Chapter 1- Real Numbers

Exercise- 1.1

Question 1: Use Euclid's division algorithm to find the HCF of:
i) 135 and 225
ii) 196 and 38220
iii) 867 and 225

Answer: i) As we can see, question 225 is more significant than 135. Hence, by Euclid's division algorithm, we have,
225 = 135 × 1 + 90
Now, the remainder 90 ≠ 0,
Thus again using division lemma for 90, we get, 135 = 90 × 1 + 45
Again, 45 ≠ 0, repeating the above step for 45, we get,
90 = 45 × 2 + 0

Now the remainder is zero.
Since, in the last step, the divisor is 45, therefore, HCF (225,135) = HCF (135, 90) = HCF (90, 45) = 45.
Hence, the HCF of 225 and 135 is 45.

ii) In this question, 38220 is more significant than 196, therefore the by applying Euclid's division algorithm and taking 38220 as a divisor, we get,
38220 = 196 × 195 + 0
As the remainder is 0, HCF (196, 38220) = 196.
Hence, the HCF of 196 and 38220 is 196.

iii) Here, 867 is greater than 225. BY applying Euclid's division algorithm on 867, we get,
867 = 225 × 3 + 102
Remainder 102 ≠ 0, therefore taking 225 as the divisor and applying the division lemma method, we get, 225 = 102 × 2 + 51
Again, 51 ≠ 0. Now 102 is the new divisor, so repeating the same step we get,
102 = 51 × 2 + 0

The remainder is now zero, so, the divisor is 51, therefore, HCF (867,225) = HCF(225,102) = HCF(102,51) = 51.
Hence, the HCF of 867 and 225 is 51.

Question 2: Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

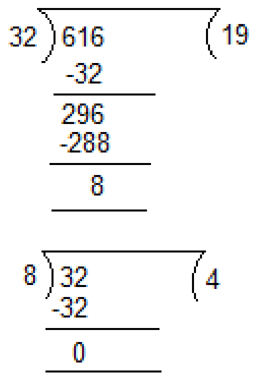
Answer: Let "a" be any positive integer and b = 6.
Then, by Euclid's algorithm, a = 6q + r, for some integer q ≥ 0, and r = 0, 1, 2, 3, 4, 5, as 0≤r<6.

Now, putting r = 0, 1, 2, 3, 4 and 5, we get,
a = 6q, 6q+1, 6q+2, 6q+3, 6q+4, 6q+5

But a = 6q, 6q+2, 6q+4 are even.
Hence, when a is odd, it is of form 6q+1, 6q+3, and 6q+5 for some integer q.

Question 3: An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer: Maximum number of columns = HCF(616, 32)
For finding the HCF, we need to apply Euclid's division algorithm
So, on applying Euclid's division algorithm, we have,
$616 = 32 \times 19 + 8$
Since the remainder is 8, which is not equal to 0, so again we apply Euclid's division algorithm to 32 and 8, we get,
$32 = 8 \times 4 + 0$.



The remainder is 0. Hence when the divisor is 8, the rest is 0.
So, the HCF (616, 32) is 8.
Hence, the maximum number of columns in which they can march is 8.

Question 4: Use Euclid's division lemma to show that the square of any positive integer is either of form 3m or 3m + 1 for some integer m.

Answer: Let x be any positive integer and y = 3 [given].
By using Euclid's division algorithm, we get,
$x = 3q + r$ for some integer $q \geq 0$ and $r = 0, 1, 2$, as $r \geq 0$ and $r < 3$.

Therefore, x = 3q, 3q+1 and 3q+2
Now as per the question given, by squaring both the sides, we get,
$x^2 = (3q)^2 = 9q^2 = 3 \times 3q^2$

Let $3q^2 = m$
Therefore, $x^2 = 3m$

……………………………………………………………………………………..(1)
$x^2 = (3q + 1)^2 = (3q)^2+1^2+2×3q×1 = 9q^2 + 1 +6q = 3(3q^2+2q) +1$

Now substitute, $3q^2+2q = m$, we get,
$x^2= 3m + 1$ ………………………………………………………………………….…………
(2)
$x^2= (3q + 2)^2 = (3q)^2+2^2+2×3q×2 = 9q^2 + 4 + 12q = 3 (3q^2 + 4q + 1)+1$

Again, substituting, $3q^2+4q+1 = m$, we get,
$x^2= 3m + 1$…………………………………………………………………………..…………
(3)
Hence, from equations (1), (2) and (3), we conclude that the square of any positive integer is either of form 3m or (3m + 1) for some integer m.

