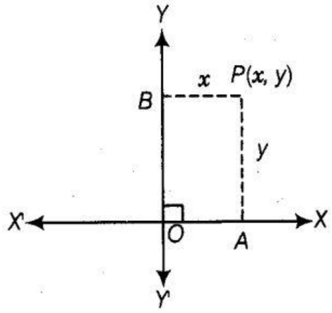
Chapter 7:Cordinate Geometry
Exercise 7.1

Question 1:
The distance of the point P(2, 3) from the X-axis is
(a) 2 (b) 3 (c) 1 (d) 5
Solution:
(b) We know that, if (x, y) is any point on the cartesian plane in first quadrant.
Then, x = Perpendicular distance from Y-axis
and y = Perpendicular distance from X-axis



Distance of the point P(2, 3) from the X-axis = Ordinate of a point P(2, 3)= 3.

Question 2:
The distance between the points A(0, 6) and 5(0,- 2) is
(a) 6 (b) 8 (c) 4 (d) 2
Solution:
(b) v Distance between the points (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0, y_1 = 6$ and $x_2 = 0, y_2 = -2$

$\therefore$ Distance between $A(0, 6)$ and $B(0, -2)$,

$$AB = \sqrt{(0-0)^2 + (-2-6)^2}$$
$$= \sqrt{0 + (-8)^2} = \sqrt{8^2} = 8$$

Question 3:
The distance of the point P(- 6, 8) from the origin is
(a) 8 (b) 2√7 (c) 10 (d) 6
Solution:
(c) ∴Distance between the points $(x_{1,}y_2)$and (x_2, y_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = -6, y_1 = 8$ and $x_2 = 0, y_2 = 0$

$\therefore$ Distance between $P(-6, 8)$ and origin *i.e.*, $O(0, 0)$,

$$PO = \sqrt{[0-(-6)]^2 + (0-8)^2}$$
$$= \sqrt{(6)^2 + (-8)^2}$$
$$= \sqrt{36 + 64} = \sqrt{100} = 10$$

Question 4:
The distance between the points (0, 5) and (- 5, 0) is
(a) 5 (b) $5\sqrt{2}$ (c)$2\sqrt{5}$ (d) 10
Solution:
(b) ∴ Distance between the points (x_1,y_1) and (x_2, y_2),

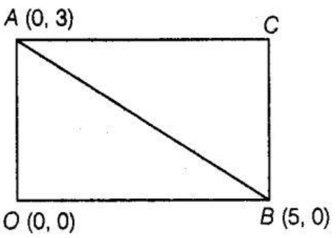
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0$, $y_1 = 5$ and $x_2 = -5$, $y_2 = 0$
∴ Distance between the points (0, 5) and (−5, 0)

$$= \sqrt{(-5-0)^2 + (0-5)^2}$$

$$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$

Question 5:
If AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0) and B(5, 0), then the length of its diagonal is
(a) 5 (b) 3 (c) $\sqrt{34}$ (d) 4
Solution:
(c)



Now, length of the diagonal AB = Distance between the points A(0, 3) and B(5, 0).
∴ Distance between the points (x,, y,) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

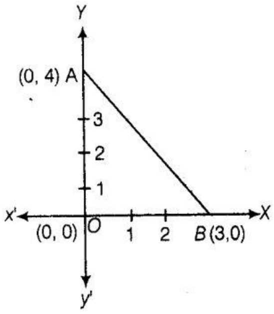
Here, $x_1 = 0$, $y_1 = 3$ and $x_2 = 5$, $y_2 = 0$
∴ Distance between the points $A(0, 3)$ and $B(5, 0)$

$$AB = \sqrt{(5-0)^2 + (0-3)^2}$$

$$= \sqrt{25+9} = \sqrt{34}$$

Hence, the required length of its diagonal is $\sqrt{34}$.

Question 6:
The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is
(a) 5 (b)
12 (c)11 (d)$7+\sqrt{5}$
Solution:
(b) we Further, adding all the distance of a triangle to get the perimeter of a triangle.We plot the vertices of a triangle i.e., (0, 4), (0,0) and (3,0) on the paper shown as given below



Now,perimeter of ΔAOB=Sum of the length of all its sides = d(AO) + d(OB) + d(AB)

∴ Distance between the points (x_1,y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

= Distance between $A(0, 4)$ and $O(0, 0)$ + Distance between $O(0, 0)$ and $B(3, 0)$ + Distance between $A(0, 4)$ and $B(3, 0)$

$= \sqrt{(0-0)^2 + (0-4)^2} + \sqrt{(3-0)^2 + (0-0)^2} + \sqrt{(3-0)^2 + (0-4)^2}$

$= \sqrt{0+16} + \sqrt{9+0} + \sqrt{(3)^2 + (4)^2} = 4 + 3 + \sqrt{9+16}$

$= 7 + \sqrt{25} = 7 + 5 = 12$

Hence, the required perimeter of triangle is 12.

Question 7:

The area of a triangle with vertices A(3,0), B(7, 0) and C(8, 4) is

(a) 14 (b) 28 (c) 8 (d) 6

Solution:

(c) Area of Δ ABC whose Vertices A≡(x_1,y_1),B≡(x_2,y_2) and C≡(x_3, y_3) are given by

$$\Delta = \left|\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$$

Here, $x_1 = 3, y_1 = 0, x_2 = 7, y_2 = 0, x_3 = 8$ and $y_3 = 4$

$$\therefore \quad \Delta = \left|\frac{1}{2}[3(0-4) + 7(4-0) + 8(0-0)]\right| = \left|\frac{1}{2}(-12 + 28 + 0)\right| = \left|\frac{1}{2}(16)\right| = 8$$

Hence, the required area of AABC is 8.

Question 8:

The points (- 4, 0), (4, 0) and (0, 3) are the vertices of a

(a) right angled triangle (b) isosceles triangle

(c) equilateral triangle (d) scalene triangle

Solution:

(b) Let A(- 4, 0), B(4, 0), C(0, 3) are the given vertices.

Now, distance between A (-4, 0) and B (4, 0),

$$AB = \sqrt{[4-(-4)]^2 + (0-0)^2}$$

$$\left[\because \text{ distance between two points } (x_1, y_1) \text{ and } (x_2, y_2), d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$$= \sqrt{(4+4)^2} = \sqrt{8^2} = 8$$

Distance between $B(4, 0)$ and $C(0, 3)$,

$$BC = \sqrt{(0-4)^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Distance between $A(-4, 0)$ and $C(0, 3)$,

$$AC = \sqrt{[0-(-4)]^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$\because \quad BC = AC$

Hence, ΔABC is an isosceles triangle because an isosceles triangle has two sides equal.

Question 9:
The point which divides the line segment joining the points (7, – 6) and (3, 4) in ratio 1: 2 internally lies in the
(a) I quadrant (b) II quadrant (c) III quadrant (d) IV quadrant
Solution:
(d) If P(x, y) divides the line segment joining $A(x_1,y_2)$ and $B(x_2, y_2)$ internally in the ratio

$$m:n, \text{ then } x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

Given that, $x_1 = 7, y_1 = -6, x_2 = 3, y_2 = 4, m = 1$ and $n = 2$

$$\therefore \quad x = \frac{1(3) + 2(7)}{1+2}, y = \frac{1(4) + 2(-6)}{1+2} \quad \text{[by section formula]}$$

$$\Rightarrow \quad x = \frac{3+14}{3}, y = \frac{4-12}{3}$$

$$\Rightarrow \quad x = \frac{17}{3}, y = -\frac{8}{3}$$

So, $(x, y) = \left(\frac{17}{3}, -\frac{8}{3}\right)$ lies in IV quadrant.

[since, in IV quadrant, x-coordinate is positive and y-coordinate is negative]

Question 10:
The point which lies on the perpendicular bisector of the line segment joining the points A(-2, – 5) and B(2, 5) is
(a) (0,0) (b) (0, 2) (c) (2, 0) (d)(-2,0)
Solution:
(a) We know that, the perpendicular bisector of the any line segment divides the^jjpe segment into two equal parts i.e., the perpendicular bisector of the line segment always passes through the mid-point of the line segment.
Mid-point of the line segment joining the points A (-2, -5) and S(2, 5)

$$= \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0, 0)$$

$$\left[\text{since, mid-point of any line segment which passes through the points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$$

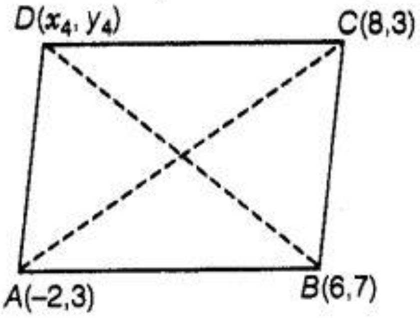
Hence, (0, 0) is the required point lies on the perpendicular bisector of the lines segment.

Question 11:
The fourth vertex D of a parallelogram ABCD whose three vertices are A(- 2, 3), B(6, 7) and C(8, 3) is
(a) (0,1) (b) (0,-1) (c) (-1,0) (d) (1,0)
Solution:
(b) Let the fourth vertex of parallelogram, D≡(x_4,y_4) and L, M be the middle points of AC and BD, respectively,

$$\text{Then, } L \equiv \left(\frac{-2+8}{2}, \frac{3+3}{2}\right) \equiv (3, 3)$$

$$\left[\text{since, mid-point of a line segment having points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$$

$$\text{and} \quad M \equiv \left(\frac{6+x_4}{2}, \frac{7+y_4}{2}\right)$$



Since, ABCD is a parallelogram, therefore diagonals AC and BD will bisect each other. Hence, L and M are the same points.

$$\therefore \quad 3 = \frac{6+x_4}{2} \text{ and } 3 = \frac{7+y_4}{2}$$

$$\Rightarrow \quad 6 = 6 + x_4 \text{ and } 6 = 7 + y_4$$

$$\Rightarrow \quad x_4 = 0 \text{ and } y_4 = 6 - 7$$

$$\therefore \quad x_4 = 0 \text{ and } y_4 = -1$$

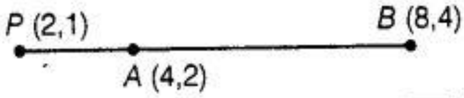
Hence, the fourth vertex of parallelogram is D s (x_4, y_4) s (0,-1).

Question 12:
If the point P(2,1) lies on the line segment joining points A(4, 2) and 6(8, 4), then

(a)AP = $\frac{1}{3}$AB (b) AP = PB (c)PB = $\frac{1}{3}$AB (d)AP = $\frac{1}{2}$AB

Solution:

(d) Given that, the point P(2,1) lies on the line segment joining the points A(4,2) and 8(8, 4), which shows in the figure below:



Now, distance between $A(4, 2)$ and $(2, 1)$, $AP = \sqrt{(2-4)^2 + (1-2)^2}$

$$\left[\because \text{ distance between two points two points } (x_1, y_1) \text{ and } B(x_2, y_2), d = \sqrt{(x_2 - x_1)^2 + (x_2 - y_1)^2}\right]$$

$$= \sqrt{(-2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$$

Distance between $A(4, 2)$ and $B(8, 4)$,

$$AB = \sqrt{(8-4)^2 + (4-2)^2}$$

$$= \sqrt{(4)^2 + (2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

Distance between $B(8, 4)$ and $P(2, 1)$, $BP = \sqrt{(8-2)^2 + (4-1)^2}$

$$= \sqrt{6^2 + 3^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$$

$$\therefore \qquad AB = 2\sqrt{5} = 2AP \Rightarrow AP = \frac{AB}{2}$$

Hence, required condition is AP = $\frac{AB}{2}$

Question 13:

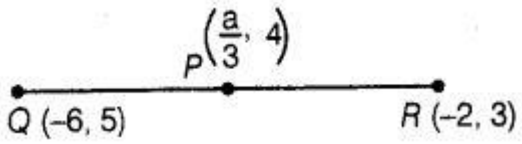
If P($\frac{1}{2}$,4) is the mid-point of the line segment joining the points Q(- 6, 5) and fl(- 2, 3), then the value of a is

(a)-4 (b) -12 (c) 12 (d) -6

Solution:

(b) Given that, P($\frac{1}{2}$,4) is the mid-point of the line segment joining the points Q(-6, 5) and

R (-2, 3), which shows in the figure given below



$\therefore$ Mid-point of $QR = P\left(\dfrac{-6-2}{2}, \dfrac{5+3}{2}\right) = P(-4, 4)$

$\left[\text{since, mid-point of line segment having points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\dfrac{(x_1+x_2)}{2}, \dfrac{(y_1+y_2)}{2}\right)\right]$

But mid-point $P\left(\dfrac{a}{3}, 4\right)$ is given.

$\therefore \qquad \left(\dfrac{a}{3}, 4\right) = (-4, 4)$

On comparing the coordinates, we get

$$\frac{a}{3} = -4$$

$\therefore \qquad a = -12$

Hence, the required value of a is -12.

