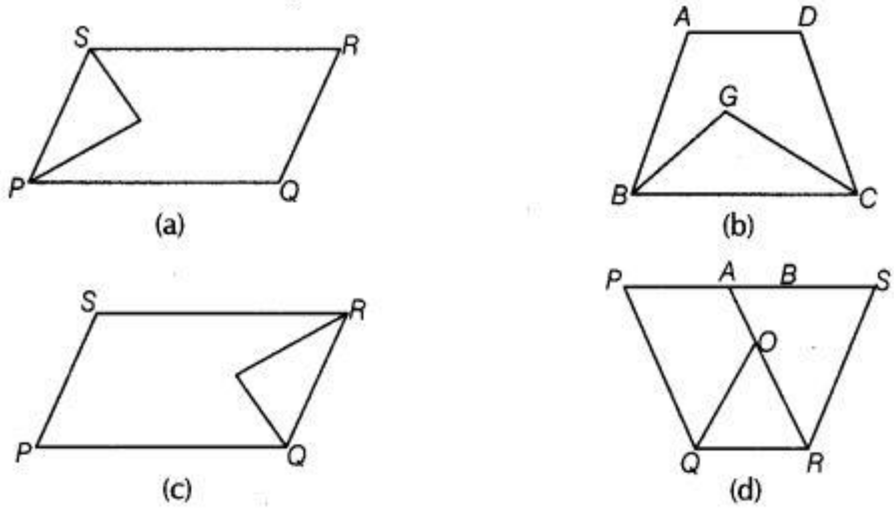
<u>Chapter 9: Areas of a parallelogram</u>
<u>Exercise 9.1 (MCQ)</u>

Question 1:
The median of a triangle divides it into two
(a) triangles of equal area (b) **congruent triangles**
(c) **right-angled triangles** (d) **isosceles triangles**

Answer: **(a)** We know that a median of a triangle is a line segment joining a vertex to the mid-point of the opposite side. Thus, a median of a triangle divides it into two triangles of equal area.

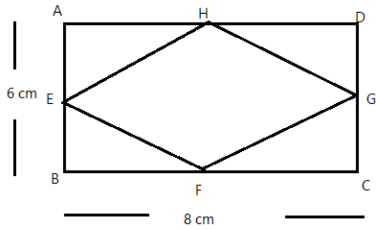
Question 2:
In which of the following figures, you find two polygons on the same base and between the same parallels?



Answer: **(d)** In figures (a), (b) and (c) there are two polygons on the same base but they are not between the same parallels.
In figure (d), there are two polygons (PQRA and BQRS) on the same base and between the same parallels.

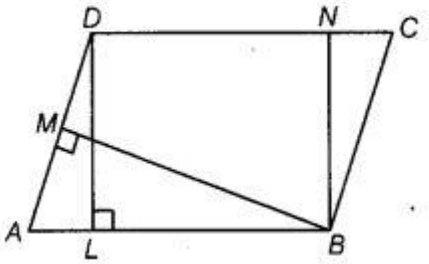
Question 3:
The figure obtained by joining the mid-points of the adjacent sides of a rectangle of sides 8 cm and 6 cm, is
(a) a rectangle of area 24 cm^2
(b) **a square of area 25 cm^2**
(c) **a trapezium of area 24 cm^2**
(d) **a rhombus of area 24 cm^2**

Answer: (d)
Here, length of rectangle ABCD = 8 cm and breadth of rectangle ABCD = 6.cm
Let E, F, G and H are the mid-points of the sides of rectangle ABCD, then EFGH is a rhombus.



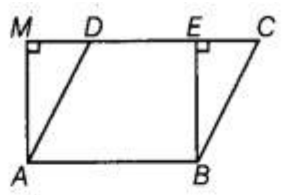
Length of ABCD = 8CM
Breadth of ABCD = 6 cm
Let, E, F, G, H are the mid-points of sides of the rectangle ABCD, hence EFGH is a rhombus and the diagonals of the rhombus is EG and FH
Here, EG = BC = 8cm
and HF = AB = 6cm
Area of rhombus = $\frac{Product\ of\ diagonals}{2} = \frac{8\times6}{2}$ = 24 cm^2

Question 4:
In the figure, the area of parallelogram ABCD is
(a) AB x BM **(b)** BC x BN **(c)** DC x DL **(d)** AD x DL



Thinking Process
Use the formula, area of parallelogram =Base x Altitude to get the required result
Solution:
(c) We know that area of a parallelogram is the product of any side and the corresponding altitude (or height).
Here, when AB is base, then the height is DL.
Area of parallelogram = AB x DL and when AD is base, then the height is BM.
Area of parallelogram = AD x BM When DC is base, then the height is DL.
Area of parallelogram = DC x DL and when BC is base, then the height is not given.
Hence, the option (c) is correct.

Question 5:
In the figure, if parallelogram ABCD and rectangle ABEM are of equal area, then



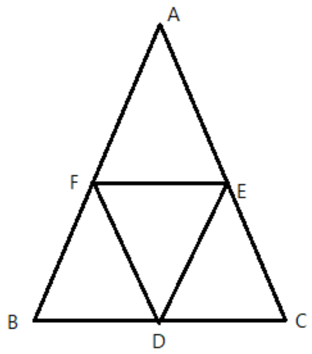
[sides of rectangle]

(a) perimeter of ABCD = perimeter of ABEM
(b) **the perimeter of ABCD < perimeter of ABEM**
(c) **the perimeter of ABCD > perimeter of ABEM**
(d) **the perimeter of ABCD = ½ (perimeter of ABEM)**

