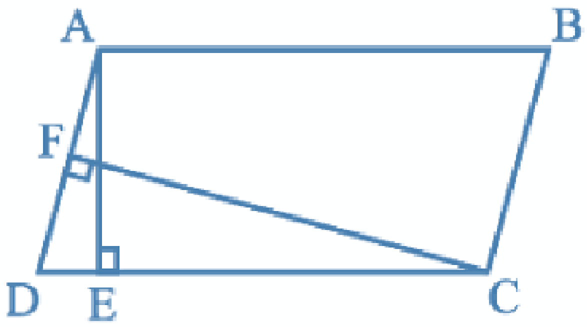
## Chapter 9: Area of triangles and parallelogram

Q.1. In Figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.




Solution:

Given,

AB = CD = 16 cm (Opposite sides of a ||gm are equal)

CF = 10 cm and AE = 8 cm

Now,

Area of parallelogram = Base × Altitude

= CD × AE = AD × CF

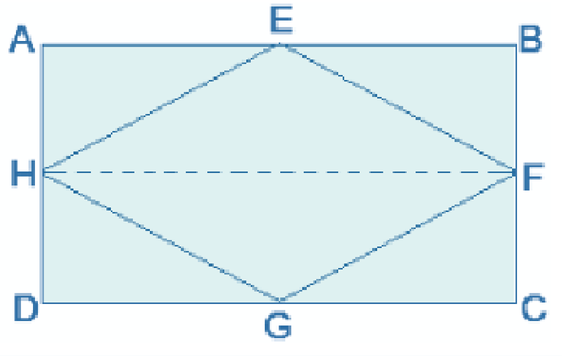
⇒ 16 × 8 = AD × 10

⇒ AD = 128/10

⇒ AD = 12.8 cm

**Q.2. If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD, show that ar (EFGH) = 1/2 ar(ABCD).**

Solution:




Given,

E, F, G and H are the mid-points of the sides of a parallelogram ABCD, respectively.

To Prove: ar (EFGH) = ½ ar(ABCD)

Construction: H and F are joined.

Proof:

AD || BC and AD = BC (Opposite sides of a ||gm)

⇒ ½ AD = ½ BC

Also,

AH || BF and and DH || CF

⇒ AH = BF and DH = CF (H and F are midpoints)

Therefore, ABFH and HFCD are parallelograms.

Now,

As we know, ΔEFH and ||gm ABFH, both lie on the same FH the common base and in-between the same parallel lines AB and HF.

∴ area of EFH = ½ area of ABFH — (i)

And,

area of GHF = ½ area of HFCD — (ii)

Adding (i) and (ii),

area of ΔEFH + area of ΔGHF = ½ area of ABFH + ½ area of HFCD

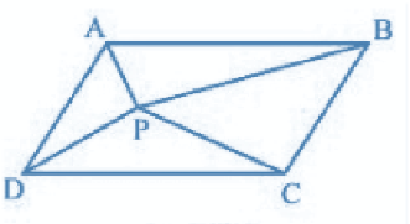
⇒ ar (EFGH) = ½ ar(ABCD)

**Q.3. In Figure, P is a point in the interior of a parallelogram ABCD. Show that**

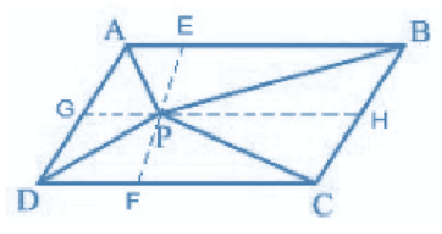
**(i) ar(APB) + ar(PCD) = ½ ar(ABCD)**

**(ii) ar(APD) + ar(PBC) = ar(APB) + ar(PCD)**

**[Hint: Through P, draw a line parallel to AB.]**




Solution:




(i) A line GH is drawn parallel to AB passing through P.

In a parallelogram,

AB || GH (by construction) — (i)

∴,

AD || BC ⇒ AG || BH — (ii)

From equations (i) and (ii),

ABHG is a parallelogram.

Now,

ΔAPB and ||gm ABHG are lying on the same base AB and in-between the same parallel lines AB and GH.

∴ ar(ΔAPB) = ½ ar(ABHG) — (iii)

also,

ΔPCD and ||gm CDGH are lying on the same base CD and in-between the same parallel lines CD and GH.

∴ ar(ΔPCD) = ½ ar(CDGH) — (iv)

Adding equations (iii) and (iv),

ar(ΔAPB) + ar(ΔPCD) = ½ {ar(ABHG) + ar(CDGH)}

⇒ ar(APB) + ar(PCD) = ½ ar(ABCD)

(ii) A line EF is drawn parallel to AD passing through P.

