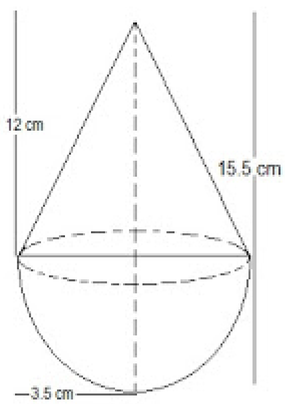
The diameter of the hemisphere (D) = 14 cm
The radius of the hemisphere (r) = 7 cm
The height of the cylinder (h) = (13-7) = 6 cm

And, the radius of the hollow hemisphere = 7 cm

Now,
The inner surface area of the vessel = Curved Surface Area of the cylindrical part + Curved Surface Area of hemispherical part
or, $(2\pi rh + 2\pi r^2)$ cm^2
$= 2\pi r(h + r)$ cm^2
$= 2 \times \left(\frac{22}{7}\right) \times 7(6 + 7)$ cm^2
$= 572$ cm^2

Question 3: A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of the same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Answer:



The radius of the cone and the hemisphere (r) = 3.5 cm or $\frac{7}{2}$ cm
The total height of the toy is given as 15.5 cm.

So, the height of the cone (h) = 15.5 - 3.5 = 12 cm
Slant height of the cone (l) = $\sqrt{h^2 + r^2}$
or, l = $\sqrt{12^2 + 3.5^2}$
or, l = $\sqrt{12^2 + \left(\frac{7}{2}\right)^2}$
or, l = $\sqrt{\frac{625}{4}}$
or, l = $\frac{25}{2}$

The curved surface area of cone = πrl

Or, $\frac{22}{7} \times \frac{7}{2} \times \frac{25}{2} = \frac{275}{2}$ cm^2

Also, the curved surface area of the hemisphere = 2πr^2

= $2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$ = 77 cm^2

Now, the Total surface area of the toy = Curved Surface Area of cone + Curved Surface Area of the hemisphere

= $\frac{275}{2}$ + 77 cm^2
= $\frac{275+154}{2}$ cm^2

= $\frac{249}{2}$ cm^2 = 214.5cm^2

So, the total surface area of the toy is 214.5cm^2

Question 4: A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Answer: The total surface area of solid
= surface area of cubical block + Curved Surface Area of hemisphere – Area of base of hemisphere

Total Surface Area of solid = 6 × (side)^2 + 2πr^2 - πr^2

= 6 × (side)^2 + πr^2
= 6 × (7)^2 + $\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$
= (6 × 49) + $\frac{77}{2}$
= 294 + 38.5 = 332.5 cm^2

So, the surface area of the solid is 332.5 cm^2

Question 5: A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Answer: Now, the diameter of hemisphere = Edge of the cube = l

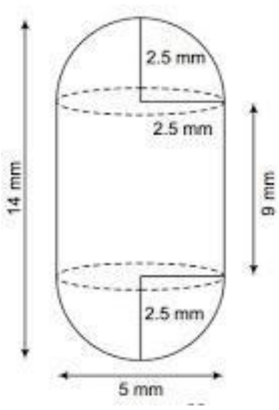
So, the radius of hemisphere = $\frac{l}{2}$

The total surface area of solid
= surface area of cube + Curved Surface Area of the hemisphere – Area of the base of hemisphere

Total Surface Area of remaining solid = 6 $(\text{edge})^2 + 2\pi r^2 - \pi r^2$
= $6l^2\ \pi r^2$
= $6l^2 + \pi\left(\frac{l}{2}\right)^2$
= $6l^2 + \left(\frac{\pi l^2}{4}\right)$
= $\frac{l^2}{4}(24+\pi)$ sq. units

Question 6: A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm, and the diameter of the capsule is 5 mm. Find its surface area.

Answer: Two hemisphere and one cylinder are shown in the figure given below.