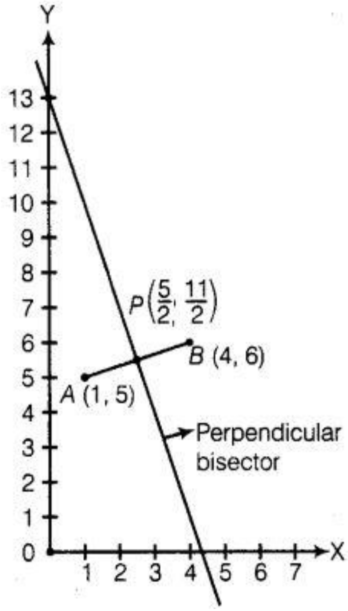
Question 14:

The perpendicular bisector of the line segment joining the points A(1,5) and 8(4,6) cuts the y-axis at

(a) (0,13) (b) (0,-13) (c) (0,12) (d) (13,0)

Solution:

(a) Firstly, we plot the points of the line segment on the paper and join them.



We know that, the perpendicular bisector of the line segment AB bisect the segment AB, i.e., perpendicular bisector of line segment AB passes through the mid-point of AB.

$\therefore \qquad$ Mid-point of $AB = \left(\dfrac{1+4}{2}, \dfrac{5+6}{2}\right)$

$\Rightarrow \qquad P = \left(\dfrac{5}{2}, \dfrac{11}{2}\right)$

$\left[\because \text{mid-point of line segment passes through the points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)\right]$

Now, we draw a straight line on paper passes through the mid-point P. We see that

the perpendicular bisector cuts the Y-axis at the point (0,13).
Hence, the required point is (0,13).

Alternate Method

We know that, the equation of line which passes through the points (x_1, y_1) and (x_2, y_2) is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \quad \text{... (i)}$$

Here, $x_1 = 1, y_1 = 5$ and $x_2 = 4, y_2 = 6$

So, the equation of line segment joining the points $A(1, 5)$ and $B(4, 6)$ is

$$(y - 5) = \frac{6 - 5}{4 - 1}(x - 1)$$

$$\Rightarrow \quad (y - 5) = \frac{1}{3}(x - 1)$$

$$\Rightarrow \quad 3y - 15 = x - 1$$

$$\Rightarrow \quad 3y = x - 14 \Rightarrow y = \frac{1}{3}x - \frac{14}{3} \quad \text{...(ii)}$$

$\therefore$ Slope of the line segment, $m_1 = \frac{1}{3}$

If two lines are perpendicular to each other, then the relation between its slopes is

$$m_1 \cdot m_2 = -1 \quad \text{...(iii)}$$

where, m_1 = Slope of line 1

and = Slope of line 2

Also, we know that the perpendicular bisector of the line segment is perpendicular on the line segment.

Let slope of line segment is m_2.

From Eq. (iii),

$$m_1 \cdot m_2 = \frac{1}{3} \cdot m_2 = -1$$

$\Rightarrow$ $$m_2 = -3$$

Also we know that the perpendicular bisector is passes through the mid-point of line segment.

$\therefore$ Mid-point of line segment $= \left(\frac{1+4}{2}, \frac{5+6}{2}\right) = \left(\frac{5}{2}, \frac{11}{2}\right)$

Equation of perpendicular bisector, which has slope (-3) and passes through the point $\left(\frac{5}{2}, \frac{11}{2}\right)$, is

$$\left(y - \frac{11}{2}\right) = (-3)\left(x - \frac{5}{2}\right)$$

[since, equation of line passes through the point (x_1, y_1) and having slope m $(y - y_1) = m(x - x_1)$]

$\Rightarrow$ $$(2y - 11) = -3(2x - 5)$$

$\Rightarrow$ $$2y - 11 = -6x + 15$$

$\Rightarrow$ $$6x + 2y = 26$$

$\Rightarrow$ $$3x + y = 13 \quad \text{...(iv)}$$

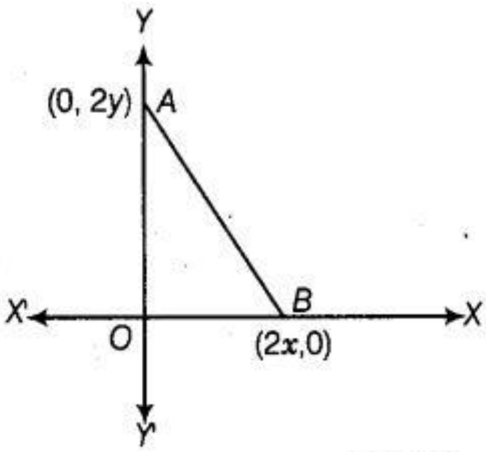
If the perpendicular bisector cuts the Y-axis, then put $x = 0$ in Eq. (iv),

$$3 \times 0 + y = 13 \Rightarrow y = 13$$

So, the required point is (0,13).

Question 15:

The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the figure is



(a) (x, y) (b) (y, x) (c) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (d) $\left(\frac{y}{2}, \frac{x}{2}\right)$

Solution:

(a) Let the coordinate of the point which is equidistant from the three vertices 0(0, 0), A(0,2y) and B(2x, 0) is P(h,k).

Then, PO = PA = PB

$\Rightarrow$ $(PO)^2 = (PA)^2 = (PB)^2$ … (i)

By distance formula,

$$\left[\sqrt{(h-0)^2+(k-0)^2}\right]^2=\left[\sqrt{(h-0)^2+(k-2y)^2}\right]^2=\left[\sqrt{(h-2x)^2+(k-0)^2}\right]^2$$

$\Rightarrow \quad h^2+k^2=h^2+(k-2y)^2=(h-2x)^2+k^2 \qquad \text{... (ii)}$

Taking first two equations, we get

$$h^2+k^2=h^2+(k-2y)^2$$

$\Rightarrow \quad k^2=k^2+4y^2-4yk \Rightarrow 4y(y-k)=0$

$\Rightarrow \quad y=k \qquad [\because y\neq 0]$

Taking first and third equations, we get

$$h^2+k^2=(h-2x)^2+k^2$$

$\Rightarrow \quad h^2=h^2+4x^2-4xh$

$\Rightarrow \quad 4x(x-h)=0$

$\Rightarrow \quad x=h \qquad [\because x\neq 0]$

$\therefore \quad$ Required points $=(h,k)=(x,y)$

Question 16:

If a circle drawn with origin as the centre passes through $\left(\frac{13}{2},0\right)$, then the point which does not lie in the interior of the circle is

(a) $\left(\frac{-3}{4},1\right)$ (b) $\left(2,\frac{7}{3}\right)$ (c) $\left(5,\frac{-1}{2}\right)$ (d) $\left(-6,\frac{5}{2}\right)$

Solution:

(d) It is given that, centre of circle in (0,0) and passes through the point $\left(\frac{13}{2}, 0\right)$.

$\therefore$ Radius of circle = Distance between (0, 0) and $\left(\frac{13}{2}, 0\right)$

$$= \sqrt{\left(\frac{13}{2} - 0\right)^2 + (0 - 0)^2} = \sqrt{\left(\frac{13}{2}\right)^2} = \frac{13}{2} = 6.5$$

A point lie outside on or inside the circles of the distance of it from the centre of the circle is greater than equal to or less than radius of the circle.

Now, to get the correct option we have to check the option one by one.

(a) Distance between (0,0) and $\left(\frac{-3}{4}, 1\right) = \sqrt{\left(\frac{-3}{4} - 0\right)^2 + (1 - 0)^2}$

$$= \sqrt{\frac{9}{16} + 1} = \sqrt{\frac{25}{16}} = \frac{5}{4} = 1.25 < 6.5$$

So, the point $(-\frac{3}{4}, 1)$ lies interior to the circle.

(b) Distance between (0,0) and $\left(2, \frac{7}{3}\right) = \sqrt{(2-0)^2 + \left(\frac{7}{3} - 0\right)^2}$

$$= \sqrt{4 + \frac{49}{9}} = \sqrt{\frac{36+49}{9}}$$

$$= \sqrt{\frac{85}{9}} = \frac{9.22}{3} = 3.1 < 6.5$$

So, the point $\left(2, \frac{7}{3}\right)$ lies inside the circle.

(c) Distance between (0,0) and $\left(5, \frac{-1}{2}\right) = \sqrt{(5-0)^2 + \left(-\frac{1}{2} - 0\right)^2}$

$$= \sqrt{25 + \frac{1}{4}} = \sqrt{\frac{101}{4}} = \frac{10.04}{2}$$

$\Rightarrow$ $$= 5.02 < 6.5$$

So, the point $\left(5, -\frac{1}{2}\right)$ lies inside the circle.

(d) Distance between (0,0) and $\left(-6, \frac{5}{2}\right) = \sqrt{(-6-0)^2 + \left(\frac{5}{2} - 0\right)^2}$

$$= \sqrt{36 + \frac{25}{4}} = \sqrt{\frac{144+25}{4}}$$

$$= \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5$$

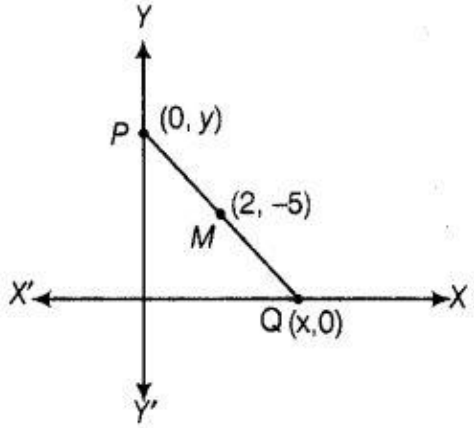
So, the point $\left(-6, \frac{5}{2}\right)$ lis an the circle *i.e.*, does not lie interior to the circle.

Question 17:
A line intersects the y-axis and X-axis at the points P and Q, respectively. If (2, – 5) is the mid-point of PQ, then the coordinates of P and Q are,
respectively ’
(a) (0,-5) and (2, 0) (b) (0, 10) and (- 4, 0)
(c) (0, 4) and (- 10, 0) (d) (0, – 10) and (4, 0)
Solution:
(d) Let the coordinates of P and 0 (0, y) and (x, 0), respectively.

So, the mid-point of $P(0, y)$ and $Q(x, 0)$ is $M\left(\frac{0+x}{2}, \frac{y+0}{2}\right)$

$\left[\because \text{ mid-point of a line segment having points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$

But it is given that, mid-point of PQ is $(2, -5)$.



$\therefore \quad 2 = \frac{x+0}{2}$

and $\quad -5 = \frac{y+0}{2}$

$\Rightarrow \quad 4 = x \text{ and } -10 = y$

$\Rightarrow \quad x = 4 \text{ and } y = -10$

So, the coordinates of P and Q are (0, -10) and (4, 0).

Question 18:
The area of a triangle with vertices (a, b + c) , (b, c + a) and (c, a + b) is
(a) $(a + b + c)^2$ (b) 0 (c) (a + b + c) (d) abc
Solution:
(b) Let the vertices of a triangle are, A ≡ (x_1, y_1) ≡ (a, b + c)
B ≡ (x_2, y_2) ≡ (b,c + a) and C = (x_3, y_3) ≡ (c, a + b)

$\because$ Area of $\Delta ABC = \Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$\therefore \quad \Delta = \frac{1}{2}[a(c + a - a - b) + b(a + b - b - c) + c(b + c - c - a)]$$

$$= \frac{1}{2}[a(c - b) + b(a - c) + c(b - a)]$$

$$= \frac{1}{2}(ac - ab + ab - bc + bc - ac) = \frac{1}{2}(0) = 0$$

Hence, the required area of triangle is. 0.

Question 19:
If the distance between the points (4, p) and (1, 0) is 5, then the value of pis
(a) 4 only (b) ±4 (c) – 4 only (d) 0

Solution:
(b) According to the question, the distance between the points (4, p) and (1, 0) = 5

i.e., $$\sqrt{(1-4)^2+(0-p)^2}=5$$

$$\left[\because \text{distance between the points } (x_1, y_1) \text{ and } (x_2, y_2), d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\right]$$

$\Rightarrow$ $$\sqrt{(-3)^2+p^2}=5$$

$\Rightarrow$ $$\sqrt{9+p^2}=5$$

On squaring both the sides, we get

$$9+p^2=25$$

$\Rightarrow$ $$p^2=16 \Rightarrow p=\pm 4$$

Hence, the required value of p is ± 4,

Question 20:
If the points A(1, 2), B(0, 0) and C(a, b) are collinear, then
(a) a = b (b) a = 2b (c) 2a = b (d) a = – b
Solution:
(c) Let the given points are B = (x_1,y_1) = (1,2),
B = (x_2,y_2) = (0,0) and C3 = (x_3,y_3)= (a, b).

$\because$ Area of ΔABC $$\Delta=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

$\therefore$ $$\Delta=\frac{1}{2}[1(0-b)+0(b-2)+a(2-0)]$$

$$=\frac{1}{2}(-b+0+2a)=\frac{1}{2}(2a-b)$$

Since, the points $A(1,2)$, $B(0,0)$ and $C(a,b)$ are collinear, then area of ΔABC should be equal to zero.

i.e., area of $\Delta ABC=0$

$\Rightarrow$ $$\frac{1}{2}(2a-b)=0$$

$\Rightarrow$ $$2a-b=0$$

$\Rightarrow$ $$2a=b$$

Hence, the required relation is 2a = b.

