Chapter 7: Coordinate geometry

2016

Short Answer Type Questions I [2 Marks]

Question 1.
Find the ratio in which the y-axis divides the line segment joining the points A(5, -6) and B(-l, -4). Also, find the coordinates of the point of division.
Solution:
Let the point on y-axis be $P(0, y)$ and $AP : PB = k : 1$.

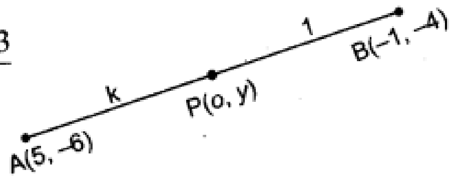
$\therefore$ Co-ordinates of P given by: $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

Then, taking x-axis of A, B; $\frac{5 \times 1 + k(-1)}{k+1} = 0 \Rightarrow \frac{5-k}{k+1} = 0 \Rightarrow k = 5$

Hence the required ratio is 5 : 1

Now, taking y-axis, $y = \frac{(-4)(5) + (1)(-6)}{5+1} = \frac{-13}{3}$

Hence point on y-axis is $\left(0, \frac{-13}{3}\right)$



Question 2.
The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q(2, -5) and R(-3, 6), find the coordinates of P.
Solution:
Let the required point be $(2y, y)$. Let $Q(2, -5)$ and $R(-3, 6)$ are given points.

Now, $PQ = PR \Rightarrow \sqrt{(2y-2)^2 + (y+5)^2} = \sqrt{(2y+3)^2 + (y-6)^2}$

[$\because$ using Distance formula, $\sqrt{(x-2)^2 + (y+5)^2} = \sqrt{(x+3)^2 + (y-6)^2}$]

Squaring both sides we get

$4y^2 + 4 - 8y + y^2 + 10y + 25 = 4y^2 + 9 + 12y + y^2 - 12y + 36$

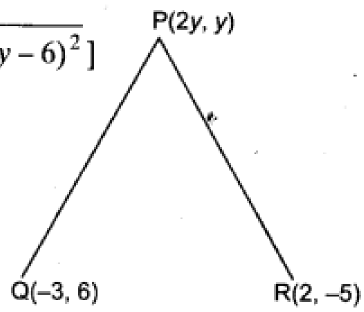
$\Rightarrow \quad 2y + 29 = 45$

$\Rightarrow \quad 2y = 45 - 29 = 16$

$\Rightarrow \quad y = 8$

$\Rightarrow \quad 2y = 16$

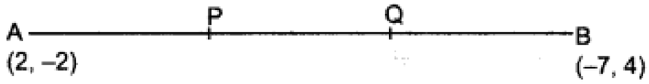
Hence coordinates of P are (16, 8)



Question 3.
Let P and Q be the points of trisection of the line segment joining the points A(2, -2) and B(-7, 4) such that P is nearer to A. Find the coordinates of P and Q.

Solution:

Let A(2, – 2), B(– 7, 4) be given points. Let $P(x, y)$, $Q(x', y')$ are point of trisection.



P divides AB in the ratio 1 : 2

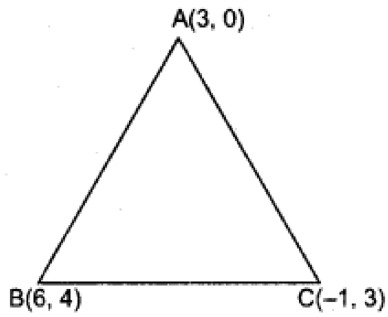
Coordinates of P are $\left(\frac{2\times 2+1(-7)}{1+2}, \frac{(-2)(2)+1(4)}{1+2}\right)$ or (–1, 0)

Q is mid point of PB. So using mid point formula coordinates of Q are $\left(\frac{-1-7}{2}, \frac{0+4}{2}\right)$ or (–4, 2)

Question 4.
Prove that the points (3,0), (6,4) and (-1,3) are the vertices of a right-angled isosceles triangle.
Solution:

Let the triangle be ΔABC as shown in figure. Distances are:

Using distance formula,



$AB = \sqrt{(3-6)^2+(0-4)^2} = 5$

$BC = \sqrt{(6+1)^2+(4-3)^2} = 5\sqrt{2}$

$CA = \sqrt{(-1-3)^2+(3-0)^2} = 5$

Here, AB = AC $\Rightarrow \Delta ABC$ is isosceles triangle

Consider, $AB^2 + AC^2 = (5)^2 + (5)^2 = 25 + 25 = 50$

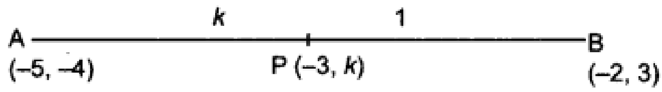
$\Rightarrow$ and, $BC^2 = (5\sqrt{2})^2 = 50$

$\therefore$ Here, $AB^2 + AC^2 = BC^2$

$\Rightarrow \Delta ABC$ is a right angled triangle.

[$\because$ In right Δ, using Pythagoras theorem $(H)^2 = (P)^2 + (B)^2$]
where H = hypotenuse, B = base, P = perpendiculars

Question 5.
Find the ratio in which the point (-3, k) divides the line-segment joining the points (-5, -4) and (-2,3). Also, find the value of k.
Solution:



Let P divides AB in k : 1.

Then $-3 = \frac{k\times(-2)+1(-5)}{k+1}$ [Using section formula, $(x, y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$]

$\Rightarrow -3k-3 = -2k-5$

$\Rightarrow -k = -2$

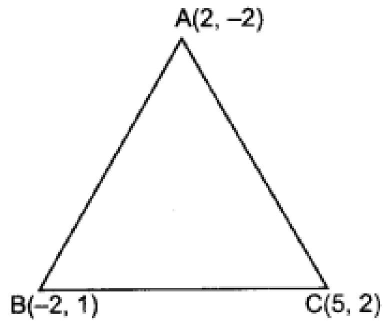
$\Rightarrow k = 2$

Hence the required ratio is 2 : 1

Question 6.
Prove that the points (2, -2), (-2, 1) and (5, 2) are the vertices of a right-angled

triangle. Also, find the area of this triangle.
Solution:

Let A(2, –2), B(–2, 1) and C(5, 2) be the given points. So,
Using Distance formula



$AB^2 = (2+2)^2 + (-2-1)^2 = 16 + 9 = 25$

$\therefore AB = 5$

$BC^2 = (-2-5)^2 + (1-2)^2 = 49 + 1 = 50$

$BC = 5\sqrt{2}$

$AC^2 = (5-2)^2 + (2+2)^2 = 9 + 16 = 25$

$AC = 5$

$\therefore BC^2 = AC^2 + AB^2$, so ΔABC is a right angled triangle in which BC is hypotenuse.

$$\therefore \quad ar(\Delta ABC) = \frac{1}{2} \times AB \times AC = \frac{1}{2} \times 5 \times 5 = \frac{25}{2} \text{ sq. units}$$

Short Answer Type Questions II [3 Marks]

Question 7.
In the figure, ABC is a triangle coordinates of whose vertex A are (0, -1). D and E respectively are the mid-points of the sides AB and AC and their coordinates are (1, 0) and (0,1) respectively. If F is the mid-point of BC, find the areas of ∆ABC and ∆DEF.
Solution:

Let coordinates of B are (x, y). Then using mid point formula we

$$\frac{x+0}{2} = 1 \quad \Rightarrow \quad x = 2$$

$$\frac{y-1}{2} = 0 \quad \Rightarrow \quad y = 1$$

Coordinates of B are (2,1)

Let coordinates of C are (p, q)

Similarly coordinates of C we have

$$\frac{p+0}{2} = 0 \quad \Rightarrow \quad p = 0$$

$$\frac{q-1}{2} = 1 \quad \Rightarrow \quad q = 3$$

Coordinates of C are (0, 3)

Area of ΔABC

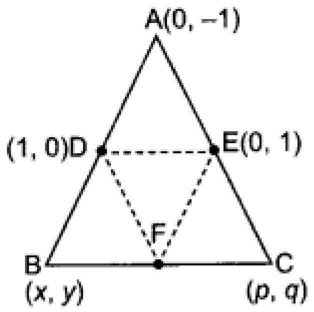
$$\Rightarrow \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = \frac{1}{2}[0(1-3)+2(3+1)+0(-1-1)]$$

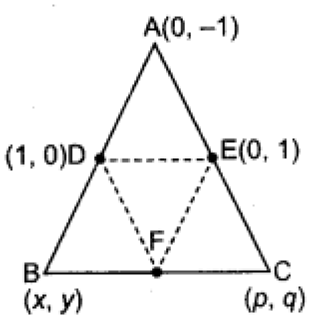
$$= \frac{1}{2} \times 8 = 4 \text{ sq. units}$$

Coordinates of F are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$ i.e. $(1, 2)$ [$\because$ Using mid-point formula]

$$\text{Area of } \Delta DEF = \frac{1}{2}[1(1-2)+0(2-0)+1(0-1)] = \frac{1}{2}[-1+0-1]$$

$$= \frac{1}{2} \times (-2) = [-1] = 1 \text{ sq. units}$$ [$\because$ Area cannot be negative]





Question 8.
If the point P(x, y) is equidistant from the points A (a + b, b – a) and B(a -b,a+ b). Prove that bx = ay.
Solution:

Given, $PA = PB \Rightarrow PA^2 = PB^2$

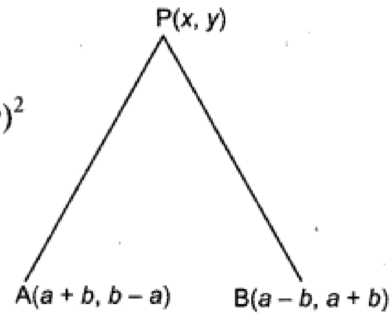
Applying distance formula,

$$\Rightarrow (a+b-x)^2 + (b-a-y)^2 = (a-b-x)^2 + (a+b-y)^2$$

$$\Rightarrow (a+b)^2 + x^2 - 2ax - 2bx + (b-a)^2 + y^2 - 2by + 2ay$$

$$= (a-b)^2 + x^2 - 2ax + 2bx + (a+b)^2 + y^2 - 2ay - 2by$$

$$\Rightarrow 4ay = 4bx \Rightarrow ay = bx \text{ or } bx = ay$$ Hence proved.

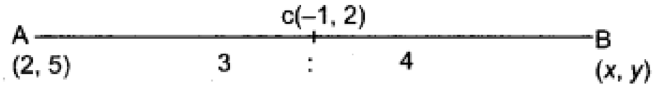


Question 9.
If the point C(-l, 2) divides internally the line-segment joining the points A(2, 5) and B(x,y) in the ratio 3: 4, find the value of $x^2 + y^2$.

Solution:

Using section formula,

$$-1 = \frac{3 \times x + 4 \times 2}{3+4}$$



$$-1 = \frac{3x+8}{7}$$

$$3x + 8 = -7 \Rightarrow 3x = -15 \Rightarrow x = -5$$

Similarly, $2 = \frac{3 \times y + 4 \times 5}{3+4}$

$$14 = 3y + 20 \Rightarrow 3y = -6 \Rightarrow y = -2$$

Hence; $x^2 + y^2 = (-5)^2 + (-2)^2 = 25 + 4 = 29$

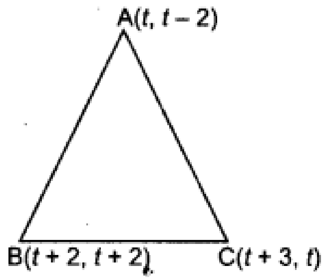
Long Answer Type Questions [4 Marks]

Question 10.

Prove that the area of a triangle with vertices (t, t – 2), (t + 2, t + 2) and (t + 3, t) is independent of t.

Solution:

Given vertices of triangle are $\{t, t-2\}$, $\{t+2, t+2\}$, $\{t+3, t\}$



Let (x_1, y_1), (x_2, y_2), (x_3, y_3) are vertices of the triangle.

Area of the triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[t(t+2-t) + (t+2)\{t-t+2\} + (t+3)\{t-2-t-2\}]$

$= \frac{1}{2}[2t + 2t + 4 - 4t - 12]$

$= \frac{1}{2} \times (-8) = 4$ sq units, since area can't be negative.

Hence, area is independent of t.

Question 11.

In fig., the vertices of AABC are A(4, 6), B(l, 5) and C(7, 2). A line-segment DE is drawn to intersect the sides AB and AC

at D and E respectively such that AD/AC=AE/AC=1/3. Calculate the area of ∆ADE and compare it with an area of ∆ABC.

Solution:

$$\text{Given:}\quad \frac{AD}{AB} = \frac{1}{3}$$

$$3AD = AB$$

$$\therefore \quad 3AD = AD + DB$$

$$2AD = DB$$

$$\frac{AD}{DB} = \frac{1}{2}$$

$$\text{Similarly,}\quad \frac{AE}{EC} = \frac{1}{2}$$

Calculated using section formula

Coordinates of D are $\left(\dfrac{1(1)+2(4)}{1+2}, \dfrac{1(5)+2(6)}{1+2}\right)$ i.e. $\left(3, \dfrac{17}{3}\right)$

Coordinates of E are $\left(\dfrac{1(7)+2(4)}{1+2}, \dfrac{1(2)+z(6)}{1+2}\right)$ i.e. $\left(5, \dfrac{14}{3}\right)$

$$\text{Area of } \Delta ADE = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$$

$$= \frac{1}{2}\left[4\left(\frac{17}{3}-\frac{14}{3}\right)+3\left(\frac{14}{3}-6\right)+6\left(6-\frac{17}{3}\right)\right]$$

$$= \frac{1}{2}\left[4+3\left(\frac{-4}{3}\right)+5\left(\frac{1}{3}\right)\right]$$

$$= \frac{1}{2}\left[4-4+\frac{5}{3}\right] = \frac{5}{6} \text{ sq. units}$$

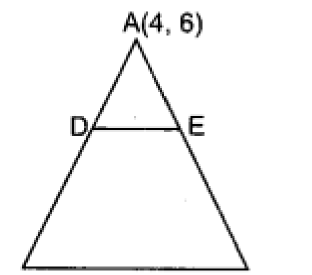
$$\text{Area of } \Delta ABC = \frac{1}{2}[4(5-2) + 1(2-6) + 7(6-5)]$$

$$= \frac{1}{2}[4 \times 3 + (-4) + 7 \times 1]$$

$$= \frac{1}{2}[12 - 4 + 7]$$

$$= \frac{1}{2} \times 15 = \frac{15}{2} \text{ sq. units}$$

$$\text{Hence,}\quad \frac{\text{Area } (\Delta ADE)}{\text{Area } (\Delta ABC)} = \frac{5/6}{15/12} = \frac{5}{6} \div \frac{15}{12} = \frac{5}{6} \times \frac{12}{15} = \frac{2}{3}$$



Question 12.
The coordinates of points A, B and C are (6,3), (-3,5) and (4, -2) respectively. P(JC, y) is any point in the plane. Show that $\frac{ar(\Delta PBC)}{ar(\Delta ABC)} = \left|\frac{x+y-2}{7}\right|.$
Solution:

Taking points P, B, C. Firstly,

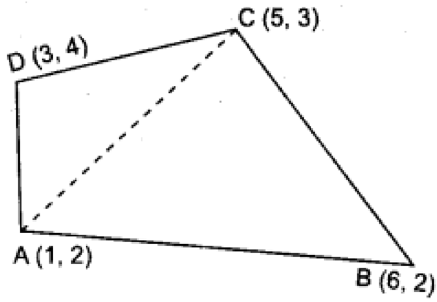
$$\text{area}(\Delta PBC) = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$$
$$= \frac{1}{2}[x(7) - 3(-2-y) + 4(y-5)]$$
$$= \frac{1}{2}[7x + 7y - 14] \text{ sq. units}$$

Now,
$$\text{area}(\Delta ABC) = \frac{1}{2}[6 \times 7 - 3(-5) + 4(3-5)]$$
$$= \frac{1}{2}[42 + 15 - 8] = \frac{1}{2} \times 49 \text{ sq. units}$$

Hence,
$$\left|\frac{\text{area}(\Delta PBC)}{\text{area}(\Delta ABC)}\right| = \left|\frac{7x+7y-14}{49}\right| = \left|\frac{x+y-2}{7}\right|$$

Question 13.
Find the area of the quadrilateral ABCD, the coordinate whose vertices are A(1, 2), B(6,2), C(5,3) and D(3,4).
Solution:

$$\text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$$
$$= \frac{1}{2}[1(2-3) + 6(3-2) + 5(2-2)]$$
$$= \frac{1}{2}[-1 + 6 + 0] = \frac{5}{2} \text{ sq. units}$$

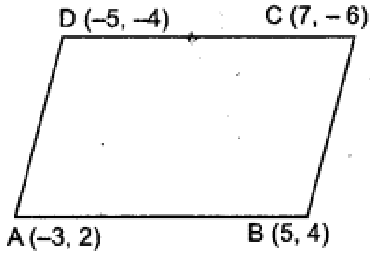


$$\text{Now, Area of } (\Delta ACD) = \frac{1}{2}[1(3-4) + 5(4-2) + 3(2-3)]$$
$$= \frac{1}{2}[-1 + 10 - 3]$$
$$= \frac{1}{2} \times 6 = 3 \text{ sq. units}$$

$$\text{Hence, Area (quadrilateral ABCD)} = \frac{5}{2} + 3 = \frac{11}{2} \text{ sq. units}$$

Question 14.
Find the area of a quadrilateral ABCD, the coordinates of whose vertices are A(—3,2), B(5,4), C(7, -6) and D(-5, -4).
Solution:

$$\text{Area of } \Delta ABD = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$
$$= \frac{1}{2}[-3(8)+5(-6)+(-5)(2-4)]$$
$$= \frac{1}{2}[-24-30+10]$$
$$= \frac{1}{2}\times(-44) = (-22) = 22 \text{ sq. units}$$



Since area can't be negative

$$\text{area of } \Delta BCD = \frac{1}{2}[5(-2)+7(-8)-5(10)]$$
$$= \frac{1}{2}[-10-56-50]$$
$$= \frac{1}{2}(-116) = (-58) = 58 \text{ sq. units}$$

Since area cannot be negative.

$$\text{Area of quadrilateral ABCD} = \text{Area }(\Delta ABD) + \text{area }(\Delta BCD)$$
$$= 22 + 58 = 80 \text{ sq. units.}$$

2015

Short Answer Type Questions I [2 Marks]

Question 15.