Here, the diameter of the capsule = 5 mm
Therefore, Radius = $\frac{5}{2}$ = 2.5 mm
Now, the length of the capsule = 14 mm

So, the length of the cylinder = 14 - (2.5 + 2.5) = 9 mm

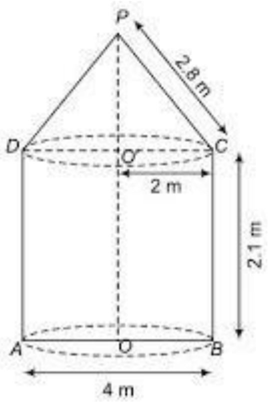
Now, the surface area of a hemisphere = $2\pi r^2 = 2 \times \frac{22}{7} \times 2.5 \times 2.5 = \frac{275}{7}$ mm^2

Now, the surface area of the cylinder = $2\pi rh$
= $2 \times \frac{22}{7} \times 2.5 \times 9$
= $\frac{22}{7} \times 45$
= $\frac{990}{7}$ mm^2

Thus, the required surface area of medicine capsule will be
= 2×surface area of hemisphere + surface area of the cylinder
$= 2 \times \frac{275}{7} \times \frac{990}{7}$
$= \frac{550}{7} + \frac{990}{7} = \frac{1540}{7} = 220\ mm^2$

Question 7: A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2. (Note that the base of the tent will not be covered with canvas.)

Answer



Diameter = 4 m
Slant height of the cone (l) = 2.8 m
Radius of the cone (r) = Radius of cylinder = $\frac{4}{2}$ = 2 m

Height of the cylinder (h) = 2.1 m

So, the required surface area of tent = surface area of cone + surface area of the cylinder
= πrl + 2πrh

= πr(l + 2h)

$= \frac{22}{7} \times 2(2.8 + 2 \times 2.1)$

$= \frac{44}{7}(2.8 + 4.2)$

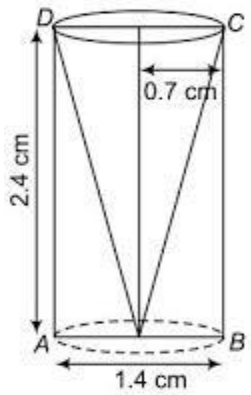
$= \frac{44}{7} \times 7 = 44\ m^2$

The cost of the canvas of the tent at the rate of ₹500 per m^2 will be = Surface area × cost per m^2
or, 44 × 500 = ₹22000

So, Rs. 22000 will be the total cost of the canvas.

Question 8: From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same size and same diameter is hollowed. Find the total surface area of the remaining solid to the nearest cm^2.

Answer:



The diameter of the cylinder = diameter of conical cavity = 1.4 cm
So, the radius of the cylinder = radius of the conical cavity = $\frac{1.4}{2}$ = 0.7
Also, the height of the cylinder = size of the conical cavity = 2.4 cm

Slant height of the cone (l) = $\sqrt{h^2 + r^2}$
$= \sqrt{2.4^2 + 0.7^2}$
$= \sqrt{5.76 + 0.49}$
$= \sqrt{6.25}$ = 2.5cm

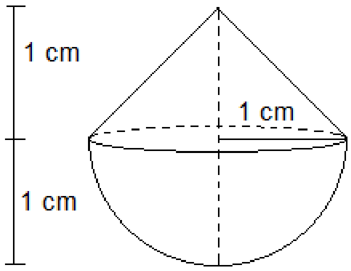
Now, the Total Surface Area of remaining solid
= surface area of conical cavity + Total Surface Area of the cylinder
= $\pi rl + (2\pi rh + \pi r^2)$
= $\pi r(l + 2h + r)$
= $\frac{22}{7} \times 0.7(2.5 + 4.8 + 0.7)$
= $2.2 \times 8 = 17.6$ cm^2

So, the total surface area of the remaining solid is 17.6 cm^2

Exercise 13.2

Question 1: A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is similar to its radius. Find the volume of the solid in terms of π.

Answer:



Here r = 1 cm and h = 1 cm.