In the parallelogram,

AD || EF (by construction) — (i)

AB || CD ⇒ AE || DF — (ii)

From equations (i) and (ii),

AEDF is a parallelogram.

Now,

ΔAPD and ||gm AEFD are lying on the same base AD and in-between the same parallel lines AD and EF.

∴ ar(ΔAPD) = ½ ar(AEFD) — (iii)

also,

ΔPBC and ||gm BCFE is lying on the same base BC and in-between the same parallel lines BC and EF.

∴ ar(ΔPBC) = ½ ar(BCFE) — (iv)

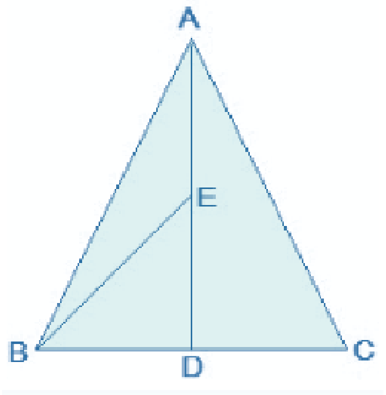
Adding equations (iii) and (iv),

ar(ΔAPD) + ar(ΔPBC) = ½ {ar(AEFD) + ar(BCFE)}

⇒ ar(APD) + ar(PBC) = ar(APB) + ar(PCD)

**Q.4. In a triangle ABC, E is the mid-point of median AD. Show that ar(BED) = 1/4 ar(ABC).**

Solution:




ar(BED) = ½ × BD × DE

Since, E is the mid-point of AD,

AE = DE

Since, AD is the median on side BC of triangle ABC,

BD = DC

DE = ½ AD — (i)

BD = ½ BC — (ii)

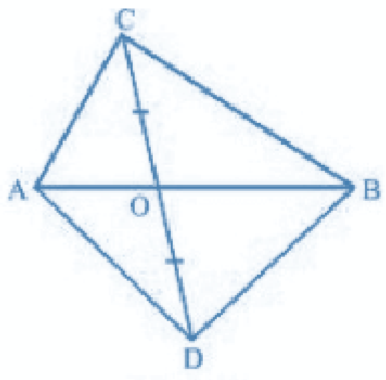
From (i) and (ii), we get,

ar(BED) = (1/2) × (½) BC × (1/2)AD

⇒ ar(BED) = (½) × (½) ar(ABC)

⇒ ar(BED) = ¼ ar(ABC)

**Q.5. In the figure, ABC and ABD are two triangles on the same base AB. If line- segment CD is bisected by AB at O, show that: ar(ABC) = ar(ABD).**




Solution:

In ΔABC, AO is the median. (CD is bisected by AB at O)

∴ ar(AOC) = ar(AOD) — (i)

also,

ΔBCD, BO is the median. (CD is bisected by AB at O)

∴ ar(BOC) = ar(BOD) — (ii)

Adding (i) and (ii),

ar(AOC) + ar(BOC) = ar(AOD) + ar(BOD)

⇒ ar(ABC) = ar(ABD)

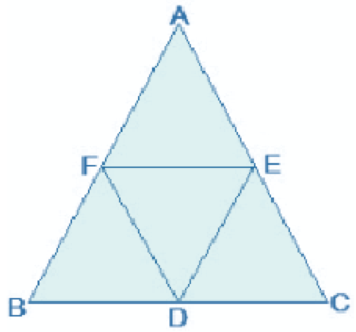
**6. D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Show that**

**(i) BDEF is a parallelogram.**

**(ii) ar(DEF) = ¼ ar(ABC)**

**(iii) ar (BDEF) = ½ ar(ABC)**

Solution:




(i)In ΔABC,

EF || BC and EF = ½ BC (by midpoint theorem)

also,

BD = ½ BC (D is the midpoint)

So, BD = EF

also,

BF and DE are parallel and equal to each other.

Therefore, the pair opposite sides are equal in length and parallel to each other.

∴ BDEF is a parallelogram.

(ii) Proceeding from the result of (i),

BDEF, DCEF, AFDE are parallelograms.

Diagonal of a parallelogram divides it into two triangles of equal area.