Exercise 7.2 Very Short Answer Type Questions

Question 1:
Δ ABC with vertices A(0 ,- 2,0), B(2, 0) and C(0,2) is similar to ΔDEF with vertices D (- 4, 0), E(4, 0) and F(0, 4).
Solution:
True

∴ Distance between $A(2,0)$ and $B(2, 0)$, $AB = \sqrt{[2-(2)]^2 + (0-0)^2} = 4$

[∵ distance between the points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$]

Similarly, distance between $B(2, 0)$ and $C(0, 2)$, $BC = \sqrt{(0-2)^2 + (2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$

In ΔABC, distance between $C(0, 2)$ and $A-(2, 0)$,

$$CA = \sqrt{[0-(2)^2] + (2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$$

Distance between $F(0, 4)$ and $D(-4, 0)$, $FD = \sqrt{(0+4)^2 + (0-4)^2} = \sqrt{4^2 + (-4)^2} = 4\sqrt{2}$

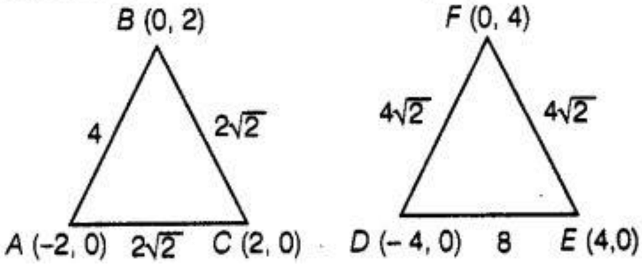
Distance between $F(0, 4)$ and $E(4, 0)$, $FE = \sqrt{(4-0)^2 + (0-4)^2} = \sqrt{4^2 + 4^2} = 4\sqrt{2}$

and distance between $E(4, 0)$ and $D(-4, 0)$, $ED = \sqrt{[4-(-4)]^2 + (0)^2} = \sqrt{8^2} = 8$

Now, $\quad \frac{AB}{OE} = \frac{4}{8} = \frac{1}{2}, \quad \frac{AC}{DF} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}, \quad \frac{BC}{EF} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$

∴ $$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

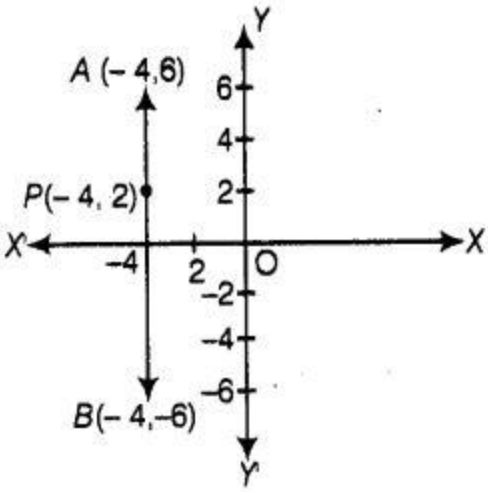
Here, we see that sides of ΔABC and ΔFDE are propotional.



Hence, both the triangles are similar.

[by SSS rule]

Question 2:
The point P(- 4, 2) lies on the line segment joining the points A(- 4, 6) and B(- 4, – 6).
Solution:
True
We plot all the points P(-4,2), A(-4, 6) and B(-4, – 6) on the graph paper,



From the figure, point P(- 4,2) lies on the line segment joining the points A(- 4,6) and B(- 4, – 6),

Question 3:
The points (0, 5), (0, -9) and (3, 6) are collinear.
Solution:
False

Here, $x_1 = 0, x_2 = 0, x_3 = 3$ and $y_1 = 5, y_2 = -9, y_3 = 6$

$\because$ Area of triangle $\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$\therefore \quad \Delta = \frac{1}{2}[0(-9-6) + 0(6-5) + 3(5+9)]$$
$$= \frac{1}{2}(0 + 0 + 3 \times 14) = 21 \neq 0$$

If the area of triangle formed by the points (0, 5), (0 – 9) and (3, 6) is zero, then the points are collinear.
Hence, the points are non-collinear.

Question 4:
Point P(0, 2) is the point of intersection of y-axis and perpendicular bisector of line segment joining the points A(-l, 1) and B(3, 3).
Solution:
False
We know that, the points lies on perpendicular bisector of the line segment joining the two
points is equidistant from these two points.

$$\therefore \quad PA = \sqrt{[-4-(4)]^2 + (6-2)^2}$$
$$= \sqrt{(0)^2 + (4)^2} = 4$$
$$PB = \sqrt{[-4-4]^2 + (-6-2)^2} \sqrt{(0)^2 + (-8)^2} = 8$$
$$\because \quad PA \neq PB$$

So, the point P does not lie on the perpendicular bisctor of AB.

Alternate Method

Slope of the line segment joining the points $A(-1, 1)$ and $B(3, 3)$, $m_1 = \frac{3-1}{3 \div 1} = \frac{2}{4} = \frac{1}{2}$

$$\left[\because m = \frac{y_2 - y_1}{x_2 - x_1}\right]$$

Since, the perpendicular bisector is perpendicular to the line segment, so its slope,

$$m_2 = -\frac{1}{(y_2)} = -2$$

[by perpendicularity condition, $m_1 m_2 = -1$]

Also, the perpendicular bisector passing through the mid-point of the line segment joining the points $A(-1, 1)$ and $B(3, 3)$.

$$\therefore \quad \text{Mid-point} = \left(\frac{-1+3}{2}, \frac{1+3}{2}\right) = (1, 2)$$

[since, mid-point of the line segment joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$]

Now, equation of perpendicular bisector have slope (-2) and passes through the point $(1, 2)$ is

$$(y-2) = (-2)(x-1)$$

$$\Rightarrow \quad y - 2 = -2x + 2$$

$$\Rightarrow \quad 2x + y = 4 \quad \text{... (i)}$$

[since, the equation of line is $(y - y_1) = m(x - x_1)$]

If the perpendicular bisector cuts the Y-axis, then put $x = 0$ in Eq. (i), we get

$$2 \times 0 + y = 4$$

$$\Rightarrow \quad y = 4$$

Hence, the required intersection point is (0, 4).

Question 5:

The points A(3,1), B(12, – 2) and C(0, 2) cannot be vertices of a triangle.

Solution:

True

Let $A \equiv (x_1, y_1) \equiv (3, 1), B \equiv (x_2, y_2) \equiv (12, -2)$

and $C \equiv (x_3, y_3) = (0, 2)$

$$\therefore \quad \text{Area of } \Delta ABC\ \Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[3 - (2-2) + 12(2-1) + 0\{1-(-2)\}]$$

$$= \frac{1}{3}[3(-4) + 12(1) + 0]$$

$$= \frac{1}{3}(-12 + 12) = 0$$

$$\because \quad \text{Area of } \Delta ABC = 0$$

Hence, the points A(3,1), B(12, -2) and C(0,2) are collinear.

So, the points A(3,1), B(12, – 2) and C(0,2) cannot be the vertices of a triangle.

Question 6:

The points A(4, 3), B(6, 4), C(5, – 6) and D(- 3, 5) are vertices of a parallelogram.

Solution:

False

Now, distance between A (4, 3) and B (6, 4), $AB = \sqrt{(6-4)^2 + (4-3)^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$

$[\because$ distance between the points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}]$

Distance between B (6,4) and C (5, − 6), $BC = \sqrt{(5-6)^2 + (-6-4)^2}$

$$= \sqrt{(-1)^2 + (-10)^2}$$

$$= \sqrt{1 + 100} = \sqrt{101}$$

Distance between C (5, − 6) and D (− 3, 5), $CD = \sqrt{(-3-5)^2 + (5+6)^2}$

$$= \sqrt{(-8)^2 + 11^2}$$

$$= \sqrt{64 + 121} = \sqrt{185}$$

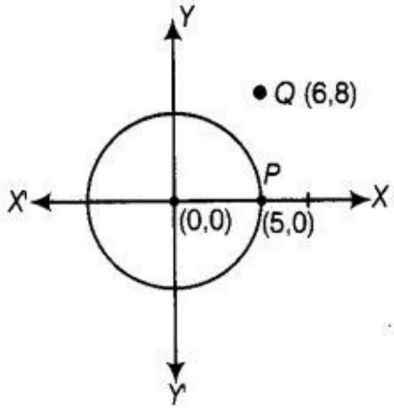
Distance between D (− 3, 5) and A (4, 3), $DA = \sqrt{(4+3)^2 + (3-5)^2}$

$$= \sqrt{7^2 + (-2)^2}$$

$$= \sqrt{49 + 4} = \sqrt{53}$$

In parallelogram, opposite sides are equal. Here, we see that all sides AB, BC, CD and DA are different.
Hence, given vertices are not the vertices of a parallelogram.

Question 7:
A circle has its centre at the origin and a point P (5, 0) lies on it. The point Q (6, 8) lies outside the circle.
Solution:
True
First,we draw a circle and a point from the given information



Now, distance between origin *i.e.,* $O(0, 0)$ and P (5, 0), $OP = \sqrt{(5-0)^2 + (0-0)^2}$

$$\left[\because \text{Distance between two points } (x_1, y_1) \text{ and } (x_2, y_2),\ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$= \sqrt{5^2 + 0^2} = 5 =$ Radius of circle and distance between origin $O(0, 0)$

and Q (6, 8), $OQ = \sqrt{(6-0)^2 + (8-0)^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

We know that, if the distance of any point from the centre is less than/equal to/ more than the radius, then the point is inside/on/outside the circle, respectively.
Here, we see that, OQ > OP
Hence, it is true that point Q (6, 8), lies outside the circle.

Question 8:
The point A (2, 7) lies on the perpendicular bisector of the line segment joining the points P (5, – 3) and Q (0, – 4).
Solution:
False
If A (2,7) lies on perpendicular bisector of P(6,5) and Q (0, – 4), then AP=AQ

$$\therefore \quad AP = \sqrt{(6-2)^2 + (5-7)^2} = \sqrt{(4)^2 + (-2)^2} = \sqrt{16+4} = \sqrt{20}$$

and
$$A = \sqrt{(0-2)^2 + (-4-7)^2} = \sqrt{(-2)^2 + (-11)^2} = \sqrt{4+121} = \sqrt{125}$$

So, A does not lies on the perpendicular bisector of PQ.
Alternate Method
If the point A (2,7) lies on the perpendicular bisector of the line segment, then the point A satisfy the equation of perpendicular bisector.
Now, we find the equation of perpendicular bisector. For this, we find the slope of perpendicular bisector.

$$\therefore \text{ Slope of perpendicular bisector} = \frac{-1}{\text{Slope of line segment joining the points } (5,-3) \text{ and } (0,-4)}$$

$$= \frac{-1}{\dfrac{-4-(-3)}{0-5}} = 5 \qquad \left[\because \text{ slope} = \frac{y_2 - y_1}{x_2 - x_1}\right]$$

[since, perpendicular bisector is perpendicular to the line segment, so its slopes have the condition, $m_1 \cdot m_2 = -1$]
Since, the perpendicular bisector passes through the mid-point of the line segment joining the points (5, – 3) and (0, – 4).

$$\therefore \quad \text{Mid-point of } PQ = \left(\frac{5+0}{2}, \frac{-3-4}{2}\right) = \left(\frac{5}{2}, \frac{-7}{2}\right)$$

So, the equation of perpendicular bisector having slope $\frac{1}{3}$ and passes through the mid-point $\left(\frac{5}{2}, \frac{-7}{2}\right)$ is,

$$\left(y + \frac{7}{2}\right) = 5\left(x - \frac{5}{2}\right)$$

$[\because$ equation of line is $(y - y_1) = m(x - x_1)]$

$$\Rightarrow \quad 2y + 7 = 10x - 25$$
$$\Rightarrow \quad 10x - 2y - 32 = 0$$
$$\Rightarrow \quad 10x - 2y = 32$$
$$\Rightarrow \quad 5x - y = 16 \qquad \text{...(i)}$$

Now, check whether the point A (2, 7) lie on the Eq. (i) or not.

$$5 \times 2 - 7 = 10 - 7 = 3 \neq 16$$

Hence, the point A (2,7) does not lie on the perpendicular bisector of the line segment.

Question 9:
The point P(5, – 3) is one of the two points of trisection of line segment joining the points A(7, – 2) and B(1, – 5).
Solution:
True
Let P (5,-3) divides the line segment joining the points A (7,-2) and B (1 ,-5) in the ratio k: 1 internally.
By section formula, the coordinate of point P will be

$$\left(\frac{k(1)+(1)(7)}{k+1}, \frac{k(-5)+1(-2)}{k+1}\right)$$

i.e., $$\left(\frac{k+7}{k+1}, \frac{-5k-2}{k+1}\right)$$

Now, $$(5,-3) = \left(\frac{k+7}{k+1}, \frac{-5k-2}{k+1}\right)$$

$\Rightarrow$ $$\frac{k+7}{k+1} = 5$$

$\Rightarrow$ $$k+7 = 5k+5$$

$\Rightarrow$ $$-4k = -2$$

$\therefore$ $$k = \frac{1}{2}$$

So the point P divides the line segment AB in ratio 1: 2. Hence, point P in the point of trisection of AB.