Now, Volume of solid = Volume of conical part + Volume of hemispherical part
We know that the volume of cone = $\frac{1}{3}\pi r^2 h$

And, the volume of hemisphere =. $\frac{2}{3}\pi r^3$
So, the volume of the solid will be = $\frac{1}{3}\pi (1)^2 [1 + 2(1)]$ cm^3

$= \frac{1}{3}\pi \times [3]$ cm^3

$= \pi$ cm^3

Question 2: Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends using a thin aluminium sheet. The diameter of the model is 3 cm, and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Answer: Given, height of cylinder = (12 – 4)cm = 8 cm
Radius = 1.5 cm and height of cone = 2 cm

Now, the total volume of the air contained will be = Volume of cylinder + 2 × (Volume of the cone)
Therefore, Total volume = $\pi r^2 h + [2 \times (\frac{1}{3}\pi r^2 h)] = 18\pi + 2(1.5\pi) = 66$ cm^3.

Question 3: A Gulab jamun contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 Gulab jamuns, each shaped like a cylinder with two hemispherical ends with a length of 5 cm and diameter of 2.8 cm (see figure).

Fig. 13.15

Answer: The total height of a gulab jamun = 5 cm.
Diameter = 2.8 cm hence, radius = 1.4 cm

The height of the cylindrical part = 5 cm – (1.4 + 1.4) cm =2.2 cm

Now, the total volume of One Gulab Jamun = Volume of Cylinder + Volume of two hemispheres
= $\pi r^2 h + \frac{4}{3}\pi r^3$
= $4.312\pi + \frac{10.976}{3}\pi$
= 25.05 cm^3

We know that the volume of sugar syrup = 30% of the total volume
So, volume of sugar syrup in 45 gulab jamuns = 45 × 30%(25.05 cm^3) = 45 × 7.515 = 338.184 cm^3

Question 4: A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm, and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Fig.).

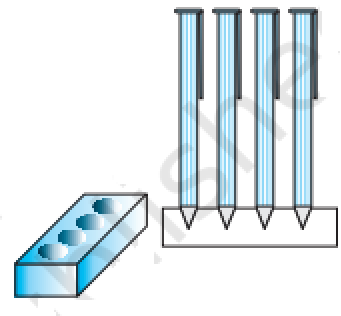


Fig. 13.16

Answer: Volume of cuboid = l x w x h
So, the volume of the cuboid = 15 x 10 x 3.5 = 525 cm^3

Volume of cone = (⅓)$\pi r^2 h$
Hence, radius (r) = 0.5 cm and depth (h) = 1.4 cm

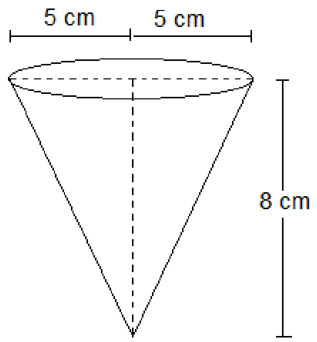
Volume of 4 cones = 4 x (⅓)$\pi r^2 h$ = 1.46 cm^2

Now, volume of wood = Volume of cuboid – 4 x volume of cone = 525-1.46 = 523.54 cm^2

Question 5: A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Answer: For the cone, radius = 5 cm,, height = 8 cm

Also, Radius of sphere = 0.5 cm



It is known that,

The volume of cone = volume of water in the cone

$= \frac{1}{3}\pi r^2 h = (\frac{200}{3})\pi\ cm^3$

Now,

Total volume of water overflown= $(\frac{1}{4})\times(\frac{200}{3})\ \pi = (\frac{50}{3})\pi$

The volume of lead shot

$= (\frac{4}{3})\pi r^3$

$= (\frac{1}{6})\ \pi$

Now,

The number of lead shots = Total Volume of Water overflown/ Volume of Lead shot

$= \frac{(\frac{50}{3})\pi}{(\frac{1}{6})\pi}$

$= (\frac{50}{3})\times 6 = 100$

There will be 100 lead shots dropped in water

Question 6: A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass.

