Chapter 5 – Arithmetic Progression
Exercise – 5.1

Question 1: In which of the following situations, does the list of numbers involved make as arithmetic progression and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.
(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.
(iii) The cost of digging a well after every meter of digging, when it costs ₹ 150 for the first meter and rises by ₹ 50 for each subsequent meter.
(iv) The amount of money in the account every year, when ₹ 10000 is deposited at compound interest at 8% per annum.

Answer: (i) We can write the given condition as;
Taxi fare for 1 km = 15
Taxi fare for first 2 kms = 15+8 = 23
Taxi fare for first 3 kms = 23+8 = 31
Taxi fare for first 4 kms = 31+8 = 39
And so on……
Thus, 15, 23, 31, 39 … forms an A.P. because every next term is 8 more than the preceding term.

(ii) Let the volume of air in a cylinder, initially, be V litres.
In each stroke, the vacuum pump removes $\frac{1}{4}$th of air remaining in the cylinder at a time. Or we can say, after every stroke, $1-\frac{1}{4}=\frac{3}{4}$th part of air will remain.

Therefore, volumes will be V, $3\frac{V}{4}$, $\left(\frac{3V}{4}\right)^2$, , $\left(\frac{3V}{4}\right)^3$…………………

We can conclude here, the adjacent terms of this series do not have the common difference between them. Therefore, this series is not an A.P.

(iii) We can write the given statement as;
Cost of digging a well for first metre = Rs.150
Cost of digging a well for first 2 metres = Rs.150+50 = Rs.200
Cost of digging a well for first 3 metres = Rs.200+50 = Rs.250
Cost of digging a well for first 4 metres =Rs.250+50 = Rs.300

And so on…………

Hence, 150, 200, 250, 300 … forms an A.P. with a common difference of 50 between each term.

(iv) We know that if Rs. P is deposited at r% compound interest per annum for n years, the amount of money will be: $P(1 + \frac{r}{100})^n$ [By using Compound interest rule]

Therefore, after each year, the amount of money will be;

$10000(1 + \frac{8}{100})$, $10000(1 + \frac{8}{100})^2$, $10000(1 + \frac{8}{100})^3$……

Clearly, the terms of this series do not have the common difference between them. Therefore, this is not an A.P.

Question 2: Write first four terms of the AP, when the first term a and the common difference d are given as follows:
(i) a = 10, d = 10
(ii) a = -2, d = 0
(iii) a = 4, d = -3
(iv) a = -1, d = $\frac{1}{2}$
(v) a = -1.25, d = -0.25

Answer: (i) Given, a = 10 and d = 10
a_1 = 10,
a_2 = 10 +10 = 20
a_3 = 20 + 10 = 30
a_4 = 30 + 10 = 40
hence , the first four terms of the AP be 10, 20, 30, 40.

(ii) Given , a = (- 2) and d = 0
As the difference between the two terms is (- 2)

(iii) a_1 = 4, d = - 3
$a_2 = a_1 + d = 4 – 3 = 1$
$a_3 = a_2 + d = 1 – 3 = - 2$
$a_4 = a_3 + d = - 2 – 3 = - 5$
Hence, the first four terms be 4, 1, -2, -5

(iv) $a_1 = -1$, $d = \frac{1}{2}$
$a_2 = a_1 + d = \frac{-1}{1} + \frac{1}{2} = \frac{-1}{2}$
$a_3 = a_2 + d = \frac{-1}{2} + \frac{1}{2} = 0$
$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$

The four terms of the AP are $\frac{1}{2}$, $\frac{-1}{2}$, 0 , $\frac{1}{2}$

(v) a_1 = -1.25, d = -0.25
$a_2 = a_1$ + d = -1.25 – 0.25 = -1.50
$a_3 = a_2$ + d = -1.50 + 0.25 = -1.75
$a_4 = a_3$ + d = -1.75 – 0.25 = -2
Hence, the four terms are, -0.25, -1.50, -1.75, -2

Question 3: For the following APs, write the first term and the common difference:
(i) 3, 1, -1, -3, ……
(ii) -5, -1, 3, 7, ……
(iii) $\frac{1}{3}, \frac{5}{3}, \frac{5}{3}, \frac{13}{3}$, ……..
(iv) 0.6, 1.7, 2.8, 3.9, …….

Answer: Let the first term be "a" and the common difference be "d"

(i) a = 3; $d = t_2 - t_1$ = 1 – 3 = - 2

(ii) a = -5; $d = t_2 - t_1$ = - 1 – (- 5) = 4

(iii) a = $\frac{1}{3}$; d = $t_2 - t_1 = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) a = 0.6; d = = $t_2 - t_1$ = 1.7 – 0.6 = 1.1

Question 4: Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.
(i) 2, 4, 8, 16, …….
(ii) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$, …….
(iii) -1.2, -3.2, -5.2, -7.2, ……

Answer: (i) Given series is 2, 4, 8, 16, ……………..
$a_2 - a_1$ = 4 – 2 = 2
$a_3 - a_2$ = 8 – 4 = 4
Since, $a_2 - a_1 \neq a_3 - a_2$
thus, the given number is not an AP

(ii) Given series is 2, $\frac{5}{2}$, 3, $\frac{7}{2}$, ……………..
$a_2 - a_1 = \frac{5}{2} - 2 = \frac{1}{2}$
$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$
Since, $a_2 - a_1 = a_3 - a_2$
thus, the given number is not an AP.

Therefore, d = $\frac{1}{2}$ and the given series are in A.P.
Hence, the next three terms are;

$a_5 = \frac{7}{2} + \frac{1}{2} = 4$

$a_6 = 4 + \frac{1}{2} = \frac{9}{2}$

$a_7 = \frac{9}{2} + \frac{1}{2} = 5$

(iii) Given series is -1.2, -3.2, -5.2, -7.2,………………
$a_2 - a_1 = -3.2 - (-1.2) = -2$
$a_3 - a_2 = -5.2 - (-3.2) = -2$

Since, $a_2 - a_1 = a_3 - a_2$
Thus the given series is an AP

Hence, the next three terms are,
$a_5 = -7.2 + (-2) = (-9.2)$

$a_6 = -9.2 + (-2) = (-11.2)$

$a_7 = (-11.2) + (-2) = (-13.2)$

Exercise 5.2

Question 1: Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the A.P.