If A(5, 2), B(2, -2) and C(-2, t) are the vertices of a right-angled triangle with ∠B = 90°, then find the value of t.

Solution:

Using distance formula in right triangle ABC,

$AB^2 = (5-2)^2 + (2-(-2))^2 = 9 + 16 = 25$

$AC^2 = (5-(-2))^2 + (2-t)^2 = 49 + 4 - 4t + t^2 = t^2 - 4t + 53$

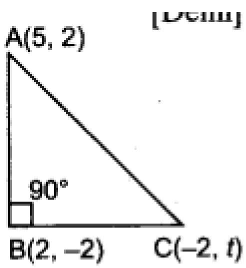
$BC^2 = (2+2)^2 + (-2-t)^2 = 16 + t^2 + 4t + 4 = t^2 + 4t + 20$

Now ΔABC is a right triangle, right angled at B.

So, $AC^2 = AB^2 + BC^2$

$$t^2 - 4t + 53 = 25 + t^2 + 4t + 20$$
$$8t = 8 \quad \Rightarrow \quad t = \frac{8}{8} = 1$$

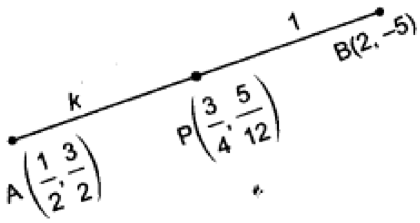
Hence, $t = 1$



(By Pythagoras theorem)

Question 16.
Find the ratio in which the point P P(3/4,5/12) divides the line segment joining the points A(1/2,3/2) and 3(2, -5).
Solution:
Let point P divides the line segment AB in the ratio $k : 1$. Using section formula,

then the coordinates of P are $\left(\dfrac{2k+\frac{1}{2}}{k+1}, \dfrac{-5k+\frac{3}{2}}{k+1}\right)$



A.T.Q. $\qquad \dfrac{2k+\frac{1}{2}}{k+1} = \dfrac{3}{4}$

$\Rightarrow \qquad 8k + 2 = 3k + 3$

$\Rightarrow \qquad 5k = 1$

$\Rightarrow \qquad k = \dfrac{1}{5}$

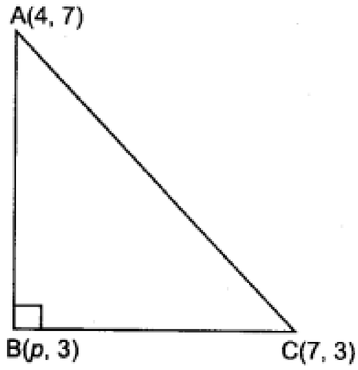
Hence, P divides the line segment AB in the ratio 1 : 5.

Question 17.
The points A(4,7), B(p, 3) and C(7,3) are the vertices of a right triangle, right-angled at B. Find the value of p.

Solution:

In right ΔABC,

Using distance formula,



$$AC = \sqrt{3^2 + (-4)^2} = 5$$
$$AB = \sqrt{(p-4)^2 + 16}$$
$$BC = \sqrt{(p-7)^2 + 0}$$

Now, $AC^2 = AB^2 + BC^2$ [$\because$ Pythagoras theorem]

$\Rightarrow$ $25 = (p-4)^2 + 16 + (p-7)^2$

$25 = p^2 - 8p + 16 + 16 + p^2 - 14p + 49$

$\Rightarrow$ $2p^2 - 22p + 56 = 0$

$\Rightarrow$ $p^2 - 11p + 28 = 0$

$(p-4)(p-7) = 0 \quad \Rightarrow \quad p = 4$ or $p = 7$

If $p = 7$, then $B = (7, 3)$

It coincides with C

$\therefore$ $p \neq 7$

Hence, $p = 4$

Question 18.

Find the relation between x and y if the points A(x, y), B(-5, 7) and C(-4, 5) are collinear

Solution:

$\because$ A, B and C are collinear. So, area (ΔABC) = 0

$\therefore$ $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow$ $\frac{1}{2}[x(7-5) + (-5)(5-y) + (-4)(y-7)] = 0$

$2x - 25 + 5y - 4y + 28 = 0$

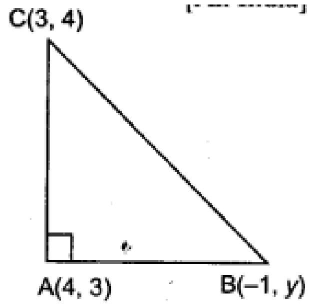
$\Rightarrow$ Required relation between x & y is $2x + y + 3 = 0$

Question 19.

If A(4,3), B(-l,y) and C(3,4) are the vertices of right triangle ABC, right-angled at A, then find the value.

Solution:

In right ΔCAB, using distance formula,



$$BC^2 = (3+1)^2 + (4-y)^2 = 16 + (4-y)^2$$
$$AB^2 = (-1-4)^2 + (y-3)^2 = 25 + (y-3)^2$$
$$AC^2 = (4-3)^2 + (3-4)^2 = 2$$

Also $BC^2 = AB^2 + AC^2$ [$\because$ Pythagoras theorem]

$\Rightarrow$ $16 + (4-y)^2 = 25 + (y-3)^2 + 2$

$16 + 16 + y^2 - 8y = 25 + y^2 - 6y + 9 + 2$

$-2y = 4 \quad \Rightarrow \quad y = -2$

Question 20.

Show that the points (a, a), (-a, -a) and (-√3a, √3 a) are the vertices of an equilateral

triangle.
Solution:
Let $P(a, a)$, $Q(-a, -a)$, $R(-\sqrt{3}a, \sqrt{3}a)$. Using distance formula,

$$PQ = \sqrt{(a+a)^2+(a+a)^2} = \sqrt{4a^2+4a^2} = 2\sqrt{2}a$$

$$QR = \sqrt{(-a+\sqrt{3}a)^2+(-a-\sqrt{3}a)^2}$$

$$= \sqrt{a^2+3a^2-2\sqrt{3}a^2+a^2+3a^2+2\sqrt{3}a^2} = \sqrt{8a^2} = 2\sqrt{2}a$$

$$RP = \sqrt{(a+\sqrt{3}a)^2+(a-\sqrt{3}a)^2}$$

$$= \sqrt{a^2+3a^2+2\sqrt{3}a^2+a^2+3a^2-2\sqrt{3}a^2} = \sqrt{8a^2} = 2\sqrt{2}a$$

$\Rightarrow$ Here, $PQ = QR = RP$

$\therefore$ P, Q, R are vertices of an equilateral triangle.

Question 21.
For what values of k are the points (8,1), (3, -2k) and (k, -5) collinear?
Solution:
For collinear points area of Δ made by these points will be zero.

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$\Rightarrow \quad 8(-2k+5)+3(-5-1)+k(1+2k) = 0$$

$$\Rightarrow \quad -16k+40-18+k+2k^2 = 0$$

$$2k^2-15k+22 = 0$$

$$2k^2-11k-4k+22 = 0$$

$$\Rightarrow \quad k(2k-11)-2(2k-11) = 0$$

$$(2k-11)(k-2) = 0$$

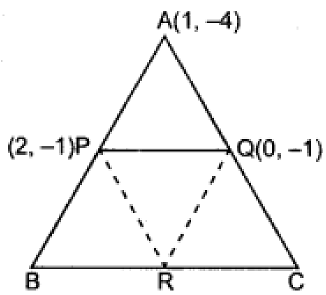
$$\Rightarrow \quad k = 2, k = \frac{11}{2}$$

Short Answer Type Questions II [3 Marks]

Question 22.
Find the area of the triangle ABC with A(l, -4) and mid-points of sides through A being (2, -1) and (0, -1).
Solution:
I. Firstly, $ar(\Delta APQ) = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$$= \frac{1}{2}|1(-1+1)+2(-1+4)+0(-4+1)|$$

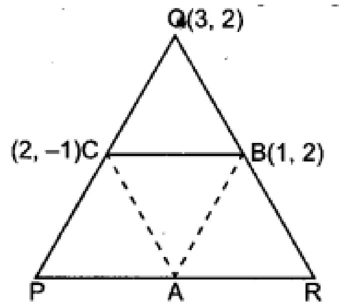
$$= \frac{1}{2}|0+6+0| = 3 \text{ sq. units}$$



$\because$ P, Q and R are the mid point of sides AB, AC and BC respectively

So, $ar(\Delta ABC) = 4\, ar(\Delta APQ) = 4 \times 3 = 12$ sq. units.

Question 23.
Find the area of the triangle PQR with Q (3, 2) and the mid-points of the sides through Q being (2, -1) and (1,2).
Solution:



$$\text{Firstly, ar}(\Delta QCB) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$
$$= \frac{1}{2}|3(-1-2) + 2(2-2) + 1(2+1)|$$
$$= \frac{1}{2}|-9 + 0 + 3| = 3 \text{ sq.units}$$

$\because$ A, B and C are the mid-points of sides PR, RQ and QP respectively.

$$\text{So,} \quad \text{ar}(\Delta QPR) = 4 \times \text{ar}(\Delta QCB)$$
$$= 4 \times 3 = 12 \text{ sq. units}$$

Question 24.
If the coordinates of points A and B are (-2, -2) and (2, – 4) respectively, find the coordinates of P such that AP = 3/7 AB, where P lies on the line segment AB.
Solution:

$$\text{Given,} \quad AP = \frac{3}{7}AB \Rightarrow AP : PB = 3 : 4$$

$\Rightarrow$ P divides AB in the ratio 3 : 4
Using section formula,

$$\therefore \quad \text{Coordinates of point P are} = \left(\frac{3\times 2 + 4\times(-2)}{3+4}, \frac{3\times(-4) + 4\times(-2)}{3+4}\right) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$$

Question 25.
Find the coordinates of a point P on the line segment joining A(l, 2) and B(6,7) such that AP=2/5AB
Solution:

$$\text{Given,} \quad AP = \frac{2}{5}AB$$
$$\therefore \quad AP : PB = 2 : 3 \Rightarrow \text{P divides AB in ratio } 2 : 3.$$

Using section formula,

$$\text{Coordinates of point P are} \left(\frac{2\times 6 + 3\times 1}{2+3}, \frac{2\times 7 + 3\times 2}{2+3}\right) \text{ i.e. } (3, 4)$$

$\therefore$ Coordinates of P are (3, 4)

Question 26.
Point A lies on the line segment PQ joining P(6, -6) and Q(-4, -1) in such a way that PA/PQ=2/5. If point P also lies on the line 3x + k(y + 1) = 0, find the value of k
Solution:

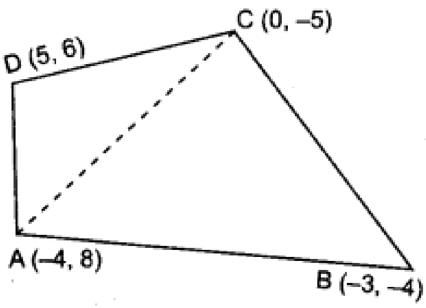
Coordinates of P are (6, –6). Given that:

$\because$ P(6, –6) lies on the line. So,

$$3x + k(y + 1) = 0$$
$$\Rightarrow \quad 3 \times 6 + k(-6 + 1) = 0$$
$$\Rightarrow \quad 18 - 5k = 0$$
$$\Rightarrow \quad k = \frac{18}{5}.$$

Long Answer Type Questions [4 Marks]

Question 27.
If A(-4,8), B(-3, -4), C(0, – 5) and D(5,6) are the vertices of a quadrilateral ABCD, find its area.
Solution:



$$\text{Firstly, ar}(\Delta ABC) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$
$$= \frac{1}{2}|-4(-4 + 5) - 3(-5 - 8) + 0(8 + 4)|$$
$$= \frac{1}{2}|-4 + 39 + 0| = \frac{1}{2} \times 35$$
$$= \frac{35}{2} \text{ sq. units}$$
$$\text{Now, ar}(\Delta ACD) = \frac{1}{2}|-4(-5 - 6) + 0(6 - 8) + 5(8 + 5)|$$
$$= \frac{1}{2}|44 + 0 + 65| = \frac{109}{2} \text{ sq. units}$$

So, ar(quadrilateral ABCD) = ar(ΔABC) + ar(ΔACD)

$$= \frac{35}{2} + \frac{109}{2} = \frac{144}{2} = 72 \text{ sq. units}$$

Question 28.
If P(-5, -3), Q (-4, -6), R(2, -3) and S(l, 2) are the vertices of a quadrilateral PQRS, find its area.

Solution:

Firstly, $\text{ar}(\Delta PQR) = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$= \frac{1}{2}|-5(-6+3)-4(-3-3)+2(-3+6)|$

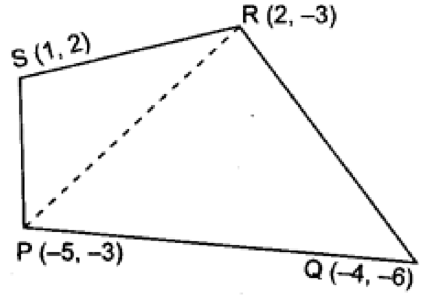
$= \frac{1}{2}|15+0+6| = \frac{21}{2}$ sq. units

Now, $\text{ar}(\Delta PRS) = \frac{1}{2}|-5(-3-2)+2(2+3)+1(-3+3)|$

$= \frac{1}{2}|25+10+0| = \frac{35}{2}$ sq. units

So, $\text{ar}(\text{quad } PQRS) = \text{ar}(\Delta PQR) + \text{ar}(\Delta PRS)$

$= \frac{21}{2}+\frac{35}{2} = \frac{56}{2} = 28$ sq. units.