Answer: The height of the big cylinder (H) = 220 cm
Radius of the base (R) = $\frac{24}{12}$ = 12 cm

So, the volume of the big cylinder = $\pi R^2 H = \pi(12)^2 \times 220\ cm^3 = 99565.8\ cm^3$

Now, the height of smaller cylinder (h) = 60 cm and radius of the base (r) = 8 cm

So, the volume of the smaller cylinder = $\pi r^2 h$
= π(8)2×60 cm^3
= 12068.5 cm^3

Therefore, the volume of iron = Volume of the big cylinder+ Volume of the small cylinder
= 99565.8 + 12068.5
=111634.5 cm^3

We know, Mass = Density x volume

So, the mass of the pole = 8×111634.5 = 893 Kg (approx.)

Question 7: A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the cylinder's radius is 60 cm and its height is 180 cm.

Answer: Given,
Radius of cone = 60 cm,
Height of cone = 120 cm
Radius of cylinder = 60 cm
Height of cylinder = 180 cm
Radius of hemisphere = 60 cm

Now, Total volume of solid = Volume of Cone + Volume of the hemisphere
Volume of cone = $\pi \times 12^2 \times 103$ cm^3
= $144 \times 10^3 \pi$ cm^3

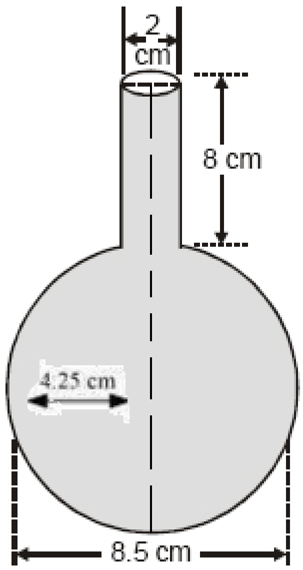
So, Total volume of solid = $144 \times 10^3 \pi$ cm^3 -(⅔) × π × 10^3 cm^3
The volume of hemisphere = (⅔) × π × 10^3 cm^3
Volume of cylinder = $\pi \times 60^2 \times 180 = 648000 = 648 \times 10^3 \pi$ cm^3

Now, the volume of water left will be = Volume of the cylinder – Volume of solid
= $(648-288) \times 10^3 \times \pi = 1.131$ m^3

Question 8: A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the spherical part's diameter is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

Answer: For the cylinder part, Height (h) = 8 cm and Radius (R) = $\frac{2}{2}$ cm = 1 cm

For the spherical part, Radius (r) = $\frac{8.5}{2}$ = 4.25 cm



Now, the volume of this vessel = Volume of cylinder + Volume of a sphere
$= \pi \times (1)^2 \times 8 + \frac{4}{3}\pi (4.25)^3$
$= 346.51\ cm^3$

Exercise 13.3

Question 1: A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Answer: It is given that radius of the sphere (R) = 4.2 cm
Also, radius of the cylinder (r) = 6 cm

Let the height of cylinder = h

The sphere is melted into a cylinder. [Given]
So, Volume of Sphere = Volume of Cylinder

Therefore, $\frac{4}{3} \times \pi \times R^3$
$= \pi \times r^2 \times h.$

h = 2.74 cm

Question 2. Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Answer: For Sphere 1:

Radius (r_1) = 6 cm
Hence, Volume (V_1) = $\frac{4}{3} \times \pi \times r_1^3$

<u>For Sphere 2:</u>

Radius (r_2) = 8 cm
Hence, Volume (V_2) = $\frac{4}{3} \times \pi \times r_2^3$

<u>For Sphere 3:</u>

Radius (r_3) = 10 cm
Hence, Volume (V_3) = $\frac{4}{3} \times \pi \times r_3^3$

Also, let the radius of the resulting sphere be "r."

