Chapter 1 Real Numbers
Exercise 1.1

Choose the correct answer from the given four options in the following questions:

1. For some integer m, every even integer is of the form:
(A) m
(B) m +1
(C) 2m
(D) 2m+1

Solution: (C)2m
Even integers are those integers which are divisible by 2.
Hence, we can say that every integer which is a multiple of 2 must be an even integer. Therefore, let us conclude that, for an integer 'm', every even integer must be of the form, $2 \times m = 2m$.

2. For some integer q, every odd integer is of the form
(A) q
(B) q +1
(C) 2g
(D) 2q +1

Solution: (D) 2q+1
Odd integers are those integers which are not divisible by 2.
Hence, we can say that every integer which is a multiple of 2 must be an even integer, while 1 added to every integer which is multiplied by 2 is an odd integer. Therefore, let us conclude that, for an integer 'q', every odd integer must be of the form $(2 \times q)+1 = 2q+1$.

3. $n^2 – 1$ is divisible by 8, if n is
(A) an integer
(B) a natural number
(C) an odd integer
(D) an even integer

Solution: (C) an odd integer
Let $x = n^2 – 1$
In the above equation, n can be either even or odd.
Let us assume that n= even.
So, when n = even i.e., n = 2k, where k is an integer,

We get, $x = (2k)^2-1$
$\Rightarrow x = 4k^2 – 1$

At k = -1, $x = 4(-1)^2 – 1 = 4 – 1 = 3$, is not divisible by 8.
At k = 0, $x = 4(0)^2 – 1 = 0 – 1 = -1$, is not divisible by 8

Let us assume that n= odd:
So, when n = odd i.e., n = 2k + 1, where k is an integer,

We get, $x = 2k + 1$
$\Rightarrow x = (2k+1)^2 - 1$
$\Rightarrow x = 4k^2 + 4k + 1 - 1$
$\Rightarrow x = 4k^2 + 4k$
$\Rightarrow x = 4k(k+1)$

At $k = -1$, $x = 4(-1)(-1+1) = 0$ which is divisible by 8.
At $k = 0$, $x = 4(0)(0+1) = 0$ which is divisible by 8 .
At $k = 1$, $x = 4(1)(1+1) = 8$ which is divisible by 8.

From the above two observation, we can conclude that, if n is odd, n^2-1 is divisible by 8.

4. If the HCF of 65 and 117 is expressible in the form 65m – 117, then the value of m is
(A) 4
(B) 2
(C) 1
(D) 3

Solution: (B) 2
Let us find the HCF of 65 and 117,
$117 = 1\times65 + 52$
$65 = 1\times 52 + 13$
$52 = 4 \times13 + 0$

Hence, we get the HCF of 65 and 117 = 13.

According to the question,
$65m - 117 = 13$
$65m = 117+13 = 130$
$\therefore m = \frac{130}{65} = 2$

5. The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is

(A) 13
(B) 65
(C) 875
(D) 1750

Solution: (A) 13
According to the question, we have to find the largest number which divides 70 and 125, leaving remainders 5 and 8.

The largest number that divides 65 and 117 is also the Highest Common Factor of 65 and 117
Therefore, the required number is the HCF of 65 and 117,
Factors of 65 = 1, 5, 13, 65
Factors of 117 = 1, 3, 9, 13, 39, 117
Common Factors = 1, 13
Highest Common factor (HCF) = 13 i.e., the largest number which divides 70 and 125, leaving remainders 5 and 8, respectively = 13

Question 6: If two positive integers a and b are written as a = x^3y^2 and b = xy^3, where x, y are prime numbers, then HCF (a, b) is
(a) xy **(b) xy^2** **(c)x^3y^3** **(d) xy^2**

Solution: (b)
Given that, a = $x^3y^2 = x \times x \times x \times y \times y$
and, b = $xy^3 = x \times y \times y \times y$
Therefore, HCF of a and b = HCF (x^3y^2, xy^3) = $x \times y \times y = xy^2$ [Since, HCF is the product of the smallest power of each common prime factor involved in the numbers]

Question 7: **If two positive integers p and q can be expressed' as p=ab^2 and q = a^3b; where 0, b being prime numbers, then LCM (p, q) is equal to**
(a) ab **(b) a^2b^2** **(c) a^3b^2** **(d) a^3b^3**