Question 29.
Find the values of k so that the area of the triangle with vertices (1, -1), (-4,2k) and (-k, – 5) is 24 sq. units.
Solution:

Given that, Area of Δ = 24 sq. units

$\Rightarrow \quad \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)| = 24$

$|1(2k+5)-4(-5+1)-k(-1-2k)| = 48$

$\Rightarrow \quad |2k+5+16+k+2k^2| = 48$

$\Rightarrow \quad |2k^2+3k+21| = 48$

$\Rightarrow \quad 2k^2+3k+21 = \pm 48$

$\Rightarrow \quad 2k^2+3k+21 = 48$ or $2k^2+3k+21 = -48$

$2k^2+3k-27 = 0$ | $2k^2+3k+69 = 0$

$2k^2+9k-6k-27 = 0$ | Discriminant, D $= (3)^2-4\times 2\times 69\ [\because b^2-4ac]$

$k(2k+9)-3(2k+9) = 0$ | $= -\text{ve}$

$(2k+9)(k-3) = 0$ | $\therefore$ No solution

$\Rightarrow \quad k = \frac{-9}{2}$ or $k = 3$

Question 30.
Find the values of k for which the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k – 1, 5k) are collinear.

Solution:

$\because$ The points A(k + 1, 2k), B(3k, 2k + 3) and C(5k – 15k) are collinear. So, ar (ΔABC) = 0

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\frac{1}{2}[(k + 1)(2k + 3 - 5k) + 3k(5k - 2k) + (5k - 1)(2k - 2k - 3)] = 0$$

$$\Rightarrow \quad (k + 1)(-3k + 3) + 3k \times 3k + (5k - 1)(-3) = 0$$

$$\Rightarrow \quad -3k^2 + 3k - 3k + 3 + 9k^2 - 15k + 3 = 0$$

$$\Rightarrow \quad 6k^2 - 15k + 6 = 0$$

$$\Rightarrow \quad 2k^2 - 5k + 2 = 0$$

$$2k^2 - 4k - k + 2 = 0$$

$$2k(k - 2) - 1(k - 2) = 0$$

$$\Rightarrow \quad (k - 2)(2k - 1) = 0$$

$$\Rightarrow \quad k = 2 \text{ or } k = \frac{1}{2}.$$

Question 31.

The base BC of an equilateral triangle ABC lies on the y-axis. The coordinates of point C are (0, -3). The origin is the mid-point of the base. Find the coordinates of points A and B. Also, find the coordinates of another point D such that BACD is a rhombus.

Solution:

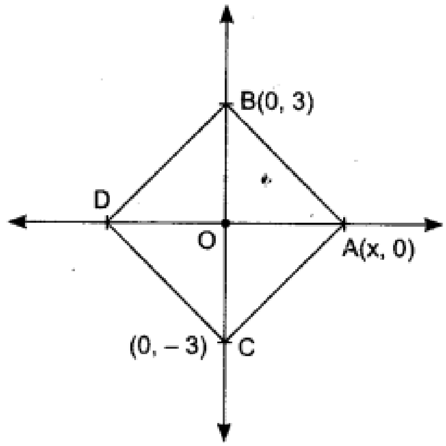
Given that, $\because$ O is mid point of BC and coordinates of C are (0, – 3)

$\therefore$ coordinate of B are (0, 3)

Now AO will be the perpendicular bisector of BC. Therefore A will lie on x-axis. let coordinates of A are (x, 0)

Now, in equilateral ΔABC, AB= BC

Using distance formula,



$$\Rightarrow \quad \sqrt{(x-0)^2 + (0-3)^2} = 6$$

$$\sqrt{x^2 + 9} = 6$$

$$x^2 + 9 = 36 \Rightarrow x^2 = 27$$

$$x = \pm 3\sqrt{3}$$

$\therefore$ coordinates of A are $(3\sqrt{3}, 0)$ or $(-3\sqrt{3}, 0)$

When A is $(3\sqrt{3}, 0)$ then D will be $(-3\sqrt{3}, 0)$ so that BACD is a rhombus, since opposite sides are equal.

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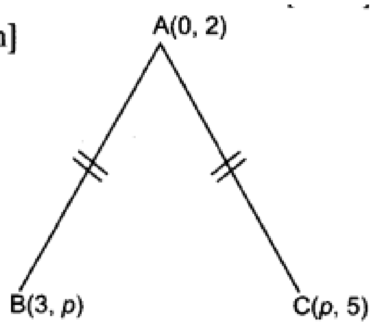
Short Answer Type Questions II [3 Marks]

Question 32.
If point A(0, 2) is equidistant from points B(3, p) and C(p, 5), find p. Also, find the length of AB.
Solution:



Here, $AB = AC$ [Given]

$\Rightarrow AB^2 = AC^2$

Using distance formula,

$\Rightarrow (3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

$\Rightarrow 9 + p^2 - 4p + 4 = p^2 + 9$

$\Rightarrow -4p + 4 = 0$

$\Rightarrow 4p = 4$

$\Rightarrow p = 1$

So, point B is (3, 1); point C is (1, 5)

Now, length of AB $= \sqrt{(3-0)^2 + (1-2)^2}$ [$\because$ use distance formula]

$= \sqrt{9+1} = \sqrt{10}$ units

Question 33.
If the points A(-2,1), B (a, b) and C(4, -1) are collinear and a – b = 1, find the value of a and b.
Solution:

Since, the points A(–2, 1), B(a, b) and C(4, –1) are collinear,

So, area of triangle, $ar(\Delta ABC) = 0$

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \frac{1}{2}|-2(b+1) + a(-1-1) + 4(1-b)| = 0$$

$\Rightarrow -2b - 2 - 2a + 4 - 4b = 0 \quad \Rightarrow 2a + 6b = 2$

$\Rightarrow a + 3b = 1$ ...(*i*)

Also, given that $a - b = 1$ ...(*ii*)

On solving the equations (*i*) and (*ii*), we get

$a = 1, b = 0$

Question 34.
If the points P(-3,9), Q(a, b) and R(4, -5) are collinear and a + b = 1, find the value of a and b.

Solution:

Since, the points P(–3, 9), Q(a, b) and R(4, –5) are collinear,

So, area of triangle $\quad ar(\Delta PQR) = 0$

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}\,|-3(b+5) + a(-5-9) + 4(9-b)| = 0$$

$$\Rightarrow \quad -3b - 15 - 14a + 36 - 4b = 0 \quad \Rightarrow 14a + 7b - 21 = 0$$

$$\Rightarrow \quad 2a + b = 3 \quad \ldots(i)$$

Also, given that $\quad a + b = 1 \quad \ldots(ii)$

On solving the equations (i) and (ii), we get

$a = 2, b = -1$

Question 35.

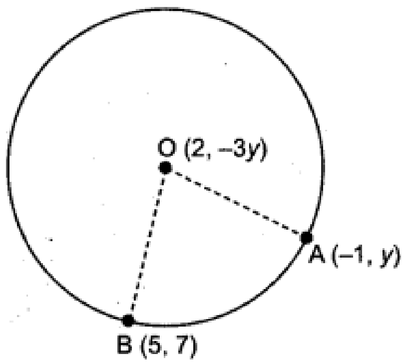
Points A(-l, y) and B(5,7) lie on a circle with centre 0(2, -3y). Find the values. Hence, find the radius of the circle.

Solution:

Given, O is the centre of the circle and the points A and B lie on the circle.

So, $\quad OA = OB\ (= r) \quad$ [∵ radius of same circle]

$$\Rightarrow \quad OA^2 = OB^2$$



Using distance formula,

$$\Rightarrow \quad (2+1)^2 + (-3y - y)^2 = (2-5)^2 + (-3y-7)^2$$

$$\Rightarrow \quad 9 + 16y^2 = 9 + 9y^2 + 42y + 49$$

$$\Rightarrow \quad 7y^2 - 42y - 49 = 0$$

$$\Rightarrow \quad y^2 - 6y - 7 = 0$$

$$\Rightarrow \quad (y-7)(y+1) = 0$$

$$\Rightarrow \quad y = -1 \text{ or } 7$$

When $y = -1$, then co-ordinates are: O(2, 3) and A(–1, –1)

$$\text{Radius of circle, } r = OA = \sqrt{(2+1)^2 + (3+1)^2} = \sqrt{9+16} = 5 \text{ units}$$

When $y = 7$, then coordinates are: O(2, – 21) and A(–1, 7)

$$\text{Radius of circle, } r = OA = \sqrt{(2+1)^2 + (-21-7)^2} = \sqrt{9+784} = \sqrt{793} \text{ units.}$$

Question 36.

If the point A(-l, -4); B(b, c) and C(5, -1) are collinear and 2b+ c = 4, find the value of b and c.

Solution:

Since, the points A(–1, –4), B(b, c) and C(5, –1) are collinear,

So, area of triangle, ar (ΔABC) = 0

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}|-1(c + 1) + b(-1 + 4) + 5(-4 - c)| = 0$$

$$\Rightarrow \quad -c - 1 + 3b - 20 - 5c = 0 \quad \Rightarrow 3b - 6c = 21$$

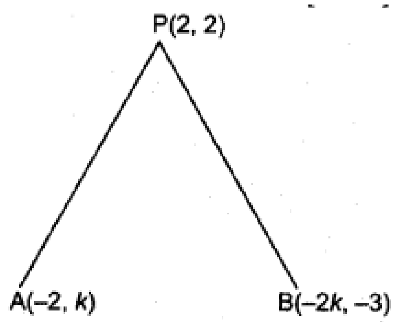
$$\Rightarrow \quad b - 2c = 7 \quad \ldots(i)$$

Also, given that $2b + c = 4$ ...(ii)

Question 37.

If the point P(2, 2) is equidistant from the points A(-2, k) and B(-2k, -3), find k.

Solution:



Since, given that PA = PB $\Rightarrow$ PA^2 = PB^2

Using distance formula,

$$\Rightarrow \quad (+2 + 2)^2 + (2 - k)^2 = (2 + 2k)^2 + (2 + 3)^2$$

$$\Rightarrow \quad 16 + 4 - 4k + k^2 = 4 + 8k + 4k^2 + 25$$

$$\Rightarrow \quad 16 - 4k + k^2 = 8k + 4k^2 + 25$$

$$\Rightarrow \quad 3k^2 + 12k + 9 = 0$$

$$\Rightarrow \quad k^2 + 4k + 3 = 0$$

$$\Rightarrow \quad (k + 3)(k + 1) = 0$$

$$\Rightarrow \quad k = -1 \text{ or } -3$$

When $k = -1$, then point A is (–2, –1)

$$AP = \sqrt{(2+2)^2 + (2+1)^2} = \sqrt{16+9} = 5 \text{ units}$$

When $k = -3$, then point A is (–2, –3)

$$AP = \sqrt{(2+2)^2 + (2+3)^2} = \sqrt{16+25} = \sqrt{41} \text{ units}$$

Question 38.

If the point P(k – 1, 2) is equidistant from the points A(3, k) and B(k, 5), find the values of k.

Solution:

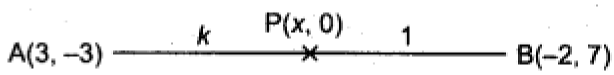
Given that point P($k-1$, 2) is equidistant from the points A(3, k) and B(k, 5), so

$$\therefore \quad AP = BP \quad \Rightarrow AP^2 = BP^2$$

Using distance formula,

$$\Rightarrow \quad (k-1-3)^2 + (2-k)^2 = (k-1-k)^2 + (2-5)^2$$
$$\Rightarrow \quad (k-4)^2 + (2-k)^2 = (-1)^2 + (-3)^2$$
$$\Rightarrow \quad k^2 - 8k + 16 + 4 - 4k + k^2 = 1 + 9$$
$$\Rightarrow \quad 2k^2 - 12k + 10 = 0$$
$$\Rightarrow \quad k^2 - 6k + 5 = 0$$
$$\Rightarrow \quad k^2 - 5k - k + 5 = 0$$
$$\Rightarrow \quad k(k-5) - 1(k-5) = 0$$
$$\Rightarrow \quad (k-1)(k-5) = 0$$
$$\Rightarrow \text{ Either} \quad k - 1 = 0 \text{ or } k - 5 = 0$$
$$\Rightarrow \quad k = 1 \text{ or } 5$$

Question 39.
Find the ratio in which the line segment joining the points A(3, -3) and B(-2, 7) is divided by the x-axis. Also, find the coordinates of the point of division.
Solution:



Let point P(x, 0) on x-axis divides the join of A(3, –3) and B(–2, 7) in the ratio $k : 1$

Then coordinates of P are $\left(\dfrac{-2k+3}{k+1}, \dfrac{7k-3}{k+1}\right)$

If point P lies on x-axis, then y coordinate of P is 0.

$$\Rightarrow \quad \frac{7k-3}{k+1} = 0 \quad \Rightarrow \quad 7k - 3 = 0$$
$$\Rightarrow \quad k = \frac{3}{7}$$

$\therefore$ Ratio is $\frac{3}{7} : 1$, i.e. $3 : 7$.

Putting $\quad k = \frac{3}{7}$ in (i), we get

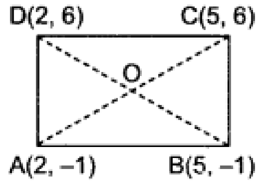
the coordinates of point P $= \left(\dfrac{-\frac{6}{7}+3}{\frac{3}{7}+1}, 0\right)$, i.e. $\left(\frac{3}{2}, 0\right)$.

Question 40.
Prove that the diagonals of a rectangle ABCD, with vertices A(2, – 1), B(5, – 1), C(5,6) and D(2,6), are equal and bisect each other.

Solution:

Given; A(2, –1), B(5, –1), C(5, 6) and D(2, 6) are the vertices of a rectangle ABCD.

Using distance formula,



$$\therefore \quad AC = \sqrt{(5-2)^2+(6+1)^2} = \sqrt{9+49} = \sqrt{58} \text{ units}$$

$$BD = \sqrt{(5-2)^2+(-1-6)^2} = \sqrt{9+49} = \sqrt{58} \text{ units}$$

$\therefore \quad AC = BD$, i.e. diagonals are equal

Now, the coordinates of mid-point of AC are $\left(\frac{2+5}{2}, \frac{6-1}{2}\right)$, i.e. $\left(\frac{7}{2}, \frac{5}{2}\right)$

The coordinates of mid-point of BD are $\left(\frac{5+2}{2}, \frac{-1+6}{2}\right)$, i.e. $\left(\frac{7}{2}, \frac{5}{2}\right)$

As the coordinates of the mid-points of AC and BD are same, hence diagonals bisect each other.

Question 41.

Find a point P on the y-axis which is equidistant from the points A(4,8) and B(- 6, 6). Also, find the distance AP.

Solution:

Let point $P(0, y)$ on y-axis is equidistant from the points A(4, 8) and B(–6, 6).

$$\therefore \quad AP = BP \quad \Rightarrow \quad AP^2 = BP^2$$

Using distance formula,

$$\Rightarrow \quad (4-0)^2 + (8-y)^2 = (-6-0)^2 + (6-y)^2$$

$$\Rightarrow \quad 16 + 64 - 16y + y^2 = 36 + 36 - 12y + y^2$$

$$\Rightarrow \quad -4y = -8 \quad \Rightarrow y = 2$$

$\therefore$ Point is P(0, 2)

Distance $AP = \sqrt{(4-0)^2+(8-2)^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$ units

Question 42.

Find the value(s) of k for which the points (3k – 1, k – 2), (k, k-1) and (k – 1, – k – 2) are collinear

Solution:

Since the points $(3k-1, k-2)$, $(k, k-7)$ and $(k-1, -k-2)$ are collinear, so area of triangle formed by these points is zero.

$$\Rightarrow \quad \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]=0$$

$$\Rightarrow \quad \frac{1}{2}[(3k-1)(k-7+k+2)+k(-k-2-k+2)+(k-1)(k-2-k+7)]=0$$

$$\Rightarrow \quad \frac{1}{2}[(3k-1)(2k-5)+k(-2k)+(k-1)(5)]=0$$

$$\Rightarrow \quad \frac{1}{2}[6k^2-15k-2k+5-2k^2+5k-5]=0$$

$$\Rightarrow \quad \frac{1}{2}[4k^2-12k]=0$$

$$\Rightarrow \quad 2k^2-6k=0$$

$$\Rightarrow \quad 2k(k-3)=0$$

$\Rightarrow$ Either $\quad k=0$ or $k-3=0$

$\Rightarrow \quad k=0$ or $k=3$.

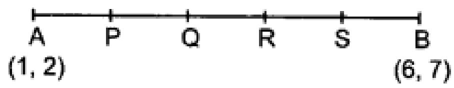
Question 43.

points P, Q, R and S divide the line segment joining the points A(l, 2) and B(6, 7) in 5 equal parts. Find the coordinates of the points P, Q and R.