Answer: **(c)** In rectangle ABEM, AB = EM [sides of rectangle] and in parallelogram ABCD, CD = AB
On adding, both equations, we get
AB + CD = EM + AB ……………………………………………….(1)
We know that the perpendicular distance between two parallel sides of a parallelogram is always less than the length of the other parallel sides.
BE < BC and AM < AD
[since, in a right-angled triangle, the hypotenuse is greater than the other side**]**
On adding both above inequalities, we get
SE + AM <BC + AD or BC + AD> BE + AM
On adding AB + CD on both sides, we get
AB + CD + BC + AD> AB + CD + BE + AM
or, AB+BC + CD + AD> AB+BE + EM+ AM [∴ CD = AB = EM]
The perimeter of parallelogram ABCD > perimeter of rectangle ABEM

Question 6:
The mid-point of the sides of a triangle along with any of the vertices as the fourth point make a parallelogram of area equal to
(a) ½ ar (ABC)
(b) **1/3 ar (ABC)** (c) **¼ ar (ABC)** (d) **ar (ABC)**

Answer: (a)
We know that, if D, E and F are respectively the mid-points of the sides BC, CA and AB od triangle ABC, then all four triangles have equal area i.e.,
ar(ΔAFE) = ar(ΔBFD) = ar(ΔEDC) = ar(ΔDEF) ……………………………..(1)
Area of ΔDEF = $\frac{1}{4}$Area of ΔABC ………………………………………………..(2)



If we take D as the fourth vertex, then the area of the parallelogram AFDE,
= area of ΔAFE + area of ΔDEF

= area of ΔDEF + area of ΔDEF
= 2 ar(ΔDEF) ……………………[using eq (1)]
= $2 \times \frac{1}{4}$ ar (ΔABC) ……………….[Using eq(2)]
= $\frac{1}{2}$ ar (ΔABC)

Question 7: Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is
(a) 1 : 2 (b) **1 : 1** (c) **2 : 1** (d) **3 : 1**

Answer: **(b)** We know that parallelogram on equal bases and between the same parallels are equal in area. So, the ratio of their areas is 1 :1.

Question 8:
ABCD is a quadrilateral whose diagonal AC divides it into two parts, equal in area, then ABCD
(a) is a rectangle
(b) **is always a rhombus**
(c) **is a parallelogram**
(d) **need not be any of (a), (b) or (c)**

Answer: **(d)** Here, ABCD need not be any of rectangle, rhombus and parallelogram because if ABCD is a square, then its diagonal AC also divides it into two parts which are equal in area.

Question 9:
If a triangle and a parallelogram are on the same base and between the same parallels, then the ratio of the area of the triangle to the area of a parallelogram

is

(a) 1 : 3 (b) **1:2** (c) **3 : 1** (d) **1 : 4**

Answer: (b) We know that, if a parallelogram and a triangle are on the same base and between the same parallels, then the area of the triangle is half the area of the parallelogram.

Area of the triangle = $\frac{1}{2}$ × area of the parallelogram

or, $\frac{area\ of\ triangle}{area\ of\ parallelogram} = \frac{1}{2}$

area of a triangle: area of parallelogram = 1: 2

Exercise 9.2: Very Short Answer Type Questions

Write whether True or False and justify your answer.

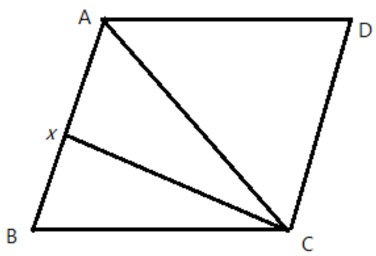
Question 1:

ABCD is a parallelogram and X is the mid-point of AB. If ar (AXCD) = 24 cm^2, then ar (ABC) = 24 cm^2.

Answer: **False**

Given, ABCD is a parallelogram and ar (AXCD) = 24 cm^2

Let the area of parallelogram ABCD is 2y cm^2 and join AC.



We know that diagonal divides the area of a parallelogram into two equal areas.

Thus, let, ar(ΔABC) = ar(ΔACD) = y

Also, X is the mid-point of AB.

so, ar(ΔACX) = ar(ΔBCX) [Since, X is the median in (ΔABC)]

= $\frac{1}{2}$ar(ΔABC) = $\frac{1}{2}$y

Now, ar(AXCD) = ar(ΔADC) + ar(ΔACX)

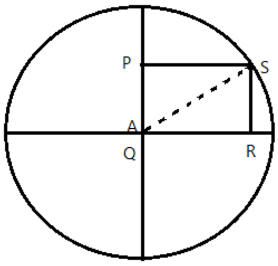
24 = y + $\frac{y}{2}$

or, 24 = $\frac{3y}{2}$
or, y = $\frac{24 \times 2}{3}$ = 16cm^2

Hence, ar(ΔABC) = 16cm^2

Question 2:
PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm and A is any point on PQ. If PS = 5 cm, then ar (ΔPAS) = 30 cm^2.

Answer: true.
Given, PS = 5 cm
radius of circle = SQ = 13 cm



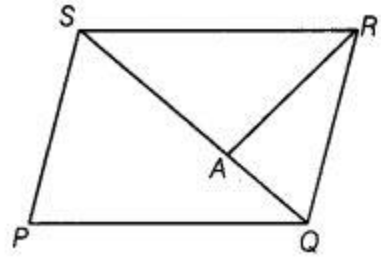
In right-angled triangle SPQ,
SQ^2 = PQ^2 + PS^2 [By Pythagoras theorem]
13^2 = PQ^2 + 5^2
or, PQ^2 = 169 – 25 = 144
or, PQ = 12 cm

Now, ar(ΔAPS) = $\frac{1}{2} \times base \times height$
$= \frac{1}{2} \times PS \times PQ$
$= \frac{1}{2} \times 5 \times 12$
= 30cm^2

So, the given statement is true, if A coincides with Q.