a	**d**	**n**	**a_n**
7	**3**	**8**	**…(i)**…..
-18	**…(ii)**….	**1**0	**0**
…(iii)……	**-**3	**1**8	**-5**
-18.9	**2.5**	**…(iv)……….**	**3.6**
3.5	**0**	**105**	**……(v)……**

d

Answer: (i) a = 7, d = 3 and n = 8
Therefore, $a_n = (n - 1)d$

$a_8 = 7 + (8 - 1)3 = 7 + 21 = 28$

(ii) a = (-18), n = 10 and $a_n = 0$
Therefore, $a_n = (n - 1)d$

or, 0 = (-18) + (10 – 1)d
or, 0 = (-18) + 9d
or, d = 2

(iii) Here, d = -3, n = 18 and a_n

Question 2: Choose the correct choice in the following and justify:
(i) 30^th^ term of the A.P: 10,7, 4, …, is
(A) 97 (B) 77 (C) −77 (D) −87

(ii) 11^th^ term of the A.P. -3, -1/2, ,2 …. is
(A) 28 (B) 22 (C) – 38 (D) $-48\frac{1}{2}$

Answer: (i) 10, 7, 4………..
a = 10, d = 7 – 10 = -3, n = 30
As, a_n = a + (n – 1)d
or, a_{30} = a + (30 – 1)d
=a + 29d
= 10 + 29(-3)
= - 77

Hence, the correct option be (C).

(ii) -3, $-\frac{1}{2}$, 2, …………………..
a = -3, n = 11
d = $-\frac{1}{2} - (-3) = -\frac{1}{2} + 3 = \frac{5}{2}$
a_n = a + (n – 1)d

a_{11} = a + (11 – 1)d = a + 10d

a_{11} = -3 + 10 x $\frac{5}{2}$ = -3 + 25 = 22

Hence, correct option is (B).

Question 3: . In the following APs find the missing term in the boxes.

(i) 2, □, 26

(ii) □, 13, □, 3

(iii) 5, □, □, $9\frac{1}{2}$

(iv) –4, □, □, □, □, 6

(v) □, 38, □, □, □, –22

Answer: (i) a = 2, $t_3 = 26$
Then, t_3 = a + (3 – 1)d
26 = 2 + 2d => d = 12
$t_2 = t_3 - d = 26 - 12 = 14$
Hence, the complete sequence is 2, 14, 26

(ii) Here , $t_2 = 13$ and $t_4 = 3$
Then, t_2 = a + (2 – 1)d
13 = a + d

and t_4 = a + (4 – 1)d
3 = a + 3d

Subtracting equation (i) from equation (ii),

we get: d= -5

Putting d=-5 in equation (i), we get:

a= 13+5 = 18

t_3 = a + (3 – 1)d = 18 + 2x(-5) = 18-10 =8

Hence, the complete sequence is 18, 13, 8, 3

(iii) a = 5, $t_4 = 9\frac{1}{2} = \frac{19}{2}$

Then, t_4 = a + (4 – 1)d

$\frac{19}{2} = 5 + 3d$
or, d = $\frac{9}{6} = \frac{3}{2}$
$t_2 = t_3 - d = \frac{19}{2} - \frac{3}{2} = \frac{16}{2} = 8$

and $t_3 = t_4 - d$ = 8- $\frac{16-3}{2} = \frac{13}{2} = 6\frac{1}{2}$

Hence, the complete sequence is 5, $6\frac{1}{2}$, 8, $9\frac{1}{2}$

(iv) Here, a = -(4) and t_6 = 6
Then, t_6 = a +(6 – 1)d
or, 6 = -4 + 5d
or, d = 2
Therefore,
t_2 = a + d = -4 + 2 = -2
t_3 = a + 2d = -4 + 4 = 0
t_4 = a + 3d = -4 + 6 = 2
t_5 = a + 4d = -4 + 8 = 4
Hence, the complete series is , -4, -2, 0, 2, 4, 6

(v) Here, t_2 = 38 and t_6 = -22
Then , t_2 = a + (2 – 1)d
or, 38 = a + d……………………………………………(1)

and, t_6 = a + 5d
or, -22 = a + 5d……………………………………………..(2)
Subtracting eq. (1) from (2), we get d = -15
Now, putting d = -15 in eq (1), we get
a = 38 + 15 = 53

Hence, t_3 = a + 2d = 53 + 2(-15) = 53 – 30 = 23
t_4 = a + 3d = 53 + 3(-15) = 53 – 45 = 8
t_5 = a + 4d = 53 – 60 = -7

Therefore, the full series is 53, 38, 23, 8, -7, - 22

Question 4. Which term of the A.P. 3, 8, 13, 18, … is 78?

Answer: Given, 3, 8, 13, 18,…………………….
a = 3, d = 8 – 3 = 5
Let the nth term is 78
a_n = 78
a + (n – 1)d = 78

or, 3 + (n – 1)5 = 78
or, (n – 1)5 = 78 – 3
or, (n – 1)5 = 75
or, n – 1 = 15
or, n = 16
Hence, a_{16} = 78

Question 5. Find the number of terms in each of the following A.P.

(i) 7, 13, 19, …, 205

(ii) 18, $15\frac{1}{2}$, 13, … -47

Answer: (i) Here, a = 7, d = 13 – 7 = 6, l = 205
Applying the formula l = a + (n – 1)d, we get,
205 = 7 + (n – 1)6
or, (n – 1) = $\frac{198}{6}$ = 33
or, n = 33 + 1 = 34
Hence, the number is 34.

(ii) Here, a = 18
d = $15\frac{1}{2} - 18 = \frac{31-36}{2} = -\frac{5}{2}$
and l = -47
$- 47 = 18 + (n - 1)(-\frac{5}{2})$
or, (n – 1) = 26
or, n = 27
Hence, the number of the AP is 27

Question 6. Check whether -150 is a term of the A.P. 11, 8, 5, 2, …

Answer: For the given series, A.P. 11, 8, 5, 2..

a = 11, d = 8−11 = −3

Let −150 be the n^{th} term of this A.P.

Therefore, $a_n = a+(n-1)d$
or, -150 = 11+(n -1)(-3)
Or, -150 = 11-3n +3
Or, -164 = -3n
or, n = 164/3

Clearly, n is not an integer but a fraction.
Hence, – 150 is not a term of this A.P.

Question 7: Find the 31^{st} term of an A.P. whose 11^{th} term is 38 and the 16^{th} term is 73.

Answer: Given that, $a_{11} = 38$ and $a_{16} = 73$

We know that,
$a_n = a+(n-1)d$
or, $a_{11} = a+(11-1)d$
or, 38 = a+10d ………………………………………..(1)

In the same way,
$a_{16} = a +(16-1)d$
or, 73 = a+15d ………………………………………..(2)

On subtracting equation (1) from (2), we get
35 = 5d
d = 7

From equation (1) we can write,
38 = a+10×(7)
or, 38 − 70 = a
or, a = −32

$a_{31} = a +(31-1) d$
= − 32 + 30 (7)

$= -32 + 210$
$= 178$

Hence, 31^{st} term is 178.