Solution:

Line segment that joins points A(1, 2) and B(6, 7) is divided by points P, Q, R, S into 5 equal parts

$\therefore \quad AP = PQ = QR = RS = SB$



Use section formula $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

P divides the join of A and B in ratio 1 : 4.

$\therefore$ The coordinates of P are $\left(\frac{6+4}{1+4}, \frac{7+8}{1+4}\right)$, i.e. P(2, 3).

R divides the join of A and B in the ratio 3 : 2

$\therefore$ The coordinates of R are $\left(\frac{18+2}{3+2}, \frac{21+4}{3+2}\right)$, i.e. R(4, 5).

Now, Q is the midpoint PR

$\therefore$ The coordinates of Q are $\left(\frac{12+3}{5}, \frac{14+6}{5}\right)$, i.e. (3, 4)

Question 44.

Find the value(s) of p for which the points (p + 1, 2p – 2), (p – 1 ,p) and (p -6, 2p – 6) are collinear.

Solution:

Since, the points $(p + 1, 2p - 2)$, $(p - 1, p)$ and $(p - 3, 2p - 6)$ are collinear, so, area of triangle formed by these points is zero.

$$\Rightarrow \quad \frac{1}{2}[(p + 1)(p - 2p + 6) + (p - 1)(2p - 6 - 2p + 2) + (p - 3)(2p - 2 - p)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(p + 1)(6 - p) + (p - 1)(-4) + (p - 3)(p - 2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(6p - p^2 + 6 - p - 4p + 4 + p^2 - 2p - 3p + 6] = 0$$

$$\Rightarrow \quad \frac{1}{2}[-4p + 16] = 0 \Rightarrow -2p + 8 = 0$$

$$\Rightarrow \quad -2p = -8 \Rightarrow p = 4$$

Question 45.

Find the value(s) of p for which the points (3p + 1, p), (p + 2, p – 5) and (p + 1, -p) are collinear.

Solution:

Since the points $(3p + 1, p)$, $(p + 2, p - 5)$ and $(p + 1, -p)$ are collinear, so area of the triangle formed by these points is zero.

$$\Rightarrow \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(3p + 1)(p - 5 + p) + (p + 2)(-p - p) + (p + 1)(p - p + 5)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(3p + 1)(2p - 5) + (p + 2)(-2p) + 5(p + 1)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(6p^2 - 15p + 2p - 5 - 2p^2 - 4p + 5p + 5] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(4p^2 - 12p] = 0$$

$$\Rightarrow \quad 2p^2 - 6p = 0$$

$$\Rightarrow \quad 2p(p - 3) = 0$$

$\Rightarrow$ Either $p = 0$ or $p - 3 = 0 \Rightarrow p = 0, 3$

Long Answer Type Questions [4 Marks]

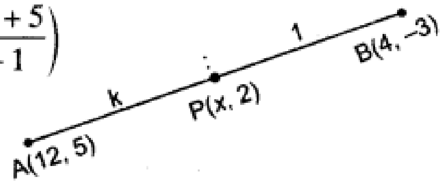
Question 46.

Find the ratio in which the point P(x, 2) divides the line segment joining the points A(12,5) and B(4, -3). Also, find the value of x.

Solution:

Let point P(x, 2) divides AB in the ratio k : 1. Using section formula,

then the coordinates of P in terms of k is $P\left(\frac{4k+12}{k+1}, \frac{-3k+5}{k+1}\right)$



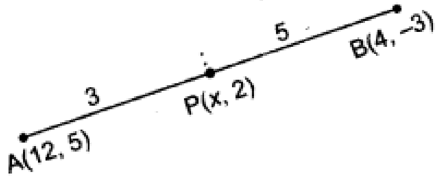
A.T.Q. $\frac{-3k+5}{k+1} = 2$

$\Rightarrow$ $-3k + 5 = 2k + 2$

$\Rightarrow$ $-5k = -3$

$\Rightarrow$ $k = \frac{3}{5}$

Thus, P divides AB in the ratio 3 : 5



Now, $x = \frac{3 \times 4 + 5 \times 12}{3+5} = \frac{12+60}{8} = \frac{72}{8} = 9$

Hence $x = 9$

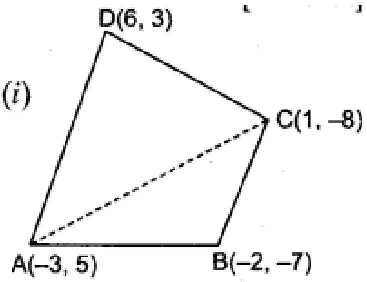
Question 47.

If A(-3,5), B(-2, -7), C(l, -8) and D(6,3) are the vertices of a quadrilateral ABCD, find its area.

Solution:

Area of quadrilateral ABCD

= Area of triangle ABC + Area of triangle ACD ...(i)



Now, $\text{ar(ABC)} = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

$= \frac{1}{2}[-3(-7+8) - 2(-8-5) + 1(5+7)]$

$= \frac{1}{2}[-3+26+12] = \frac{35}{2}$ sq. units ...(ii)

Also, $\text{ar(ACD)} = \frac{1}{2}[-3(-8-3) + 1(3-5) + 6(5+8)]$

$= \frac{1}{2}[33-2+78] = \frac{109}{2}$ sq. units ...(iii)

From (i), (ii), (iii), we get

$\text{ar(ABCD)} = \frac{35}{2} + \frac{109}{2} = \frac{144}{2} = 72$ sq. units

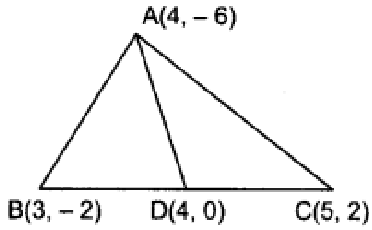
Question 48.

A(4, -6), B(3, -2) and C(5, 2) are the vertices of an AABC and AD is its median. Prove that the median AD divides AABC into two triangles of equal areas.

Solution:

A(4, –6), B(3, –2) and C(5, 2) are the vertices of ΔABC.

$\because$ AD is the median, so, D is the mid-point of BC.



$\therefore$ Coordinates of D $= \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4, 0)$

$$\text{Now,}\quad ar(ABD) = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = 0$$

$$= \left|\frac{1}{2}[3(0+6)+4(-6+2)+4(-2-0)]\right|$$

$$= \left|\frac{1}{2}[18-16-8]\right| = \frac{1}{2}|-6| = 3 \text{ sq. units}$$

$$ar(ADC) = \left|\frac{1}{2}[4(2+6)+5(-6-0)+4(0-2)]\right|$$

$$= \left|\frac{1}{2}[32-30-8]\right| = \frac{1}{2}|-6| = 3 \text{ sq. units}$$

$\therefore$ Clearly, ar(ABD) = ar(ADC).

Thus, median AD divides ΔABC into 2 triangles of equal areas.

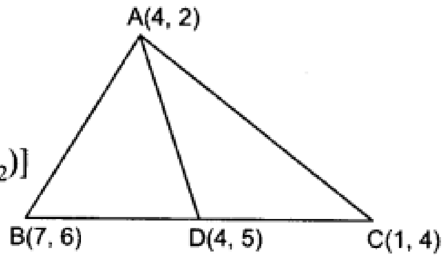
Question 49.

If A(4,2), B(7,6) and C(l, 4) are the vertices of an ΔABC and AD is its median, prove that the median AD divides ΔABC into two triangles of equal areas

Solution:

Given; A(4, 2), B(7, 6) and C(1, 4) are the vertices of a triangle ABC and AD is the median.

$\therefore$ D is the mid-point of BC as AD is median



$\therefore$ The coordinates of D are $\left(\frac{7+1}{2}, \frac{6+4}{2}\right)$, i.e. (4, 5).

$$\text{Now,}\quad ar(ABD) = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$$

$$= \left|\frac{1}{2}[4(6-5)+7(5-2)+4(2-6)]\right|$$

$$= \left|\frac{1}{2}[4+21-16]\right| = \frac{9}{2} \text{ sq. units}$$

$$ar(ADC) = \left|\frac{1}{2}[4(5-4)+4(4-2)+1(2-5)]\right| = \left|\frac{1}{2}[4+8-3]\right| = \frac{9}{2} \text{ sq. units}$$

$\therefore$ Clearly, ar(ABD) = ar(ADC).

Thus, median AD divides ΔABC into 2 triangles of equal areas.

Question 50.

The mid-point P of the line segment joining the points A(- 10, 4) and B(- 2, 0) lies on the line segment joining the points C(- 9, – 4) and D(- 4, y). Find the ratio in which P divides CD. Also, find the value of y.

Solution:

$\because$ P is the mid-point of the line segment joining A(–10, 4) and B(–2, 0).

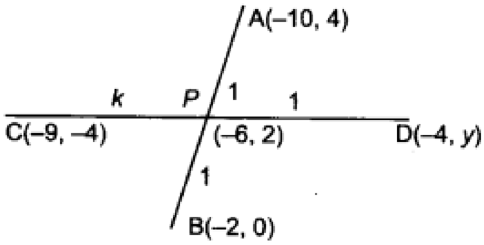
$\therefore$ The coordinates of P are $\left(\frac{-10-2}{2}, \frac{4+0}{2}\right)$, i.e. P(–6, 2). ...(*i*)

Let P(–6, 2) divides the join of C(–9, –4) and D(–4, y) in the ratio $k : 1$. Using section formula,

$\therefore$ The coordinates of P are $\left(\frac{-4k-9}{k+1}, \frac{ky-4}{k+1}\right)$ ...(*ii*)

$\therefore$ From (*i*), (*ii*)

A.T.Q. $\frac{-4k-9}{k+1} = -6$ and $\frac{ky-4}{k+1} = 2$ ...(*iii*)



Consider, $\frac{-4k-9}{k+1} = -6$

$\Rightarrow$ $-4k-9 = -6k-6$

$\Rightarrow$ $2k = 3$

$\Rightarrow$ $k = \frac{3}{2}$

So, P divides CD in the ratio 3 : 2

From (*iii*), $\frac{\frac{3}{2}y-4}{\frac{3}{2}+1} = 2$

$\Rightarrow$ $\frac{3y-8}{3+2} = 2 \Rightarrow 3y-8 = 10$

$\Rightarrow$ $3y = 18 \Rightarrow y = 6$

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Short Answer Type Questions II [3 Marks]

Question 51.

Prove that the points (7,10), (-2,5) and (3, -4) are the vertices of an isosceles right triangle.

Solution:

Let A (7, 10); B(–2, 5); C(3, –4) be vertices of isosceles right triangle.

Now, using distance formula,

$$AB = \sqrt{(7+2)^2+(10-5)^2} = \sqrt{81+25} = \sqrt{106} \text{ units}$$

$$BC = \sqrt{(-2-3)^2(5+4)^2} = \sqrt{25+81} = \sqrt{106} \text{ units}$$

$$CA = \sqrt{(3-7)^2+(-4-10)^2} = \sqrt{16+196} = \sqrt{212} \text{ units}$$

Clearly,

$\therefore \quad (\sqrt{212})^2 = (\sqrt{106})^2 + (\sqrt{106})^2$

$\Rightarrow \quad AC^2 = AB^2 + BC^2$

$\Rightarrow \quad \angle ABC = 90°$ [$\because$ Follows converse of Pythagoras theorem]

Here, AB = BC and $\angle ABC = 90°$

So, ΔABC is an isosceles right triangle.

Question 52.

Find, the ratio in which the y-axis divides the line segment joining the points (-4, -6) and (10,12). Also, find the coordinates of the point of division.

Solution:

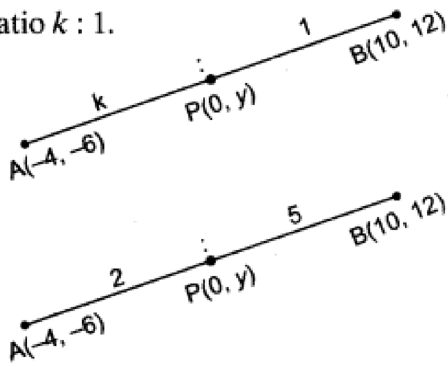
Let point P (0, y) which lies on y-axis divides AB in the ratio $k : 1$.

Then by section formula, in x-co-ordinates,

$$\frac{10k-4}{k+1} = 0 \quad \Rightarrow 10k - 4 = 0$$

$$\Rightarrow \quad k = \frac{4}{10} = \frac{2}{5}$$

Hence, point P divides AB in the ratio 2 : 5



$$\text{Now, } y = \frac{2 \times 12 - 5 \times (-6)}{2+5} = \frac{24-30}{7} = -\frac{6}{7}.$$

[Applying section formula for y-coordinates]

Hence, Coordinates of P are $\left(0, \frac{-6}{7}\right)$.

Question 53.

Prove that the points A(0, -1), B(-2, 3), C(6, 7) and D(8, 3) are the vertices of a rectangle ABCD?

Solution:

Using distance formula,

$$AB = \sqrt{(-2-0)^2+(3+1)^2} = \sqrt{4+16} = \sqrt{20} \text{ units}$$

$$BC = \sqrt{(6+2)^2+(7-3)^2} = \sqrt{64+16} = \sqrt{80} \text{ units}$$

$$CD = \sqrt{(8-6)^2+(3-7)^2} = \sqrt{4+16} = \sqrt{20} \text{ units}$$

$$DA = \sqrt{(8-0)^2+(3+1)^2} = \sqrt{64+16} = \sqrt{80} \text{ units}$$

Also $AC = \sqrt{(6-0)^2+(7+1)^2} = \sqrt{36+64} = \sqrt{100} = 10$ units

and $BD = \sqrt{(8+2)^2+(3-3)^2} = \sqrt{100+0} = 10$ units

Since, $AB = CD$ and $BC = DA$

and diagonal $AC = BD$

In quadrilateral ABCD, opposite sides are equal and both the diagonals are equal. Therefore, ABCD is a rectangle.

Question 54.
Show that the points (-2,3) (8,3) and (6, 7) are the vertices of a right triangle.
Solution:

Using distance formula,

Consider A(–2, 3), B(8, 3) and C(6, 7)

Now, $AB^2 = (8+2)^2 + (3-3)^2 = 100$ units

$BC^2 = (6-8)^2 + (7-3)^2 = 4 + 16 = 20$ units

$AC^2 = (6+2)^2 + (7-3)^2 = 64 + 16 = 80$ units

Clearly, $100 = 20 + 80$

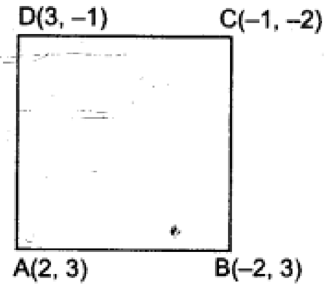
$\Rightarrow AB^2 = BC^2 + AB^2$

So, by converse of Pythagoras theorem, ΔABC is a right triangle.

Question 55.
Prove that the points A(2, 3), B(-2, 2), C(-l, -2) and D(3, -1) are the vertices of a square ABCD.
Solution:

Using distance formula,

$$AB = \sqrt{(-2-2)^2+(2-3)^2} = \sqrt{(-4)^2+(-1)^2}$$
$$= \sqrt{16+1} = \sqrt{17} \text{ units}$$
$$BC = \sqrt{(-2+1)^2+(2+2)^2} = \sqrt{(-1)^2+(4)^2}$$
$$= \sqrt{1+16} = \sqrt{17} \text{ units}$$
$$CD = \sqrt{(-1-3)^2+(-2+1)^2} = \sqrt{(-4)^2+(-1)^2}$$
$$= \sqrt{16+1} = \sqrt{17} \text{ units}$$
$$DA = \sqrt{(3-2)^2+(-1-3)^2} = \sqrt{(1)^2+(-4)^2}$$
$$= \sqrt{1+16} = \sqrt{17} \text{ units}$$



Also, $$AC = \sqrt{(-1-2)^2+(-2-3)^2} = \sqrt{(-3)^2+(-5)^2}$$
$$= \sqrt{9+25} = \sqrt{34} \text{ units}$$
$$BD = \sqrt{(-2-3)^2+(2+1)^2} = \sqrt{(-5)^2+(3)^2}$$
$$= \sqrt{25+9} = \sqrt{34} \text{ units}$$

Since, AB = BC = CD = DA and AC = BD

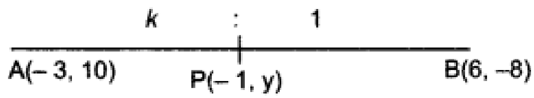
$\Rightarrow$ ABCD is a square, because all sides are equal. Diagonals are also equal.