Question 3:
PQRS is a parallelogram whose area is 180 cm^2 and A is any point on the diagonal QS. The area of ΔASR = 90 cm^2.
Solution:
False
Given, area of parallelogram PQRS = 180 cm^2 and QS is it's diagonal which divides

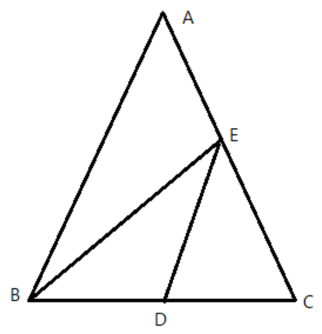
it into two triangles of equal area.



Question 4:
ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Then, ar (ΔBDE) = ¼ ar (ΔABC).

Solution:

True
Given, ΔABC and ΔBDE are two equilateral triangles.
Area of equilateral ΔABC = $\frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4}(BC)^2$ …………………………………..(1)
Also given, D is the midpoint of BC.



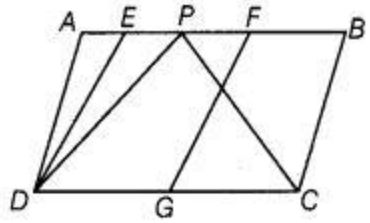
BC = DC = $\frac{1}{2}$BC …………………………………..(2)
Now, the area of an equilateral ΔBDE
$= \frac{\sqrt{3}}{4} \times (\text{side})^2$
$= \frac{\sqrt{3}}{4} \times (BD)^2$
$= \frac{\sqrt{3}}{4} \times (\frac{1}{2}BC)^2$ [From eq (2)]
$= \frac{\sqrt{3}}{4} \times \frac{1}{4}BC^2$

$= \frac{1}{4}\left(\frac{\sqrt{3}}{4}BC^2\right)$

area of an equilateral ΔBDE = $\frac{1}{4}$ ar (ΔABC)

Question 5:

In the figure, ABCD and EFGD are two parallelograms and G is the mid-point of CD. Then, ar (ΔDPC) = ½ ar (EFGD).

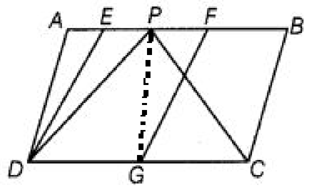


Answer: **False**

In the given figure, join PG. Since G is the mid-point of CD.

Thus, PG is a median of ΔDPC and it divides the triangle into parts of equal areas.

Then, ar(ΔDPC) = ar (ΔGPC) = $\frac{1}{2}$ar (ΔDPC)…………………………………………..(1)



Also, we know that, if a parallelogram and a triangle lie on the same base and between the same parallels then the area of a triangle is equal to half of the area of a parallelogram.

here, parallelogram EFGH and triangle DPG lie on the same base DG and between the same parallels DG and EF.

So, ar(ΔDPG) = $\frac{1}{2}$ ar(ΔEFGD)………………………………………(2)

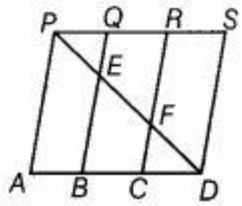
from eq(1) and eq(2), $\frac{1}{2}$ ar(ΔDPC) = $\frac{1}{2}$ ar(ΔEFGD)

or, ar(ΔDPC) = ar(ΔEFGD)

Exercise 9.3: Short Answer Type Questions

Question 1:

In the figure, PSDA is a parallelogram. Points Q and R are taken on PS such that PQ = QR= RS and PA || QB || RC. Prove that ar (PQE) =ar (CFD).



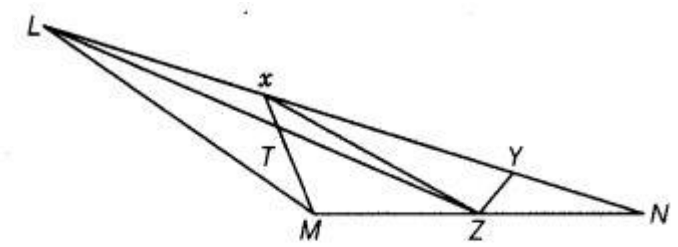
Thinking Process

1. **Firstly, use the formula, area of parallelogram = Base x Altitude**
2. **Further, proving that A PQE = AD CF, by ASA congruent rule.**
3. **At the end use the property that congruent figures have the same area.**

Answer: Given In a parallelogram PSDA, points 0 and R are on PS such that
PQ = QR = RS and PA || QB || RC.
To prove ar (PQE) = ar (CFD)
Proof In parallelogram PABQ,
and PA||QB [given]
So, PABQ is a parallelogram.
PQ = AB …(i)
Similarly, QBCR is also a parallelogram.
QR = BC …(ii)
and RCDS is a parallelogram.
RS =CD …(iii)
Now, PQ=QR = RS …(iv)
From Eqs. (i), (ii) (iii) and (iv),
PQ || AB [∴ in parallelogram PSDA, PS || AD]
In ΔPQE and ΔDCF, ∠QPE = ∠FDC
[since PS || AD and PD is transversal, then alternate interior angles are equal]
PQ=CD [from Eq. (v)]
and ∠PQE = ∠FCD
[∴ ∠PQE = ∠PRC corresponding angles and ∠PRC = ∠FCD alternate interior angles]
ΔPQE = ΔDCF [by ASA congruence rule]
∴ ar (ΔPQE) = ar (ΔCFD) [since,congruent figures have equal area]
Hence proved.

Question 2:

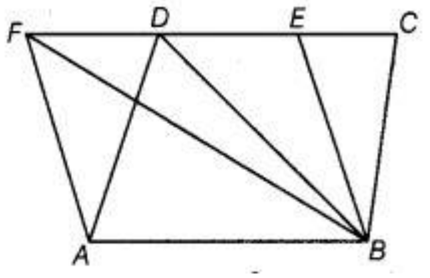
X and Y are points on the side LN of the triangle LMN such that LX = XY = YN. Through X, a line is drawn parallel to LM to meet MN at Z (see figure). Prove that ar (ΔLZY) = ar (MZYX).