Solution: (c)
Given that, p = $ab^2 = a \times b \times b$
and, q = $a^3b = a \times a \times a \times b$
Thus, LCM of p and q, = LCM (ab^2 , a^3b) = $a \times b \times b \times a \times a = a^3b^2$ [since, LCM is the product of the greatest power of each prime factor involved in the numbers]

Question 8: The product of a non-zero rational and an irrational number is
(a) always irrational **(b) always rational**
(c) rational or irrational **(d) one**

Solution: (a)
Product of an non-zero rational and an irrational number is always irrational i.e., $\frac{3}{4} \times \sqrt{2} = \frac{3\sqrt{2}}{4}$ (irrational)

Question 9: The least number that is divisible by all the numbers from 1 to 10 (both inclusive)
(a) 10 **(b) 100** **(c) 504** **(d) 2520**

Solution: (d) Factors of 1 to 10 numbers,

$1 = 1$
$2 = 1 \times 2$
$3 = 1 \times 3$
$4 = 1 \times 2 \times 2$
$5 = 1 \times 5$
$6 = 1 \times 2 \times 3$
$7 = 1 \times 7$
$8 = 1 \times 2 \times 2 \times 2$
$9 = 1 \times 3 \times 3$
$10 = 1 \times 2 \times 5$

LCM of number 1 to 10 = LCM(1,2,3,4,5,6,7,8,9,10)
$= 1 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$

Question 10: The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after

(a) one decimal place
(b) two decimal places
(c) three decimal places
(d) four decimal places

Solution:

(d) Rational number $= \frac{14587}{1250} = \frac{14587}{2^1 \times 5^4}$

$$= \frac{14587}{10 \times 5^3} \times \frac{(2)^3}{(2)^3}$$

$$= \frac{14587 \times 8}{10 \times 1000}$$

$$= \frac{116696}{10000} = 11.6696$$

2	1250
5	625
5	125
5	25
5	5
	1

Hence, given rational number will terminate after four decimal places.

Exercise 1.2

1. Write whether every positive integer can be of the form 4q + 2, where q is an integer. Justify your answer.

Solution: No, every positive integer cannot be of the form 4q + 2, where q is an integer.

All the numbers of the form 4q + 2, where ‘q’ is an integer, are even numbers which are not divisible by ‘4’.

For example, When q = 1,
4q + 2 = 4(1) + 2 = 6.

When q = 2,
4q + 2 = 4(2) + 2 = 10

When q = 0,
4q + 2 = 4(0) + 2 = 2 and so on.

So, any number which is of the form 4q + 2 will give only even numbers which are not multiples of 4.

Hence, every positive integer cannot be written in the form 4q + 2.

2. "The product of two consecutive positive integers is divisible by 2". Is this statement true or false? Give reasons.

Solution: Yes, the statement "the product of two consecutive positive integers is divisible by 2" is true.

Let the two consecutive positive integers = a, a + 1
According to Euclid's division lemma,

We have, a = bq + r, where 0 ≤ r < b

For b = 2, we have a = 2q + r, where 0 ≤ r < 2 ……………………………………….(1)

Substituting r = 0 in equation (1), we get,
a = 2q, is divisible by 2.
a + 1 = 2q + 1, is not divisible by 2.

Substituting r = 1 in equation (1), we get,
a = 2q + 1, is not divisible by 2.
a + 1 = 2q + 1+1 = 2q + 2, is divisible by 2.

Thus, we can conclude that, for 0 ≤ r < 2, one out of every two consecutive integers is divisible by 2. So, the product of the two consecutive positive numbers will also be even. Hence, the statement "product of two consecutive positive integers is divisible by 2" is true.

3. "The product of three consecutive positive integers is divisible by 6". Is this statement true or false? Justify your answer.

Solution: Yes, the statement "the product of three consecutive positive integers is divisible by 6" is true.

Consider the 3 consecutive numbers 2, 3, 4

$$\frac{2 \times 3 \times 4}{6} = \frac{24}{6} = 4$$

Now, consider another 3 consecutive numbers 4, 5, 6

$$\frac{4 \times 5 \times 6}{6} = \frac{120}{6} = 20$$

Now, consider another 3 consecutive numbers 7, 8, 9

$$\frac{7 \times 8 \times 9}{6} = \frac{504}{6} = 84$$

Hence, the statement “product of three consecutive positive integers is divisible by 6” is true.