Question 56.

Find the ratio in which point P(-l, y) lying on the line segment joining points A(-3,10) and B(6, -8) divides it. Also, find the value of y.

Solution:

Let point $P(-1, y)$ divides AB in ratio $k : 1$.



Using sections formula for x-coordinates;

$$\therefore \quad -1 = \frac{6k-3}{k+1}$$
$$-k-1 = 6k-3$$
$$-7k = -2$$
$$k = \frac{2}{7}$$

Hence point P divides AB in the ratio 2 : 7.

Now, again using section formula for y-coordinates,

$$y = \frac{-8k+10}{k+1}$$
$$\Rightarrow \quad y = \frac{-8\left(\frac{2}{7}\right)+10}{\frac{2}{7}+1} = \frac{\frac{-16+70}{7}}{\frac{2+7}{7}} = \frac{70-16}{2+7} = \frac{54}{9} = 6$$

$\therefore$ Coordinates of P are $(-1, 6)$.

Question 57.
Prove that the points A(2, -1), B(3, 4), C(-2, 3) and D(-3, -2) are the vertices of a rhombus ABCD. Is ABCD a square?
Solution:

Using distance formula,

$$AB = \sqrt{(3-2)^2+(4+1)^2} = \sqrt{(1)^2+(5)^2} = \sqrt{1+25} = \sqrt{26} \text{ units}$$

$$BC = \sqrt{(3+2)^2+(4-3)^2} = \sqrt{(5)^2+(1)^2} = \sqrt{25+1} = \sqrt{26} \text{ units}$$

$$CD = \sqrt{(-2+3)^2+(3+2)^2} = \sqrt{(1)^2+(5)^2} = \sqrt{25+1} = \sqrt{26} \text{ units}$$

$$DA = \sqrt{(-3-2)^2+(-2+1)^2} = \sqrt{(-5)^2+(-1)^2} = \sqrt{25+1} = \sqrt{26} \text{ units}$$

Also,

$$AC = \sqrt{(-2-2)^2+(3+1)^2} = \sqrt{(-4)^2+(4)^2} = \sqrt{16+16} = \sqrt{32} \text{ units}$$

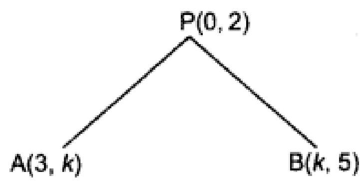
$$BD = \sqrt{(3+3)^2+(4+2)^2} = \sqrt{(6)^2+(6)^2} = \sqrt{36+36} = \sqrt{72} \text{ units}$$

$\because$ AB = BC = CD = DA. But AC $\neq$ BD

$\Rightarrow$ ABCD is a rhombus, not a square.

Question 58.
Find that value of k for which the point (0, 2) is equidistant from two points (3, k) and (k, 5).
Solution:

Consider point P(0, 2) is equidistant from A(3, k) and B(k, 5)



Given that, PA = PB

Using distance formula,

$$\sqrt{(0-3)^2+(2-k)^2} = \sqrt{(k-0)^2+(5-2)^2}$$

$$\sqrt{9+4+k^2-4k} = \sqrt{k^2+9}$$

Squaring both sides

$$k^2-4k+13 = k^2+9$$

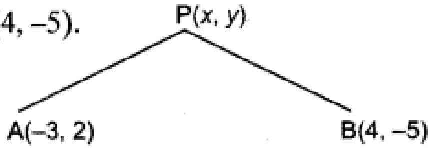
$$k^2-4k+13-9-k^2 = 0$$

$$-4k = -4 \quad \Rightarrow k = 1$$

Question 59.
If the point P(x,y) is equidistant from two points A (-3,2) and B (4, -5), prove that y = x- 2.

Solution:

Point $P(x, y)$ is equidistant from the points $A(-3, 2)$ and $B(4, -5)$.



$\therefore \quad PA = PB$ [Given]

Using distance formula,

$$\Rightarrow \quad \sqrt{(-3-x)^2+(2-y)^2} = \sqrt{(4-x)^2+(-5-y)^2}$$

Squaring both sides, we get

$$9 + x^2 + 6x + 4 + y^2 - 4y = 16 + x^2 - 8x + 25 + y^2 + 10y$$

$$\Rightarrow \quad 6x - 4y + 13 = -8x + 10y + 41$$

$$\Rightarrow \quad -4y - 10y = 41 - 13 - 8x - 6x$$

$$\Rightarrow \quad -14y = -14x + 28$$

$$\Rightarrow \quad -14y = -14(x-2)$$

$$\Rightarrow \quad y = x - 2$$

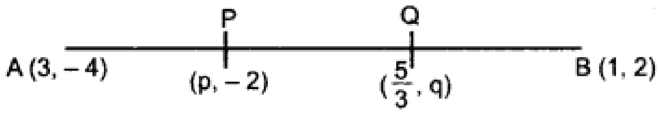
Question 60.

The line segment AB joining the points A(3, – 4), and B(l, 2) is trisected at the points P(p, – 2) and Q(5/3, q). Find the values of p and q.

Solution:

Now, again $AP : PB = 1 : 2$. Using distance formula,

$$\therefore \quad p = \frac{1\times 1+2\times 3}{1+2}$$



$$\Rightarrow \quad p = \frac{7}{3}$$

Also, $\quad AQ : QB = 2 : 1$. Again using section formula,

$$\Rightarrow \quad q = \frac{2\times 2+1\times -4}{1+2} = 0 \quad \Rightarrow \quad q = 0$$

Question 61.

If point A (x, y) is equidistant from two points P (6, -1) and Q (2,3), prove that y = x – 3.

Solution:

Point $A(x, y)$ is equidistant from $P(6, -1)$ and $Q(2, 3)$. Using distance formula,

$$PA = AQ$$

$$\Rightarrow \quad \sqrt{(6-x)^2+(-1-y)^2} = \sqrt{(2-x)^2+(3-y)^2}$$

Squaring both sides, we get

$$(6-x)^2 + (-1-y)^2 = (2-x)^2 + (3-y)^2$$

$$\Rightarrow \quad 36 + x^2 - 12x + 1 + y^2 + 2y = 4 + x^2 - 4x + 9 + y^2 - 6y$$

$$\Rightarrow \quad -12x + 2y + 37 = -4x - 6y + 13$$

$$\Rightarrow \quad 2y + 6y = 13 - 4x + 12x - 37$$

$$\Rightarrow \quad 8y = 8x - 24$$

$$\Rightarrow \quad y = x - 3$$

Hence, proved.

Question 62.
If the point R (x, y) is equidistant from two points P (- 3, 4) and Q (2, – 1), prove that y = x + 2.
Solution:
Point $R(x, y)$ is equidistant from the points $P(-3, 4)$ and $Q(2, -1)$. Using distance formula,

$$\begin{aligned} \therefore \quad PR &= RQ \\ \sqrt{(x+3)^2+(y-4)^2} &= \sqrt{(x-2)^2+(y+1)^2} \end{aligned}$$

Squaring both sides, we get

$$\begin{aligned} (x+3)^2+(y-4)^2 &= (x-2)^2+(y+1)^2 \\ \Rightarrow \quad x^2+y^2+6x-8y+9+16 &= x^2+y^2-4x+2y+4+1 \\ \Rightarrow \quad 6x-8y+25 &= -4x+2y+5 \\ \Rightarrow \quad -8y-2y &= -4x-6x+5-25 \\ \Rightarrow \quad -10y &= -10x-20 \\ \Rightarrow \quad -10y &= -10(x+2) \\ \Rightarrow \quad y &= x+2 \end{aligned}$$

Hence, proved.

Long Answer Type Questions [4 Marks]

Question 63.
If the area of AABC formed by A(x, y), B(l, 2) and C(2, 1) is 6 square units, then prove that x +y= 15.
Solution:
Point $R(x, y)$ is equidistant from the points $P(-3, 4)$ and $Q(2, -1)$. Using distance formula,

$$\begin{aligned} \therefore \quad PR &= RQ \\ \sqrt{(x+3)^2+(y-4)^2} &= \sqrt{(x-2)^2+(y+1)^2} \end{aligned}$$

Squaring both sides, we get

$$\begin{aligned} (x+3)^2+(y-4)^2 &= (x-2)^2+(y+1)^2 \\ \Rightarrow \quad x^2+y^2+6x-8y+9+16 &= x^2+y^2-4x+2y+4+1 \\ \Rightarrow \quad 6x-8y+25 &= -4x+2y+5 \\ \Rightarrow \quad -8y-2y &= -4x-6x+5-25 \\ \Rightarrow \quad -10y &= -10x-20 \\ \Rightarrow \quad -10y &= -10(x+2) \\ \Rightarrow \quad y &= x+2 \end{aligned}$$

Hence, proved.

Question 64.
Find the value of x for which the points (x – 1), (2,1) and (4,5) are collinear

Solution:

Since the point A(x, –1), B(2, 1) and C(4, 5) are collinear

So, area of triangle, ar(ΔABC) = 0

$$\Rightarrow \quad \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}\,|x(1-5)+2(5+1)+4(-1,-1)| = 0$$

$$\Rightarrow \quad |-4x+12-8| = 0$$

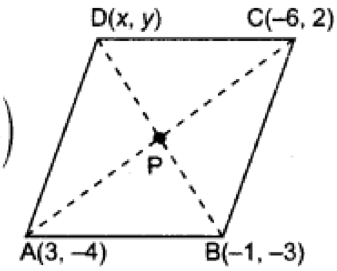
$$\Rightarrow \quad |-4x+4| = 0 \quad \Rightarrow -4x+4=0 \Rightarrow 4x=4$$

$$\Rightarrow \quad x = 1$$

Question 65.

The three vertices of a parallelogram ABCD are A(3, -4), B(-l, -3) and C(-6, 2). Find the coordinates of vertex D and find the area of parallelogram ABCD.

Solution:



Given ABCD is a parallelogram

Let coordinates of point D be (x, y).

The coordinates of mid-point of AC $= \left(\frac{-6+3}{2}, \frac{2-4}{2}\right) = \left(\frac{-3}{2}, -1\right)$

The coordinates of mid-point of BD $= \left(\frac{x-1}{2}, \frac{y-3}{2}\right)$

Since, diagonals of a parallelogram bisect each other, so, P is the mid-point of AC as well as BD.

$$\Rightarrow \quad \left(\frac{x-1}{2}, \frac{y-3}{2}\right) = \left(\frac{-3}{2}, -1\right)$$

Comparing both sides, we get

$$\Rightarrow \quad \frac{x-1}{2} = \frac{-3}{2} \text{ and } \frac{y-3}{2} = -1$$

$$\Rightarrow \quad x-1 = -3 \text{ and } y-3 = -2$$

$$\Rightarrow \quad x = -2 \text{ and } y = 1$$

Therefore, coordinates of D are (– 2, 1).

Now,
$$\text{ar}(\Delta ABC) = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

$$= \frac{1}{2}\,|3(-3-2)-1(2+4)-6(-4+3)|$$

$$= \frac{1}{2}\,|-15-6+6| = \frac{15}{2} \text{ sq. units}$$

$$\text{Area of parallelogram(ABCD)} = 2\text{ ar }(\Delta ABC)$$

$$= 2\times\frac{15}{2} = 15 \text{ sq. units}$$

Question 66.

If the points A(l, -2), B(2,3), C(-3,2) and D(-4, -3) are the vertices of parallelogram

ABCD, then taking AB as the base, find the height of this parallelogram
Solution:

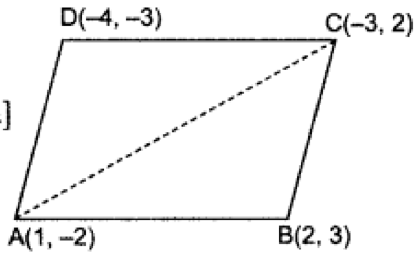
Using distance formula,

$$AB = \sqrt{(2-1)^2 + \{3-(-2)\}^2} = \sqrt{26} \text{ units}$$

$$\text{Area } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}|1(3-2) + 2\{2-(-2)\} + (-3)(-2-3)|$$

$$= \frac{1}{2} \times 24 = 12 \text{ sq. units}$$



$$\therefore \quad \text{Area of } ||\text{gm } ABCD = 2 \times \text{area of } \Delta ABC$$

$$= 2 \times 12 = 24 \text{ sq. units}$$

$$\text{Now, area } ||\text{gm } ABCD = \text{Base} \times h \quad [\because \text{By formula}]$$

$$= AB \times h$$

$$\Rightarrow \quad AB \times h = 24$$

$$\Rightarrow \quad h = \frac{24}{\sqrt{26}}$$

$$\Rightarrow \quad h = \frac{24}{26} \times \sqrt{26}$$

$$\Rightarrow \text{Height of parallelogram} = \frac{12}{13}\sqrt{26} \text{ units}$$

Question 67.
For the AABC formed by the points A(4, -6), B(3, -2) and C(5,2), verify that the median divides the triangle into two triangles of equal area.
Solution:

Consider AD is median of ΔABC.

Here D is mid point of BC as AD is median

$$\therefore \quad \text{Coordinate of D are } x = \frac{3+5}{2} = 4$$

$$y = \frac{-2+2}{2} = 0$$

Coordinates of D are (4, 0). Now

B(3,

$$\text{Area of } \Delta ABD = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

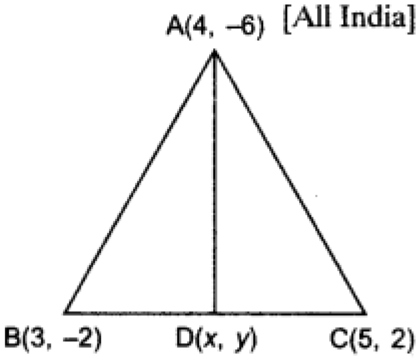
$$= \frac{1}{2}|4(0+2) + 4(-2+6) + 3(-6-0)|$$

$$= \frac{1}{2}|8 + 16 - 18| = \left|\frac{6}{2}\right| = 3 \text{ sq. units}$$

Now, Area of $\Delta ACD = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2}|4(0-2) + 4(2+6) + 5(-6-0)|$$

$$= \frac{1}{2}|-8 - 32 - 30| = \frac{1}{2}|32 - 38| = \frac{1}{2}|-6| = 3 \text{ sq. units}$$



Clearly, ar ΔABD = ar ΔACD.

Thus, median AD divides triangle in q triangles of equal area.

Question 68.
Find the area of a parallelogram ABCD if three of its vertices are A(2, 4), B(2 + √3,5) and C(2, 6).

Solution:

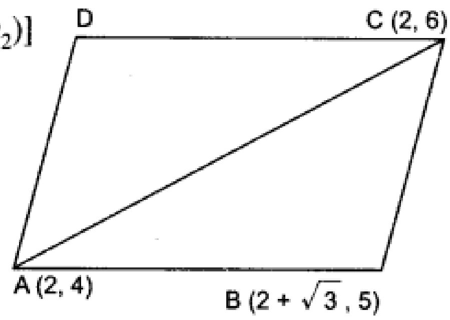
ABCD is a parallelogram, A(2, 4), B($2 + \sqrt{3}$, 5), C(2, 6) form the vertices of ΔABC.

$\therefore$ Area of $\Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2}|2(5-6) + (2+\sqrt{3})(6-4) + 2(4-5)|$$

$$= \frac{1}{2}|(-2 + 4 + 2\sqrt{3} - 2|$$

$$= \frac{1}{2}|2\sqrt{3}| = \sqrt{3} \text{ sq. units.}$$



Diagonal AC divides the parallelogram in two triangles of equal area.

$\therefore$ Area of parallelogram ABCD = 2(Area of ΔABC) = $2(\sqrt{3}) = 2\sqrt{3}$ sq. units.

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Question 69.
If the area of the triangle formed by points A (x,y), B (1,2) and C (2,1) is 6 square units, then show that x + y = 15.