Question 5: Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Answer: Let x be any positive integer and y = 3.
By using Euclid's division algorithm, we get,
x = 3q+r, where q≥0 and r = 0, 1, 2, as r ≥ 0 and r < 3.

Hence, by putting the value of r, we get,
x = 3q,
x = 3q + 1,
x = 3q + 2
Now, taking the cube of all the three above expressions, we get,

Case 1: When r = 0, then, $x^2= (3q)^3 = 27q^3= 9(3q^3)= 9m$; where $m = 3q^3$

Case 2: When r = 1, then, $x^3 = (3q+1)^3 = (3q)^3 +1^3+3×3q×1(3q+1) = 27q^3+1+27q^2+9q$
or, $x^3 = 9(3q^3+3q^2+q)+1$

Putting $(3q^3+3q^{2+}q) = m$, we get,
$x^3 = 9m+1$

Case 3: When r = 2, then,

$x^3 = (3q+2)^3= (3q)^3+2^3+3×3q×2(3q+2) = 27q^3+54q^2+36q+8$
or, $x^3 = 9(3q^3+6q^2+4q)+8$

Putting $(3q^3+6q^2+4q) = m$, we get,
$x^3 = 9m+8$

Therefore, from all the three cases solved earlier, we can conclude that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

<u>Exercise 1.2</u>

Question 1: Express each number as a product of its prime factors:
(i) 140
(ii) 156
(iii) 3825
(iv) 5005
(v) 7429

Answer: (i) 140 = 2 × 2 × 5 × 7 × 1 = 2^2×5×7

(ii) 156 = 2 × 2 × 13 × 3 × 1 = 2^2× 13 × 3

(iii) 3825 = 3 × 3 × 5 × 5 × 17 × 1 = 3^2×5^2×17

(iv) 5005 = 5 × 7 × 11 × 13 × 1 = 5 × 7 × 11 × 13

(v) 7429 = 17 × 19 × 23 × 1 = 17 × 19 × 23

Question 2: Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.
(i) 26 and 91
(ii) 510 and 92
(iii) 336 and 54

Answer:
(i)26 = 2 × 13 × 1

91 = 7 × 13 × 1

Hence, LCM (26, 91) = 2 × 7 × 13 × 1 = 182 and HCF (26, 91) = 13

Now, product of given two numbers = 26 × 91 = 2366
And Product of LCM and HCF = 182 × 13 = 2366

Hence, LCM × HCF = product of the 26 and 91.

(ii) 510 = 2 × 3 × 17 × 5 × 1

92 = 2 × 2 × 23 × 1

Hence, LCM(510, 92) = 2 × 2 × 3 × 5 × 17 × 23 = 23460 and HCF (510, 92) = 2

Now, product of the two numbers = 510 × 92 = 46920
And Product of LCM and HCF = 23460 × 2 = 46920

Hence, LCM × HCF = product of the 510 and 92.

(iii) 336 = 2 × 2 × 2 × 2 × 7 × 3 × 1

54 = 2 × 3 × 3 × 3 × 1

Hence, LCM(336, 54) = = 3024 and HCF(336, 54) = 2×3 = 6

Now, a product of the given two numbers = 336 × 54 = 18144
And Product of LCM and HCF = 3024 × 6 = 18144

Hence, LCM × HCF = product of the 336 and 54.

Question 3: Find the LCM and HCF of the following integers by applying the prime factorisation method.
(i) 12, 15 and 21
(ii) 17, 23 and 29
(iii) 8, 9 and 25

Answer: (i) From the prime factorisation of,
12=2×2×3
15=5×3
21=7×3
Therefore, HCF(12,15,21) = 3
LCM(12,15,21) = 2 × 2 × 3 × 5 × 7 = 420

(ii) From the prime factorisation of,
17=17×1
23=23×1
29=29×1
Therefore, HCF(17,23,29) = 1
LCM(17,23,29) = 17 × 23 × 29 = 11339

(iii) From the prime factorisation of,
8=2×2×2×1
9=3×3×1
25=5×5×1
Therefore, HCF(8,9,25)=1
LCM(8,9,25) = 2×2×2×3×3×5×5 = 1800

Question 4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Answer: Since we know that, HCF×LCM=Product of the two given numbers
Hence, 9 × LCM = 306 × 657
or, LCM = $\frac{306 \times 657}{9}$ = 22338

Hence, LCM(306,657) = 22338

Question 5. Check whether 6^n can end with the digit 0 for any natural number n.

Answer: Any number of the form 6^n that ends with the digit zero (0), should be divisible by 5 because any number with 0 or 5 in its unit place is divisible by 5.

Prime factorisation of $6^n = (2\times3)^n$

We see the prime factorisation of 6^n doesn't have prime number 5.