Question 10:
The points A (-6,10), B(- 4, 6) and C(3, -8) are collinear such that
AB $=\frac{2}{9}$ AC.
Solution:
True
If the area of triangle formed by the points (x_1,y_2), (x_2, y_2) and (x_3, y_3) is zero, then the points are collinear,

$\because$ Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Here, $x_1 = -6, x_2 = -4, x_3 = 3$ and $y_1 = 10, y_2 = 6, y_3 = -8$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}[-6\{6-(-8)\} + (-4)(-8-10) + 3(10-6)]$

$$= \frac{1}{2}[-6(14) + (-4)(-18) + 3(4)]$$

$$= \frac{1}{2}(-84 + 72 + 12) = 0$$

So, given points are collinear.

Now, distance between $A(-6, 10)$ and $B(-4, 6)$, $AB = \sqrt{(-4+6)^2 + (6-10)^2}$

$$= \sqrt{2^2 + 4^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$\left[\because \text{distance between the points } (x_1, y_1) \text{ and } (x_2, y_2), d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

Distance between $A(-6, 10)$ and $C(3, -8)$, $AC = \sqrt{(3+6)^2 + (-8-10)^2}$

$$= \sqrt{9^2 + 18^2} = \sqrt{81 + 324}$$

$$= \sqrt{405} = \sqrt{81 \times 5} = 9\sqrt{5}$$

$\therefore$ $AB = \frac{2}{9} AC$

which is the required relation.

Question 11:
The point P(- 2, 4) lies on a circle of radius 6 and centre (3, 5).
Solution:
False
If the distance between the centre and any point is equal to the radius, then we say that point lie on the circle.
Now, distance between P (-2,4) and centre (3, 5)

$$= \sqrt{(3+2)^2 + (5-4)^2}$$

$$= \sqrt{5^2 + 1^2}$$

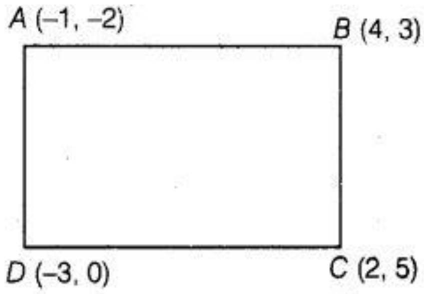
$$= \sqrt{25+1} = \sqrt{26}$$

$\left[\because \text{distance between the points } (x_1, y_1) \text{ and } (x_2, y_2), d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

which is not equal to the radius of the circle.
Hence, the point P(-2, 4) does not lies on the circle.

Question 12:
The points A(- 1, – 2), B (4, 3), C (2, 5) and D (- 3, 0) in that order form a rectangle.
Solution:
True

Distance between $A(-1, -2)$ and $B(4, 3)$,



$$AB = \sqrt{(4+1)^2 + (3+2)^2}$$
$$= \sqrt{5^2 + 5^2} = \sqrt{25+25} = 5\sqrt{2}$$

Distance between $C(2, 5)$ and $D(-3, 0)$,

$$CD = \sqrt{(-3-2)^2 + (0-5)^2}$$
$$= \sqrt{(-5)^2 + (-5)^2}$$
$$= \sqrt{25+25} = 5\sqrt{2}$$

$$\left[\because \text{distance between the points } (x_1, y_1) \text{ and } (x_2, y_2), d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

Distance between $A(-1, -2)$ and $D(-3, 0)$,

$$AD = \sqrt{(-3+1)^2 + (0+2)^2} = \sqrt{(-2)^2 + 2^2} = \sqrt{4+4} = 2\sqrt{2}$$

and distance between $B(4, 3)$ and $C(2, 5)$, $BC = \sqrt{(4-2)^2 + (3-5)^2}$

$$= \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = 2\sqrt{2}$$

We know that, in a rectangle, opposite sides and equal diagonals are equal and bisect each other.

Since, $AB = CD$ and $AD = BC$

Also, distance between $A(-1, -2)$ and $C(2, 5)$, $AC = \sqrt{(2+1)^2 + (5+2)^2}$

$$= \sqrt{3^2 + 7^2} = \sqrt{9+49} = \sqrt{58}$$

and distance between $D(-3, 0)$ and $B(4, 3)$, $DB = \sqrt{(4+3)^2 + (3-0)^2}$

$$= \sqrt{7^2 + 3^2} = \sqrt{49+9} = \sqrt{58}$$

Since, diagonals AC and BD are equal.

Hence, the points A (-1, – 2), B (4, 3), C (2, 5)and D (- 3 0)form a rectangle.

Exercise 7.3 Short Answer Type Questions

Question 1:

Name the type of triangle formed by the points A (-5, 6), B(- 4, – 2) and C(7, 5).

Solution:

To find the type of triangle, first we determine the length of all three sides and see whatever condition of triangle is satisfy by these sides.

Now, using distance formula between two points,

$$AB = \sqrt{(-4+5)^2 + (-2-6)^2}$$
$$= \sqrt{(1)^2 + (-8)^2}$$
$$= \sqrt{1+64} = \sqrt{65} \qquad [\because d = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}]$$
$$BC = \sqrt{(7+4)^2 + (5+2)^2} = \sqrt{(11)^2 + (7)^2}$$
$$= \sqrt{121+49} = \sqrt{170}$$
and
$$CA = \sqrt{(-5-7)^2 + (6-5)^2} = \sqrt{(-12)^2 + (1)^2}$$
$$= \sqrt{144+1} = \sqrt{145}$$

We see that, $AB \neq BC \neq CA$
and not hold the condition of Pythagoras in a ΔABC.
i.e., $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$
Hence, the required triangle is scalene because all of its sides are not equal i.e., different to each other.

Question 2:
Find the points on the X-axis which are at a distance of 2√5 from the point (7, -4). How many such points are there?
Solution:
We know that, every point on the X-axis in the form (x, 0). Let P(x, 0) the point on the X-axis
have 2√5 distance from the point Q (7, – 4).

By given condition, $PQ = 2\sqrt{5}$ $\quad [\because \text{distance formula} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}]$
$$\Rightarrow (PQ)^2 = 4 \times 5$$
$$\Rightarrow (x-7)^2 + (0+4)^2 = 20$$
$$\Rightarrow x^2 + 49 - 14x + 16 = 20$$
$$\Rightarrow x^2 - 14x + 65 - 20 = 0$$
$$\Rightarrow x^2 - 14x + 45 = 0$$
$$\Rightarrow x^2 - 9x - 5x + 45 = 0 \qquad \text{[by factorisation method]}$$
$$\Rightarrow x(x-9) - 5(x-9) = 0$$
$$\Rightarrow (x-9)(x-5) = 0$$
$$x = 5, 9$$

Hence, there are two points lies on the axis, which are (5, 0) and (9, 0), have 2V5 distance from the point (7, – 4).

Question 3:
What type of quadrilateral do the points A (2, -2), B (7, 3) C (11, – 1) and D (6, – 6) taken in that order form?
Solution:
To find the type of quadrilateral, we find the length of all four sides as well as two diagonals and see whatever condition of quadrilateral is satisfy by these sides as well as diagonals. Now, using distance formula between two points,

sides, $AB = \sqrt{(7-2)^2 + (3+2)^2}$

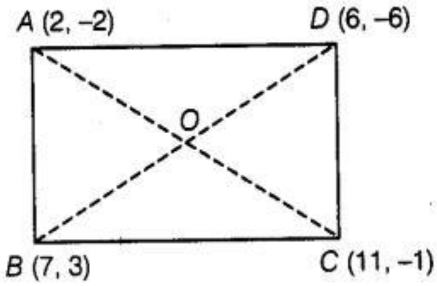
$$= \sqrt{(5)^2 + (5)^2} = \sqrt{25+25}$$

$$= \sqrt{50} = 5\sqrt{2}$$

$\left[\text{since, distance between two points } (x_1, y_1) \text{ and } (x_2, y_2) = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}\right]$

$$BC = \sqrt{(11-7)^2 + (-1-3)^2} = \sqrt{(4)^2 + (-4)^2}$$

$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$



$$CD = \sqrt{(6-11)^2 + (-6+1)^2}$$

$$= \sqrt{(-5)^2 + (-5)^2}$$

$$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$

and $DA = \sqrt{(2-6)^2 + (-2+6)^2}$

$$= \sqrt{(-4)^2 + (4)^2} = \sqrt{16+16}$$

$$= \sqrt{32} = 4\sqrt{2}$$

Diagonals, $AC = \sqrt{(11-2)^2 + (-1+2)^2}$

$$= \sqrt{(9)^2 + (1)^2} = \sqrt{81+1} = \sqrt{82}$$

and $BD = \sqrt{(6-7)^2 + (-6-3)^2}$

$$= \sqrt{(-1)^2 + (-9)^2}$$

$$= \sqrt{1+81} = \sqrt{82}$$

Here, we see that the sides AB = CD and BC = DA
Also, diagonals are equal i.e., AC = BD
which shows the quadrilateral is a rectangle.

Question 4:
Find the value of a, if the distance between the points A(- 3, – 14) and B (a, – 5) is 9 units.
Solution:
According to the question,
Distance between A (- 3, -14) and 8 (a, – 5), AB = 9

$\left[\because \text{distance between two points } (x_1, y_1) \text{ and } (x_2, y_2), d = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}\right]$

$\Rightarrow \quad \sqrt{(a+3)^2 + (-5+14)^2} = 9$

$\Rightarrow \quad \sqrt{(a+3)^2 + (9)^2} = 9$

On squaring both the sides, we get

$$(a+3)^2 + 81 = 81$$

$\Rightarrow \quad (a+3)^2 = 0 \Rightarrow a = -3$

Hence, the required value of a is – 3.

Question 5:
Find a point which is equidistant from the points A (- 5, 4) and B (- 1, 6). How many

such points are there?
Solution:
Let P (h, k) be the point which is equidistant from the points A (- 5, 4) and B (-1, 6).

$$\therefore \quad PA = PB \quad \left[\because \text{by distance formula, distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$$\Rightarrow \quad (PA)^2 = (PB)^2$$

$$\Rightarrow \quad (-5-h)^2 + (4-k)^2 = (-1-h)^2 + (6-k)^2$$

$$\Rightarrow \quad 25 + h^2 + 10h + 16 + k^2 - 8k = 1 + h^2 + 2h + 36 + k^2 - 12k$$

$$\Rightarrow \quad 25 + 10h + 16 - 8k = 1 + 2h + 36 - 12k$$

$$\Rightarrow \quad 8h + 4k + 41 - 37 = 0$$

$$\Rightarrow \quad 8h + 4k + 4 = 0$$

$$\Rightarrow \quad 2h + k + 1 = 0 \quad \ldots\text{(i)}$$

$$\text{Mid-point of } AB = \left(\frac{-5-1}{2}, \frac{4+6}{2}\right) = (-3, 5)$$

$$\left[\because \text{mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\right]$$

At point (– 3, 5), from Eq. (i),

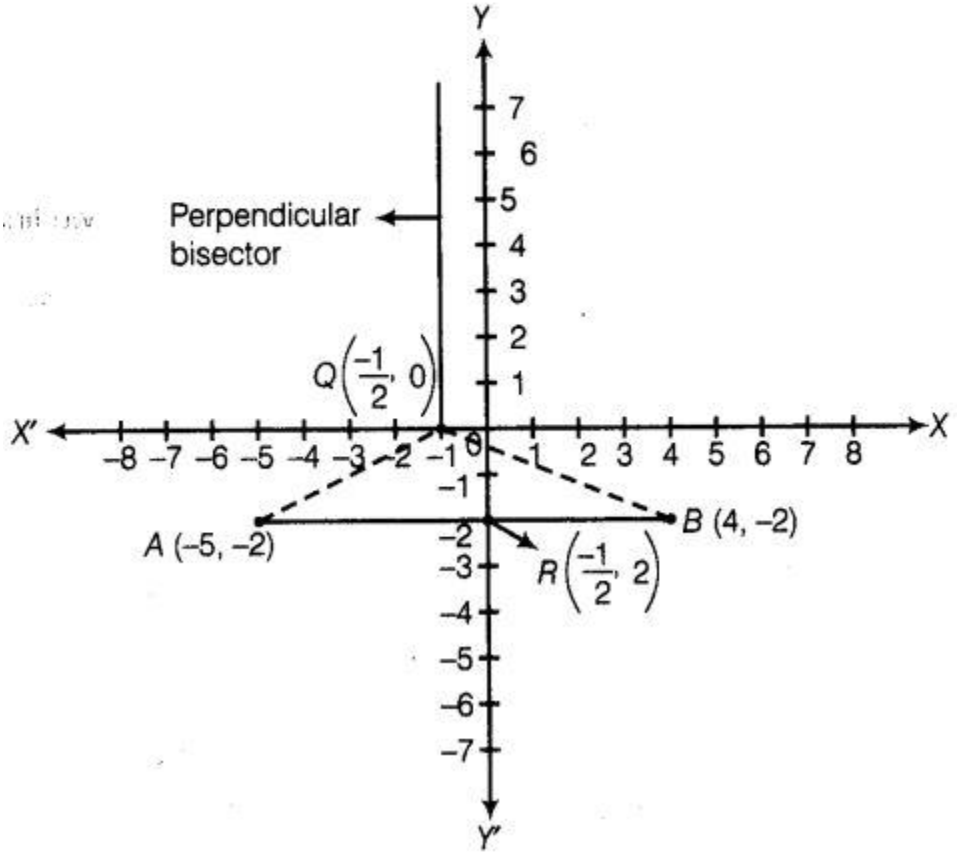
$$2h + k = 2(-3) + 5$$

$$= -6 + 5 = -1$$

$$\Rightarrow \quad 2h + k + 1 = 0$$

So, the mid-point of AB satisfy the Eq. (i). Hence, infinite number of points, in fact all points which are solution of the equation 2h + k +1 = 0, are equidistant from the points A and B.
Replacing h, k by x, y in above equation, we have 2 x +y+1= 0

Question 6:
Find the coordinates of the point Q on the x-axis which lies on the perpendicular bisector of the line segment joining the points A (- 5, – 2) and B (4, – 2). Name the type of triangle formed by the point Q, A and B.
Solution:
Firstly, we plot the points of the line segment on the paper and join them.