Solution:

The points are A(x, y), B(1, 2) and C(2, 1)

Area of $\Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 6$ sq. units [Given]

$$\Rightarrow \quad \frac{1}{2}|x(2-1) + 1(1-y) + 2(y-2)| = 6$$

$$\Rightarrow \quad |x + 1 - y + 2y - 4| = 12$$

$$\Rightarrow \quad |x + y - 3| = 12$$

$$\Rightarrow \quad x + y - 3 = 12$$

$$\Rightarrow \quad x + y = 15$$

Hence, proved.

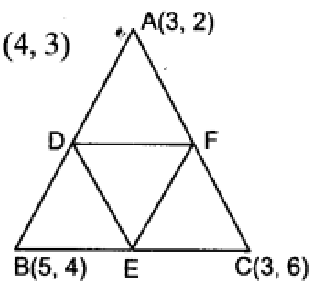
Question 70.

Find the area of the triangle formed by joining the mid-points of the sides of a triangle whose vertices are (3,2), (5,4) and (3, 6).

Solution:

Consider, the points are A(3, 2), B(5, 4) and C(3, 6) form the vertices of ΔABC.

Let D, E and F be the mid-points of the sides AB, BC and AC respectively of the triangle ABC.



$\therefore$ Coordinates of D are $\left(\frac{3+5}{2}, \frac{2+4}{2}\right)$, i.e. the coordinates of D are (4, 3)

Coordinates of E are $\left(\frac{5+3}{2}, \frac{4+6}{2}\right)$

$\Rightarrow$ Coordinates of E = (4, 5)

Coordinates of F are $\left(\frac{3+3}{2}, \frac{6+2}{2}\right)$

$\Rightarrow$ Coordinates of F are (3, 4).

Coordinates of D(4, 3), E(4, 5), F(3, 4)

Now, Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of $\Delta DEF = \frac{1}{2}|4(5-4) + 4(4-3) + 3(3-5)| = \frac{2}{2} = 1$ sq. unit

Question 71.

If the area of the triangle formed by joining the points A (x, y), B (3, 2) and C (- 2, 4) is 10 square units, then show that 2x + 5y + 4 = 0.

Solution:

We know that,

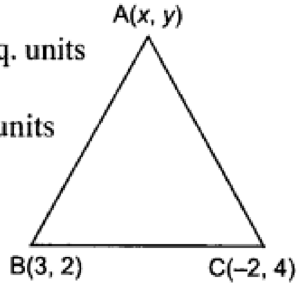
$$\text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 10 \text{ sq. units}$$

$$= \frac{1}{2}|x(2-4) + (3)(4-y) - 2(y-2)| = 10 \text{ sq. units}$$

$$20 = (-2x - 5y + 16)$$

$$-2x - 5y + 16 - 20 = 0$$

$$\Rightarrow \quad 2x + 5y + 4 = 0 \quad \text{Hence proved}$$



2010

Very Short Answer Type Questions [1 Mark]

Question 72.
If a point A(0, 2) is equidistant from the points B(3, p) and C(p, 5), then find the value of p.

Solution:

Given points are A(0, 2), B(3,p), C(p, 5)

According to question,

$$AB = AC$$

Using distance formula

$$\sqrt{(3-0)^2 + (p-2)^2} = \sqrt{(p-0)^2 + (5-2)^2}$$

Squaring both sides

$$9 + p^2 + 4 - 4p = p^2 + 9$$

$$4 - 4p = 0$$

$$p = 1$$

Question 73.
Find the value of k, if the point P(2,4) is equidistant from the points A(5, k) and B(k, 7).

Solution:

Now, given that, $AP = PB$. Using distance formula,

$$\Rightarrow \quad \sqrt{(5-2)^2 + (k-4)^2} = \sqrt{(k-2)^2 + (7-4)^2}$$

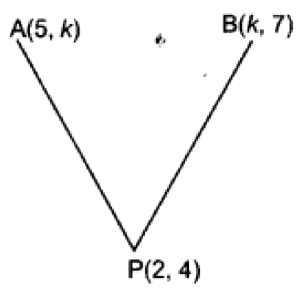
Squaring both sides we get

$$\Rightarrow \quad (3)^2 + (k-4)^2 = (k-2)^2 + (3)^2$$

$$\Rightarrow \quad 9 + k^2 - 8k + 16 = k^2 - 4k + 4 + 9$$

$$\Rightarrow \quad -4k = -12$$

$$\Rightarrow \quad k = 3$$



Question 74.
Find the ratio in which the line segment joining the points (1,-3) and (4,5) is divided by the x-axis.

Solution:

Let the ratio in which the line segment joining (1, –3) and (4, 5) is divided by x-axis be $k : 1$

Therefore, the coordinates of the point of division is $\left(\frac{4k+1}{k+1}, \frac{5k-3}{k+1}\right)$

[$\because$ Using section formula]

We know that y-coordinate of any point on x-axis is 0.

$$\therefore \quad \frac{5k-3}{k+1} = 0$$

$$5k - 3 = 0$$

$$5k = 3$$

$$k = \frac{3}{5}$$

Ratio = $k : 1 = 3 : 5$

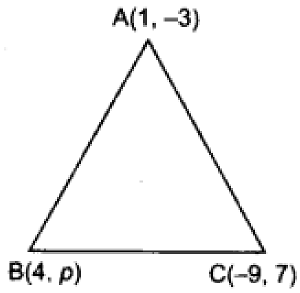
Short Answer Type Questions II [3 Marks]

Question 75.

If the vertices of a triangle are (1, – 3) (4,p) and (-9,7) and its area is 15 sq. units, find the value(s) of p.

Solution:

Let the triangle be ABC with vertices A(1, –3), B(4, p), C(–9, 7).



Area of ΔABC = 15 sq. units.

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 15$$

$$\frac{1}{2}[1(p-7) + 4(7+3) - 9(-3-p)] = 15$$

$$p - 7 + 40 + 27 + 9p = 30$$

$$p = -3.$$

Question 76.

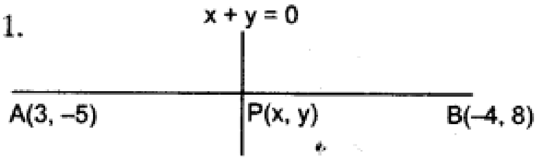
A point P divides the line segment joining the points A(3, -5) and B(-4, 8) such AP/PB=k/1. If P lies on the line x + y = 0, then find the value of K.

Solution:

AB is a line with A(3, –5) and B(–4, 8).

P(x, y) is any point on AB such that AP : PB = K : 1.

Using section formula.



$$(x, y) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

Here, $x_1 = 3,\ \ x_2 = -4,\ \ y_1 = -5, y_2 = 8,\ m_1 = K,\ m_2 = 1.$

$$(x, y) = \left(\frac{K(-4) + 1(3)}{K + 1}, \frac{K(8) + 1(-5)}{K + 1}\right)$$

$$(x, y) = \left(\frac{-4K + 3}{K + 1}, \frac{8K - 5}{K + 1}\right)$$

On equating the coordinates both sides, we get

$$x = \frac{-4K + 3}{K + 1}, \quad y = \frac{8K - 5}{K + 1}$$

Given that, $x + y = 0$

$$\frac{-4K + 3}{K + 1} + \frac{8K - 5}{K + 1} = 0$$

$$\frac{-4K + 3 + 8K - 5}{K + 1} = 0$$

$$4K - 2 = 0$$

$$4K = 2$$

$$K = \frac{2}{4} = \frac{1}{2}$$

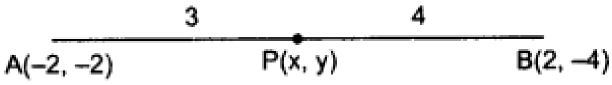
$$K = \frac{1}{2}.$$

Question 77.

Find the coordinates of a point P, which lies on the line segment joining the points A(-2, -2) and B(2, -4) such that AP =3/7 AB.

Solution:

Given that, $AP = \frac{3}{7} AB \Rightarrow \frac{AP}{AB} = \frac{3}{7} \Rightarrow AP : PB = 3 : 4$



Using section formula,

$$x = \frac{3(2)+4(-2)}{3+4} = \frac{6-8}{7} = -\frac{2}{7}$$

$$y = \frac{3(-4)+4(-2)}{3+4} = \frac{-12-8}{7} = \frac{-20}{7}$$

Coordinates of P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$

Question 78.

Find the area of the quadrilateral ABCD whose vertices are A(-3, -1), B(-2, -4), C(4, –1) and D(3, 4).

Solution:

Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

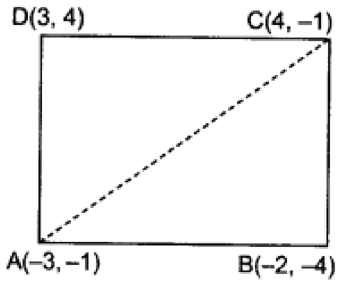
where area of triangle is $\frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

$$= \frac{1}{2}[-3(-4+1) + (-2)(-1+1) + 4(-1+4)] + \frac{1}{2}[-3(-1-4) + 4(4+1) + 3(-1+1)]$$

$$= \frac{1}{2}[-3(-3) + (-2)(0) + 4(3)] + \frac{1}{2}[-3(-5) + 4(5) + 3(0)]$$

$$= \frac{1}{2}[9 + 0 + 12] + \frac{1}{2}[15 + 20] = \frac{1}{2} \times 21 + \frac{1}{2} \times 35$$

$$= \frac{1}{2} \times 56 = 28 \text{ Sq. units.}$$



Question 79.

If the points A(x, y), B(3, 6) and C(-3,4) are collinear, show that x – 3y + 15 = 0.

Solution:

If A, B and C are collinear then area of $\Delta ABC = 0$

$$\Rightarrow \frac{1}{2}\,|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$$

$$\Rightarrow \frac{1}{2}[x(6-4)+3(4-y)+(-3)(y-6)]=0$$

$$\Rightarrow \quad 2x+12-3y-3y+18=0$$

$$\Rightarrow \quad 2x-6y+30=0$$

$$\Rightarrow \quad x-3y+15=0$$

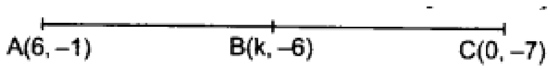
Hence, proved.

Question 80.

Find the value of£, for which the points A(6, -1), B(& – 6) and C(0, -7) are collinear.

Solution:

Given points are $A(6,-1)$, $B(k,-6)$, $C(0,-7)$.



As A, B, C are collinear points, so, $Ar\ \Delta ABC = 0$

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$$

i.e., $$\frac{1}{2}|6(-6+7)+k(-7+1)+0(-1+6)|=0$$

$$\frac{1}{2}|6-6k+0|=0$$

$$\frac{1}{2}|6-6k|=0$$

$$|6-6k|=0$$

$$6-6k=0$$

$$6(1-k)=0$$

$$1-k=0$$

$$\therefore \quad k=1.$$

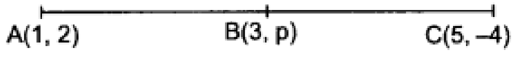
Question 81.

Find the value, if the points A(l, 2), B(3,p) and C(5, -4) are collinear.

Solution:

If three points are collinear, then area bounded by them is zero.

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]=0$$



i.e., $$\frac{1}{2}|1(p+4)+3(-4-2)+5(2-p)|=0$$

$$\frac{1}{2}|p+4-18+10-5p|=0$$

$$|-4p-4|=0$$

$$4p+4=0$$

$$p=-1$$

Question 82.
Find the area of the triangle whose vertices are (-7, -3), (1, -7) and (3,0).
Solution:

The area of the $\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

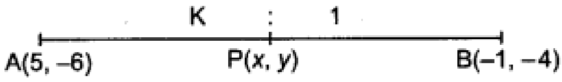
Here,
$$x_1 = -7, \quad x_2 = 1, \quad x_3 = 3$$
$$y_1 = -3, \quad y_2 = -7, \ y_3 = 0$$

$$\text{Area of the triangle} = \frac{1}{2}[-7(-7-0) + 1\{0-(-3)\} + 3\{-3-(-7)\}]$$
$$= \frac{1}{2}[(-7)(-7) + (1)(3) + 3(-3+7)]$$
$$= \frac{1}{2}[49 + 3 + (3 \times 4)]$$
$$= \frac{1}{2}[49 + 3 + 12] = \frac{1}{2} \times 64 = 32 \text{ sq. units.}$$

Question 83.
Find the ratio in which the y-axis divides the line segment joining the points (5, -6) and (-1, -4). Also, find the coordinates of the point of intersection.
Solution:

Let the ratio be K : 1. Then by the section formula, the coordinates of the point which divides the line segment in the ratio K : 1 are $\left(\frac{-K+5}{K+1}, \frac{-4K-6}{K+1}\right)$ [$\because$ Using section formula]

This point lies on the y-axis and we know that on the y-axis, x is 0.



$$\therefore \quad \frac{-K+5}{K+1} = 0$$
$$-K + 5 = 0$$
$$K = 5$$

The ratio is 5 : 1.

On putting the value of K = 5, we get the point of intersection $\left(\frac{-5+5}{5+1}, \frac{-4\times5-6}{5+1}\right)$

$$\Rightarrow \quad \left(0, -\frac{26}{6}\right)$$
$$\Rightarrow \quad \left(0, \frac{-13}{3}\right)$$

$\therefore$ coordinates of point of intersection are $\left(0, \frac{-13}{3}\right)$
>

Question 84.
Find the value of y for which the points (5, -4), (3, -1) and (1, y) are collinear.

Solution:

The points are collinear so area of triangle = 0

So, Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Here,

$$x_1 = 5, \quad x_2 = 3, \quad x_3 = 1$$
$$y_1 = -4, \quad y_2 = -1, \quad y_3 = y$$

$$\text{Area of triangle} = \frac{1}{2}[5(-1-y) + 3(y+4) + 1(-4+1)]$$
$$= \frac{1}{2}[-5 - 5y + 3y + 12 - 3]$$
$$= \frac{1}{2}[-5y + 3y - 5 + 9]$$
$$= \frac{1}{2}[-2y + 4] = \frac{1}{2} \times 2(-y + 2) = (-y + 2)$$

As per condition, area of triangle must be zero.

$$-y + 2 = 0$$
$$-y = -2$$
$$y = 2$$

Question 85.

For what value of k, (k > 0), is the area of the triangle with vertices (-2, 5), (k, -4) and {2k + 1,10) equal to 53 sq. units?

Solution:

Vertices of the triangle are $(-2, 5)$, $(k, -4)$ and $(2k + 1, 10)$.