Now, Volume of resulting sphere = $V_1 + V_2 + V_3$
or, $\frac{4}{3} \times \pi \times r^3 = \frac{4}{3} \times \pi \times r_1^3 + \frac{4}{3} \times \pi \times r_2^3 + \frac{4}{3} \times \pi \times r_3^3$
or, $r^3 = 6^3+8^3+10^3$
or, $r^3 = 1728$
or, r = 12 cm

Question 3. A 20 m deep well with diameter of 7 m is dug, and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Answer: Radius = $\frac{7}{2}$ m [As, D = 7m]
Also, Depth (h) = 20 m

As per the question, volume of the earth dug out will be equal to the volume of the cylinder
Hence, Volume of Cylinder = $\pi r^2 \times h = 22 \times 7 \times 5$ m^3

Let the height of the platform be = H

The volume of soil from well (cylinder) = Volume of soil used to make such platform
or, $\pi \times r^2 \times h$ = Area of platform × Height of the platform

We know that the dimension of the platform is = 22 × 14

So, Area of platform = 22×14 m^2

Therefore, $\pi \times r^2 \times h = 22 \times 14 \times H$
or, H = 2.5 m

Question 4: A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Answer: Depth (h_1) of well = 14 m [Given]
The diameter of the circular end of the well = 3 m
hence, radius (r_1) = $\frac{3}{2}$ m

Width of the embankment = 4 m

From the figure, it can be said that the embankment will be a cylinder having an outer radius (r_2) as $4 + \frac{3}{2} = \frac{11}{2}$ m and inner radius (r_1) as $\frac{3}{2}$ m

Now, let the height of embankment be h_2

Therefore, Volume of soil dug from well = Volume of earth used to form embankment
$\pi \times r_1^2 \times h = \pi \times (r_2^2 - r_1^2) \times h_2$
or, $\frac{22}{7} \times \frac{9}{4} \times 14 = \frac{2}{7} \times \left(\frac{121}{4} - \frac{9}{4}\right) \times h_2$
or, $h_2 = 1.125$ m

Question 5: A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Answer: Number of cones will be = Volume of cylinder ÷ Volume of ice cream cone
For the cylinder part,
radius = $\frac{12}{2}$ = 6 cm and height = 15 cm

Hence, volume of cylinder = $\pi \times r^2 \times h = 540\pi$

For the ice cone part, radius of conical part = $\frac{6}{2}$ = 3 cm
height = 12 cm
and radius of hemispherical part = $\frac{6}{2}$ = 3 cm

Now, Volume of ice cream cone = Volume of conical part + Volume of hemispherical part

$= (\frac{1}{3}) \times \pi \times r^2 \times h + (\frac{2}{3}) \times \pi \times r^3$
$= 36\pi + 18\pi$
$= 54\pi$

Hence, Number of cones = $(540\pi \div 54\pi) = 10$

Question 6. How many silver coins, 1.75 cm in diameter and thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

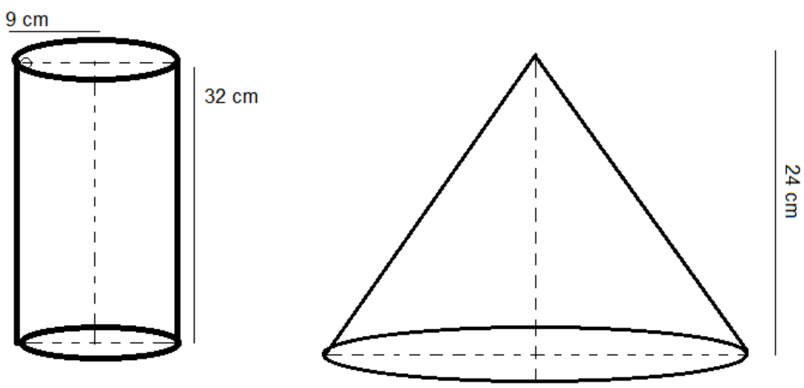
Answer: The height (h_1) of the cylinder = 2 mm = 0.2 cm
And radius (r) of circular end of coins = $\frac{1.75}{2}$ = 0.875 cm

Now, the number of coins to be melted to form the required cuboids be “n.”
So, Volume of n coins = Volume of cuboids
or, $n \times \pi \times r^2 \times h_1 = l \times b \times h$

or, n × π × $(0.875)^2$× 0.2 = 5.5 × 10 × 3.5
Or, n = 400

Question 7: A cylindrical bucket, 32 cm high and a radius of base 18 cm, is filled with sand. This bucket is emptied on the ground, and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Answer:



Height (h_1) of cylindrical bucket = 32 cm
And radius (r_1) of the circular end of the bucket = 18 cm

Now, height of the conical heap ((h_2) = 24 cm
And let "r_2" be the radius of the circular end of the conical heap.