∴ ar(ΔBFD) = ar(ΔDEF) (For ||gm BDEF) — (i)

also,

ar(ΔAFE) = ar(ΔDEF) (For ||gm DCEF) — (ii)

ar(ΔCDE) = ar(ΔDEF) (For ||gm AFDE) — (iii)

From (i), (ii) and (iii)

ar(ΔBFD) = ar(ΔAFE) = ar(ΔCDE) = ar(ΔDEF)

⇒ ar(ΔBFD) + ar(ΔAFE) + ar(ΔCDE) + ar(ΔDEF) = ar(ΔABC)

⇒ 4 ar(ΔDEF) = ar(ΔABC)

⇒ ar(DEF) = ¼ ar(ABC)

(iii)Area (||gm BDEF) = ar(ΔDEF) + ar(ΔBDE)

⇒ ar(BDEF) = ar(ΔDEF) + ar(ΔDEF)

⇒ ar(BDEF) = 2× ar(ΔDEF)

⇒ ar(BDEF) = 2× ¼ ar(ΔABC)

⇒ ar(BDEF) = (1/2)ar(ΔABC)

**Q. 7. In the figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD.**

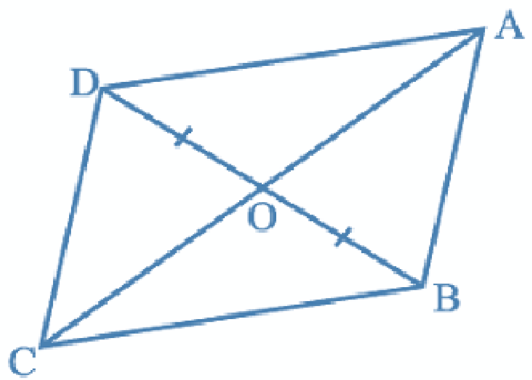
**If AB = CD, then show that:**

**ar (DOC) = ar (AOB)**

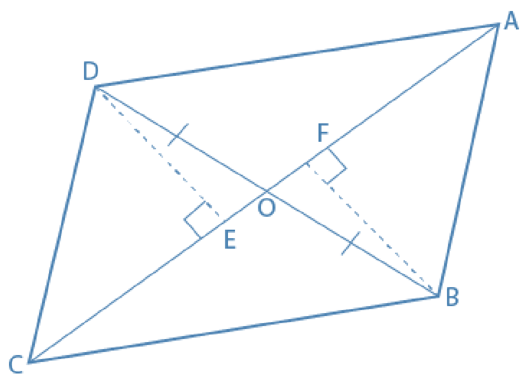
**ar (DCB) = ar (ACB)**

**DA || CB or ABCD is a parallelogram.**

**[Hint: From D and B, draw perpendiculars to AC.]**




Solution:




Given: OB = OD and AB = CD

Construction: DE ⊥ AC and BF ⊥ AC are drawn.

Proof:

(i) In ΔDOE and ΔBOF,

∠DEO = ∠BFO (Perpendiculars)

∠DOE = ∠BOF (Vertically opposite angles)

OD = OB (Given)

∴, ΔDOE ≅ ΔBOF (by AAS congruence criterion)

∴, DE = BF (By CPCT) — (1)

also, ar(ΔDOE) = ar(ΔBOF) (Congruent triangles) — (2)

Now,

In ΔDEC and ΔBFA,

∠DEC = ∠BFA (Perpendiculars)

CD = AB (Given)

DE = BF (From eq.1)

∴, ΔDEC ≅ ΔBFA (by RHS congruence criterion)

∴, ar(ΔDEC) = ar(ΔBFA) (Congruent triangles) — (3)

Adding (2) and (3),

ar(ΔDOE) + ar(ΔDEC) = ar(ΔBOF) + ar(ΔBFA)

⇒ ar (DOC) = ar (AOB)

(ii) ar(ΔDOC) = ar(ΔAOB)

Adding ar(ΔOCB) in LHS and RHS, we get,

⇒ ar(ΔDOC)+ar(ΔOCB)=ar(ΔAOB)+ar(ΔOCB)

⇒ ar(ΔDCB) = ar(ΔACB)

(iii) When two triangles have the same base and equal areas, the triangles will be in between the same parallel lines

ar(ΔDCB) = ar(ΔACB)

DA || BC — (4)

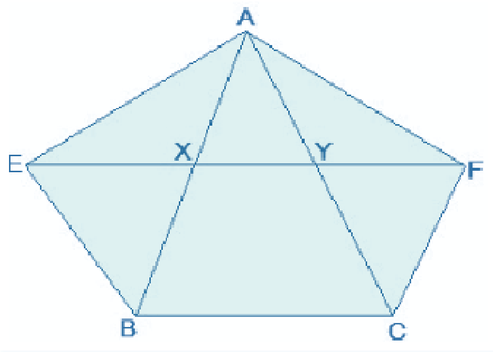
For quadrilateral ABCD, one pair of opposite sides are equal (AB = CD) and the other pair of opposite sides are parallel.

∴, ABCD is a parallelogram.