Question 8: An A.P. consists of 50 terms of which 3^{rd} term is 12 and the last term is 106. Find the 29^{th} term.

Answer: Given that, $a_3 = 12$ and $a_{50} = 106$

We know that,
$a_n = a+(n-1)d$
or, $a_3 = a+(3-1)d$
or, $12 = a+2d$ ……………………………………(1)

In the same way,
$a_{50} = a+(50-1)d$
$106 = a+49d$ …………………………….(2)

On subtracting equation (1) from (2), we get
$94 = 47d$
or, $d = 2$

From equation (1) we get,
$12 = a+2(2)$
or, $a = 12-4 = 8$
or, $a_{29} = a+(29-1)\,d$
or, $a_{29} = 8+(28)2$
or, $a_{29} = 8+56 = 64$

Therefore, 29^{th} term is 64.

Question 9. If the 3^{rd} and the 9^{th} terms of an A.P. are 4 and − 8 respectively. Which term of this A.P. is zero.

Answer: Given that, $a_3 = 4$ and $a_9 = -8$

We know that,
$a_n = a+(n-1)d$

Hence, $a_3 = a+(3-1)d$

$4 = a+2d$ ………………………………………(1)
or, $a_9 = a+(9-1)d$
or, $-8 = a+8d$ ………………………………………………….(2)

On subtracting equation (1) from (2) we will get here,
$-12 = 6d$
or, $d = -2$

From equation (1) we get,
$4 = a+2(-2)$
or, $4 = a-4$
or, $a = 8$

Let n^{th} term of this A.P. be zero.
$a_n = a+(n-1)d$
or, $0 = 8+(n-1)(-2)$
or, $0 = 8-2n+2$
or, $2n = 10$
or, $n = 5$

Hence, 5^{th} term of this A.P. is 0.

Question 10. If 17^{th} term of an A.P. exceeds its 10^{th} term by 7. Find the common difference.

Answer: We know that, for an A.P series; $a_n = a+(n-1)d$

Hence, $a_{17} = a+(17 - 1)d$
or, $a_{17} = a +16d$

In the same way, $a_{10} = a+ 9d$

According to the question, $a_{17} - a_{10} = 7$

Therefore,
$(a + 16d) - (a + 9d) = 7$
or, $7d = 7$
or, $d = 1$

Therefore, the common difference is 1.

Question 11. Which term of the A.P. 3, 15, 27, 39,.. will be 132 more than its 54^{th} term?

Answer: Given A.P. is 3, 15, 27, 39,……….where, $a = 3$ and $d = 15 - 3 = 12$

We know that, $a_n = a+(n-1)d$

Therefore, $a_{54} = a+(54-1)d$
or, $3+(53)(12)$
or, $3+636 = 639$
or, $a_{54} = 639$

Now we have to find the term of this A.P. which is 132 more than a_{54}, i.e.771.

Let n^{th} term be 771.
or, $a_n = a+(n-1)d$
or, $771 = 3+(n -1)12$
or, $768 = (n-1)12$
or, $(n -1) = 64$
or, $n = 65$

Therefore, 65^{th} term was 132 more than 54^{th} term.

Question 12. Two APs have the same common difference. The difference between their 100^{th} term is 100, what is the difference between their 1000^{th} terms?

Answer: Let, the first term of two APs be a_1 and a_2 respectively and the common difference be d

For the first A.P., $a_n = a+(n-1)d$

Therefore, $a_{100} = a_1 + (100 - 1)d = a_1 + 99d$
or, $a_{1000} = a_1 + (1000 - 1)d$
or, $a_{1000} = a_1 + 999d$

For second A.P., $a_n = a+(n-1)d$

Therefore, $a_{100} = a_2+(100-1)d = a_2+99d$
or, $a_{1000} = a_2+(1000-1)d = a_2+999d$

The difference between 100^{th} term of the two APs = 100 [Given]

Therefore, $(a_1+99d) - (a_2+99d) = 100$

$a_1 - a_2 = 100$……………………………………………………………(1)

Difference between 1000^{th} terms of the two Aps
$(a_1 + 999d) - (a_2 + 999d) = a_1 - a_2$

From equation (1), $a_1 - a_2 = 100$

Hence, the difference between 1000^{th} terms of the two A.P. will be 100.

Question 13. How many three digit numbers are divisible by 7?

Answer: Three digits numbers that are divisible by 7 are, 105, 112, 119,……….994
Here, a = 105, d = 112 – 105 = 7
$t_n = 994$
Therefore, $t_n = a + (n - 1)d$
or, 994 = 105 + (n – 1) 7
or, 889 = (n – 1)7
or, n – 1 = 127
or, n = 128

Hence, there are 128, 3-digit numbers divisible by 7

Question 14. How many multiples of 4 lie between 10 and 250?

Answer: The multiples pf 4 between 10 and 250 be , 12, 16, 20, ….248
Here, a = 12, d = 16 – 12 = 4, and $a_n = 248$
$a_n = a + (n - 1)d$
or, 248 = 12 + (n – 1) 4
or, 248 – 12 = (n – 1)4
or, 236 = (n – 1)4
or, n = 60

Question 15. For what value of n, are the n^{th} terms of two APs 63, 65, 67, and 3, 10, 17, … equal?

Answer: Given two APs as; 63, 65, 67,… and 3, 10, 17,….

From first AP, 63, 65, 67, … we get, a = 63, d = 65 − 63 = 2
We know, n^{th} term of this A.P. = a_n = a+(n−1)d

a_n= 63+(n−1)2 = 63+2n−2
or, a_n = 61+2n ……………………………………………..(1)

From second AP, 3, 10, 17, … we have, a = 3, d = 10 − 3 = 7

We know that, n^{th} term of this A.P. = 3+(n−1)7 [Given]

Hence, a_n = 3+7n−7
or, a_n = 7n−4 ……………………………………………………..(2)

Given, n^{th} term of these A.P.s are equal to each other.
Hence, 61 + 2n = 7n – 4
or, 61+4 = 5n
or, 5n = 65
or, n = 13

Therefore, 13^{th} terms of both these A.P.s are equal to each other.