As Volume of sand in the cylindrical bucket = Volume of sand in a conical heap
$\pi \times r_1^2 \times h_1 = (⅓) \times \pi \times r_2^2 \times h_2$
or, $\pi \times 18^2 \times 32 = (⅓) \times \pi \times r_2^2 \times 24$
or, r_2 = 36 cm

And, Slant height (l) = $\sqrt{36^2 + 24^2}$ = 12√13 cm.

Question 8: Water in a canal, 6 m wide and 1.5 m deep, is flowing at a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Answer: Breadth (b) of the cuboid = 6 m and Height (h) = 1.5 m [Given]

Again, the speed of canal = 10 km/hr
Hence, the length of the canal covered in 1 hour = 10 km
or, length of canal covered in 60 minutes = 10 km

or, length of canal covered in 1 min = $\frac{1}{60}$ x 10 km
And, length of canal covered in 30 min (l) = $\frac{30}{60}$ x 10 = 5km = 5000 m

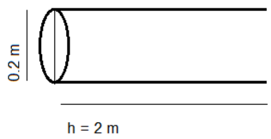
We know that the canal is in cuboidal shape. So,
Volume of canal = l x b x h
= 5000 x 6 x 1.5 m^3
= 45000 m^3

Now, Volume of water in the canal = Volume of area irrigated = Area irrigated x Height
Area irrigated = 56.25 hectares
Hence, volume of canal = l x b x h

45000 = Area irrigated x 8 cm
or, 45000 = Area irrigated x $\frac{8}{100}$m
or, Area irrigated = 562500 m^2 = 56.25 hectares.

Question 9: A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Answer:



Radius (r_1) of circular end pipe = $\frac{20}{200}$ = 0.1 m
Area of the cross-section = πr_1^2 = $\pi(0.1)^2$ = 0.01π m^2
Speed of the water = 3 km/hr = $\frac{3000}{60}$ = 50m/min

Volume of the water that flows in 1 minute = 50 × 0.01π = 0.5 π m^2

The volume of water that flows int minutes = t × 0.5π m^3
The volume of water that flows int minutes = t × 0.5π m^3
Radius of circular end of cylindrical tank (r_2) = $\frac{10}{2}$ = 5 m
Depth of cylindrical tank (h_2) = 2 m

Let the tank be filled in t minutes.
Hence, the volume of water that flows int minutes from pipe = Volume of water in the tank

$t \times 0.5\pi = \pi \times r_2^2 \times h_2$
or, $t = \frac{5^2 \times 2}{0.5}$
or, t = 100 minutes

Exercise 13.4

Question 1: A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Answer: Radius of the upper base (r_1) $= \frac{4}{2} = 2$ cm
And, radius of lower the base (r_2) $= \frac{2}{2} = 1$ cm
And given height = 14 cm

Now, Capacity of glass = Volume of a frustum of a cone
or, capacity of glass $= \frac{1}{3} \times \pi \times h(r_1^2 + r_2^2 + r_1r_2)$
$= \frac{1}{3} \times \pi \times (14)\ (2^2 + 1^2 + (2)(1))$
Hence, the capacity of the glass $= 102\frac{2}{3}$ cm^3

Question 2: The slant height of a cone's frustum is 4 cm, and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the surface area of the frustum.