**Q. 8. XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that**

**ar(ΔABE) = ar(ΔACF)**

Solution:




Given: XY || BC, BE || AC and CF || AB

To show: ar(ΔABE) = ar(ΔACF)

Proof:

BCYE is a ||gm as ΔABE and ||gm BCYE are on the same base BE and between the same parallel lines BE and AC.

∴, ar(ABE) = ½ ar(BCYE) … (1)

Now,

CF || AB and XY || BC

⇒ CF || AB and XF || BC

⇒ BCFX is a parallelogram

As ΔACF and ||gm BCFX are on the same base CF and in-between the same parallel AB and FC.

∴, ar (ΔACF)= ½ ar (BCFX) … (2)

But,

||gm BCFX and ||gm BCYE are on the same base BC and between the same parallels BC and EF.

∴, ar (BCFX) = ar(BCYE) … (3)

From (1) , (2) and (3) , we get

ar (ΔABE) = ar(ΔACF)

⇒ ar(BEYC) = ar(BXFC)

As the parallelograms are on the same base BC and in-between the same parallels EF and BC…..(4)

Also,

△AEB and ||gm BEYC is on the same base BE and in-between the same parallels BE and AC.

⇒ ar(△AEB) = ½ ar(BEYC) ……(5)

Similarly,

△ACF and ||gm BXFC on the same base CF and between the same parallels CF and AB.

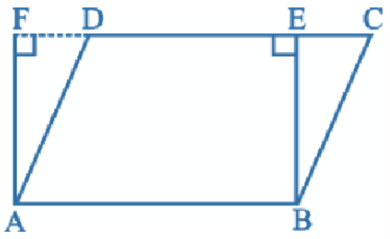
⇒ ar(△ ACF) = ½ ar(BXFC) ……..(6)

From (4), (5) and (6),

ar(△AEB) = ar(△ACF)

**Q.9. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.**

Solution:




Given,

||gm ABCD and a rectangle ABEF have the same base AB and equal areas.

To prove,

The perimeter of ||gm ABCD is greater than the perimeter of rectangle ABEF.

Proof,

As we know, the opposite sides of a||gm and rectangle are equal.

AB = DC [As ABCD is a ||gm]

and, AB = EF [As ABEF is a rectangle]

DC = EF … (i)

Adding AB on both sides, we get,

⇒ AB + DC = AB + EF … (ii)

As we know, the perpendicular segment is the shortest of all the segments that can be drawn to a given line from a point not lying on it.

BE < BC and AF < AD

⇒ BC > BE and AD > AF

⇒ BC + AD > BE + AF … (iii)

Adding (ii) and (iii), we get

AB + DC + BC + AD > AB + EF + BE + AF

⇒ AB + BC + CD + DA > AB + BE + EF + FA

⇒ perimeter of ||gm ABCD > perimeter of rectangle ABEF.

The perimeter of the parallelogram is greater than that of the rectangle.

**Q.10. Diagonals AC and BD of a quadrilateral ABCD intersect each other at E. Show that ar(ΔAED) × ar(ΔBEC) = ar (ΔABE) × ar (ΔCDE).**

**[Hint: From A and C, draw perpendiculars to BD.]**

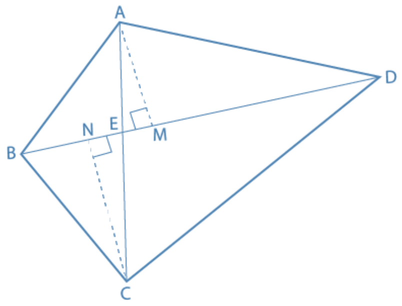
Solution:

Given: The diagonal AC and BD of the quadrilateral ABCD, intersect each other at point E.

Construction:

From A, draw AM perpendicular to BD

From C, draw CN perpendicular to BD




To Prove: ar(ΔAED) ar(ΔBEC) = ar (ΔABE) ar (ΔCDE)

Proof:

ar(ΔABE) = ½ ×BE×AM………….. (i)

ar(ΔAED) = ½ ×DE×AM………….. (ii)

Dividing eq. (ii) by (i) , we get,

ar(ΔAED)/ar(ΔABE) = [1/2×DE×AM]/ [1/2×BE×AM]

= DE/BE ………..(iii)

Similarly,

ar(ΔCDE)/ar(ΔBEC) = DE/BE …….(iv)

From eq. (iii) and (iv), we get;

ar(ΔAED)/ar(ΔABE) = ar(ΔCDE)/ar(ΔBEC)

Therefore, ar(ΔAED) × ar(ΔBEC) = ar (ΔABE) × ar (ΔCDE)