4. Write whether the square of any positive integer can be of the form 3m + 2, where m is a natural number. Justify your answer.

Solution: No, the square of any positive integer cannot be written in the form 3m + 2 where m is a natural number.

According to Euclid’s division lemma, a positive integer ‘a’ can be written in the form of bq + r.
a = bq + r, where b, q and r are any integers,

For b = 3,
a = 3(q) + r, where, r can be an integers,

For r = 0, 1, 2, 3……….
3q + 0, 3q + 1, 3q + 2, 3q + 3……. are positive integers,

$(3q)^2 = 9q^2 = 3(3q^2) = 3m$ (where $3q^2 = m$)
$(3q + 1)^2 = (3q + 1)^2 = 9q^2 + 1 + 6q = 3(3q^2 + 2q) + 1 = 3m + 1$ (Where, $m = 3q^2+2q$)
$(3q + 2)^2 = (3q + 2)^2 = 9q^2 + 4 + 12q = 3(3q^2 + 4q) + 4 = 3m + 4$ (Where, $m = 3q^2+2q$)
$(3q + 3)^2 = (3q + 3)^2 = 9q^2 + 9 + 18q = 3(3q^2 + 6q) + 9 = 3m + 9$ (Where, $m = 3q^2+2q$)

Hence, there is no positive integer whose square can be written in the form 3m + 2 where m is a natural number.

5. A positive integer is of the form 3q + 1, q being a natural number. Can you write its square in any form other than 3m + 1, i.e., 3m or 3m + 2 for some integer m? Justify your answer.

Solution: No.

Consider the positive integer 3q + 1, where q is a natural number.

$(3q + 1)^2 = 9q^2 + 6q + 1$
$= 3(3q^2 + 2q) + 1$
$= 3m + 1$, (where m is an integer which is equal to $3q^2 + 2q$.

Thus $(3q + 1)^2$ cannot be expressed in any other form apart from 3m + 1.

Question 6: The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75. What is HCF (525, 3000)? Justify your answer.

Solution:
Since, the HCF (525, 3000) = 75
By Euclid’s Lemma,
3000 = 525(5) + 375………[dividend = divisor × quotient + remainder]
525 = 375(1) + 150
375 = 150(2) + 75
150 = 75(2) + 0
and the numbers 3, 5, 15, 25 and 75 divides the numbers 525 and 3000 that mean these terms are common in both 525 and 3000. So, the highest common factor among these is 75.

Question 7: Explain why3 x 5 x 7 + 7 is a composite number,

Solution: We have, $3 \times 5 \times 7 + 7$ = 105 + 7 = 112
Now, 112 = 2 × 2 × 2 × 2 × 7 = $2^4 \times 7$
So, it is the product of prime factors 2 and 7. i.e., it has more than two factors.
Hence, it is a composite number.

Question 8: Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.

Solution:
No, because HCF is always a factor of LCM but here 18 is not a factor of 380.

Question 9: Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non-terminating(repeating) decimal expansion. Give reasons for your answer.

Solution:
Yes, after simplification denominator has factor 5^3 -2^2 and which is of the type 2^m . 5″. So, this is terminating decimal.

$$\frac{987}{10500} = \frac{47}{500} = \frac{47}{5^3 . 2^2} \times \frac{2}{2} = \frac{94}{5^3 \times 2^3} = \frac{94}{10^3} = \frac{94}{1000} = 0.094$$

Question 10: A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of q, when this number is expressed in the form p/q ? Give reasons.

Solution:
327.7081 is terminating decimal number. So, it represents a rational number and also its denominator must have the form 2^m x5^n

Hence, 327.7081 = $\frac{3277081}{10000} = \frac{p}{q}$
Therefore, q = 10^4 = 2×2×2×2×5×5×5×5 = $2^4 \times 5^4 = (2 \times 5)^4$
Hence, the prime factors of q is 2 and 5.