We know that, the perpendicular bisector of the line segment AB bisect the segment AB, i.e.,perpendicular bisector of the line segment AB passes through the mid-point of AB.

$$\therefore \qquad \text{Mid-point of } AB = \left(\frac{-5+4}{2}, \frac{-2-2}{2}\right)$$

$$\Rightarrow \qquad R = \left(-\frac{1}{2}, -2\right)$$

[$\because$ mid-point of a line segment passes through the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$]

Now, we draw a straight line on paper passes through the mid-point R. We see that perpendicular bisector cuts the X-axis at the point $Q\left(-\frac{1}{2}, 0\right)$.

Hence, the required coordinates of $Q \equiv \left(-\frac{1}{2}, 0\right)$

Alternate Method

(i) To find the coordinates of the point of 0 on the X-axis. We find the equation of perpendicular bisector of the line segment AS.

Now, slope of line segment AB,

Let $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-2)}{4-(-5)} = \frac{-2+2}{4+5} = \frac{0}{9}$

$\Rightarrow$ $m_1 = 0$

Let the slope of perpendicular bisector of line segment is m_2.

Since, perpendicular bisector is perpendicular to the line segment AB.

By perpendicularity condition of two lines,

$$m_1 \cdot m_2 = -1$$

$$\Rightarrow \quad m_2 = \frac{-1}{m_1} = \frac{-1}{0}$$

$$\Rightarrow \quad m_2 = \infty$$

Also, we know that, the perpendicular bisector is always passes through the mid-point of the line segment.

$\therefore$ Mid-point $= \left(\frac{-5+4}{2}, \frac{-2-2}{2}\right) = \left(\frac{-1}{2}, -2\right)$

$\left[\because \text{ mid-point} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$

To find the equation of perpendicular bisector of line segment, we find the slope and a point through which perpendicular bisector is pass.

Now, equation of perpendicular bisector having slope ∞ and passing through the point $\left(\frac{-1}{2}, -2\right)$ is,

$$(y+2) = \infty\left(x + \frac{1}{2}\right) \qquad [\because (y - y_1) = m_2(x - x_1)]$$

$$\Rightarrow \quad \frac{y+2}{x+\frac{1}{2}} = \infty = \frac{1}{0} \Rightarrow x + \frac{1}{2} = 0$$

$$\therefore \quad x = \frac{-1}{2}$$

So, the coordinates of the point Q is $\left(\frac{-1}{2}, 0\right)$ on the X-axis which lies on the perpendicular bisector of the line segment joining the point AB.

To know the type of triangle formed by the points Q, A and B. We find the length of all three sides and see whatever condition of triangle is satisfy by these sides.

Now, using distance formula between two points,

$$AB = \sqrt{(4+5)^2 + (-2+2)^2} = \sqrt{(9)^2 + 0} = 9$$

[$\because$ distance between two points (x_1, y_1) and $(x_2, y_2) = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$]

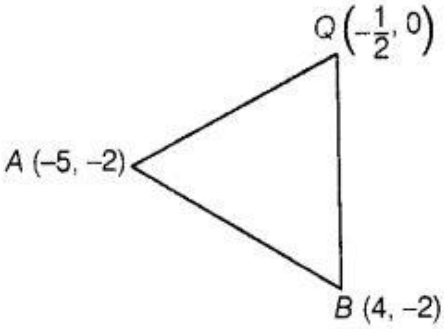
$$BQ = \sqrt{\left(\frac{-1}{2} - 4\right)^2 + (0+2)^2}$$

$$= \sqrt{\left(\frac{-9}{2}\right)^2 + (2)^2} = \sqrt{\frac{81}{4} + 4} = \sqrt{\frac{97}{4}} = \sqrt{\frac{97}{2}}$$

and $$QA = \sqrt{\left(-5 + \frac{1}{2}\right) + (-2-0)^2}$$

$$= \sqrt{\left(\frac{-9}{2}\right)^2 + (2)^2}$$

$$= \sqrt{\frac{81}{4} + 4} = \sqrt{\frac{97}{4}} = \sqrt{\frac{97}{2}}$$



We see that, $BQ = QA \neq AB$

which shows that the triangle formed by the points Q, A and 6 is an isosceles.

Question 7:

Find the value of m, if the points (5,1), (- 2, – 3) and (8, 2m) are collinear.

Solution:

Let A ≡ (x_1,y_1) s (5,1), B = (x_2, y_2) = (- 2, – 3), C s (x_3, y_3) = (8,2m)

Since, the points A ≡ (5,1), B ≡ (- 2, – 3) and C ≡ (8,2m) are collinear.

$\therefore$ Area of $\Delta ABC = 0$

$$\Rightarrow \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[5(-3-2m) + (-2)(2m-1) + 8\{1-(-3)\}] = 0$$

$$\Rightarrow \quad \frac{1}{2}(-15 - 10m - 4m + 2 + 32) = 0$$

$$\Rightarrow \quad \frac{1}{2}(-14m + 19) = 0 \Rightarrow m = \frac{19}{14}$$

Hence, the required value of m is $\frac{19}{14}$.

Question 8:

If the point A(2, – 4) is equidistant from P(3, 8) and Q(- 10, y), then find the value of y. Also, find distance PQ.

Solution:

According to the question,

A (2, – 4) is equidistant from P (3, 8) = 0 (-10, y) is equidistant from A (2, – 4) *i.e.,* $$PA = QA$$

$$\Rightarrow \quad \sqrt{(2-3)^2 + (-4-8)^2} = \sqrt{(2+10)^2 + (-4-y)^2}$$

[$\because$ distance between two points (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$]

$$\Rightarrow \quad \sqrt{(-1)^2 + (-12)^2} = \sqrt{(12)^2 + (4+y)^2}$$

$$\Rightarrow \quad \sqrt{1+144} = \sqrt{144 + 16 + y^2 + 8y}$$

$$\Rightarrow \quad \sqrt{145} = \sqrt{160 + y^2 + 8y}$$

On squaring both the sides, we get

$$145 = 160 + y^2 + 8y$$

$$\Rightarrow \quad y^2 + 8y + 160 - 145 = 0$$

$$\Rightarrow \quad y^2 + 8y + 15 = 0$$

$$\Rightarrow \quad y^2 + 5y + 3y + 15 = 0$$

$$\Rightarrow \quad y(y+5) + 3(y+5) = 0$$

$$(y+5)(y+3) = 0$$

If $y + 5 = 0$, then $y = -5$
If $y + 3 = 0$, then $y = -3$

$$\therefore \quad y = -3, -5$$

Now, distance between $P(3, 8)$ and $Q(-10, y)$,

$$PQ = \sqrt{(-10-3)^2 + (y-8)^2} \quad \text{[putting } y = -3]$$

$$\Rightarrow \quad = \sqrt{(-13)^2 + (-3-8)^2}$$

$$= \sqrt{169 + 121} = \sqrt{290}$$

Again, distance between $P(3, 8)$ and $(-10, y)$, $PQ = \sqrt{(-13)^2 + (-5-8)^2}$ [putting $y = -5$]

$$= \sqrt{169 + 169} = \sqrt{338}$$

Hence, the values of y are -3, – 5 and corresponding values of PQ are √290 and √338 = 1342, respectively.

Question 9:
Find the area of the triangle whose vertices are (-8,4), (-6,6) and (- 3, 9).
Solution:
Given that, the vertices of triangles

Let $(x_1, y_1) \rightarrow (-8, 4)$
$(x_2, y_2) \rightarrow (-6, 6)$
and $(x_3, y_3) \rightarrow (-3, 9)$

We know that, the area of triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3)

$$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3 + (y_1 - y_2)]$$

$$\therefore \quad = \frac{1}{2}[-8(6-9) - 6(9-4) + (-3)(4-6)]$$

$$= \frac{1}{2}[-8(-3) - 6(5) - 3(-2)] = \frac{1}{2}(24 - 30 + 6)$$

$$= \frac{1}{2}(30 - 30) = \frac{1}{2}(0) = 0$$

Hence, the required area of triangle is 0.

Question 10:
In what ratio does the X-axis divide the line segment joining the points (- 4, – 6) and (- 1, 7)? Find the coordinates of the points of division.
Solution:
Let the required ratio be λ: 1. So, the coordinates of the point M of division A (- 4, – 6) and B(-1,7) are

$$\left[\frac{\lambda x_2 + 1 \cdot x_1}{\lambda + 1}, \frac{\lambda y_2 + 1 \cdot y_1}{\lambda + 1}\right]$$

Here, $x_1 = -4$, $x_2 = -1$ and $y_1 = -6$, $y_2 = 7$

i.e., $$\left(\frac{\lambda(-1) + 1(-4)}{\lambda + 1}, \frac{\lambda(7) + 1 \cdot (-6)}{\lambda + 1}\right) = \left(\frac{-\lambda - 4}{\lambda + 1}, \frac{7\lambda - 6}{\lambda + 1}\right)$$

But according to the question, line segment joining $A(-4, -6)$ and $B(-1, 7)$ is divided by the *X*-axis. So, *y*-coordinate must be zero.

$$\therefore \quad \frac{7\lambda - 6}{\lambda + 1} \Rightarrow 7\lambda - 6 = 0$$

$$\therefore \quad \lambda = \frac{6}{7}$$

So, the required ratio is 6 : 7 and the point of division *M* is $\left[\frac{-\frac{6}{7} - 4}{\frac{6}{7} + 1}, \frac{7 \times \frac{6}{7} - 6}{\frac{6}{7} + 1}\right]$

i.e., $$\left(\frac{\frac{-34}{7}}{\frac{13}{7}}, \frac{6 - 6}{\frac{13}{7}}\right) \text{ i.e., } \left(\frac{-34}{13}, 0\right).$$

Hence, the required point of division is $\left(\frac{-34}{13}, 0\right)$.

Question 11:
Find the ratio in which the point P($\frac{3}{4}$,$\frac{5}{12}$) divides the line segment joinnig the points A($\frac{1}{2}$,$\frac{3}{2}$) and B(2,5).
Solution:
Let P($\frac{3}{4}$,$\frac{5}{12}$) divide AB internally in the ratio m:n
using the section formula ,we get

$$\left(\frac{3}{4}, \frac{5}{12}\right) = \left(\frac{2m - \frac{n}{2}}{m + n}, \frac{-5m + \frac{3}{2}n}{m + n}\right)$$

[$\because$ internal section formula, the coordinates of point *P* divides the line segment joining the point (x_1, y_1) and (x_2, y_2) in the ratio $m_1 : m_2$ internally is $\left(\frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}, \frac{m_2 y_1 + m_1 y_2}{m_1 + m_2}\right)$]

On equating, we get

$$\frac{3}{4}=\frac{2m-\frac{n}{2}}{m+n} \quad \text{and} \quad \frac{5}{12}=\frac{-5m+\frac{3}{2}n}{m+n}$$

$$\Rightarrow \quad \frac{3}{4}=\frac{4m-n}{2(m+n)} \quad \text{and} \quad \frac{5}{12}=\frac{-10m+3n}{2(m+n)}$$

$$\Rightarrow \quad \frac{3}{2}=\frac{4m-n}{m+n} \quad \text{and} \quad \frac{5}{6}=\frac{-10m+3n}{m+n}$$

$$\Rightarrow \quad 3m+3n=8m-2n \text{ and } 5m+5n=-60m+18n$$

$$\Rightarrow \quad 5n-5m=0 \text{ and } 65m-13n=0$$

$$\Rightarrow \quad n=m \text{ and } 13(5m-n)=0$$

$$\Rightarrow \quad n=m \text{ and } 5m-n=0$$

Since, $m=n$ does not satisfy.

$$\therefore \quad 5m-n=0$$

$$\Rightarrow \quad 5m=n$$

$$\therefore \quad \frac{m}{n}=\frac{1}{5}$$

Hence, the required ratio is 1 : 5.