Then, Ar(Triangle) = 53 sq. units

$$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 53$$
$$\Rightarrow \frac{1}{2}|-2(-4-10) + k(10-5) + (2k+1)(5-(-4))| = 53$$
$$\Rightarrow |28 + 5k + 18k + 9| = 106$$
$$\Rightarrow |23k + 37| = 106 \Rightarrow 23k + 37 = \pm 106$$
$$\Rightarrow 23k + 37 = 106 \text{ or } 23k + 37 = -106$$
$$\Rightarrow 23k = 69 \text{ or } 23k = -143$$
$$k = 3 \text{ or } k = \frac{-143}{23}$$

$\because$ Given, $k > 0$ $\therefore$ $k = 3$

2011

Short Answer Type Questions I [2 Marks]

Question 86.
Find that value(s) of x for which the distance between the points P(JC, 4) and Q(9,10) is 10 units.
Solution:

Given that, $\quad PQ = 10$ units

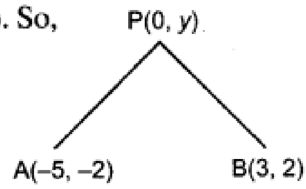
$\Rightarrow \quad PQ^2 = 100$

Using distance formula,

$$\Rightarrow \quad (x-9)^2 + (4-10)^2 = 100$$
$$\Rightarrow \quad (x-9)^2 + 36 = 100$$
$$\Rightarrow \quad (x-9)^2 = 64$$
$$\Rightarrow \quad x-9 = \pm 8$$
$$\Rightarrow \quad x-9 = 8 \quad \text{or} \quad x-9 = -8$$
$$\Rightarrow \quad x = 17 \quad \text{or} \quad x = 1.$$

Question 87.
Find the point -axis which is equidistant from the points (-5, -2) and (3, 2).
Solution:

Let point $P(0, y)$ on y-axis be equidistant from $A(-5, -2)$ and $B(3, 2)$. So,



$$PA = PB$$
$$\Rightarrow \quad PA^2 = PB^2$$

Using distance formula,

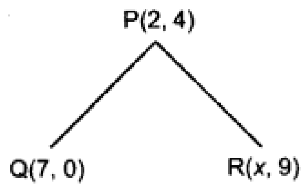
$$\Rightarrow \quad 5^2 + (y+2)^2 = (-3)^2 + (y-2)^2$$
$$\Rightarrow \quad 25 + y^2 + 4y + 4 = 9 + y^2 - 4y + 4$$
$$\Rightarrow \quad 8y = -16 \quad \Rightarrow \quad y = -2$$

Hence, required points is $(0, -2)$

Question 88.
If P(2,4) is equidistant from Q(7, 0) and R(x, 9), find the values of x. Also, find the distance PQ.
Solution:

Given: $\quad PQ = PR$

$\Rightarrow \quad PQ^2 = PR^2$



Using distance formula,

$$\Rightarrow \quad (2-7)^2 + 4^2 = (2-x)^2 + (4-9)^2$$
$$\Rightarrow \quad 25 + 16 = (2-x)^2 + 25$$
$$\Rightarrow \quad (2-x)^2 = 16$$
$$\Rightarrow \quad 2-x = \pm 4$$
$$\Rightarrow \quad 2-x = 4 \quad \text{or} \quad 2-x = -4$$
$$\Rightarrow \quad x = -2 \qquad x = 6$$

$$\text{Distance, } PQ = \sqrt{(7-2)^2 + (0-4)^2}$$
$$= \sqrt{25+16} = \sqrt{41} \text{ units}$$

Question 89.
Find the value of k, if the points P(5,4), Q(7, k) and R(9, -2) are collinear
Solution:

Since points P, Q, R are collinear.

$$\text{So,} \quad \text{area of } \Delta PQR = 0$$

$$\Rightarrow \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

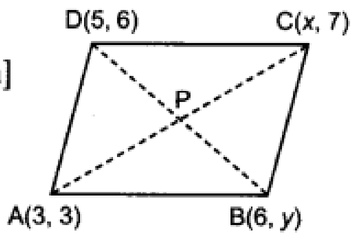
$$\Rightarrow \quad \frac{1}{2}[5(k+2) + 7(-2-4) + 9(4-k)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[5k + 10 - 42 + 36 - 9k] = 0$$

$$\Rightarrow \quad -4k + 4 = 0$$

$$\Rightarrow \quad 4k = 4 \Rightarrow k = 1$$

Question 90.
If (3, 3), (6, y), (x, 7) and (5, 6) are the vertices of a parallelogram taken in order, find the values of x and y.
Solution:



ABCD is a ||gm, [Given]

P is the mid-point of AC and BD. [$\because$ Property of parallelogram]

Taking AC, Coordinates of point P is $\left(\frac{x+3}{2}, 5\right)$ [$\because$ Using mid-point formula]

Taking BD, Also, coordinates of point P is $\left(\frac{11}{2}, \frac{y+6}{2}\right)$ [$\because$ Using mid-point formula]

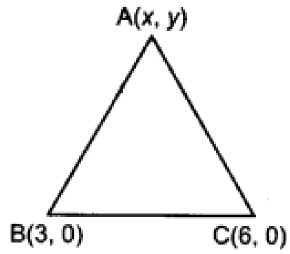
A.T.Q $\quad \frac{x+3}{2} = \frac{11}{2}$ and $\frac{y+6}{2} = 5$ [$\because$ As P is mid-point of AC & BD]

$x + 3 = 11 \qquad y + 6 = 10$

$x = 8 \qquad y = 4$

Question 91.
If two vertices of an equilateral triangle are (3,0) and (6,0), find the third vertex.
Solution:

Let ABC be an equilateral Δ.



Let coordinates of points A, B and C are (x, y), $(3, 0)$ and $(6, 0)$ respectively.

Now $\quad AB = AC = BC$

[∵ In equilateral Δ, all sides are equal]

$\Rightarrow \quad AB^2 = AC^2 = BC^2$

Using distance formula,

$\Rightarrow \quad (x-3)^2 + y^2 = (x-6)^2 + y^2 = 3^2$

$\Rightarrow \quad (x-3)^2 + y^2 = (x-6)^2 + y^2 \quad ...(i) \quad$ and $(x-3)^2 + y^2 = 3^2 \quad ...(ii)$

$\Rightarrow$ Solving (i); $(x-3)^2 = (x-6)^2$

$\Rightarrow \quad x^2 - 6x + 9 = x^2 - 12x + 36$

$\Rightarrow \quad 6x = 27$

$\Rightarrow \quad x = \frac{27}{6} = \frac{9}{2}$

Put $x = \frac{9}{2}$ in eqn (ii) we get

$$\left(\frac{9}{2} - 3\right)^2 + y^2 = 3^2$$

$\Rightarrow \quad \left(\frac{3}{2}\right)^2 + y^2 = 3^2$

$\Rightarrow \quad \frac{9}{4} + y^2 = 9$

$\Rightarrow \quad y^2 = 9 - \frac{9}{4} = \frac{27}{4} \quad \Rightarrow \quad y = \frac{3\sqrt{3}}{2}$

Hence, the third vertex is $\left(\frac{9}{2}, \frac{3\sqrt{3}}{2}\right)$

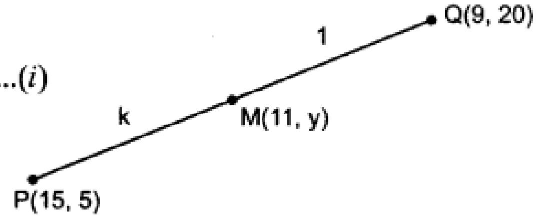
Question 92.

Point M(11,y) lies on the line segment joining the points P(15,5) and Q(9,20). Find the ratio in which point M divides the line segment PQ. Also, find the value

Solution:

Let point M divides the line segment PQ in the ratio $k : 1$

Then, Using section formula to calculate coordinates of M, and equating with given M-coordinates.



$$\frac{9k+15}{k+1} = 11 \text{ and } \frac{20k+5}{k+1} = y \quad ...(i)$$

$$\Rightarrow \quad 9k + 15 = 11k + 11$$

$$\Rightarrow \quad 2k = 4 \Rightarrow k = 2$$

Hence, point M divides the line segment PQ in the ratio 2 : 1. Then, using (i),

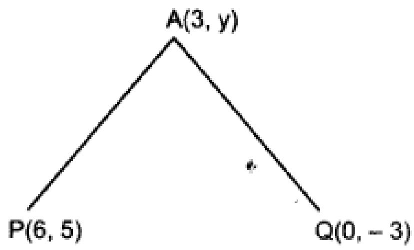
$$y = \frac{20 \times 2 + 5}{2+1} = \frac{45}{3} = 15$$

$$\therefore \quad y = 15$$

Question 93.

Point A(3,y) is equidistant from the points P(6,5) and Q(0, -3). Find the value of y.

Solution:



Given: $\quad AQ = AP$

$$\Rightarrow \quad Q^2 = AP^2$$

Using distance formula,

$$\Rightarrow \quad (3-0)^2 + (y+3)^2 = (3-6)^2 + (y-5)^2$$

$$\Rightarrow \quad 9 + y^2 + 6y + 9 = 9 + y^2 - 10y + 25$$

$$\Rightarrow \quad 6y + 9 = -10y + 25$$

$$\Rightarrow \quad 16y = 16 \Rightarrow y = 1$$

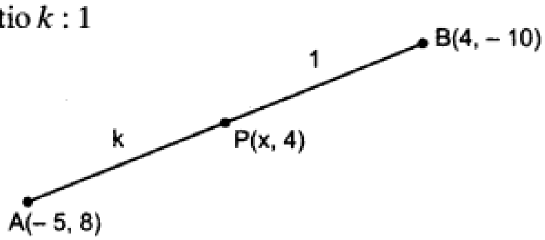
Question 94.

Point P(x, 4) lies on the line segment joining the points A(-5,8) and B(4, -10). Find the ratio in which point P divides the line segment AB. Also, find the value of x

Solution:

Let point P divides the line segment AB in the ratio $k : 1$

Using distance formula



A.T.Q., $\frac{4k-5}{k+1} = x$ and $\frac{-10k+8}{k+1} = 4$

$\Rightarrow$ $-10k + 8 = 4k + 4$

$14k = 4$

$\Rightarrow$ $k = \frac{4}{14} = \frac{2}{7}$

$k = \frac{2}{7}$

So, point P divides the line segment AB in the ratio 2 : 7

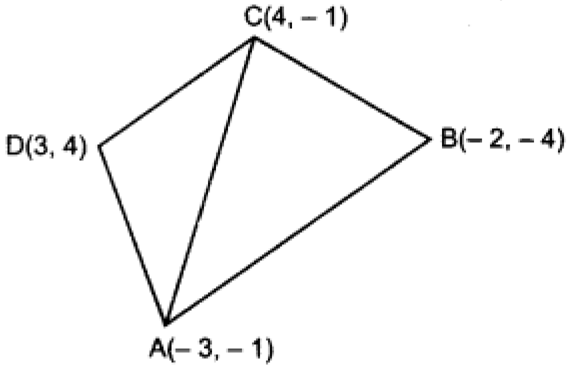
Now, $x = \frac{4.\left(\frac{2}{7}\right)-5}{\frac{2}{7}+1} = \frac{8-35}{2+7} = \frac{-27}{9} = -3$

$x = 3$

Question 95.

Find the area of the quadrilateral ABCD, whose vertices are A(-3, -1), B(-2, -4), C(4, -1) and D(3, 4).

Solution:



Firstly, $\text{ar}(\Delta ABC) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2}[-3(-4+1) - 2(-1+1) + 4(-1+4)]$$

$$= \frac{1}{2}[9 - 0 + 12] = \frac{21}{2} \text{ sq. units}$$

$$\text{ar}(\Delta ACD) = \frac{1}{2}[-3(-1-4) + 4(4+1) + 3(-1+1)]$$

$$= \frac{1}{2}[15 + 20 + 0] = \frac{35}{2} \text{ sq. units}$$

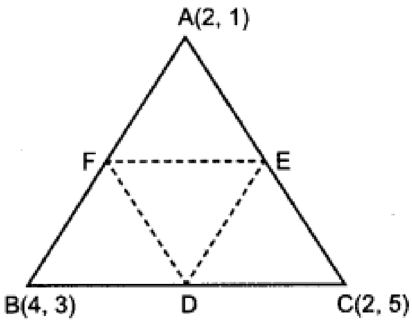
Now, $\text{ar(quadrilateral } ABCD) = \text{ar}(\Delta ABC) + \text{ar}(\Delta ACD)$

$$= \frac{21}{2} + \frac{35}{2} = \frac{56}{2} = 28 \text{ sq. units}$$

Question 96.

Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are A(2,1), B(4,3) and C(2,5).

Solution:



. D, E, F are the mid-points of the sides BC, CA and AB respectively.

So, coordinates of the points D, E, F are as

Using mid-point formula,

$$D\left(\frac{4+2}{2}, \frac{3+5}{2}\right); E\left(\frac{2+2}{2}, \frac{5+1}{2}\right); F\left(\frac{4+2}{2}, \frac{3+1}{2}\right)$$

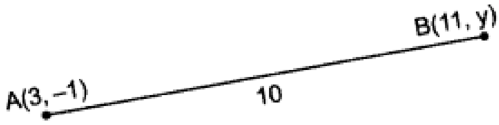
i.e. D(3, 4); E(2, 3); F(3, 2)

Now, $\text{ar}(\Delta DEF) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2}[3(3-2) + 2(2-4) + 3(4-3)]$$

$$= \frac{1}{2}[3 - 4 + 3] = \frac{2}{2} = 1 \text{ sq. unit}$$

Question 97.
Find the value of y for which the distance between the points A(3, -1) and B(11,y) is 10 units.
Solution:



Given that: $AB = 10$ units

$\Rightarrow \quad AB^2 = 100$

Using distance formula,

$\Rightarrow \quad (11-3)^2 + (y+1)^2 = 100$

$\Rightarrow \quad 64 + (y+1)^2 = 100$

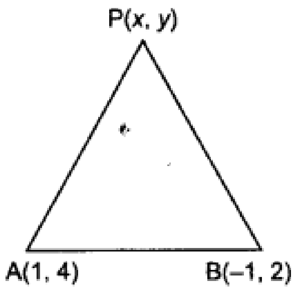
$\Rightarrow \quad (y+1)^2 = 36$

$\Rightarrow \quad y + 1 = \pm 6$

$\Rightarrow \quad y + 1 = 6 \quad \text{or} \quad y + 1 = -6$

$\Rightarrow \quad y = 5 \quad \text{or} \quad y = -7.$

Question 98.
Find a relation between it and y such that the point P(x, y) is equidistant from the points A(1, 4) and B(-1, 2).
Solution:



Given that: $PA = PB$

$\Rightarrow \quad PA^2 = PB^2$

Using distance formula,

$\Rightarrow \quad (x-1)^2 + (y-4)^2 = (x+1)^2 + (y-2)^2$

$\Rightarrow \quad x^2 - 2x + 1 + y^2 - 8y + 16 = x^2 + 2x + 1 + y^2 - 4y + 4$

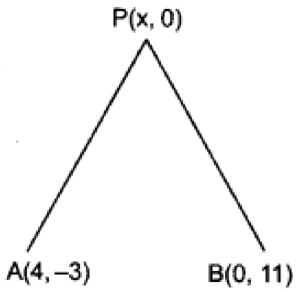
$\Rightarrow \quad -2x - 8y + 17 = 2x - 4y + 5$

$\Rightarrow \quad 4x + 4y - 12 = 0$

$\Rightarrow$ Required relation: $x + y - 3 = 0.$

Question 99.
Find a point on the x-axis which is equidistant from A(4, -3) and B(0,11).
Solution:

Let point P(x, 0) on x-axis be equidistant from the points A and B.



Then, $PA = PB$

$\Rightarrow \quad PA^2 = PB^2$

Using distance formula,

$\Rightarrow \quad (x-4)^2 + (0+3)^2 = (x-0)^2 + (0-11)^2$

$\Rightarrow \quad x^2 - 8x + 16 + 9 = x^2 + 121$

$\Rightarrow \quad -8x = 121 - 25$

$\Rightarrow \quad -8x = 96$

$\Rightarrow \quad x = \frac{96}{-8} = -12 \quad \Rightarrow \quad x = -12$

Hence, coordinates of required point are (–12, 0)

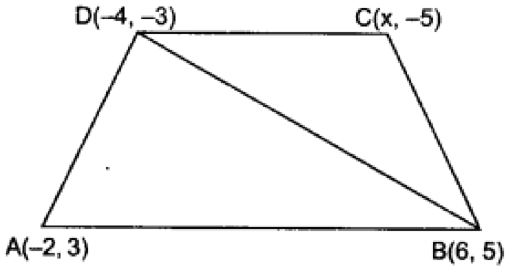
Question 100.
If A(-2,3), B(6,5), C(x, – 5) and D(-4, – 3) are the vertices of a quadrilateral ABCD of area 80 sq. units, then find a positive value of x.
Solution:
ar (quad ABCD) = ar (ΔABD) + ar (ΔBCD)

$$80 = \left|\frac{1}{2}[-2(5+3)+6(-3-3)-4(3-5)]\right| + \left|\frac{1}{2}[6(-5+3)+x(-3-5)-4(5+5)]\right|$$

$$\left[\because \text{Area of triangle} = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\right]$$

$$\Rightarrow \quad 80 = \left|\frac{1}{2}[-16-36+8]\right| + \left|\frac{1}{2}[-12-8x-40]\right|$$

$$\Rightarrow \quad 80 = \left|\frac{1}{2}(-44)\right| + \left|\frac{1}{2}(-8x-52)\right|$$



$$\Rightarrow \quad 80 = 22 + |26 + 4x|$$

$$\Rightarrow \quad |26x + 4x| = 58 \Rightarrow 26 + 4x = \pm 58$$

$$[\because |x| = a \Rightarrow x = \pm a]$$

$$\Rightarrow \quad 26 + 4x = 58 \text{ or } 26 + 4x = -58$$

$$\Rightarrow \quad 4x = 32 \text{ or } 4x = -84$$

$$\Rightarrow \quad x = 8 \text{ or } x = -21 \quad \therefore \text{Positive value of } x = 8$$

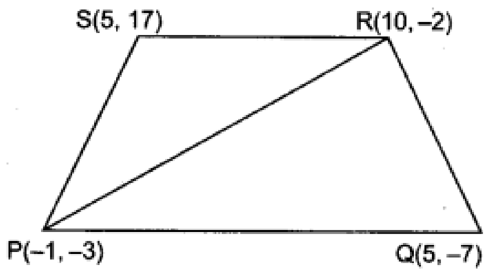
Question 101.
Find the area of the quadrilateral PQRS whose vertices are P(-l, – 3), Q(5, – 7), R(10, — 2) and S(5,17).
Solution:
ar (quad PQRS) = ar (ΔPQR) + ar (ΔPRS)

$$= \frac{1}{2}[-1(-7+2)+5(-2+3)+10(-3+7)] + \frac{1}{2}[-1(-2-17)+10(17+3)+5(-3+2)]$$

$$\left[\because \text{Area of triangle} = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\right]$$

$$= \frac{1}{2}[5+5+40] + \frac{1}{2}[19+200-5]$$

$$= 25 + 107 = 132 \text{ sq. units.}$$



Question 102.
Find the area of the quadrilateral ABCD whose vertices are A(3, – 1), B(9, -5), C(14,0) and D(9,19).