Exercise 1.3

1. Show that the square of any positive integer is either of the form 4q or 4q + 1 for some integer q.

Solution: According to Euclid's division lemma, $a = bq + r$

According to the question, when $b = 4$.
$a = 4k + r, 0 < r < 4$

When $r = 0$, we get, $a = 4k$
$a^2 = 16k^2 = 4(4k^2) = 4q$, where $q = 4k^2$

When $r = 1$, we get, $a = 4k + 1$
$a^2 = (4k + 1)^2 = 16k^2 + 1 + 8k = 4(4k + 2) + 1 = 4q + 1$, where $q = k(4k + 2)$

When $r = 2$, we get, $a = 4k + 2$
$a^2 = (4k + 2)^2 = 16k^2 + 4 + 16k = 4(4k^2 + 4k + 1) = 4q$, where $q = 4k^2 + 4k + 1$

When $r = 3$, we get, $a = 4k + 3$
$a^2 = (4k + 3)^2 = 16k^2 + 9 + 24k = 4(4k^2 + 6k + 2) + 1 = 4q + 1$, where $q = 4k^2 + 6k + 2$

Therefore, the square of any positive integer is either of the form $4q$ or $4q + 1$ for some integer q.

Hence Proved.

2. Show that cube of any positive integer is of the form 4m, 4m + 1 or 4m + 3, for some integer m.

Solution: Let a be any positive integer and $b = 4$.
According to Euclid Division Lemma,
$a = bq + r$ $[0 \le r < b]$
$a = 3q + r$ $[0 \le r < 4]$

According to the question, the possible values of r are, $r = 0$, $r = 1$, $r = 2$, $r = 3$

When $r = 0$,
$a = 4q + 0$
$a = 4q$

Taking cubes on LHS and RHS, we have,
$a^3 = (4q)^3$
$a^3 = 4\ (16q^3)$
$a^3 = 4m$……………..[where m is an integer $= 16q^3$]

When $r = 1$,
$a = 4q + 1$

Taking cubes on LHS and RHS, we have,

$a^3 = (4q + 1)^3$
$a^3 = 64q^3 + 1^3 + 3 \times 4q \times 1\ (4q + 1)$
$a^3 = 64q^3 + 1 + 48q^2 + 12q$
$a^3 = 4\ (16q^3 + 12q^2 + 3q) + 1$
$a^3 = 4m + 1$………………….[where m is an integer $= 16q^3 + 12q^2 + 3q$]

When $r = 2$, $a = 4q + 2$

Taking cubes on LHS and RHS, we have,
$a^3 = (4q + 2)^3$
$a^3 = 64q^3 + 2^3 + 3 \times 4q \times 2\ (4q + 2)$
$a^3 = 64q^3 + 8 + 96q^2 + 48q$

$a^3 = 4(16q^3 + 2 + 24q^2 + 12q)$
$a^3 = 4m$……………………….[where m is an integer $= 16q^3 + 2 + 24q^2 + 12q$]

When r = 3, a = 4q + 3

Taking cubes on LHS and RHS, we have,

$a^3 = (4q + 3)^3$
$a^3 = 64q^3 + 27 + 3 \times 4q \times 3(4q + 3)$
$a^3 = 64q^3 + 24 + 3 + 144q^2 + 108q$
$a^3 = 4(16q^3 + 36q^2 + 27q + 6) + 3$
$a^3 = 4m + 3$…………………..[where m is an integer $= 16q^3 + 36q^2 + 27q + 6$]

Hence, the cube of any positive integer is in the form of 4m, 4m+1 or 4m+3.

3. Show that the square of any positive integer cannot be of the form 5q + 2 or 5q + 3 for any integer q.

Solution: Let the positive integer = a
According to Euclid's division lemma, a = bm + r

According to the question, b = 5, a = 5m + r

So, r= 0, 1, 2, 3, 4
When r = 0, a = 5m.
When r = 1, a = 5m + 1.
When r = 2, a = 5m + 2.
When r = 3, a = 5m + 3.
When r = 4, a = 5m + 4.

Now, When a = 5m
$a^2 = (5m)^2 = 25m^2$
$a^2 = 5(5m^2) = 5q$, where $q = 5m^2$

When a = 5m + 1,
$a^2 = (5m + 1)^2 = 25m^2 + 10m + 1$
$a^2 = 5(5m^2 + 2m) + 1 = 5q + 1$, where $q = 5m^2 + 2m$

When a = 5m + 2,
$a^2 = (5m + 2)^2$
$a^2 = 25m^2 + 20m + 4$
$a^2 = 5(5m^2 + 4m) + 4$
$a^2 = 5q + 4$ where $q = 5m^2 + 4m$

When a = 5m + 3,
$a^2 = (5m + 3)^2 = 25m^2 + 30m + 9$
$a^2 = 5(5m^2 + 6m + 1) + 4$
$a^2 = 5q + 4$ where $q = 5m^2 + 6m + 1$

When a = 5m + 4,
$a^2 = (5m + 4)^2 = 25m^2 + 40m + 16$
$a^2 = 5(5m^2 + 8m + 3) + 1$
$a^2 = 5q + 1$ where $q = 5m^2 + 8m + 3$

Therefore, square of any positive integer cannot be of the form 5q + 2 or 5q + 3.