Thinking Process
Use the property that the triangles on the same base and between the same two parallel lines are equal in area. Further, prove the required result.

Answer: Given X and Y are points on the side LN such that
LX = XY = YN and XZ || LM To prove ar (ΔLZY) = ar (MZYX)
Proof Since, ΔXMZ and ΔXLZ are on the same base XZ and between the same parallel lines LM and XZ.
Then, ar (ΔXMZ) = ar (ΔXLZ) …(i)
On adding ar (ΔXYZ) both sides of Eq. (i), we get
ar (ΔXMZ) + ar (ΔXXZ) = ar (ΔXLZ) + ar (ΔXYZ)
or, ar (MZYX) = ar (ΔLZY) Hence proved.

Question 3:
The area of the parallelogram ABCD is 90 cm^2. Find
1.ar (ABEF**)**
2.ar (ΔABD)
3.ar (ΔBEF)

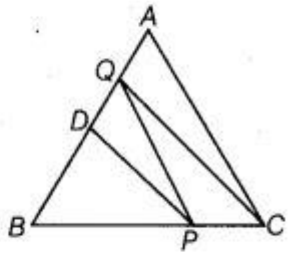


Answer:
Given, area of parallelogram, ABCD = 90 cm^2.

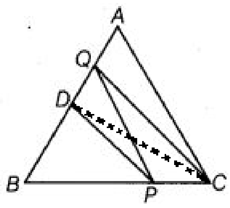
1. We know that parallelograms on the same base and between the same parallel are equal in areas.
   Here, parallelograms ABCD and ABEF are on the same base AB and between the same parallels AB and CF.
   So, ar (ΔBEF) = ar (ABCD) = 90 cm^2
2. We know that, if a triangle and a parallelogram are on the same base and between the same parallels, then the area of a triangle is equal to half of the area of the parallelogram.
   Here, ΔABD and parallelogram ABCD are on the same base AB and between the same parallels AB and CD.
   So, ar (ΔABD) = ½ ar (ABCD)
   = ½ x 90 = 45 cm^2 [∴ ar (ABCD) = 90 cm^2]

3. Here, ABEF and parallelogram ABEF are on the same base EF and between the same parallels AB and EF.
   ar (ΔBEF) = ½ ar (ABEF)
   = ½ x 90 = 45 cm^2 [∴ ar (ABEF) = 90 cm^2, from part (i)]

Question 4:
In ΔABC, D is the mid-point of AB and P is any point on BC. If CQ || PD meets AB in Q (shown in the figure), then prove that ar (ΔBPQ) = ½ ar (ΔABC).



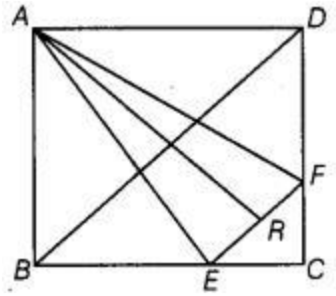
Answer:



Given: In triangle ABC, D is the midpoint of AB and P is any point on BC
CQ ‖PD means AB in Q
To prove: ar(ΔBPQ) = $\frac{1}{2}$ar(ΔBEF)
Construction: Join PQ and CD,
Proof: since D is the mid-point of AB. So, the CD is the median of (ΔABC)
We know that a median of a triangle divides it into two triangles of equal areas.
ar(ΔBCD) = $\frac{1}{2}$(ΔABC)
or, ar(ΔBPD) + ar(ΔDPC) = $\frac{1}{2}$ar(ΔABC)…………………………………………..(1)
Now, (ΔDPQ) and (ΔDPC) are on the same base DP and between the same parallel lines DP and CQ.
So, ar(ΔDPQ) = ar(ΔDPC)………………………………………………………(2)
On putting (1) and (2), we get
ar(ΔBPD) + ar(ΔDPQ) = $\frac{1}{2}$ar(ΔABC)
or, ar(ΔBPQ) = $\frac{1}{2}$ar(ΔABC)

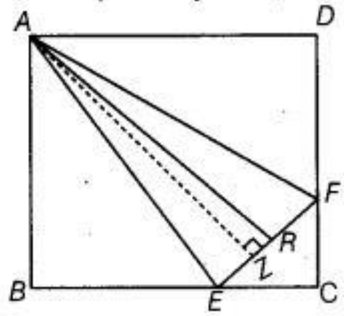
Question 5:
ABCD is a square. E and F are respectively the mid-points of BC and CD. If R is

the mid-point of EF, prove that ar (ΔAER) = ar (ΔAFR).



Answer:

Given In square $ABCD$, E and F are the mid-points of BC and CD respectively. Also, R is the mid-point of EF.



To prove ar (AER) = ar (AFR).

Construction Draw $AN \perp EF$.

Proof ∵ $\text{ar}(\Delta AER) = \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times ER \times AN$

$= \frac{1}{2} \times FR \times AN$

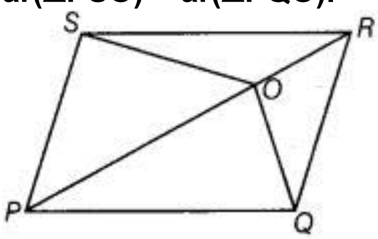
[∵ R is the mid-point of EF, so $ER = FR$]

$= \text{ar}(\Delta AFR)$

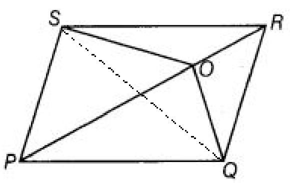
Hence proved.