Hence, we see that for any natural number n, 6^n is not divisible by five, so 6^n cannot end with the digit 0 for any natural number n.

Question 6. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Answer: From the definition of a composite number, we know that a number is composite when it has factors other than 1 and itself. Therefore,

$7 \times 11 \times 13 + 13$
$=13(7\times11\times1+1)$
$= 13(77+1)$
$= 13\times78$
$= 13\times3\times2\times13$

Hence, 7 × 11 × 13 + 13 is a composite number.

Now,
$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$
$=5(7\times6\times4\times3\times2\times1+1)$
$= 5(1008+1)$
$= 5\times1009$

Hence, 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 is a composite number.

Question 7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet again at the starting point?

Answer: Since, Both Sonia and Ravi move towards the same direction simultaneously, to find the time when they will be meeting again at the starting point we have to find the LCM of 18 and 12.

Therefore, $LCM(18,12) = 2\times3\times3\times2\times1=36$

Hence, Sonia and Ravi will meet each other again at the starting point after 36 minutes.

Exercise 1.3

Question 1: Prove that √5 is irrational.

Answer: To prove √5 is irrational we need to assume, that √5 is a rational number.
i.e. √5 = $\frac{x}{y}$ (where, x and y are co-primes)
or, y√5= x

Squaring both the sides,
$(y\sqrt{5})^2 = x^2$
or, $5y^2 = x^2$……………………………………………………………………(1)

As, x^2 is divisible by 5, so x is also divisible by 5.
Let us assume, x = 5k, for some value of k.
Substitute the value of x in equation (1), we get,
$5y^2 = (5k)^2$

T Since, our assumption about is rational is incorrect.

Hence, √5 is an irrational number.

Question 2. Prove that 3 + 2√5 + is irrational.

Answer: To prove $3 + 2\sqrt{5}$ is an irrational we need to assume 3 + 2√5 is rational.
3 + 2√5 = $\frac{x}{y}$ (y ≠ 0)

$2\sqrt{5} = \frac{x}{y} - 3$
or, $\sqrt{5} = \frac{1}{2}\left(\frac{x}{y} - 3\right)$

Since x and y are integers, thus, $\frac{1}{2}\left(\frac{x}{y} - 3\right)$ is a rational number.
Hence, √5 is a rational number. But this contradicts the fact that √5 is irrational.
So, we can conclude that 3 + 2√5 is irrational.

Question 3. Prove that the following are irrationals:
(i) $\frac{1}{\sqrt{2}}$
(ii) 7√5
(iii) 6 + √2

Answer: Let us assume that $1/\sqrt{2}$ is a rational number.

let, $\frac{1}{\sqrt{2}} = \frac{x}{y}$

or, $\sqrt{2} = \frac{y}{x}$

Since x and y are integers, hence, √2 is a rational number, which contradicts the fact that √two is irrational.

Hence, we can conclude that $\frac{1}{\sqrt{2}}$ is irrational.

(ii) Let us assume 7√5 is a rational number.

let, $7\sqrt{5} = \frac{x}{y}$

or, $\sqrt{5} = \frac{x}{7y}$

Since x and y are integers, √5 is a rational number, which contradicts the fact that √5 is irrational.

Hence, we can conclude that 7√5 is irrational.

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(iii) Let us assume 6 +√2 is a rational number.

let, $6 + \sqrt{2} = \frac{x}{y}$

or, $\sqrt{2} = \frac{x}{y} - 6$

Since, x and y are integers, x/y – 6 is a rational number, and therefore, √2 is reasonable. This contradicts the fact that √2 is an irrational number.

Hence, we can conclude that 6 +√2 is irrational.

Exercise 1.4

Question 1: Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$

(ii) $\frac{17}{8}$

(iii) $\frac{64}{455}$

(iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$

(vi) $\frac{23}{2^3 5^2}$
(vii) $\frac{129}{2^2 5^7 7^5}$
(viii) $\frac{6}{15}$
(ix) $\frac{35}{50}$
(x) $\frac{77}{210}$

Answer: We know that, if the denominator has only factors of 2 and 5 or in the form of $2^m \times 5^n$ then it has a terminating decimal expansion.

(i) $\frac{13}{3125}$

By factorizing the denominator, we get, 3125 = 5 × 5 × 5 = 5^5
Since the denominator has only 5 as its factor, $\frac{13}{3125}$ has a terminating decimal expansion.

(ii) $\frac{17}{8}$
By factorizing the denominator, we get, 8 = 2×2×2 = 2^3
Since the denominator has only two as its factor, $\frac{17}{8}$ has a terminating decimal expansion.