Hence Proved.

4. Show that the square of any positive integer cannot be of the form 6m + 2 or 6m + 5 for any integer m.

Solution: Let the positive integer = a

According to Euclid's division algorithm,
$a = 6q + r$, where $0 \le r < 6$
$a^2 = (6q + r)^2 = 36q^2 + r^2 + 12qr$ [$\because (a+b)^2 = a^2 + 2ab + b^2$]
$a^2 = 6(6q^2 + 2qr) + r^2$ ……………………………………………..…(1), where,$0 \le r < 6$

When r = 0, substituting r = 0 in Eq.(1), we get
$a^2 = 6 (6q^2) = 6m$, where, $m = 6q^2$ is an integer.

When r = 1, substituting r = 1 in Eq.(1), we get
$a^2 = 6 (6q^2 + 2q) + 1 = 6m + 1$, where, $m = (6q^2 + 2q)$ is an integer.

When r = 2, substituting r = 2 in Eq(1), we get
$a^2 = 6(6q^2 + 4q) + 4 = 6m + 4$, where, $m = (6q^2 + 4q)$ is an integer.

When r = 3, substituting r = 3 in Eq.(1), we get
$a^2 = 6(6q^2 + 6q) + 9 = 6(6q^2 + 6a) + 6 + 3$
$a^2 = 6(6q^2 + 6q + 1) + 3 = 6m + 3$, where, $m = (6q + 6q + 1)$ is integer.

When r = 4, substituting r = 4 in Eq.(1) we get
$a^2 = 6(6q^2 + 8q) + 16$
$= 6(6q^2 + 8q) + 12 + 4$
$= 6(6q^2 + 8q + 2) + 4 = 6m + 4$, where, $m = (6q^2 + 8q + 2)$ is integer.

When r = 5, substituting r = 5 in Eq.(1), we get
$a^2 = 6 (6q^2 + 10q) + 25 = 6(6q^2 + 10q) + 24 + 1$
$a^2 = 6(6q^2 + 10q + 4) + 1 = 6m + 1$, where, $m = (6q^2 + 10q + 4)$ is integer.

Hence, the square of any positive integer cannot be of the form 6m + 2 or 6m + 5 for any integer m.

Hence Proved

5. Show that the square of any odd integer is of the form 4q + 1, for some integer q.

Solution: Let a be any odd integer and b = 4.
According to Euclid's algorithm, $a = 4m + r$ for some integer $m \ge 0$ and r = 0,1,2,3 because $0 \le r < 4$.

So, we have that, a = 4m or 4m + 1 or 4m + 2 or 4m + 3 So, a = 4m + 1 or 4m + 3

We know that, a cannot be 4m or 4m + 2, as they are divisible by 2.
$(4m + 1)^2 = 16m^2 + 8m + 1$
$= 4(4m^2 + 2m) + 1$
$= 4q + 1$, where q is some integer and $q = 4m^2 + 2m$.

$(4m + 3)^2 = 16m^2 + 24m + 9$
$= 4(4m^2 + 6m + 2) + 1$
$= 4q + 1$, where q is some integer and $q = 4m^2 + 6m + 2$

Therefore, Square of any odd integer is of the form $4q + 1$, for some integer q.
Hence Proved.

6. If n is an odd integer, then show that $n^2 - 1$ is divisible by 8.

Solution: We know that any odd positive integer n can be written in form $4q + 1$ or $4q + 3$.

So, according to the question,
When $n = 4q + 1$,
Then $n^2 - 1 = (4q + 1)^2 - 1 = 16q^2 + 8q + 1 - 1 = 8q(2q + 1)$, is divisible by 8.
When $n = 4q + 3$,
Then $n^2 - 1 = (4q + 3)^2 - 1 = 16q^2 + 24q + 9 - 1 = 8(2q^2 + 3q + 1)$, is divisible by 8.

So, from the above equations, it is clear that, if n is an odd positive integer $n^2 - 1$ is divisible by 8.
Hence Proved.