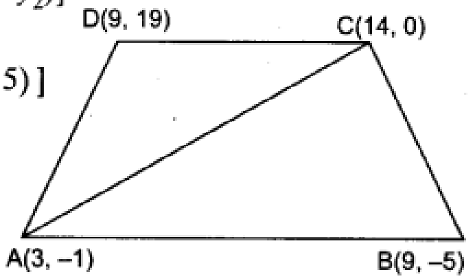
Solution:

Firstly, ar $(\Delta ABC) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2}[3(-5-0) + 9(0+1) + 14(-1+5)]$$

$$= \frac{1}{2}[-15 + 9 + 56]$$

$$= \frac{1}{2} \times 50 = 25 \text{ sq. units.}$$



Now, ar $(\Delta ACD) = \frac{1}{2}[3(0-19) + 14(19+1) + 9(-1-0)]$

$$= \frac{1}{2}[-57 + 280 - 9]$$

$$= \frac{1}{2} \times 214 = 107 \text{ sq. units.}$$

Area of quadrilateral ABCD = ar(ΔABC) + ar(ΔACD) = 25 + 107 = 132 sq. units

Question 103.

Find the coordinates of the points which divide the line segment joining A (2, – 3) and B(-4, – 6) into three equal parts.

Solution:

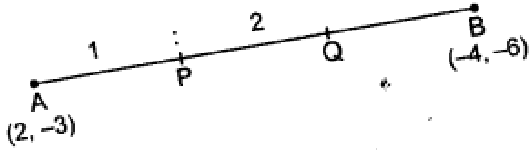
Let P and Q are the required point; which divides AB in three equal parts.

Point P divides the line segment AB in the ratio 1 : 2

So, coordinates of P are given by $\left(\frac{-4 + 2 \times 2}{1+2}, \frac{1 \times (-6) + 2 \times (-3)}{1+2}\right)$ [$\because$ Using section formula]

i.e. $\left(\frac{-4+4}{3}, \frac{-6-6}{3}\right)$

i.e. $(0, -4)$



Now point Q is the mid-point of PB.

So, coordinates of point Q are $\left(\frac{0-4}{2}, \frac{-4-6}{2}\right)$ i.e. $(-2, -5)$.

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Very Short Answer Type Questions [1 Mark]

Question 104.

If P(2, p) is the mid-point of the line segment joining the points A(6, -5) and B(-2,11), find the value of p.

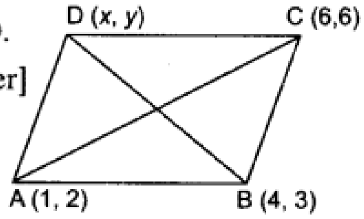
Solution:

P$(2, p)$ is mid-point of A $(6, -5)$ and B $(-2, 11)$. Using mid-point formula,

$$\text{So,} \qquad \frac{-5+11}{2} = p$$

$$\Rightarrow \qquad p = \frac{6}{2} \Rightarrow p = 3.$$

Question 105.
If A(l, 2), B(4, 3) and €(6,6) are three vertices of parallelogram ABCD, find coordinates of D.
Solution:



Let coordinates of D be (x, y) and P is mid-point of AC and BD.

[Diagonals of a parallelogram bisect each other]

$\therefore$ Using mid-point formula,

$$\left(\frac{x+4}{2}, \frac{y+3}{2}\right) = \left(\frac{1+6}{2}, \frac{2+6}{2}\right)$$

$$\Rightarrow \qquad \frac{x+4}{2} = \frac{7}{2}, \frac{y+3}{2} = \frac{8}{2}$$

$$\Rightarrow \qquad x + 4 = 7; y + 3 = 8$$

$$\Rightarrow \qquad x = 3; y = 5$$

$\therefore$ Coordinates of D are $(3, 5)$.

Question 106.
What is the distance between points A(c, 0) and B(0, -c)?
Solution:

$$\text{Distance} \quad AB = \sqrt{(0-c)^2+(-c-0)^2} \qquad [\because \text{Using distance formula}]$$

$$= \sqrt{c^2+c^2} = \sqrt{2c^2} = \sqrt{2}c \text{ units}$$

Question 107.
Find the distance between the points, A(2a, 6a) and B(2a + √3 a, 5a).
Solution:

$$\text{Distance} \quad AB = \sqrt{(2a+\sqrt{3}a-2a)^2+(5a-6a)^2} \qquad [\because \text{Using distance formula}]$$

$$= \sqrt{3a^2+a^2} = \sqrt{4a^2} = 2a \text{ units}$$

Question 108.
Find the value of k if P(4, -2) is the midpoint of the line segment joining the points A(5k, 3) and B(-k, —7).

Solution:

P(4, – 2) is mid point of A(5k, 3) and B(– k, – 7), Using mid-point formula,

$$\therefore \quad \frac{5k-k}{2} = 4 \Rightarrow 4k = 8 \Rightarrow k = 2$$

Short Answer Type Questions II [3 Marks]

Question 109.

Point P divides the line segment joining the points A(2,1) and B(5, -8) such that AP /AB=1/3. If P lies on the line 2x – y + k = 0, find the value of k.

Solution:

P is the point of intersection of line segment AB and line $2x - y + k = 0$.

Here, given that, $\frac{AP}{AB} = \frac{1}{3} \Rightarrow 3AP = AB$

$\Rightarrow \quad 3AP = AP + PB$

$\Rightarrow \quad 2AP = PB$

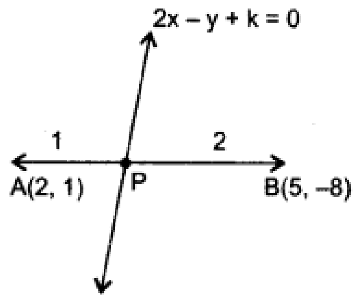
$\Rightarrow \quad \frac{AP}{PB} = \frac{1}{2} \Rightarrow AP : PB = 1 : 2$

$\Rightarrow$ P divides the line segment joining A(2, 1) and B(5, – 8) in the ratio 1 : 2.

$\therefore$ Coordinates of point P are

$$x = \frac{1\times5+2\times2}{1+2} = 3 \quad [\because \text{Using section formula}]$$

$$y = \frac{1\times(-8)+2\times1}{1+2} = -2$$



i.e. P (3, – 2)

As point P lies on the line $2x - y + k = 0$, P must satisfy it.

$\Rightarrow \quad 6 + 2 + k = 0 \Rightarrow k = -8$

Question 110.

If R(x, y) is a point on the line segment joining the points P(a, b) and Q(b, a), then prove that x + y = a + b.

Solution:

$R(x, y)$ lies on the line segment joining the points $P(a, b)$ and $Q(b, a)$. Then P, Q, R are collinear, so ar(Δ PQR) = 0

$$\Rightarrow \quad \frac{1}{2}|x(b-a)+a(a-y)+b(y-b)| = 0 \quad \left[\begin{array}{l}\text{Using formula for area of a triangle} \\ \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\end{array}\right]$$

$$\Rightarrow \quad bx - ax + a^2 - ay + by - b^2 = 0$$

$$\Rightarrow \quad b(x+y) - a(x+y) + (a^2 - b^2) = 0$$

$$\Rightarrow \quad (b-a)(x+y) - (b-a)(b+a) = 0$$

$$\Rightarrow \quad (b-a)[(x+y)-(b+a)] = 0$$

$$\Rightarrow \quad x + y = b + a$$

[Assuming $a \neq b$] Hence, proved.

Question 111.

Prove that the points P(a, b + c), Q(b, c + a) and R(c, a + b) are collinear.

Solution:

Area of Δ formed by the points $P(a, b+c)$, $Q(b, c+a)$ and $R(c, a+b)$ is given by

$$= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

or,

$$\text{ar}(\Delta\, PQR) = \frac{1}{2}|a(c+a-a-b)+b(a+b-b-c)+c(b+c-c-a)|$$

$$= \frac{1}{2}|a(c-b)+b(a-c)+c(b-a)|$$

$$\Rightarrow \quad = \frac{1}{2}|ac-ab+ab-bc+bc-ac| = 0$$

$\because$ ar(ΔPQR) = 0

Hence, points are collinear.

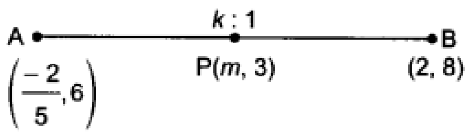
Question 112.

If the point P(m, 3) lies on the line segment joining the points A(-2/5,6) and B(2, 8), find the value of m.

Solution:

Let point P divides AB in ratio $k : 1$.

Using section formula,



$$3 = \frac{8 \times k + 6 \times 1}{k+1}$$

$$\Rightarrow \quad 3 = \frac{8k+6}{k+1} \quad \Rightarrow \quad 3k + 3 = 8k + 6 \quad \Rightarrow \quad -3 = 5k$$

$$\Rightarrow \quad k = \frac{-3}{5}$$

and,

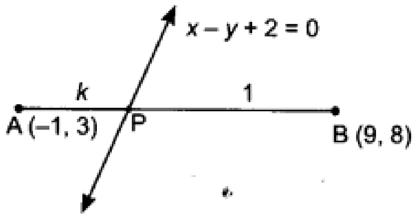
$$m = \frac{2k + \left(-\frac{2}{5}\right)}{k+1} = \frac{2\left(-\frac{3}{5}\right) - \frac{2}{5}}{\frac{-3}{5}+1} = \frac{-6-2}{2} \Rightarrow m = -\frac{8}{2} = -4 \quad \therefore\ m = -4$$

Question 113.
Point P divides the line segment joining the points A(-l, 3) and B(9, 8) such that AP/PB=k/1. If P lies on the line x -y + 2 = 0, find the value of k.
Solution:

P divides the joining of A(– 1, 3) and B(9, 8) such that $\frac{AP}{AB} = \frac{k}{1}$ i.e. AP : PB = k : 1.

Using section formula,

$\therefore$ Coordinates of P are: $\left(\frac{9k-1}{k+1}, \frac{8k+3}{k+1}\right)$



If P lies on $x - y + 2 = 0$, then P must satisfy it.

$$\frac{9k-1}{k+1} - \left(\frac{8k+3}{k+1}\right) + 2 = 0$$

$$\Rightarrow \quad 9k - 1 - 8k - 3 + 2k + 2 = 0$$

$$\Rightarrow \quad 3k - 2 = 0$$

$$\Rightarrow \quad k = \frac{2}{3}$$

Question 114.
Find the value of k, if the points A(7, -2), B(5,1) and C(3,2k) are collinear
Solution:

If points A(7, – 2), B(5, 1) and C(3, 2k) are collinear then, ar ΔABC = 0

$$\left[\because \text{Area of triangle } \frac{1}{2}\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right]$$

$$\therefore \quad \text{ar } \Delta ABC = \frac{1}{2}\,|7(1 - 2k) + 5(2k + 2) + 3(-2 - 1)| = 0$$

$$\Rightarrow \quad 7 - 14k + 10k + 10 - 9 = 0$$

$$\Rightarrow \quad -4k = -8 \qquad \Rightarrow k = 2$$

Question 115.
If the points (p, q); (m, n) and (p-m,q-n) are collinear, show that pn = qm
Solution:

If P(p, q), Q(m, n), R($p - m, q - n$) are collinear then area of triangle formed by them is zero.

Hence, $\quad$ ar ΔPQR = 0

$$\left[\because \text{Area of triangle} = \frac{1}{2}\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right]$$

$$\frac{1}{2}\,|pn - qm + mq - mn - pn + mn + pq - mq - qp + pn| = 0$$

$$\Rightarrow \quad |pn - qm| = 0$$

$$\Rightarrow \quad pn - qm = 0$$

$$\Rightarrow \quad pn = qm$$

Hence, proved.

Question 116.
Find the value of k, if the points A(8,1), B(3, -4) and C(2, k) are collinear

Solution:

Given points are A(8, 1), B(3, –4) and C(2, k).

As these points are collinear, so the area of triangle formed by these points is zero sq. units.

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\therefore \quad \frac{1}{2}[8(-4-k) + 3(k-1) + 2(1+4)] = 0$$

$$\therefore \quad -32 - 8k + 3k - 3 + 10 = 0$$

$$-5k - 25 = 0$$

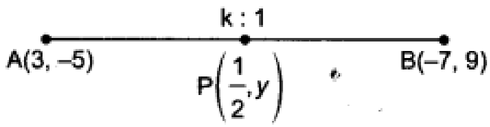
$$\therefore \quad k = -5$$

Question 117.

If point P (1/2, y)lies on the line segment joining the points A(3, -5) and B(-7,9) then find the ratio in which P divides AB. Also, find the value of y.

Solution:

Let P divides AB in the ratio k : 1.



$$\therefore \quad \left(\frac{-7k+3}{k+1}, \frac{9k-5}{k+1}\right) = \left(\frac{1}{2}, y\right) \quad ...(i)$$

$$\Rightarrow \quad \frac{-7k+3}{k+1} = \frac{1}{2}$$

$$\Rightarrow \quad -14k + 6 = k + 1$$

$$\Rightarrow \quad -15k = -5$$

$$\Rightarrow \quad k = \frac{1}{3}$$

$$\therefore \quad \text{Ratio is } k : 1, \text{ i.e. } \frac{1}{3} : 1 \Rightarrow 1 : 3$$

and, using (i),
$$y = \frac{9k-5}{k+1} = \frac{9 \times \frac{1}{3} - 5}{\frac{1}{3} + 1} = \frac{-6}{4} = \frac{-3}{2} \quad \therefore \; y = \frac{-3}{2}$$

Question 118.

Find the value of k for which the points A(9, k), B(4, -2) and C(3, -3) are collinear.

Solution:

If points A(9, k), B(4, – 2) and C(3, – 3) are collinear, so, ar (ΔABC) = 0

$$\left[\because \text{ Area of triangle} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right]$$

$$\Rightarrow \quad \frac{1}{2}|9(-2+3) + 4(-3-k) + 3(k+2)| = 0$$

$$\Rightarrow \quad |9 - 12 - 4k + 3k + 6| = 0$$

$$\Rightarrow \quad -k = -3$$

$$\Rightarrow \quad k = 3$$

Question 119.

Find the value of k for which the points A(fc, 5), B(0,1) and C(2, -3) are collinear.

Solution:

If A(k, 5), B(0, 1), C(2, –3) are collinear then ar ΔABC = 0.

$$\left[\because \text{Area of triangle} = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]\right]$$

$$\Rightarrow \quad \frac{1}{2}[k(1+3) + 0(-3-5) + 2(5-1)] = 0$$

$$\Rightarrow \quad |\,4k + 8\,| = 0$$

$$\Rightarrow \quad 4k = -8$$

$$\Rightarrow \quad k = -2$$