Question 16. Determine the A.P. whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Answer: Given, a_3 = 16
As we know, a +(3−1)d = 16
a+2d = 16 ………………………………………. .(1)

7^{th} term exceeds the 5^{th} term by 12.[Given]

a_7 − a_5 = 12
or, [a + (7 − 1)d] − [a + (5 − 1)d] = 12
or, (a + 6d) − (a + 4d) = 12
or, 2d = 12
d = 6

From equation (1), we get,
a + 2(6) = 16
or, a + 12 = 16
or, a = 4

Therefore, A.P. will be 4, 10, 16, 22, …

Question 17. Find the 20^{th} term from the last term of the A.P. 3, 8, 13, …, 253.

Answer: On reversing the AP, we get, 253, 248, …., 13, 8, 3
Now, a = 253, d = 253 – 248 = -5
Therefore, a_{20} = a + (20 − 1)d

or, $a_{20} = 253+(19)(-5)$
or, $a_{20} = 253-95$
or, $a = 158$

Therefore, 20th term from the last term of the AP 3, 8, 13, …, 253**.**is 158

Question 18. The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

Answer: We know that, the nth term of the AP is;
$a_n = a+(n-1)d$
or, $a_4 = a+(4-1)d$
or, $a_4 = a+3d$

In the same way, we get,
$a_8 = a+7d$
$a_6 = a+5d$
$a_{10} = a+9d$

Now, $a_4+a_8 = 24$ [given]
or, $a + 3d + a + 7d = 24$
or, $2a+10d = 24$
or, $a+5d = 12$ …………………………………………….(1)

And, $a_6 + a_{10} = 44$
or, $a + 5d + a + 9d = 44$
or, $2a+14d = 44$
or, $a+7d = 22$ …………………………………………..(2)

On subtracting equation (1) from (2) we get,
$2d = 22 – 12$
or, $2d = 10$
or, $d = 5$

From equation (1) we get,
$a + 5d = 12$
or, $a + 5(5) = 12$
or, $a + 25 = 12$
or, $a = -13$
or, $a_2 = a + d = - 13+5 = -8$
or $a_3 = a_2 + d = - 8+5 = -3$

Therefore, the first three terms of this A.P. are −13, −8, and −3.

Question 19. Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Answer: The incomes of Subba Rao increases every year by Rs.200 and hence, forms an AP. [Given]

Therefore, after 1995, the salaries of each year are; 5000, 5200, 5400, …

Here, a = 5000 and d = 200

Let after n^{th} year, his salary be Rs 7000 therefore, by the n^{th} term formula of AP,

$a_n = a+(n-1)\,d$
or, 7000 = 5000+(n−1)200
or, 200(n−1)= 2000
or, (n−1) = 10
or, n = 11

Therefore, in 11th year, his salary will be Rs 7000.

Question 20. Ramkali saved Rs 5 in the first week of a year and then increased her weekly saving by Rs 1.75. If in the n^{th} week, her weekly savings become Rs 20.75, find n.

Answer: Given, a = 5 and d = 1.75 and $a_n = 20.75$
As we know, $a_n = a+(n-1)d$

Hence, 20.75 = 5 + (n - 1) × 1.75
or, 15.75 = (n – 1) × 1.75
or, (n -1) = $\frac{15.75}{1.75} = \frac{1575}{175} = 9$
or, n -1 = 9
n = 10

Hence, n is 10.

Exercise 5.3

Question 1. Find the sum of the following APs.

(i) 2, 7, 12 ,…., to 10 terms.
(ii) − 37, − 33, − 29 ,…, to 12 terms
(iii) 0.6, 1.7, 2.8 ,…….., to 100 terms

Answer : (i) a = 2, d = 7−2 = 5 and n = 10

We know that, the formula for sum of nth term in AP series is,

$S_n = \frac{n}{2}[2a + (n - 1)d]$
or, $S_{10} = \frac{10}{2}[2(2) + (10 -1) \times 5]$
= 5[4 + (9) × (5)]
= 5 × 49 = 245

(ii) a = −37 and d = (−33)−(−37) = − 33 + 37 = 4 and n = 12

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{12} = \frac{12}{2}$ [2(-37) + (12 – 1) × 4]
= 6[-74 + 11 × 4]
= 6[-74 + 44]
= 6(-30) = -180

(iii) a = 0.6 and d = 1.7 − 0.6 = 1.1 and n = 100

$S_n = \frac{n}{2}$ [2a + (n – 1)d]

$S_{12} = \frac{50}{2}$ [1.2 + (99) × 1.1]
= 50[1.2 + 108.9]
= 50[110.1]
= 5505

Question 2. Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 + \cdots + 84$

(ii) **34 + 32 + 30 + ……….. + 10**
(iii) − 5 + (− 8) + (− 11) + ………… + (− 230)

Answer: (i) a = 7 and a_n = 84
d = $10\frac{1}{2} - 7 = \frac{7}{2}$

Let 84 be the n^{th} term of this A.P., then as per the n^{th} term formula,
84 = 7+(n – 1)×7/2
or, 77 = (n-1)×7/2
or, 22 = n−1
or, n = 23

We know that, sum of n term is;

$S_n = \frac{n}{2}$ (a + l) , l = 84
or, $S_n = \frac{23}{2}$ (7 + 84)

S_n = $(\frac{23}{2} \times 91) = \frac{2093}{2}$

(ii) a = 3, and d = 32−34 = −2 and a_n= 10

Let 10 be the n^{th} term of this A.P., therefore,

a_n= a +(n−1)d
or, 10 = 34+(n−1)(−2)
or, −24 = (n −1)(−2)
or, 12 = n −1
or, n = 13

We know that, sum of n terms is;

$S_n = \frac{n}{2}(a + l)$,[l = 10]

$= \frac{13}{2}(34 + 10)$

$= (\frac{13}{2} \times 44) = 13 \times 22 = 286$

(iii) $a = -5$, $a_n = -230$ and $d = (-8)-(-5) = -8+5 = -3$

Let −230 be the n^{th} term of this A.P., and by the n^{th} term formula we know,
$a_n = a+(n-1)d$
or, $-230 = -5+(n-1)(-3)$
or, $-225 = (n-1)(-3)$
or, $(n-1) = 75$
or, $n = 76$

And, Sum of n term,
$S_n = \frac{n}{2}(a + l)$
$= \frac{76}{2}[(-5) + (-230)]$
$= 38(-235)$
$= -8930$

Question 3: In an AP
(i) Given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.
(ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13}.
(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}.
(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.
(v) Given $d = 5$, $S_9 = 75$, find a and a_9.
(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.
(vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d.
(viii) Given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a.
(ix) Given $a = 3$, $n = 8$, $S = 192$, find d.
(x) Given $l = 28$, $S = 144$ and there are total 9 terms. Find a.