Question 6:

O is any point on the diagonal PR of a parallelogram PQRS (figure). Prove that ar(ΔPSO) = ar(ΔPQO).

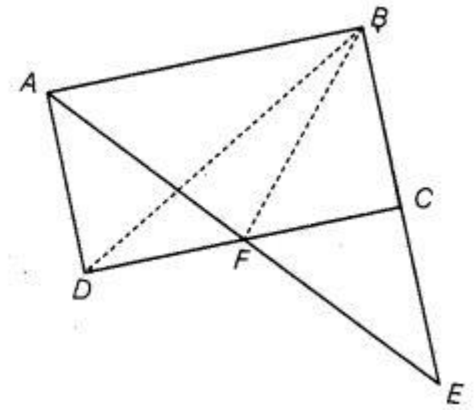


Answer:



In the parallelogram PQRS, O is any point on the diagonal PR. (Given)
To prove: ar(ΔPSO) = ar(ΔPQO).
Construction: Join SQ which intersects PR at B.
proof: We know that diagonals of a parallelogram bisect each other, so B is the midpoint of SQ.
Here, PB is a median of ar(ΔQPS) and we know that a median of a triangle divides it into two triangles of equal area.
Thus, ar(ΔBPQ) = ar(ΔBPS)……………………………………………(1)
Also, OB is the median of ar(ΔOSQ),
ar(ΔOBQ) = ar(ΔOBS).
On adding eq(1) and (2) we get,
ar(ΔBPQ) + ar(ΔOBQ) = ar(ΔBPS) = ar(ΔOBS).
or, ar(ΔPSO) = ar(ΔPQO).

Question 7:
ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F



If ar (ΔDFB) = 3 cm^2, then find the area of the parallelogram ABCD.

Answer: Given, ABCD is a parallelogram and CE = BC i.e., C is the midpoint of BE
Also, ar(ΔDFB) = $3cm^2$
Now, ΔADF and ΔDFB are on the same base DF and between parallels CD and AB
then, ar(ΔADF) = ar(ΔDFB) = $3cm^2$ …………………………..(1)
In ΔABE, by the converse of midpoint theorem,
EF = AF [C is the midpoint of BE] ……………………………..(2)
In ΔADF and ΔECF, ∠AFD = ∠CFE [vertically opposite angles]
AF = EF [from eq(2)]
And, ∠DAF = ∠CEF [Since, BE ∥AD and AE is transversal, then alternate interior angles are equal]
Therefore, ΔADF ≅ ΔECF [by ASA congruency]
then, ar(ΔADF) = ar(ΔCFE) [Since, congruent figures have equal area]
Therefore, ar(ΔCFE) = ar(ΔADF) = $3cm^2$ [eq (1)] ……………………………..(3)

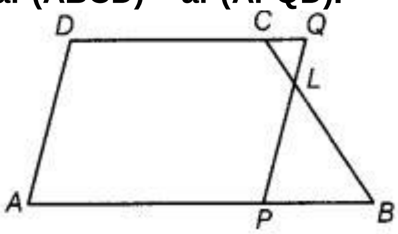
Now, in (ΔBFE), C is the midpoint of BE then,
CF is the median of (ΔBFE)

therefore, ar(ΔCEF) = ar(ΔBFC) [median of a triangle divides it into two trinagles of equal area]
or, ar(ΔBFC) = 3cm^2 ……………………………………………………………(4)

Now, ar(ΔBDC) = ar(ΔDFB) + ar(ΔBFC)
= 3 + 3 = 6 cm^2 [from eq. (1) and (4)]
we know that, diagonal of a parallelogram divides it into two congruent triangle of equal areas. Therefore, ar parallelogram ABCD = 2 ar(ΔBDC) = 2(6) = 12 cm^2

Hence, the area of parallelogram ABCD = 12cm^2

Question 8:
In trapezium ABCD, AB || DC and L is the mid-point of BC. Through L, a line PQ || AD has been drawn which meets AB in P and DC produced in Q. Prove that ar (ABCD) = ar (APQD).



∴ BL = CL

Answer:

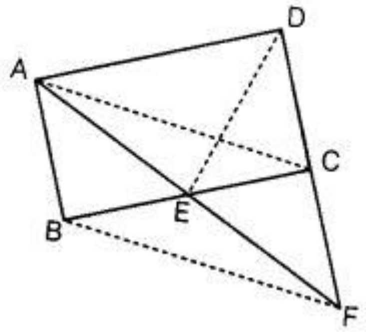
Given In trapezium *ABCD*, *AB* || *DC*, *DC* produced in *Q* and *L* is the mid-point of *BC*.

∴ $BL = CL$
To prove ar (*ABCD*) = ar (*APQD*)
Proof Since, *DC* produced in *Q* and *AB* || *DC*.
So, *DQ* || *AB*
In Δ*CLQ* and Δ*BLP*,
$CL = BL$ [since, *L* is the mid-point of *BC*]
$\angle LCQ = \angle LBP$ [alternate interior angles as *BC* is a transversal]
$\angle CQL = \angle LPB$ [alternate interior angles as *PQ* is a transversal]
∴ $\Delta CLQ \cong \Delta BLP$ [by AAS congruence rule]
Then, ar (Δ*CLQ*) = ar (Δ*BLP*) ...(i)
[since, congruent triangles have equal area]
Now, ar (*ABCD*) = ar (*APQD*) − ar (Δ*CQL*) + ar (Δ*BLP*)
= ar (*APQD*) − ar (Δ*BLP*) + ar (Δ*BLP*) [from Eq. (i)]
⇒ ar (*ABCD*) = ar (*APQD*) **Hence proved.**

Exercise 9.4: Long Answer Type Questions

Question 1:
A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F. Prove that ar (ΔADF) = ar (ΔBFC).