Question 12:
If P(9a -2, – b) divides line segment joining A(3a + 1,-3) and B(8a, 5) in the ratio 3 : 1, then find the values of a and b.
Solution:
Let P(9a – 2, – b) divides AS internally in the ratio 3:1.
By section formula,

$$9a - 2 = \frac{3(8a) + 1(3a + 1)}{3 + 1}$$

$\Big[\because$ internal section formula, the coordinates of point P divides the line segment joining the point (x_1, y_1) and (x_2, y_2) in the ratio $m_1 : m_2$ internally is $\left(\frac{m_2x_1 + m_1x_2}{m_1 + m_2}, \frac{m_2y_1 + m_1y_2}{m_1 + m_2}\right)\Big]$

and $$-b = \frac{3(5) + 1(-3)}{3 + 1}$$

$\Rightarrow$ $$9a - 2 = \frac{24a + 3a + 1}{4}$$

and $$-b = \frac{15 - 3}{4}$$

$\Rightarrow$ $$9a - 2 = \frac{27a + 1}{4}$$

and $$-b = \frac{12}{4}$$

$\Rightarrow$ $$36a - 8 = 27a + 1$$

and $$b = -3$$

$\Rightarrow$ $$36a - 27a - 8 - 1 = 0$$

$\Rightarrow$ $$9a - 9 = 0$$

$\therefore$ $$a = 1$$

Hence, the required values of a and b are 1 and – 3.

Question 13:

If (a, b) is the mid-point of the line segment joining the points A(10, – 6), B(k, 4) and a – 2b= 18, then find the value of k and the distance AB.

Solution:

Since, (a, b) is the mid-point of line segment AB.

$$\therefore \quad (a, b) = \left(\frac{10 + k}{2}, \frac{-6 + 4}{2}\right)$$

$\left[\text{since, mid-point of a line segment having points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\right]$

$$\Rightarrow \quad (a, b) = \left(\frac{10 + k}{2}, -1\right)$$

Now, equating coordinates on both sides, we get

$$\therefore \quad a = \frac{10 + k}{2} \quad \ldots(\text{i})$$

$$\text{and} \quad b = -1 \quad \ldots(\text{ii})$$

Given, $a - 2b = 18$

From Eq. (ii), $a - 2(-1) = 18$

$\Rightarrow \quad a + 2 = 18 \Rightarrow a = 16$

From Eq. (i), $16 = \frac{10 + k}{2}$

$\Rightarrow \quad 32 = 10 + k \Rightarrow k = 22$

Hence, the required value of k is 22.

$\Rightarrow \quad k = 22$

$\therefore \quad A \equiv (10, -6), B \equiv (22, 4)$

Now, distance between A (10, – 6) and B (22, 4),

$$AB = \sqrt{(22 - 10)^2 + (4 + 6)^2}$$

$\left[\because \text{distance between the points } (x_1, y_1) \text{ and } (x_2, y_2), \ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

$$= \sqrt{(12)^2 + (10)^2} = \sqrt{144 + 100}$$

$$= \sqrt{244} = 2\sqrt{61}$$

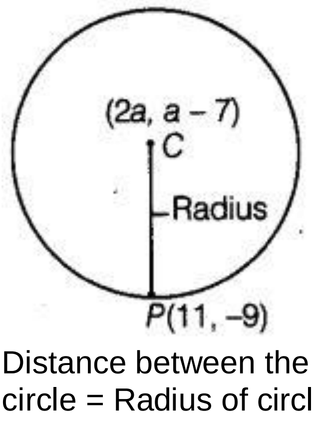
Hence, the required distance of AB is 2√61.

Question 14:

If the centre of a circle is (2a, a – 7), then Find the values of a, if the circle passes through the point (11, – 9) and has diameter 10√2 units.

Solution:

By given condition,



Distance between the centre C(2a, a – 7) and the point P(11, – 9), which lie on the circle = Radius of circle

$\therefore$ Radius of circle $= \sqrt{(11-2a)^2 + (-9-a+7)^2}$ ...(i)

[$\because$ distance between two points (x_1, y_1) and $(x_2, y_2) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$]

Given that, length of diameter $= 10\sqrt{2}$

$$\therefore \quad \text{Length of radius} = \frac{\text{Length of diameter}}{2}$$

$$= \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

Put this value in Eq. (i), we get

$$5\sqrt{2} = \sqrt{(11-2a)^2 + (-2-a)^2}$$

Squaring on both sides, we get

$$50 = (11-2a)^2 + (2+a)^2$$

$$\Rightarrow \quad 50 = 121 + 4a^2 - 44a + 4 + a^2 + 4a$$

$$\Rightarrow \quad 5a^2 - 40a + 75 = 0$$

$$\Rightarrow \quad a^2 - 8a + 15 = 0$$

$$\Rightarrow \quad a^2 - 5a - 3a + 15 = 0 \quad \text{[by factorisation method]}$$

$$\Rightarrow \quad a(a-5) - 3(a-5) = 0$$

$$\Rightarrow \quad (a-5)(a-3) = 0$$

$$\therefore \quad a = 3, 5$$

Hence, the required values of a are 5 and 3.

Question 15:

The line segment joining the points A(3, 2) and B(5,1) is divided at the point P in the ratio 1 : 2 and it lies on the line

3x – 18y + k = 0. Find the value of k.

Solution:

Given that, the line segment joining the points 4(3,2) and 6(5,1) is divided at the point P in the ratio 1 : 2.

$$\therefore \quad \text{Coordinate of point } P \equiv \left\{\frac{5(1)+3(2)}{1+2}, \frac{1(1)+2(2)}{1+2}\right\}$$

$$\equiv \left(\frac{5+6}{3}, \frac{1+4}{3}\right) \equiv \left(\frac{11}{3}, \frac{5}{3}\right)$$

$$\left[\because \text{by section formula for internal ratio} \equiv \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right)\right]$$

But the point $P\left(\frac{11}{3}, \frac{5}{3}\right)$ lies on the line $3x - 18y + k = 0$. [given]

$$\therefore \quad 3\left(\frac{11}{3}\right) - 18\left(\frac{5}{3}\right) + k = 0$$

$$\Rightarrow \quad 11 - 30 + k = 0$$

$$\Rightarrow \quad k - 19 = 0 \Rightarrow k = 19$$

Hence, the required value of k is 19.

Question 16:

If D($\frac{-1}{2}$,$\frac{5}{2}$) E(7,3) and F($\frac{7}{2}$,$\frac{7}{2}$) are the mid-points of sides of ΔABC,then find the area of the ΔABC.

Solution:

Let $A \equiv (x_1, y_1)$, $B \equiv (x_2, y_2)$ and $C \equiv (x_3, y_3)$ are the vertices of the ΔABC.

Gives, $D\left(-\frac{1}{2}, \frac{5}{2}\right)$, $E(7, 3)$ and $F\left(\frac{7}{2}, \frac{7}{2}\right)$ be the mid-points of the sides BC, CA and AB, respectively.

Since, $D\left(-\frac{1}{2}, \frac{5}{2}\right)$ is the mid-point of BC.

$$\therefore \quad \frac{x_2 + x_3}{2} = -\frac{1}{2}$$

$\left[\text{since, mid-point of a line segment having points } (x_1, y_1) \text{ and } (x_2, y_2) \text{ is} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\right]$

$$\text{and} \quad \frac{y_2 + y_3}{2} = \frac{5}{2}$$

$$\Rightarrow \quad x_2 + x_3 = -1 \quad \text{...(i)}$$

$$\text{and} \quad y_2 + y_3 = 5 \quad \text{...(ii)}$$

As $E(7, 3)$ is the mid-point of CA.

$$\therefore \quad \frac{x_3 + x_1}{2} = 7$$

$$\text{and} \quad \frac{y_3 + y_1}{2} = 3$$

$$\Rightarrow \quad x_3 + x_1 = 14 \quad \text{...(iii)}$$

$$\text{and} \quad y_3 + y_1 = 6 \quad \text{...(iv)}$$

Also, $F\left(\frac{7}{2}, \frac{7}{2}\right)$ is the mid-point of AB.

$$\therefore \quad \frac{x_1 + x_2}{2} = \frac{7}{2}$$

$$\text{and} \quad \frac{y_1 + y_2}{2} = \frac{7}{2}$$

$$\Rightarrow \quad x_1 + x_2 = 7 \quad \text{...(v)}$$

$$\text{and} \quad y_1 + y_2 = 7 \quad \text{...(vi)}$$

On adding Eqs. (i), (iii) and (v), we get

$$2(x_1 + x_2 + x_3) = 20$$

$$\Rightarrow \quad x_1 + x_2 + x_3 = 10 \quad \text{...(vii)}$$

On subtracting Eqs. (i), (iii) and (v) from Eq. (vii) respectively, we get

$$x_1 = 11, x_2 = -4, x_3 = 3$$

On adding Eqs. (ii), (iv) and (vi), we get

$$2(y_1 + y_2 + y_3) = 18$$

$$\Rightarrow \quad y_1 + y_2 + y_3 = 9 \quad \text{...(viii)}$$

On subtracting Eqs. (ii), (iv) and (vi) from Eq. (viii) respectively, we get

$$y_1 = 4,\ y_2 = 3,\ y_3 = 2$$

Hence, the vertices of ΔABC are $A\ (11, 4)$, $B\ (-4, 3)$ and $C\ (3, 2)$.

$\because$ Area of $\Delta ABC = \Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$\therefore \quad \Delta = \frac{1}{2}[11(3-2) + (-4)(2-4) + 3(4-3)]$$

$$= \frac{1}{2}[11 \times 1 + (-4)(-2) + 3(1)]$$

$$= \frac{1}{2}(11 + 8 + 3) = \frac{22}{2} = 11$$

$\therefore$ Required area of $\Delta ABC = 11$

Question 17:

If the points A (2, 9), B (a, 5) and C (5, 5) are the vertices of a ABC right angled at B, then find the values of a and hence the area of ΔABC.

Solution:

Given that, the points A (2, 9), B(a, 5) and C(5, 5) are the vertices of a ΔABC right angled at B.

By Pythagoras theorem, $AC^2 = AB^2 +$
BC^2

,,.(i)

$$\left[\because \text{ distance between two points } (x_1, y_1) \text{ and } (x_2, y_2) = \sqrt{(x_2 - x_1^2) + (y_2 - y_1)^2}\right]$$

$$= \sqrt{a^2 + 4 - 4a + 16} = \sqrt{a^2 - 4a + 20}$$

$$BC = \sqrt{(5 - a)^2 + (5 - 5)^2}$$

$$= \sqrt{(5 - a)^2 + 0} = 5 - a$$

and $$AC = \sqrt{(2 - 5)^2 + (9 - 5)^2}$$

$$= \sqrt{(-3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Put the values of AB, BC and AC in Eq. (i), we get

$$(5)^2 = (\sqrt{a^2 - 4a + 20})^2 + (5 - a)^2$$

$$\Rightarrow \quad 25 = a^2 - 4a + 20 + 25 + a^2 - 10a$$

$$\Rightarrow \quad 2a^2 - 14a + 20 = 0$$

$$\Rightarrow \quad a^2 - 7a + 10 = 0$$

$$\Rightarrow \quad a^2 - 2a - 5a + 10 = 0 \qquad \text{[by factorisation method]}$$

$$\Rightarrow \quad a(a - 2) - 5(a - 2) = 0$$

$$\Rightarrow \quad (a - 2)(a - 5) = 0$$

$$\therefore \quad a = 2, 5$$

Here, $a \neq 5$, since at $a = 5$, the length of $BC = 0$. It is not possible because the sides AB, BC and CA form a right angled triangle.

So, $a = 2$

Now, the coordinate of A, B and C becomes (2, 9), (2, 5) and (5, 5), respectively.

$$\because \quad \text{Area of } \Delta\, ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\therefore \quad \Delta = \frac{1}{2}[2(5 - 5) + 2(5 - 9) + 5(9 - 5)]$$

$$= \frac{1}{2}[2 \times 0 + 2(-4) + 5(4)]$$

$$= \frac{1}{2}(0 - 8 + 20) = \frac{1}{2} \times 12 = 6$$

Hence, the required area of ΔABC is 6 sq units.

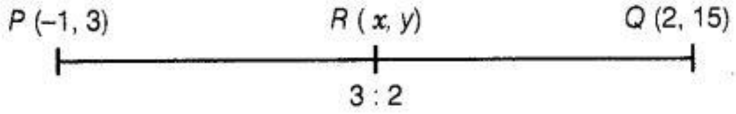
Question 18:

Find the coordinates of the point R on the line segment joining the points P(- 1, 3) and Q(2, 5) such that

PR = $\frac{3}{5}$ PQ.