7. Prove that if x and y are both odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

Solution: Let the two odd positive numbers x and y be $2k + 1$ and $2p + 1$, respectively i.e., $x^2 + y^2 = (2k + 1)^2 +(2p + 1)^2$
$= 4k^2 + 4k + 1 + 4p^2 + 4p + 1$
$= 4k^2 + 4p^2 + 4k + 4p + 2$
$= 4 (k^2 + p^2 + k + p) + 2$

Thus, the sum of square is even the number is not divisible by 4
Therefore, if x and y are odd positive integer, then $x^2 + y^2$ is even but not divisible by four.
Hence Proved

Question 8: Use Euclid's division algorithm to find the HCF of 441, 567 and 693.

Solution:
Let a = 693, b = 567 and c = 441 By Euclid's division algorithms,
$a = bq + r$
Finding the HCF,
$693 = 567 \times 1 + 126$
$567 = 126 \times 4 + 63$
$126 = 63 \times 2 + 0$
Therefore, HCF(693, 567) = 63
Let us take, c = 441, and d = 63 , and find their HCF.
$441 = 63 \times 7 + 0$
Therefore, HCF(693, 567, 441) = 63

Question 9:
Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3, respectively.
Solution:

Since, 1, 2 and 3 are the remainders of 1251, 9377 and 15628, respectively. Thus, after subtracting these remainders from the numbers.
We have the numbers, 1251-1 = 1250,9377-2 = 9375 and 15628-3 = 15625 which is divisible by the required number.
Now, required number = HCF of 1250,9375 and 15625 [for the largest number]
By Euclid's division algorithm,

a = bq + r

For largest number, a = 15625, b = 9375

15625 = 9375 x 1 + 6250
9375 = 6250 x 1 + 3125
6250 = 3125 x 2 + 0

Therefore, HCF(15625, 9375) = 3125.
Now, let c = 1250, and d = 3125
Using Euclid's division algorithm,

3125 = 1250 x 2 + 625
1250 = 625 x 2 + 0

Therefore, HCF(3125, 1250) = 625
Therefore, HCF(1250, 9375, 15625) = 625

Hence, 625 is the largest number which divides 1251,9377 and 15628 leaving remainder 1, 2 and 3, respectively.

Question 10: Prove that√3 + √5 is irrational.
Solution:
Let us suppose that √3 + √5 is rational.
Let √3 + √5 = a, where a is rational.
Therefore, $\sqrt{3} = a - \sqrt{5}$
On squaring both sides, we get, $(\sqrt{3})^2 = (a - \sqrt{5})^2$
or, 3 = $(a)^2 + 5 - 2a\sqrt{5}$
or, $2a\sqrt{5} = a^2 + 2$
or, $\sqrt{5} = \frac{a^2+2}{2a}$ which is contradiction

As the right hand side is rational number while V5 is irrational. Since, 3 and 5 are prime numbers. Hence, √3 + √5 is irrational.

Question 11: Show that 12^n cannot end with the digit 0 or 5 for any natural number n.
Solution:
If any number ends with the digit 0 or 5, it is always divisible by 5.
If 12^n ends with the digit zero it must be divisible by 5.
This is possible only if prime factorisation of 12^n contains the prime number 5.
Now, 12 = 2 x 2 x 3 = $2^2 \times 3$
or, $12^n = (2^2 \times 3)^n = 2^{2n} \times 3^n$ [since there is no term containing 5]

Hence, there is no value of n ∈ N for which 12^n ends with digit zero or five.

Question 12: On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively.What is the minimum distance each should walk, so that each can cover the same distance in complete steps?

Solution:

We have to find the LCM of 40 cm, 42 cm and 45 cm to get the required minimum distance.

For this, 40 = 2 x 2 x 2 x 5
42 = 2 x 3 x 7
45 = 3 x 3 x 5

Therefore LCM(40, 42, 45) = 2 x 3 x 5 x 2 x 2 x 3 x 7
= 30 x 12 x 7 = 210 x 12
= 2520

Minimum distance each should walk 2520 cm. So that, each can cover the same distance in
complete steps.

Question 13: Write the denominator of rational number $\frac{257}{5000}$ in the form 2^m x5^n,where m, n are non-negative integers. Hence, write its decimal expansion, without actual division

Solution:

Denominator of the rational number $\frac{257}{5000}$ is 5000.