Answer: Given ABCD is a parallelogram and E is a point on BC. AE and DC are produced to meet at F.
AB||CD and BC||AD ……………………………………(1)



$\therefore$ $AB \parallel CD$ and $BC \parallel AD$ ...(i)

To prove ar (ΔADF) = ar$(ABFC)$

Construction Join AC and DE.

Proof Since, AC is a diagonal of parallelogram $ABCD$.

So, ar (ΔABC) = ar (ΔACD) ...(ii)

Since, ΔABF and ΔABC are on the same base AB and between the same parallels AB and DF. [since, $AB \parallel DC$ and DC produced to F]

$\therefore$ ar (ΔABF) = ar (ΔABC) ...(iii)

From Eqs. (ii) and (iii),

$$\text{ar}(\Delta ABC) = \text{ar}(\Delta ACD) = \text{ar}(\Delta ABF)$$

On subtracting ar (ΔABE) from both sides of Eq. (iii), we get

$$\text{ar}(\Delta ABF) - \text{ar}(\Delta ABE) = \text{ar}(\Delta ABC) - \text{ar}(\Delta ABE)$$

$\Rightarrow$ ar (ΔBEF) = ar (ΔAEC) ...(iv)

Now, ar $(AECD)$ = ar (ACD) + ar (AEC)

= ar (ΔABC) + ar (ΔBEF) [from Eqs. (ii) and (iv)]

On adding ar (ΔCEF) both sides, we get

ar $(AECD)$ + ar (ΔCEF)

= ar (ΔABC) + ar (ΔBEF) + ar (ΔCEF)

$\Rightarrow$ ar (ΔADF) = ar $(ABFC)$ **Hence proved.**

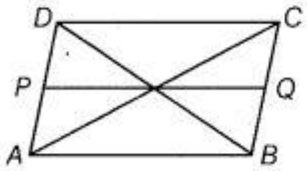
Question 2:
The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. Show that PQ divides the parallelogram into two parts of equal area.

Answer:

Given In a parallelogram *ABCD*, diagonals interect at *O* and draw a line *PQ*, which intersects *AD* and *BC*.

To prove *PQ* divides the parallelogram *ABCD* into two parts of equal area.

i.e., ar $(ABQP)$ = ar $(CDPQ)$



Proof We know that, diagonals of a parallelogram bisect each other.

$\therefore$ $OA = OC$ and $OB = OD$ ...(i)

In ΔAOB and ΔCOD,

$OA = OC$

$OB = OD$ [from Eq. (i)]

and $\angle AOB = \angle COD$ [vertically opposite angles]

$\therefore$ $\Delta AOB \cong \Delta COD$ [by SAS congruence rule]

Then, ar (ΔAOB) = ar (ΔCOD) ...(ii)

[since, congruent figures have equal area]

Now, in ΔAOP and ΔCOQ,

$\angle PAO = \angle OCQ$ [alternate interior angles]

$OA = OC$ [from Eq. (i)]

and $\angle AOP = \angle COQ$ [vertically opposite angles]

$\therefore$ $\Delta AOP \cong \Delta COQ$ [by ASA congruence rule]

$\therefore$ ar (ΔAOP) = ar (ΔCOQ) ...(iii)

[since, congruent figures have equal area]

Similarly, ar (ΔPOD) = ar (ΔBOQ) ...(iv)

Now, ar$(ABQP)$ = ar (ΔCOQ) + ar (ΔCOD) + ar (ΔPOD)

= ar (ΔAOP) + ar (ΔAOB) + ar (ΔBOQ)

[from Eqs. (ii), (iii) and (iv)]

$\Rightarrow$ ar $(ABQP)$ = ar $(CDPQ)$ **Hence proved.**

Question 3:

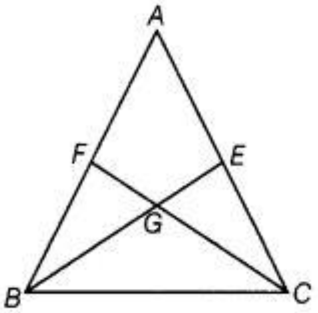
The median BE and CF of a triangle ABC intersect at G. Prove that the area of ΔGBC = area of the quadrilateral AFGE.

Answer:

Given In ΔABC, medians BE and CF intersect each other at G.

To prove $\quad ar(\Delta GBC) = ar(AFGE)$

Proof Since, BE is the median of ΔABC and we know that a median of a triangle divides it into two parts of equal area.



So, $\quad ar(\Delta ABE) = ar(\Delta CBE)$

$\Rightarrow \quad ar(\Delta ABE) = \frac{1}{2} ar(\Delta ABC) \quad$ ...(i)

Similarly, CF is the median of ΔABC.

Then, $\quad ar(\Delta ACF) = ar(\Delta BCF)$

$\Rightarrow \quad ar(\Delta BCF) = \frac{1}{2} ar(\Delta ABC) \quad$ ...(ii)

From Eqs. (i) and (ii),

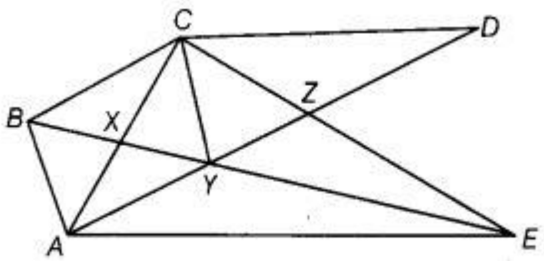
$ar(\Delta ABE) = ar(\Delta BCF) \quad$ ...(iii)

On subtracting $ar(\Delta GBF)$ from both sides of Eq. (iii), we get

$$ar(\Delta ABE) - ar(\Delta GBF) = ar(\Delta BCF) - ar(\Delta GBF)$$

$\Rightarrow \quad ar(AFGE) = ar(GBC) \quad$ **Hence proved.**

Question 4:
In figure, CD||AE and CY||BA. Prove that ar (ΔCBX) = ar (ΔAXY).