Solution:

According to the question,



Given that, $$PR = \frac{3}{5} PQ$$

$$\Rightarrow \quad \frac{PQ}{PR} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{PR + RQ}{PR} = \frac{5}{3}$$

$$\Rightarrow \quad 1 + \frac{RQ}{PR} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{PQ}{PR} = \frac{5}{3} - 1 = \frac{2}{3}$$

$$\therefore \quad RQ : PR = 2 : 3$$

or $$PR : RQ = 3 : 2$$

Suppose, $R(x, y)$ be the point which divides the line segment joining the points $P(-1, 3)$ and $Q(2, 5)$ in the ratio 3 : 2.

$$\therefore \quad (x, y) = \left\{\frac{3(2) + 2(-1)}{3 + 2}, \frac{3(5) + 2(3)}{3 + 2}\right\}$$

$$\left[\because \text{ by internal section formula, } \left\{\frac{m_2x_1 + m_1x_2}{m_1 + m_2}, \frac{m_2y_1 + m_1y_2}{m_1 + m_2}\right\}\right]$$

$$= \left(\frac{6-2}{5}, \frac{15+6}{5}\right) = \left(\frac{4}{5}, \frac{21}{5}\right)$$

Hence, the required coordinates of the point R is $\left(\frac{4}{5}, \frac{21}{5}\right)$.

Question 19:

Find the values of k, if the points A(k + 1, 2k), B(3k, 2k + 3) and C (5k – 1, 5k) are colli near.

Solution:

We know that, if three points are collinear, then the area of triangle formed by these points is zero.

Since, the points A(k + 1,2k), B(3k, 2k + 3) and C(5k -1, 5k) are collinear.

Then, area of ΔABC = 0

$$\Rightarrow \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2] = 0$$

Here, $x_1 = k + 1, x_2 = 3k, x_3 = 5k - 1$ and $y_1 = 2k_1, y_2 = 2k + 3, y_3 = 5k$

$$\Rightarrow \quad \frac{1}{2}[(k + 1)(2k + 3 - 5k) + 3k(5k - 2k) + (5k - 1)(2k - (2k + 3))] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(k + 1)(-3k + 3) + 3k(3k) + (5k - 1)(2k - 2k - 3)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[-3k^2 + 3k - 3k + 3 + 9k^2 - 15k + 3] = 0$$

$$\Rightarrow \quad \frac{1}{2}(6k^2 - 15k + 6) = 0 \quad \text{[multiply by 2]}$$

$$\Rightarrow \quad 6k^2 - 15k + 6 = 0 \quad \text{[by factorisation method]}$$
$$\Rightarrow \quad 2k^2 - 5k + 2 = 0 \quad \text{[divide by 3]}$$
$$\Rightarrow \quad 2k^2 - 4k - k + 2 = 0$$
$$\Rightarrow \quad 2k(k-2) - 1(k-2) = 0$$
$$\Rightarrow \quad (k-2)(2k-1) = 0$$

If $k - 2 = 0$, then $k = 2$

If $2k - 1 = 0$, then $k = \frac{1}{2}$

$$\therefore \quad k = 2, \frac{1}{2}$$

Hence, the required values of k are 2 and $\frac{1}{2}$

Question 20:
Find the ratio in which the line 2x+ 3y – 5 = 0 divides the line segment joining the points (8, – 9) and (2,1). Also, find the coordinates of the point of division.
Solution:
Let the line 2x + 3y – 5 = 0 divides the line segment joining the points A (8, – 9) and B (2,1) in the ratio λ: 1 at point P.

$$\therefore \quad \text{Coordinates of } P \equiv \left\{\frac{2\lambda + 8}{\lambda + 1}, \frac{\lambda - 9}{\lambda + 1}\right\}$$
$$\left[\because \text{internal division} = \left\{\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right\}\right]$$

But P lies on $2x + 3y - 5 = 0$.

$$\therefore \quad 2\left(\frac{2\lambda + 8}{\lambda + 1}\right) + 3\left(\frac{\lambda - 9}{\lambda + 1}\right) - 5 = 0$$
$$\Rightarrow \quad 2(2\lambda + 8) + 3(\lambda - 9) - 5(\lambda + 1) = 0$$
$$\Rightarrow \quad 4\lambda + 16 + 3\lambda - 27 - 5\lambda - 5 = 0$$
$$\Rightarrow \quad 2\lambda - 16 = 0$$
$$\Rightarrow \quad \lambda = 8 \Rightarrow \lambda : 1 = 8 : 1$$

So, the point P divides the line in the ratio 8 : 1.

$$\therefore \quad \text{Point of division } P \equiv \left\{\frac{2(8) + 8}{8 + 1}, \frac{8 - 9}{8 + 1}\right\}$$
$$\equiv \left(\frac{16 + 8}{9}, -\frac{1}{9}\right)$$
$$\equiv \left(\frac{24}{9}, \frac{-1}{9}\right) \equiv \left(\frac{8}{3}, \frac{-1}{9}\right)$$

Hence, the required point of division is $\left(\frac{8}{3}, \frac{-1}{9}\right)$.

Exercise 7.4 Long Answer Type Questions

Question 1:
If (- 4, 3) and (4, 3) are two vertices of an equilateral triangle, then find the

coordinates of the third vertex, given that the origin lies in the interior of the triangle.
Solution:
Let the third vertex of an equilateral triangle be (x, y). Let A (- 4, 3), B(4 3) and C (x, y).
We know that, in equilateral triangle the angle between two adjacent side is 60 and all three sides are equal.

$\therefore \quad AB = BC = CA$

$\Rightarrow \quad AB^2 = BC^2 = CA^2 \quad \text{...(i)}$

Now, taking first two parts.

$$AB^2 = BC^2$$

$\Rightarrow \quad (4+4)^2 + (3-3)^2 = (x-4)^2 + (y-3)^2$

$\Rightarrow \quad 64 + 0 = x^2 + 16 - 8x + y^2 + 9 - 6y$

$\Rightarrow \quad x^2 + y^2 - 8x - 6y = 39 \quad \text{...(ii)}$

Now, taking first and third parts,

$$AB^2 = CA^2$$

$\Rightarrow \quad (4+4)^2 + (3-3)^2 = (-4-x)^2 + (3-y)^2$

$\Rightarrow \quad 64 + 0 = 16 + x^2 + 8x + 9 + y^2 - 6y$

$\Rightarrow \quad x^2 + y^2 + 8x - 6y = 39 \quad \text{...(iii)}$

On subtracting Eq. (ii) from Eq. (iii), we get

$$\begin{array}{r} x^2 + y^2 + 8x - 6y = 39 \\ \underset{-}{x^2} \underset{+}{+} \underset{+}{y^2} \underset{+}{-} 8x \underset{-}{-} 6y = \underset{-}{39} \\ \hline 16x = 0 \end{array}$$

$\Rightarrow \quad x = 0$

On subtracting Eq. (ii) from Eq. (iii), we get

$$\begin{array}{r} x^2 + y^2 + 8x - 6y = 39 \\ \underline{\underset{-}{x^2} + \underset{+}{y^2} \underset{+}{-} 8x \underset{+}{-} 6y = \underset{-}{39}} \\ 16x = 0 \end{array}$$

$\Rightarrow \quad x = 0$

Now, put the value of x in Eq. (ii), we get

$$0 + y^2 - 0 - 6y = 39$$

$\Rightarrow \quad y^2 - 6y - 39 = 0$

$\therefore \quad y = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(-39)}}{2 \times 1}$

$$\left[\because \text{ solution of } ax^2 + bx + c = 0 \text{ is } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$\Rightarrow \quad y = \dfrac{6 \pm \sqrt{36 + 156}}{2}$

$\Rightarrow \quad y = \dfrac{6 \pm \sqrt{192}}{2}$

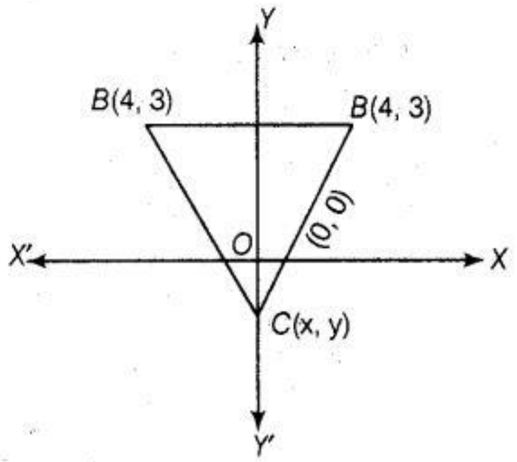
$\Rightarrow \quad y = \dfrac{6 \pm 2\sqrt{48}}{2} = 3 \pm \sqrt{48}$

$\Rightarrow \quad y = 3 \pm 4\sqrt{3}$

$\Rightarrow \quad y = 3 + 4\sqrt{3}$ or $3 - 4\sqrt{3}$

So, the points of third vertex are $(0, 3 + 4\sqrt{3})$ or $(3 - 4\sqrt{3})$

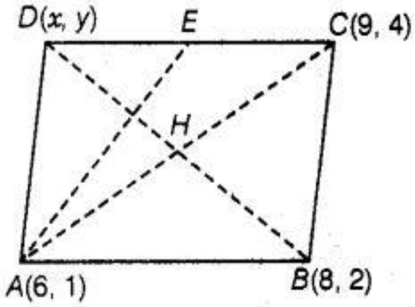
But given that, the origin lies in the interior of the ΔABC and the x-coordinate of third vertex is zero. Then, y-coordinate of third vertex should be negative.



Hence, the required coordinate of third vertex, $C \equiv (0, 3 - 4\sqrt{3})$. $\quad [\because c \not\equiv (0, 3 + 4\sqrt{3})]$

Question 2:
A(6,1), B (8, 2) and C(9, 4) are three vertices of a parallelogram ABCD. If E is the mid-point of DC, then find the area of ΔADE.
Solution:
Given that, A (6,1), B (8,2) and C (9, 4) are three vertices of a parallelogram ABCD.

Let the fourth vertex of parallelogram be (x, y).
We know that, the diagonals of a parallelogram bisect each other.



$\therefore$ Mid-point of BD = Mid-point of AC

$$\Rightarrow \quad \left(\frac{8+x}{2}, \frac{2+y}{2}\right) = \left(\frac{6+9}{2}, \frac{1+4}{2}\right)$$

$$\left[\because \text{mid-point of a line segment joining the points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$$

$$\Rightarrow \quad \left(\frac{8+x}{2}, \frac{2+y}{2}\right) = \left(\frac{15}{2}, \frac{5}{2}\right)$$

$$\therefore \quad \frac{8+x}{2} = \frac{15}{2}$$

$$\Rightarrow \quad 8 + x = 15 \Rightarrow x = 7$$

and $$\frac{2+y}{2} = \frac{5}{2}$$

$$\Rightarrow \quad 2 + y = 5 \Rightarrow y = 3$$

So, fourth vertex of a parallelogram is D (7, 3).

Now, mid-point of side $DC \equiv \left(\dfrac{7+9}{2}, \dfrac{3+4}{2}\right)$

$$E \equiv \left(8, \frac{7}{2}\right)$$

$$\left[\because \text{ area of } \Delta ABC \text{ with vertices } (x_1, y_1), (x_2, y_2) \text{ and } (x_3, y_3) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$\therefore$ Area of ΔADE with vertices A (6, 1), D (7, 3) and $E\left(8, \dfrac{7}{2}\right)$,

$$\Delta = \frac{1}{2}\left[6\left(3 - \frac{7}{2}\right) + 7\left(\frac{7}{2} - 1\right) + 8(1-3)\right]$$

$$= \frac{1}{2}\left[6 \times \left(\frac{-1}{2}\right) + 7\left(\frac{5}{2}\right) + 8(-2)\right]$$

$$= \frac{1}{2}\left(-3 + \frac{35}{2} - 16\right)$$

$$= \frac{1}{2}\left(\frac{35}{2} - 19\right) = \frac{1}{2}\left(\frac{-3}{2}\right)$$

$$= \frac{-3}{4}$$ [but area cannot be negative]

Hence, the required area of ΔADE is $\frac{3}{4}$ sq units.

Question 3:

The points $A(x_1, y_1)$, $B(x_2\ y_2)$ and $C(x_3, y_3)$ are the vertices of ΔABC.

(i) The median from A meets BC at Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that AP : PD = 2:1

(iii) Find the coordinates of points Q and R on medians BE and CF, respectively such that BQ:QE = 2:1 and CR:RF = 2:1

(iv) What are the coordinates of the centroid of the ΔABC?

Solution:

Given that, the points A (x_1, y_1), $B(x_2, y_2)$andC $(x_3, y_3)$are the vertices of ΔABC.

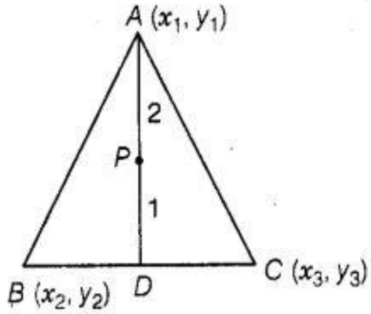
(i) We know that, the median bisect the line segment into two equal parts i.e., here D is the mid-point of BC.

$$\therefore \text{ Coordinate of mid-point of } BC = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

$$\Rightarrow \quad D \equiv \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

(ii) Let the coordinates of a point P be (x, y).