(iii) $\frac{64}{455}$
By factorising the denominator, we get, 455 = 5×7×13
Since the denominator is not in the form of $2^m \times 5^n$, 64/455 has a non-terminating decimal expansion.

(iv) $\frac{15}{1600}$

By factorising the denominator, we get, 1600 = $2^6 5^2$
Since the denominator is in the form of $2^m \times 5^n$, $15/1600$ has a terminating decimal expansion.

(v) $\frac{29}{343}$

By factorizing the denominator, we get, 343 = 7×7×7 = 7^3
Since the denominator is not in the form of $2^m \times 5^n$, 29/343 has a non-terminating decimal expansion.

(vi) $\frac{23}{2^3 5^2}$

We can see that the denominator is in the form of $2^m \times 5^n$.
Hence, $\frac{23}{2^3 5^2}$ has a terminating decimal expansion.

(vii) $\frac{129}{2^2 5^7 7^5}$

As we can see, the denominator is not in the form of $2^m \times 5^n$.
Hence, $\frac{129}{2^2 5^7 7^5}$ has a non-terminating decimal expansion.

(viii) $\frac{6}{15}$

$\frac{6}{15} = \frac{2}{5}$

Since the denominator has only five as its factor, $\frac{6}{15}$ has a terminating decimal expansion.

(ix) $\frac{35}{50}$

$\frac{35}{50} = \frac{7}{10} = \frac{7}{5 \times 2}$

Since the denominator is $2^m \times 5^n$, $\frac{35}{50}$ has a terminating decimal expansion.

(x) $\frac{77}{210}$

$\frac{77}{210} = \frac{7 \times 11}{30 \times 7} = \frac{11}{30} = \frac{11}{3 \times 5 \times 2}$

As you can see, the denominator is not in the form of $2^m \times 5^n$. Hence, $\frac{77}{210}$ has a non-terminating decimal expansion.

Question 2: Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Answer: (i) $\frac{13}{3125} = \frac{13}{5^5} = \frac{13 \times 2^5}{5^5 \times 2^5} = \frac{416}{10^5} = 0.00416$

(ii) $\frac{17}{8} = \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{2125}{10^3} = 2.125$

(iii) $\frac{64}{455}$. We will proceed by division method.

```
455)64 (0.140...
     0
    ----
    640
    455
    ----
    1850
    1820
    ----
      300
      0
    ----
      300
       :
       :
```

As we see, $\frac{64}{455}$ has a non-terminating decimal expansion.

(iv) $\frac{15}{1600}$

```
1600)15.000000(0.009375
      0
     -----
     150
       0
     -----
     1500
        0
     -----
     15000
    -14400
     -----
      6000
    - 4800
     -----
     12000
    -11200
     -----
      8000
     -8000
     -----
      0000
     -----
```

Therefore, $\frac{15}{1600}$ has a terminating decimal expansion.

(v) $\frac{29}{343}$

```
343)29.0000(0.0845...
     0
    ----
    290
      0
    ----
    2900
  - 2744
    ----
     1560
    -1372
    ----
      1880
     -1715
     ----
       165
        :
        :
```

(vi) $\frac{23}{2^3 5^2}$

$= \frac{23}{8\times25} = \frac{23}{200}$

```
200)23.000(0.115
      0
     230
    -200
      300
     -200
      1000
     -1000
      0000
```

Therefore, $\frac{23}{2^3 5^2} = 0.115$

(vii) $\frac{129}{2^2 5^7 7^5}$ has a non-terminating decimal expansion.

(viii) $\frac{6}{15} = \frac{2}{5}$

```
5 )2.0(0.4
   0
   20
  -20
   00
```

(ix) $\frac{35}{50} = \frac{7}{10}$

```
10)7.0(0.7
    0
    70
   -70
    00
```

Therefore, $\frac{35}{50} = 0.7$

(x) $\frac{77}{210} = \frac{11}{30}$

```
30)11.00(0.366..
     0
    110
    -90
     200
    -180
      200
       :
       :
```

Therefore, $\frac{77}{210}$ has a non-terminating decimal expansion.

Question 3: The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form, p q what can you say about the prime factors of q?
(i) 43. 123456789

(ii) 0. 120120012000120000…
(iii) 43. $\overline{123456789}$

Answer: (i) Since, it has a terminating decimal expansion, it is a rational number of the form p/q, and q has a factor as 2 and 5 only.

(ii) Since, it has a non-terminating and non-repeating decimal expansion, it is an irrational number.

(iii) Since it has a non-terminating and repeating decimal expansion, it is a rational number in the form of p/q and q having factors other than 2 and 5.