Answer: (i) Given that, $a = 5$, $d = 3$, $a_n = 50$
$a_n = a +(n-1)d$
or, $50 = 5+(n-1)\times 3$
or, $3(n-1) = 45$
or, $n-1 = 15$
or, $n = 16$

Now, sum of n terms,
$S_n = \frac{n}{2}(a + a_n)$
$S_n = \frac{16}{2}(5 + 50) = 440$

(ii) Given that, $a = 7$, $a_{13} = 35$

$a_n = a + (n - 1)d$,
or, $35 = 7+(13 - 1)d$
or, $12d = 28$
or, $d = \frac{28}{12} = 2.33$

Now, $S_n = \frac{n}{2}(a+a_n)$

$S_{13} = \frac{13}{2}(7+35) = 273$

(iii) Given that, $a_{12} = 37$, $d = 3$

$a_n = a+(n - 1)d$,
or, $a_{12} = a+(12-1)3$
or, $37 = a+33$
or, $a = 4$

Now, sum of nth term,

$S_n = \frac{n}{2}(a+a_n)$

$S_n = \frac{12}{2}(4+37) = 246$

(iv) Given that, $a_3 = 15$, $S_{10} = 125$

$a_n = a +(n - 1)d$,
or, $a_3 = a+(3 - 1)d$
or, $15 = a + 2d$ ……………………………..(1)

Sum of the nth term,
$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{10} = \frac{10}{2}[2a + (10 - 1)d]$
or, $125 = 5(2a + 9d)$
or, $25 = 2a+9d$ ………………………………………(2)

On multiplying equation (1) by (2), we will get;
$30 = 2a+4d$ ………………………………. …………….(3)

By subtracting equation (3) from (2), we get,
$-5 = 5d$
or, $d = -1$

From equation (1),
$15 = a+2(-1)$
or, $15 = a-2$

As, $a = 17$
or, $a_{10} = a + (10 - 1)d$
or, $a_{10} = 17 + (9)(-1)$
or, $a_{10} = 17-9 = 8$

(v) $d = 5$, $S_9 = 75$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

Therefore, the sum of first nine terms are;

$S_9 = \frac{9}{2}[2a + (9 - 1)5]$
or, $25 = 3(a + 20)$
or, $25 = 3a + 60$
or, $3a = 25 - 60$
or, $a = -\frac{35}{3}$

As we know, the n^{th} term can be written as;

$a_n = a + (n-1)d$
or, $a_9 = a + (9-1)(5)$
$= -\frac{35}{3} + 8(5)$
$= -\frac{35}{3} + 40$
$= (\frac{-35+120}{3}) = \frac{85}{3}$

(vi) Given that, $a = 2$, $d = 8$, $S_n = 90$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$90 = \frac{n}{2}[2a + (n - 1)d]$
or, $180 = n(4 + 8n - 8) = n(8n - 4) = 8n^2 - 4n$
or, $8n^2 - 4n - 180 = 0$
or, $2n^2 - n - 45 = 0$
or, $2n^2 - 10n + 9n - 45 = 0$
or, $2n(n - 5) + 9(n - 5) = 0$
or, $(n-5)(2n+9) = 0$

So, $n = 5$ (as n only be a positive integer)

Therefore, $a_5 = 8 + 5 \times 4 = 34$

(vii) Given that, $a = 8$, $a_n = 62$, $S_n = 210$

$S_n = \frac{n}{2}(a + a_n)$

$210 = \frac{n}{2}(8 + 62)$
or, $35n = 210$
or, $n = \frac{210}{35} = 6$

Now, $62 = 8 + 5d$
or, $5d = 62 – 8 = 54$
or, $d = \frac{54}{5} = 10.8$

(viii) Given that, $a_n = 4$, $d = 2$, $S_n = −14$.

$a_n = a+(n −1)d$,
or, $4 = a + (n − 1)2$
or, $4 = a + 2n – 2$
or, $a + 2n = 6$
or, $a = 6 − 2n$ …………………………………………..(1)

As we know, the sum of n terms is;

$S_n = \frac{n}{2}(a + a_n)$
or, $-14 = \frac{n}{2}(a + 4)$
or, $−28 = n (a + 4)$
or, $−28 = n (6 −2n +4)$ [From equation (1)]
or, $−28 = n (− 2n +10)$
or, $−28 = − 2n^2 + 10n$
or, $2n^2 −10n − 28 = 0$
or, $n^2 −5n −14 = 0$
or, $n^2 −7n+2n −14 = 0$
or, $n (n−7)+2(n −7) = 0$
or, $(n −7)(n +2) = 0$

Either $n − 7 = 0$ or $n + 2 = 0$ or, $n = 7$ or $n = −2$

However, n can neither be negative nor fractional.

Therefore, $n = 7$ from equation (1), we get

$a = 6−2n$
or, $a = 6−2(7)$
$= 6−14$
$= −8$

(ix) $a = 3$, $n = 8$ and $S = 192$

As we know,

$S_n = \frac{n}{2}[2a + (n – 1)d]$

Or, $192 = \frac{8}{2}[2 \times 3 + (8 – 1)d]$
or, $192 = 4[6 +7d]$
or, $48 = 6+7d$
or, $7d = 42$
or, $d = 6$

(x) l = 28,S = 144 and there are total of 9 terms.

Sum of n terms formula,, $S_n = \frac{n}{2}(a + l)$

or, $144 = \frac{9}{2}(a + 28)$

or, (16) × (2) = a + 28

or, 32 = a + 28

or, a = 4

Question 4. How many terms of the AP. 9, 17, 25 … must be taken to give a sum of 636?

Answer: Let there be n terms of the AP. 9, 17, 25, we have a = 9, d = 17−9 = 8

As, the sum of n terms, is;

$S_n = \frac{n}{2}[2a+(n-1)d]$

or, $636 = \frac{n}{2}[2×a+(8-1)×8]$

or, $636 = \frac{n}{2}[18+(n-1)×8]$

or, 636 = n [9 +4n −4]

or, 636 = n (4n +5)

or, $4n^2 + 5n - 636 = 0$

or, $4n^2 + 53n - 48n - 636 = 0$

or, n (4n + 53)−12 (4n + 53) = 0

or, (4n +53)(n −12) = 0

Either 4n + 53 = 0 or n−12 = 0

$n = (-\frac{53}{4})$ or n = 12

n cannot be negative or fraction, therefore, n = 12 only.

Question 5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference..