Now, factors of 5000 = 2 x 2 x 2 x 5 x 5 x 5 x 5 = $(2)^3 \times (5)^4$, which is of type $2^m \times 5^n$, where m = 3, n = 4 are non-negative integers.

Therefore, Rational number = $\frac{257}{5000} = \frac{257}{(2)^3\times(5)^4} \times \frac{2}{2}$

$$= \frac{514}{(2)^4\times(5)^4} = \frac{514}{(10)^4} = \frac{514}{10000} = 0.0514$$

Hence, which is the required decimal expansion of the rational $\frac{257}{5000}$ and it is also a 5000 terminating decimal number.

Question 14: Prove that √p + √q is irrational, where p and q are primes.

Solution:

Let us suppose that $\sqrt{p} + \sqrt{q}$ is rational.

Again, let $\sqrt{p} + \sqrt{q} = a$, where a is rational.

Therefore, $\sqrt{q} = a - \sqrt{p}$

On squaring both sides, we get

$q = a^2 + p - 2a\sqrt{p}$ $[\because (a-b)^2 = a^2 + b^2 - 2ab]$

Therefore, $\sqrt{p} = \frac{a^2 + p - q}{2a}$, which is a contradiction as the right hand side is rational number while $\sqrt{p}$ is irrational, since p and q are prime numbers.

Hence, $\sqrt{p} + \sqrt{q}$ is irrational.

Exercise 1.4

1. Show that the cube of a positive integer of the form 6q + r, q is an integer and r = 0, 1, 2, 3, 4, 5 is also of the form 6m + r.

Solution: 6q + r is a positive integer, where q is an integer and r = 0, 1, 2, 3, 4, 5
Then, the positive integers are of form 6q, 6q + 1, 6q + 2, 6q + 3, 6q + 4 and 6q + 5.

Taking cube on L.H.S and R.H.S,
For 6q,
$(6q)^3 = 216\ q^3 = 6(36q)^3 + 0$
$= 6m + 0$, (where m is an integer $= (36q)^3$)

For 6q + 1,
$(6q + 1)^3 = 216q^3 + 108q^2 + 18q + 1$
$= 6(36q^3 + 18q^2 + 3q) + 1$
$= 6m + 1$, (where m is an integer $= 36q^3 + 18q^2 + 3q$)

For 6q + 2,
$(6q + 2)^3 = 216q^3 + 216q^2 + 72q + 8$
$= 6(36q^3 + 36q^2 + 12q + 1) +2$
$= 6m + 2$, (where m is an integer $= 36q^3 + 36q^2 + 12q + 1$)

For 6q + 3,
$(6q + 3)^3 = 216q^3 + 324q^2 + 162q + 27$
$= 6(36q^3 + 54q^2 + 27q + 4) + 3$
$= 6m + 3$, (where m is an integer $= 36q^3 + 54q^2 + 27q + 4$)

For 6q + 4,
$(6q + 4)^3 = 216q^3 + 432q^2 + 288q + 64$
$= 6(36q^3 + 72q^2 + 48q + 10) + 4$
$= 6m + 4$, (where m is an integer $= 36q^3 + 72q^2 + 48q + 10$)

For 6q + 5,
$(6q + 5)^3 = 216q^3 + 540q^2 + 450q + 125$
$= 6(36q^3 + 90q^2 + 75q + 20) + 5$
$= 6m + 5$, (where m is an integer $= 36q^3 + 90q^2 + 75q + 20$)

Hence, the cube of a positive integer of the form 6q + r, q is an integer and r = 0, 1, 2, 3, 4, 5 is also of the form 6m + r.

2. Prove that one and only one out of n, n + 2 and n + 4 is divisible by 3, where n is any positive integer.

Solution: According to Euclid's division Lemma, let the positive integer = n and b = 3
n = 3q + r, where q is the quotient and r is the remainder

$0 < r < 3$ implies remainders may be 0, 1 and 2

Therefore, n may be in the form of 3q, 3q + 1, 3q + 2
When n = 3q
n + 2 = 3q + 2
n + 4 = 3q + 4
Here n is only divisible by 3

When $n = 3q + 1$
$n + 2 = 3q = 3$
$n + 4 = 3q + 5$
Here only n+2 is divisible by 3