Answer: Slant height (l) = 4 cm [Given]
Circumference of upper circular end of the frustum = 18 cm [Given]

Hence, $2\pi r_1 = 18$
or, $r_1 = \frac{9}{\pi}$

Similarly, the circumference of lower end of the frustum = 6 cm [Given]

Hence, $2\pi r_2 = 6$
or, $r_2 = \frac{3}{\pi}$

Now, Curved Surface Area of frustum $= \pi(r_1 + r_2) \times l$
$= \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right) \times 4$
$= 12 \times 4 = 48$ cm^2

Question 3: **A fez, the Turks' cap, is shaped like the frustum of a cone (see Fig.). If its radius on the open side is 10 cm, the radius at the upper base is 4 cm, and its slant height is 15 cm, find the area of material used for making it.**

Fig. 13.24

Answer: We have r_1= 10 cm and r_2 = 4 cm and Slant height, l = 15 cm

Therefore, the total area of material used = Curved surface area + area of closed top

$= \pi l\ (r_1 + r_2) + \pi r_2^2$

$= \left[\frac{22}{7} \times 15(10+4) + \frac{22}{7} \times 4 \times 4\right]$ cm^2

$= \frac{22}{7} \times 226$ cm^2

= 710.28 cm^2

Question 4: **A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can fill the container, at the rate of Rs. 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs. 8 per 100 cm^2.**

Answer: Given, r_1 = 20 cm,

r_2 = 8 cm

h = 16 cm

Therefore, volume of frustum of cone = $\frac{1}{3} \times \pi \times h \times (r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{314 \times 16 \times 208}{100000}$ litres

given that the rate of milk = Rs. 20/litre

So, Cost of milk = 20×volume of the frustum

= Rs $20 \times \frac{314 \times 16 \times 208}{100000}$ = Rs. 209

Therefore, the slant height (l) = $\sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{16^2 + (20-8)^2} = \sqrt{16^2 + 12^2}$

Therefore the Curved Surface Area = $\pi(r_1 + r_2) \times l$

$= \frac{314}{100}(20+8) \times 20\ cm^2$

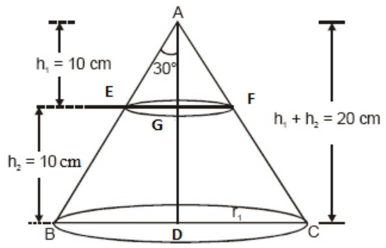
= 1758.4 cm^2

Hence, the total metal that would be required to make container will be = 1758.4 + (Area of bottom circle)
= 1758.4+201 = 1959.4 cm^2

Therefore, total cost of metal = Rs. $\left(\frac{8}{100}\right)$ x 1959.4 = Rs. 157

Question 5: A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained is drawn into a wire of diameter 1/16 cm, find the wire's length.

Answer:



Consider, $\Delta AEG,$

$\frac{EG}{AG} = \tan 30^o$

$EG = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$ cm

In ΔABD,

$\frac{BD}{AD} = \tan 30^o$

$BD = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$ cm

Radius (r_1) of upper end of frustum = $\frac{10\sqrt{3}}{3}$ cm

Radius (r_2) of lower end of container = $\frac{20\sqrt{3}}{3}$ cm

Height (r_3) of container = 10 cm

Now,
Volume of the frustum = $(\frac{1}{3})\times\pi\times h({r_1}^2+{r_2}^2+r_1r_2)$

= $\frac{1}{3} \times \pi \times 10 \times \left[\left(\frac{10\sqrt{3}}{3}\right)^2 + \left(\frac{20\sqrt{3}}{3}\right)^2 + \frac{(10\sqrt{3})(20\sqrt{3})}{3\times3}\right]$

On solving this, we get,
Volume of frustum = 22000/9 cm^3

The radius (r) of wire = $\left(\frac{1}{16}\right) \times \left(\frac{1}{2}\right) = \frac{1}{32}$ cm

Now,
Let the length of wire be “l”.
The volume of wire = Area of cross-section x Length
= $(\pi r^2) \times l$
= $\pi \left(\frac{1}{2}\right)^2 \times l$

Now, Volume of frustum = Volume of wire

$\frac{22000}{9} = \left(\frac{22}{7}\right) \times \left(\frac{1}{32}\right)^2 \times l$

Solving this we get,

l = 7964.44 m