Answer: Given that, a = 5, l = 45, $S_n = 400$

$S_n = \frac{n}{2}(a + l)$

or, $400 = \frac{n}{2}(5 + 45)$

or, $400 = \frac{n}{2}(50)$

n =16

or, l = a+(n −1)d

or, 45 = 5 +(16 −1)d
or, 40 = 15d

Common difference, d = $\frac{40}{15}$ = $\frac{8}{3}$

Question 6. The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer: a = 1 and l = 350 and d = 9

Let there be n terms in the A.P., thus the formula for last term can be written as;

l = a+(n −1)d
or, 350 = 17+(n −1)9
or, 333 = (n−1)9
or, (n−1) = 37
or, n = 38

$S_n = \frac{n}{2}$ (a + l)

$S_{38} = \frac{13}{2}$ (17 + 350)
= 19 × 367
= 6973

Thus, this A.P. contains 38 terms and the sum of the terms of this A.P. is 6973.

Question 7. Find the sum of first 22 terms of an AP in which d = 7 and 22^{nd} term is 149.

Answer : d = 7, a_{22} = 149 then, S_{22} = ?

By the formula of nth term,
a_n = a + (n − 1)d
or, a_{22} = a + (22 − 1)d
or, 149 = a + 21 × 7
or, 149 = a + 147

a = 2
Sum of n terms,
$S_n = \frac{n}{2}$ (a + a_n)
or, $S_{22} = \frac{22}{2}$ (2+149)
= 11×151
= 1661

Question 8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Answer: a_2 = 14, a_3 = 18 and, d = $a_3 - a_2$ = 18 − 14 = 4

$a_2 = a + d$
or, $14 = a + 4$
or, $a = 10$

Sum of n terms;
$S_n = \frac{n}{2}$ [2a + (n – 1)d]

$S_{51} = \frac{51}{2}$ [2×10 (51-1) 4]

$= \frac{51}{2}$ [2+(20)×4]

$= \frac{51}{2} \times 220 = 51 \times 110 = 5610$

Question 9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Answer: $S_7 = 49$, $S_{17} = 289$

We know, Sum of n terms;

$S_n = \frac{n}{2}$ [2a + (n – 1)d]

Therefore,

$S_7 = \frac{7}{2}$ [2a +(n -1)d]
$S_7 = \frac{7}{2}$ [2a + (7 -1)d]
$49 = \frac{7}{2}$ [2a + 6d]
7 = (a + 3d)
a + 3d = 7 ………………………………….(1)

In the same way,

$S_{17} = \frac{17}{2}$ [2a+(17-1)d]
or, $289 = \frac{17}{2}$ (2a +16d)
or, 17 = (a+8d)
or, a + 8d = 17 ……………………………….(2)

Subtracting equation (1) from equation (2),.

5d = 10
or, d = 2

From equation (1), we can write it as;
a + 3(2) = 7
or, a+ 6 = 7
or, a = 1

Hence,

$S_n = \frac{n}{2}$ [2a+(n-1)d]

$= \frac{n}{2}$ [2(1)+(n – 1)×2]

$= \frac{n}{2}$ (2+2n-2)

$= \frac{n}{2}$ (2n)

$= n^2$

Question 10. Show that a_1, a_2 … , a_n , … form an AP where a_n is defined as below

(i) a_n = 3+4n
(ii) a_n = 9−5n
Also find the sum of the first 15 terms in each case.

Answer: (i) a_n = 3+4n

a_1 = 3+4(1) = 7

a_2 = 3+4(2) = 3+8 = 11

a_3 = 3+4(3) = 3+12 = 15

a_4 = 3+4(4) = 3+16 = 19

We can see here, the common difference between the terms are;

$a_2 - a_1$ = 11−7 = 4

$a_3 - a_2$ = 15−11 = 4

$a_4 - a_3$ = 19−15 = 4

Hence, $a_{k+1} - a_k$ is the same value every time. Therefore, this is an AP with common difference as 4 and first term as 7.

Now, we know, the sum of nth term is;

S_n = n/2[2a+(n -1)d]

S_{15} = 15/2[2(7)+(15-1)×4]

= 15/2[(14)+56]

= 15/2(70)

= 15×35

= 525

(ii) a_n = 9−5n

a_1 = 9−5×1 = 9−5 = 4

a_2 = 9−5×2 = 9−10 = −1

a_3 = 9−5×3 = 9−15 = −6

$a_4 = 9−5×4 = 9−20 = −11$

We can see here, the common difference between the terms are;

$a_2 − a_1 = −1−4 = −5$

$a_3 − a_2 = −6−(−1) = −5$

$a_4 − a_3 = −11−(−6) = −5$

Hence, $a_{k+1} − a_k$ is same every time. Therefore, this is an A.P. with common difference as −5 and first term as 4.

Now, we know, the sum of nth term is;

$S_n = \frac{n}{2} [2a +(n-1)d]$

$S_{15} = \frac{15}{2} [2(4) +(15 -1)(-5)]$

$= \frac{15}{2} [8 +14(-5)]$

$= \frac{15}{2} (8-70)$

$= \frac{15}{2} (-62)$

= 15(-31)

= -465

Question 11. If the sum of the first n terms of an AP is $4n − n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3^{rd}, the10^{th} and the n^{th} terms.

Answer: $S_n = 4n−n^2$
$a = S_1 = 4(1) − (1)^2 = 4−1 = 3$

$S_2= 4(2)−(2)^2 = 8−4 = 4$
$a_2 = S_2 − S_1 = 4−3 = 1$

$d = 1−3 = −2$

N^{th} term, $a_n = a+(n−1)d$
$= 3+(n −1)(−2)$
$= 3−2n +2$
$= 5−2n$

Therefore, $a_3 = 5−2(3) = 5-6 = −1$

$a_{10} = 5−2(10) = 5−20 = −15$

Hence, the sum of first two terms is 4. The second term is 1.

The 3^{rd}, the 10^{th}, and the n^{th} terms are −1, −15, and 5 − 2n respectively.

Question 12. Find the sum of first 40 positive integers divisible by 6.

Answer: The positive integers that are divisible by 6 are 6, 12, 18, 24 ….

a = 6, d = 6, S_{40} = ?

By the formula of sum of n terms, we know,

$S_n = \frac{n}{2}$ [2a +(n – 1)d]

Therefore, putting n = 40, we get,

$S_{40} = \frac{40}{2}$ [2(6) + (40-1)6]

= 20[12 + (39)(6)]

= 20(12 + 234)

= 20 × 246

= 4920

Question 13. Find the sum of first 15 multiples of 8.

Answer: The multiples of 8 are 8, 16, 24, 32…

Therefore, a = 8, d = 8 and S_{15} = ?