When $n = 3q + 2$
$n + 2 = 3q + 4$
$n + 4 = 3q + 2 + 4 = 3q + 6$
Here only n+4 is divisible by 3

So, we can conclude that one and only one out of n, n + 2 and n + 4 is divisible by 3.
Hence Proved

Question 3:
Prove that one of any three consecutive positive integers must be divisible by 3.
Solution:
Any three consecutive positive integers must be of the form
n, (n + 1)and (n + 2), where n is any natural number, i.e., n = 1,2, 3…
Let, a = n, b = n+ 1 and c = n + 2
Order triplet is (a, b, c) = (n, n + 1, n + 2), where n = 1,2, 3,… …(i)
At n = 1; (a, b, c) = (1,1 + 1,1 + 2) = (1,2, 3)
At n = 2; (a, b, c) = (1,2 + 1,2 + 2) = (2, 3, 4)
At n = 3; (a, b, c) = (3, 3 + 1, 3 + 2) = (3, 4, 5)
At n = 4; (a, b, c) = (4, 4 + 1, 4 + 2) = (4, 5, 6)
At n = 5; (a, b, c) = (5, 5 + 1, 5 + 2) = (5, 6,7)
At n= 8 (a, b, c) = (6, 6+ 1, 6 + 2) = (6,7, 8)
At n = 7; (a, b, c) = (7,7 + 1,7 + 2) = (7, 8, 9)
At n = 8; (a, b, c) = (8 8 + 1, 8 + 2) = (8, 9,10)
We observe that each triplet consist of one and only one number which is multiple of 3 i.e., divisible by 3.
Hence, one of any three consecutive positive integers must be divisible by 3.

Question 4: For any positive integer n, prove that $n^3 – n$ is divisible by 6.
Solution: Let $a = n^3 – n$
or, $a = n(n^2 – 1)$
or, $a = n(n – 1)(n + 1)$
or, $a = (n – 1) n (n + 1)$…………………………………(1)
We know that,
1) If a number is completely divisible by 2 and 3, then it is also divisible by 6.
2) If the sum of digits of any number is divisible by 3, then it is also divisible by 3.
3) If one of the factor of any number is an even number, then it is also divisible by 2.

∴ $a = (n – 1) . n . (n + 1)$……………………. [from Eq. (1)]
Now, sum of the digits $= n – 1 + n + n + 1 = 3n$
= multiple of 3, where n is any positive integer,
and (n – 1) – n – (n + 1) will always be even, as one out of (n – 1) or n or (n + 1) must of even. Since, conditions II and III is completely satisfy the Eq. (1).

Hence, by condition I the number $n^3 - n$ is always divisible by 6, where n is any positiveinteger.
Hence proved.

Question 5: Show that one and only one out of n, n + 4 , n + 8, n + 12 and n + 16 is divisible by 5, where n is any positive integer.
Solution:
Given numbers are n, (n+ 4), (n + 8), (n + 12) and (n + 16), where n is any positive integer.
Then, let n = 5q, 5g + 1, 5g + 2, 5g + 3, 5q + 4 for $q \in N$ ……[by Euclid’s algorithm]
Then, in each case if we put the different values of n in the given numbers. We definitely get one and only one of given numbers is divisible by 5.
Hence, one and only one out of n, n+ 4, n+ 8, n+12 and n+16 is divisible by 5.
Alternate Method
On dividing on n by 5, let q be the quotient and r be the remainder.
Then, $n = 5q + r$, where $0 < r < 5$.
or, $5q + r$, where r = 0, 1, 2, 3, 4
or, n = 5q + 1 or
5q + 2 or
5q + 3 or
5q + 4

Case 1:
If n = 5q then n is only divisible by 5.

Case 2:
If n = 5q + 1, then n + 4 = 5q + 1 + 4 = 5q + 5 = 5(q + 1), which is only divisible by 5.
So, in this case, (n + 4) is divisible by 5

Case 3:
If n = 5q + 3, then n + 2 = 5q + 3 + 12 = 5q + 15 = 5(q + 3), which is divisible by 5.
So, in this case (n + 12) is only divisible by 5

Case 4:
If n = 5q + 4, then n + 16 = 5q + 4 +16 = 5q + 15 = 5(q + 3), which is divisible by 5.
So, in this case, (n + 16) is only divisible by 5

Hence, one and only one out of n, n + 4, n + 8, n + 12, n + 16 is divisible by 5, where n is any positive number.