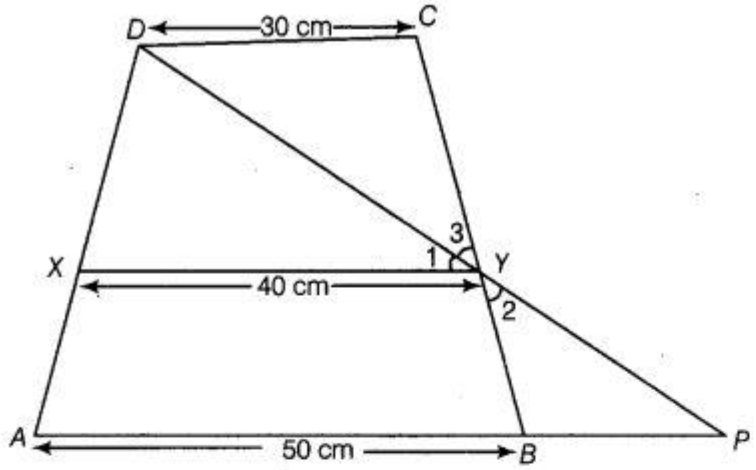


Answer:
Given In figure, CD||AE
and CY || BA
To prove ar (ΔCBX) = ar (ΔAXY) .
Proof We know that triangles on the same base and between the same parallels are equal. in areas.
Here, ΔABY and ΔABC both lie on the same base AB and between the same parallels CY and BA.
ar (ΔABY) = ar (ΔABC)
OR, ar (ABX) + ar (AXY) = ar (ABX) + ar (CBX)
OR, ar (AXY) = ar (CBX) [eliminating ar (ABX) from both sides]

Question 5:
ABCD is trapezium in which AB || DC, DC = 30 cm and AB = 50 cm. If X and Y are, respectively the mid-points of AD and BC, prove that ar (DCYX) = 7/9 ar (XYBA).
Answer:

Given In a trapezium $ABCD$, $AB \parallel DC$, $DC = 30$ cm and $AB = 50$ cm.

Also, X and Y are respectively the mid-points of AD and BC.



To prove $\quad \text{ar}(DCYX) = \frac{7}{9}\text{ar}(XYBA)$

Construction Join DY and extend it to meet produced AB at P.

Proof In ΔDCY and ΔPBY,

	$CY = BY$	[since, Y is the mid-point of BC]
	$\angle DCY = \angle PBY$	[alternate interior angles]
and	$\angle 2 = \angle 3$	[vertically opposite angles]
$\therefore$	$\Delta DCY \cong PBY$	[by ASA congruence rule]
Then,	$DC = BP$	[by CPCT]
But	$DC = 30$ cm	[given]
$\therefore$	$DC = BP = 30$ cm	
Now,	$AP = AB + BP$	
	$= 50 + 30 = 80$ cm	

In ΔADP, by mid-point theorem,

$$XY = \frac{1}{2}AP = \frac{1}{2} \times 80 = 40 \text{ cm}$$

Let distance between AB, XY and XY, DC is h cm.

Now, area of trapezium $DCYX = \frac{1}{2}h(30 + 40)$

[$\because$ area of trapezium $= \frac{1}{2}$ sum of parallel sides $\times$ distance between them]

$$= \frac{1}{2}h(70) = 35h \text{ cm}^2$$

Similarly, area of trapezium $XYBA = \frac{1}{2}h(40 + 50) = \frac{1}{2}h \times 90 = 45h \text{ cm}^2$

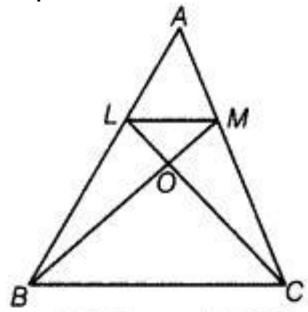
$$\therefore \quad \frac{\text{ar}(DCYX)}{\text{ar}(XYBA)} = \frac{35h}{45h} = \frac{7}{9}$$

$$\Rightarrow \quad \text{ar}(DCYX) = \frac{7}{9}\text{ar}(XYBA)$$

Hence proved.

Question 6:
In ΔABC, if L and M are the points on AB and AC, respectively such that LM || BC. Prove that ar (ΔLOB) = ar (ΔMOC).

Answer:
Given In ΔABC, L and M are points on AB and AC respectively such that LM || BC.



To prove ar (ΔLOB) = ar (ΔMOC)

Proof We know that, triangles on the same base and between the same parallels are equal in area.

Hence, ΔLBC and ΔMBC lie on the same base BC and between the same parallels BC and LM.

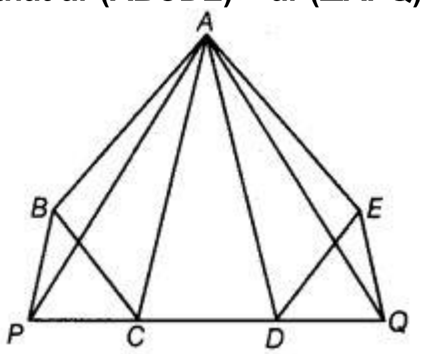
So, ar (ΔLBC) = ar (ΔMBC)

⇒ ar (ΔLOB) + ar (ΔBOC) = ar (ΔMOC) + ar (ΔBOC)

On eliminating D ar (ΔBOC) from both sides, we get

ar (ΔLOB) = ar (ΔMOC) **Hence proved.**

Question 7:
In the figure, ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that ar (ABCDE) = ar (ΔAPQ).