By the formula of sum of nth term, we know,

$S_n = \frac{n}{2}$ [2a + (n - 1)d]

$S_{15} = \frac{15}{2}$ [2(8) + (15-1)8]

= $\frac{15}{2}$ [6 +(14)(8)]

= $\frac{15}{2}$ [16 +112]

= $\frac{15}{2}$(128)

= 15 × 64

= 960

Question 14. Find the sum of the odd numbers between 0 and 50.

Answer: The odd numbers between 0 and 50 are 1, 3, 5, 7, 9 … 49.
These odd numbers are in the form of A.P.

Hence, a = 1, d = 2 and l = 49

By the formula of last term, we know,

$l = a+(n-1)\,d$
or, $49 = 1+(n-1)2$
or, $48 = 2(n-1)$
or, $n-1 = 24$
or, $n = 25$

By the formula of sum of nth term, we know,

$S_n = \frac{n}{2}(a+l)$

$S_{25} = \frac{25}{2}(1+49)$

$= \frac{25}{2}(50)$
$=(25)(25)$
$= 625$

Question 15. 15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Answer: We can see, that the given penalties are in the form of A.P.

Therefore, a = 200 and d = 50

Penalty that has to be paid if contractor has delayed the work by 30 days = S_{30}

By the formula of sum of nth term, we know,

$S_n = \frac{n}{2}[2a+(n-1)d]$

Therefore,

$S_{30}= \frac{30}{2}[2(200)+(30-1)50]$

$= 15[400+1450]$

$= 15(1850)$

$= 27750$

Therefore, the contractor has to pay Rs 27750 as penalty.

Question 16. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Answer: Let the cost of 1st prize be Rs. P.
Cost of 2nd prize = Rs. P – 20
And cost of 3rd prize = Rs. P − 40

Thus, a = P and d = −20

Given that, $S_7 = 700$

By the formula of sum of nth term, we know,

$S_n = \frac{n}{2}[2a + (n - 1)d]$
or, $\frac{7}{2}[2a + (7 - 1)d] = 700$
or, $\frac{[2a+(6)(-20)]}{2} = 100$
or, $a + 3(-20) = 100$
or, $a - 60 = 100$
or, $a = 160$

Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

Question 17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Answer: It can be observed that the number of trees planted by the students is in an AP.

1, 2, 3, 4, 5………………..12
therefore, a = 1, d = 2−1 = 1

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2(1)+(12-1)(1)]$
= 6(2+11)
= 6(13)
= 78

Therefore, number of trees planted by 1 section = 78

Number of trees planted by 3 sections = 3×78 = 234

Therefore, 234 trees will be planted by the students.

Question 18. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, ……… as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take π = 22/7)

Answer: We know,

Perimeter of a semi-circle = πr

Therefore,

$P_{1,}$ = π(0.5) = $\frac{\pi}{2}$ cm
P_2 = π(1) = π cm
P_3 = π(1.5) = $\frac{3\pi}{2}$ cm

Where, $P_{1,}$ P_2, P_3 are the lengths of the semi-circles.

Hence we got a series here, as,
$\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, 2π, ….
$P_1 = \frac{\pi}{2}$ cm
P_2 = π cm

Common difference, d = $P_2 - P_1 = \pi - \frac{\pi}{2} = \frac{\pi}{2}$
P_1= a = π/2 cm

By the sum of n term formula, we know,
$S_n = \frac{n}{2}[2a + (n - 1)d]$

Therefor, Sum of the length of 13 consecutive circles is;

$S_{13} = \frac{13}{2}[2(\frac{\pi}{2}) + (13 - 1)\frac{\pi}{2}]$

$= \frac{13}{2}[\pi + 6\pi]$

$=\frac{13}{2}(7\pi)$

$= \frac{13}{2} \times 7 \times \frac{22}{7}$

= 143 cm

Question 19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

Answer: We can see that the numbers of logs in rows are in the form of an A.P.20, 19, 18…

For the given A.P.,

First term, a = 20 and common difference, d = $a_2 - a_1$ = 19−20 = −1

Let a total of 200 logs be placed in n rows.

Thus, S_n = 200

By the sum of nth term formula,

$S_n = \frac{n}{2}$ [2a +(n -1)d]

$S_{12} = \frac{12}{2}$ [2(20)+(n -1)(-1)]

400 = n (40−n+1)

400 = n (41-n)

400 = 41n−n^2

n^2−41n + 400 = 0

n^2−16n−25n+400 = 0

n(n −16)−25(n −16) = 0

(n −16)(n −25) = 0

Either (n −16) = 0 or n−25 = 0

n = 16 or n = 25

By the nth term formula,

a_n = a+(n−1)d

a_{16} = 20+(16−1)(−1)

a_{16} = 20−15

a_{16} = 5

Similarly, the 25^{th} term could be written as;

a_{25} = 20+(25−1)(−1)

a_{25} = 20−24 = −4

It can be seen, the number of logs in 16^{th} row is 5 as the numbers cannot be negative.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16^{th} row is 5.

Question 20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?
[Hint: to pick up the first potato and the second potato, the total distance (in metres) run by a competitor is 2×5+2×(5+3)]

Answer: The distances of potatoes from the bucket are 5, 8, 11, 14…, which is in the form of AP.

Given, the distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept.

Therefore, distances to be run w.r.t distances of potatoes, could be written as;

10, 16, 22, 28, 34,……….

Hence, the first term, a = 10 and d = 16−10 = 6

S_{10} =?

By the formula of sum of n terms, we know,

$S_{10} = \frac{12}{2}$ [2(20)+(n -1)(-1)]

= 5[20+54]

= 5(74)

= 370

Therefore, the competitor will run a total distance of 370 m.

Exercise 5.4

Question 1. Which term of the AP: 121, 117, 113, . . ., is its first negative term?
[Hint: Find n for $a_n < 0$]

Answer: Given the AP series is 121, 117, 113, . . .,

Thus, a = 121, d = 117-121= -4

By the nth term formula,

$a_n = a+(n-1)d$

Therefore,

$a_n = 121+(n-1)(-4)$

$= 121-4n+4$

$=125-4n$

To find the first negative term of the series, $a_n < 0$

Therefore,

$125-4n < 0$

$125 < 4n$

$n>125/4$

$n>31.25$

Therefore, the first negative term of the series is 32^{nd} term.