Given that, the point $P(x, y)$, divide the line joining $A(x_1, y_1)$ and $D\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$ in the ratio 2 : 1, then the coordinates of P



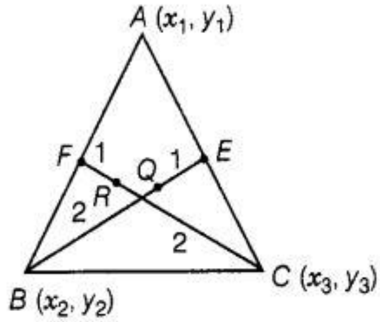
$$\equiv \left[\frac{2\cdot\left(\frac{x_2 + x_3}{2}\right) + 1\cdot x_1}{2 + 1}, \frac{2\cdot\left(\frac{y_2 + y_3}{2}\right) + 1\cdot y_1}{2 + 1}\right]$$

$$\left[\because \text{internal section formula} = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)\right]$$

$$\equiv \left(\frac{x_2 + x_3 + x_1}{3}, \frac{y_2 + y_3 + y_1}{2}\right)$$

$$\therefore \text{ So, required coordinates of point } P \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

(iii) Let the coordinates of a point Q be (p, q)



Given that, the point $Q(p, q)$, divide the line joining $B(x_2, y_2)$ and $E\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$ in the ratio 2 : 1, then the coordinates of Q

$$\equiv \left[\frac{2 \cdot \left(\frac{x_1 + x_3}{2}\right) + 1 \cdot x_2}{2 + 1}, \frac{2 \cdot \left(\frac{y_1 + y_2}{2}\right) + 1 \cdot y_2}{2 + 1}\right]$$

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

[since, BE is the median of side CA, so BE divides AC in to two equal parts.

$\therefore$ mid-point of AC = Coordinate of $E \Rightarrow E = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$]

So, the required coordinate of point $Q \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

Now, let the coordinates of a point E be (α, β). Given that, the point $R(\alpha, \beta)$, divide the line joining $C(x_3, y_3)$ and $F\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ in the ratio 2 : 1, then the coordinates of R

$$\equiv \left[\frac{2 \cdot \left(\frac{x_1 + x_2}{2}\right) + 1 \cdot x_3}{2 + 1}, \frac{2 \cdot \left(\frac{y_1 + y_2}{2}\right) + 1 \cdot y_3}{2 + 1}\right]$$

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

[since, CF is the median of side AB. So, CF divides AB in to two equal parts.

$\therefore$ mid-point of AB = coordinate of $F \Rightarrow F = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$]

So, the required coordinate of point $R \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

(iv) Coordinate of the centroid of the ΔABC

$$= \left(\frac{\text{Sum of abscissa of all vertices}}{3}, \frac{\text{Sum of ordinate of all vertices}}{3}\right)$$

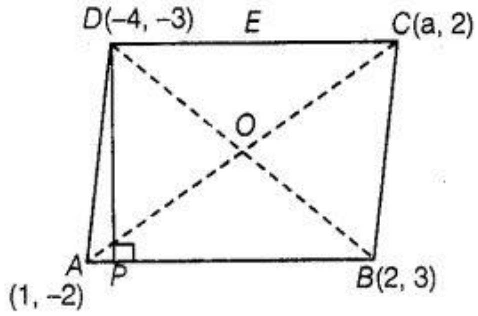
$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Question 4:

If the points A (1, – 2), B (2, 3), C(a, 2) and D(- 4, – 3) form a parallelogram, then find the value of a and height of the parallelogram taking AB as base.

Solution:

In parallelogram, we know that, diagonals are bisects each other i.e., mid-point of AC = mid-point of BD



$$\Rightarrow \quad \left(\frac{1 + a}{2}, \frac{-2 + 2}{2}\right) = \left(\frac{2 - 4}{2}, \frac{3 - 3}{2}\right)$$

$$\Rightarrow \quad \frac{1 + a}{2} = \frac{2 - 4}{2} = \frac{-2}{2} = -1$$

[since, mid-point of a line segment having points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$]

$$\Rightarrow \quad 1 + a = -2$$

$$\Rightarrow \quad a = -3$$

So, the required value of a is – 3.

Given that, AS as base of a parallelogram and drawn a perpendicular from D to AS which meet AS at P. So, DP is a height of a parallelogram.

Now, equation of base AB, passing through the points $(1, -2)$ and $(2, 3)$ is

$$\Rightarrow \quad (y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad (y + 2) = \frac{3 + 2}{2 - 1}(x - 1)$$

$$\Rightarrow \quad (y + 2) = 5(x - 1)$$

$$\Rightarrow \quad 5x - y = 7 \qquad \text{...(i)}$$

$$\text{Slope of } AB, \text{ say } m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 + 2}{2 - 1} = 5$$

Let the slope of DP be m_2.
Since, DP is perpendicular to AB.
By condition of perpendicularity,

$$m_1 \cdot m_2 = -1 \Rightarrow 5 \cdot m_2 = -1$$

$$\Rightarrow \quad m_2 = -\frac{1}{5}$$

Now, Eq. of DP, having slope $\left(-\frac{1}{5}\right)$ and passing the point $(-4, -3)$ is

$$(y - y_1) = m_2(x - x_1)$$

$$\Rightarrow \quad (y + 3) = -\frac{1}{5}(x + 4)$$

$$\Rightarrow \quad 5y + 15 = -x - 4$$

$$\Rightarrow \quad x + 5y = -19 \qquad \text{...(ii)}$$

On adding Eqs. (i) and (ii), then we get the intersection point P.

Put the value of y from Eq. (i) in Eq. (ii), we get

$$x + 5(5x - 7) = -19 \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad x + 25x - 35 = -19$$

$$\Rightarrow \quad 26x = 16$$

$$\therefore \quad x = \frac{8}{13}$$

Put the value of x in Eq. (i), we get

$$y = 5\left(\frac{8}{13}\right) - 7 = \frac{40}{13} - 7$$

$$\Rightarrow \quad y = \frac{40 - 91}{13} \Rightarrow y = \frac{-51}{13}$$

$$\therefore \quad \text{Coordinates of point } P \equiv \left(\frac{8}{13}, \frac{-51}{13}\right)$$

So, length of the height of a parallelogram,

$$DP = \sqrt{\left(\frac{8}{13} + 4\right)^2 + \left(\frac{-51}{13} + 3\right)^2}$$

[$\because$ by distance formula, distance between two points (x_1, y_1) and (x_2, y_2), is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$]

$$\Rightarrow \quad DP = \sqrt{\left(\frac{60}{13}\right)^2 + \left(\frac{-12}{13}\right)^2}$$

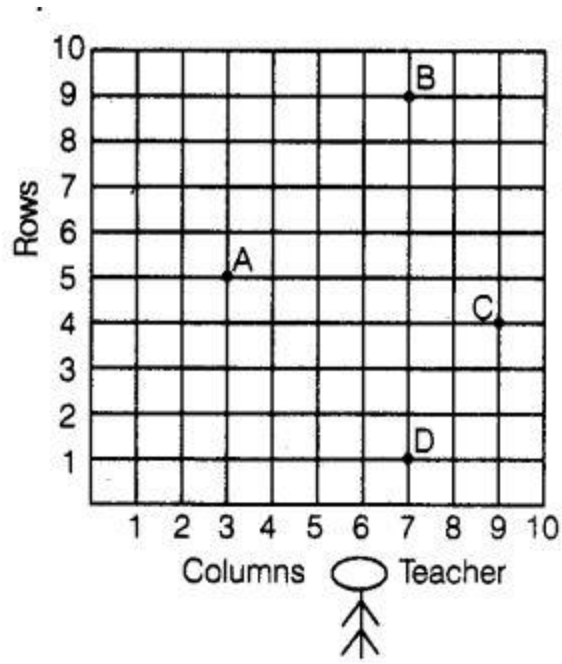
$$= \frac{1}{13}\sqrt{3600 + 144}$$

$$= \frac{1}{13}\sqrt{3744} = \frac{12\sqrt{26}}{13}$$

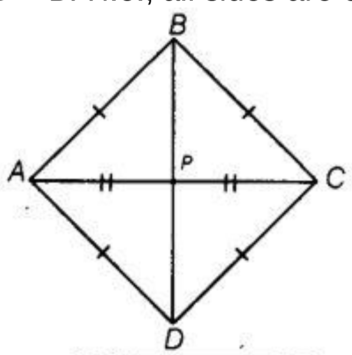
Hence, the required length of height of a parallelogram is $\frac{12\sqrt{26}}{13}$.

Question 5:

Students of a school are standing in rows and columns in their playground for a drill practice. A, B, C and D are the positions of four students as shown in figure. Is it possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B, C and D? If so, what should be his position?



Solution:
Yes, from the figure we observe that the positions of four students A, B, C and D are (3, 5), (7, 9), (11, 5) and (7,1) respectively i.e., these are four vertices of a quadrilateral. Now, we will find the type of this quadrilateral. For this, we will find all its sides.
We see that, AB = BC = CD = DA i.e., all sides are equal.



Now, $$AB = \sqrt{(7-3)^2 + (9-5)^2}$$

$$\left[\text{by distance formula, } d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$$AB = \sqrt{(4)^2 + (4)^2} = \sqrt{16+16}$$

$$AB = 4\sqrt{2}$$

$$BC = \sqrt{(11-7)^2 + (5-9)^2} = \sqrt{(4)^2 + (-4)^2}$$

$$= \sqrt{16+16} = 4\sqrt{2}$$

$$CD = \sqrt{(7-11)^2 + (1-5)^2} = \sqrt{(-4)^2 + (-4)^2}$$

$$= \sqrt{16+16} = 4\sqrt{2}$$

and $$DA = \sqrt{(3-7)^2 + (5-1)^2} = \sqrt{(-4)^2 + (4)^2}$$

$$= \sqrt{16+16} = 4\sqrt{2}$$

“>

Now, we find length of both diagonals.

$$AC = \sqrt{(11-3)^2 + (5-5)^2} = \sqrt{(8)^2 + 0} = 8$$

and $$BD = \sqrt{(7-7)^2 + (1-9)^2} = \sqrt{0 + (-8)^2} = 8$$

Here, $$AC = BD$$

Since, $AB = BC = CD = DA$ and $AC = BD$

Which represent a square. Also known the diagonals of a square bisect each other. So, P be position of Jaspal in which he is equidistant from each of the four students A, B, C and D.

$\therefore$ Coordinates of point $P \equiv$ Mid-point of AC

$$\equiv \left(\frac{3+11}{2}, \frac{5+5}{2}\right) \equiv \left(\frac{14}{2}, \frac{10}{2}\right) \equiv (7, 5)$$

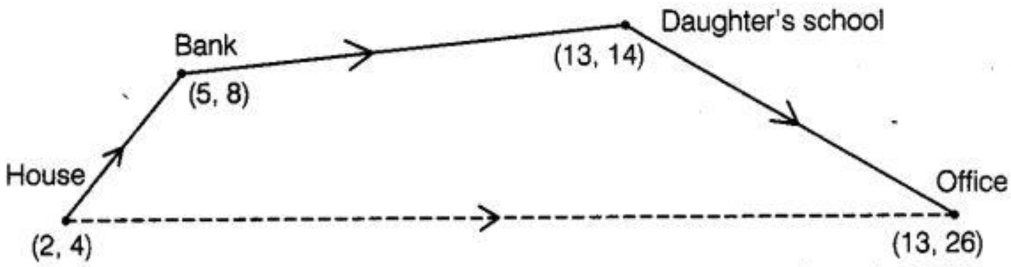
$\left[\text{since, mid-point of a line segment having points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right)\right]$

Hence, the required position of Jaspal is (7, 5).

Question 6:

Ayush starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. What is the extra distance travelled by Ayush in reaching his office? (Assume that all distance covered are in straight lines). If the house is situated at (2, 4), bank at (5, 8), school at (13,14) and office at (13, 26) and coordinates are in km.

Solution:



By given condition, we drawn a figure in which every place are indicated with his coordinates and direction also.

We know that,

distance between two points (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Now, distance between house and bank $= \sqrt{(5-2)^2 + (8-4)^2}$

$$= \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Distance between bank and daughter's school

$$= \sqrt{(13-5)^2 + (14-8)^2} = \sqrt{(8)^2 + (6)^2}$$
$$= \sqrt{64 + 36} = \sqrt{100} = 10$$

Distance between daughter's school and office $= \sqrt{(13-13)^2 + (26-14)^2}$

$$= \sqrt{0 + (12)^2} = 12$$

Total distance (House + Bank + School + Office) travelled $= 5 + 10 + 12 = 27$ units

Distance between house to offices $= \sqrt{(13-2)^2 + (26-4)^2}$

$$= \sqrt{(11)^2 + (22)^2} = \sqrt{121 + 484}$$
$$= \sqrt{605} = 24.59 \approx 24.6 \text{ km}$$

So, extra distance travelled by Ayush in reaching his office = 27 – 24.6 = 2.4 km

Hence, the required extra distance travelled by Ayush is 2.4 km.