Question 2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Answer: From the given statements, we can write,

$a_3 + a_7 = 6$ …………………………….(1)

And $a_3 \times a_7 = 8$ ……………………………..(2)

By the nth term formula,

$a_n = a+(n-1)d$

Third term, $a_3 = a+(3 -1)d$

$a_3 = a + 2d$………………………………(3)

And Seventh term, $a7= a+(7-1)d$

$a_7 = a + 6d$ ………………………………..(4)

From equation (3) and (4), putting in equation(1), we get,

$a + 2d +a + 6d = 6$
or, $2a + 8d = 6$
or, $a+4d=3$

or

$a = 3–4d$ …………………………………(5)

Again putting the eq.(iii) and (iv), in eq. (ii), we get,
$(a+2d)\times(a+6d) = 8$

Putting the value of a from equation (5), we get,
$(3–4d +2d)\times(3–4d+6d) = 8$
or $(3 –2d)\times(3+2d) = 8$
or, $3^2 – 2d^2 = 8$
or, $9 – 4d^2 = 8$
or, $4d^2 = 1$

$d = \frac{1}{2}$ or $-\frac{1}{2}$

Now, by putting both the values of d, we get,

$a = 3 – 4d = 3 – 4(\frac{1}{2}) = 3 – 2 = 1$, when $d = \frac{1}{2}$

$a = 3 – 4d = 3 – 4(-\frac{1}{2}) = 3+2 = 5$, when $d = -\frac{1}{2}$

We know, the sum of nth term of AP is;

$S_n = \frac{n}{2} [2a +(n – 1)d]$

So, when a = 1 and $d= \frac{1}{2}$

Then, the sum of first 16 terms are;

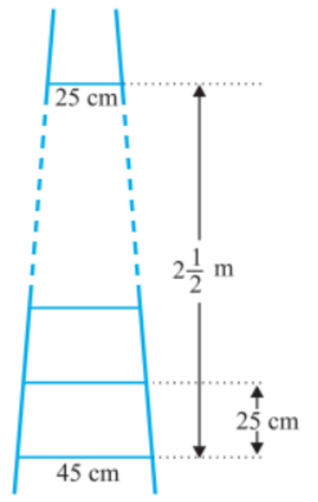
$S_{16} = \frac{16}{2} [2 +(16-1) \frac{1}{2}] = 8(2+\frac{15}{2}) = 76$

And when a = 5 and $d= -\frac{1}{2}$

Then, the sum of first 16 terms are;

$S_{16} = \frac{16}{2} [2.5+(16-1)(-\frac{1}{2})] = 8(\frac{5}{2})=20$

Question 3. A ladder has rungs 25 cm apart. (see Fig. 5.7). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}m$ apart, what is the length of the wood required for the rungs? [Hint: Number of rungs = -250/25].



Answer: Distance between the rungs of the ladder is 25cm.

Distance between the top rung and bottom rung of the ladder is =

$2\frac{1}{2}\text{m} = 2\frac{1}{2} \times 100 \text{ cm} = \frac{5}{2} \times 100 \text{ cm} = 250$ cm

Therefore, total number of rungs = $\frac{250}{25}$ + 1 = 11

As we can see from the figure, the ladder has rungs in decreasing order from top to bottom. Thus, we can conclude now, that the rungs are decreasing in an order of AP.

And the length of the wood required for the rungs will be equal to the sum of the terms of AP series formed.

So, a = 45**,** l = 25, n = 11

Now, as we know, sum of nth terms is equal to,

$S_n = \frac{n}{2}(a+l)$

$S_n = \frac{11}{2}(45+25) = \frac{11}{2}(70) = 385$ cm

Hence, the length of the wood required for the rungs is 385cm.

Question 4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x. [Hint :Sx – 1 = S49 – Sx]

Answer : Given,

Row houses are numbers from 1,2,3,4,5…….49.

Thus we can see the houses numbered in a row are in the form of AP.

So,

First term, a = 1

Common difference, d=1

Let us say the number of x^{th} houses can be represented as;

Sum of nth term of AP = $\frac{n}{2}[2a+(n-1)d]$

Sum of number of houses beyond x house = S_{x-1}

$= \frac{x-1}{2}[2*1+(x-1-1)1]$

$= \frac{x-1}{2}[2+x-2]$

$= \frac{x(x-1)}{2}$ ………………………………………………(1)

By the given condition, we get

$S_{49} – S_x = \{\frac{49}{2}[2.1+(49-1)1]\}–\{\frac{x}{2}[2.1+(x-1)1]\}$

$= 25(49) – \frac{x(x+1)}{2}$ ………………………………………..(2)

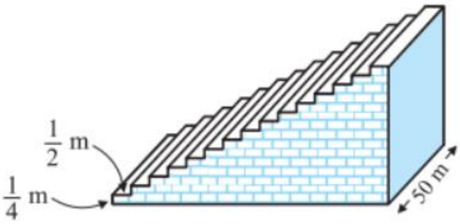
As per the given condition, eq.(1) and eq(2) are equal to each other;

Therefore, $\frac{x(x-1)}{2} = 25(49) – \frac{x(x+1)}{2}$

or, x = ±35

As we know, the number of houses cannot be a negative number. Hence, the value of x is 35.

Question 5: A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of 1 4 m and a tread of 1 2 m. (see Fig. 5.8). Calculate the total volume of concrete required to build the terrace. [Hint : Volume of concrete required to build the first step = ¼ ×1/2 ×50 m^3.]



Answer: As we can see from the given figure, the first step is ½ m wide, 2nd step is 1m wide and 3rd step is $\frac{3}{2}$m wide. Thus we can understand that the width of step by ½ m each time when height is ¼ m. And also, given length of the steps is 50m all the time. So, the width of steps forms a series AP in such a way that;

½ , 1, $\frac{3}{2}$, 2, ……..

Volume of steps = Volume of Cuboid = L × B × H

Now,

Volume of concrete required to build the first step = ¼ ×$\frac{1}{2}$ ×50 = $\frac{25}{4}$

Volume of concrete required to build the second step =¼ ×1×50 = $\frac{25}{2}$

Volume of concrete required to build the second step = ¼ ×$\frac{3}{2}$ ×50 = $\frac{75}{2}$

Now, we can see the volumes of concrete are in AP series; $\frac{25}{4}$, $\frac{25}{2}$, $\frac{75}{2}$…………

Thus, applying the AP series concept,
$a = \frac{25}{4}$, $d = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$

As we know, the sum of n terms is;

$S_n = \frac{n}{2}$ [2a+(n-1)d]
$= \frac{15}{2} (2\times(\frac{25}{4}))+(\frac{15}{2} -1) \frac{25}{4})$

Upon solving, we get,
$S_n = \frac{15}{2} (100)$

Sn = 750

Hence, the total volume of concrete required to build the terrace is 750 m^3.