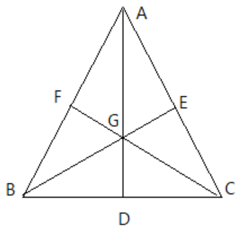


Answer:
Given that ABCDE is a pentagon.
BP || AC and EQ|| AD.
To prove ar (ABCDE) = ar (APQ)
Proof We know that triangles on the same base and between the same parallels are equal in area.
Here, ΔADQ and ΔADE lie on the same base AD and between the same parallels AD and EQ.
So, ar (ΔADQ) = ar (ΔADE) ...**(i)**
Similarly, ΔACP and ΔACB lie on the same base AC and between the same parallels AC and BP.

So, ar (ΔACP) = ar (ΔACB) …**(ii)**
On adding Eqs. **(i)** and **(ii)**, we get
ar (ΔADQ) + ar (ΔACP) = ar (ΔADE) + ar (ΔACB)
On adding ar (ΔACD) on both sides, we get
ar (ΔADQ) + ar (ΔACP) + ar (ΔACD) = ar (ΔADE) + ar (ΔACB) + ar (ΔACD)
=> ar (ΔAPQ) = ar (ABCDE) Hence proved.

Question 8:
If the medians of a AABC intersect at G, then show that ar (ΔAGB) = ar (ΔAGC) = ar (ΔBGC) = 1/3 ar(ΔABC).
Thinking Process
Use the property that the median of a triangle divides it into two triangles of equal area.
Further, apply the above property by considering different triangles and prove the required result.

Answer: Given In ΔABC, AD, BE and CF are medians and intersect at G.



To prove: ar(ΔAGB) = ar(ΔAGC) = ar(ΔBGC) = $\frac{1}{3}$ ar(ΔABC)

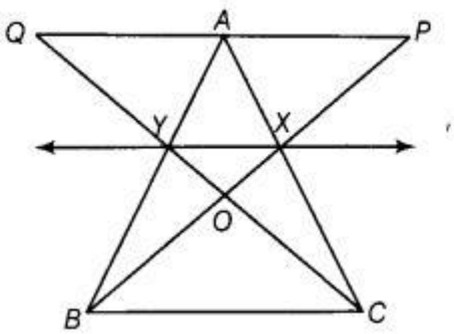
Proof: We know that a median of a triangle divides it into two triangles of equal area.
In (ΔABC), AD is a median.
Therefore, ar(ΔABD) = ar(ΔACD) ……………………………………………(1)
In (ΔBGC), GD is a median,
ar(ΔGBD) = ar(ΔGCD)……………………………………………………….(2)
On subtracting eq(2) from eq(1) we get,
ar(ΔABD) - ar(ΔGDB) = ar(ΔACD) - ar(ΔGCD)
or, ar(ΔAGB) = ar(ΔAGC)……………………………………………………..(3)

Similarly, ar(ΔAGB) = ar(ΔBGC)……………………………………………….(4)
From eq(3) and (4),
ar(ΔAGB) = ar(ΔBGC) = ar(ΔAGC)……………………………………………(5)

Now, ar(ΔABC) = ar(ΔAGB) + ar(ΔBGC) + ar(ΔAGC)
or, ar(ΔABC) = ar(ΔAGB) + ar(ΔAGB) + ar(ΔAGB) [From eq(5)]
or, ar(ΔABC) = 3 ar(ΔAGB)
or, ar(ΔAGB) = $\frac{1}{3}$ar(ΔABC)……………………………………………………..(6)

From eq(5) and (6),
ar(ΔBGC) = $\frac{1}{3}$ar(ΔABC)
ar(ΔAGC) = $\frac{1}{3}$ar(ΔABC)

Question 9:
In figure X and Y are the mid-points of AC and AB respectively, QP || BC and CYQ and BXP are straight lines. Prove that ar (ΔABP) = ar (ΔACQ).



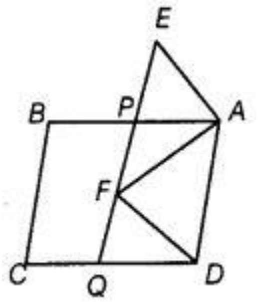
Thinking Process

1. Firstly, use the theorem that joining the mid-points of the two sides of a triangle is parallel to the third side.

2. Further, use the theorem that triangles on the same base and between the same
parallels are equal in area. Use this theorem by considering different triangles and prove the required result

Solution:
Given X and Y are the mid-points of AC and AB respectively. Also, QP|| BC and CYQ, BXP are straight lines.
To prove ar (ΔABP) = ar (ΔACQ)
Proof Since, X and Y are the mid-points” of AC and AB respectively.
So, XY || BC
We know that triangles on the same base and between the same parallels are equal in area. Here, ΔBYC and ΔBXC lie on the same base BC and between the same parallels BC and XY.
So, ar (ΔBYC) = ar (ΔBXC)
On subtracting ar (ΔBOC)from both sides, we get
ar (ΔBYC) – ar (ΔBOC) = ar (ΔBXC) – ar (ΔBOC)
=» ar (ΔBOY) = ar (ΔCOX)
On adding ar (ΔXOY) on both sides, we get
ar (ΔSOY) + ar (ΔXOY) = ar (ΔCOX) + ar (ΔXOY)
=> ar (ΔBYX) = ar (ΔCXY) …**(i)**
Hence, we observe that quadrilaterals XYAP and YXAQ are on the same base XY and between the same parallels XY and PQ.
ar (XYAP) = ar (YXAQ) …**(ii)**
On adding Eqs. **(i)** and **(ii)**, we get
ar (ΔBYX) + ar (XYAP) = ar (ΔCXY) + ar (YXAQ)
=> ar (ΔABP) = ar (ΔACQ) Hence proved.

Question 10:
In the figure, ABCD and AEFD are two parallelograms. Prove that ar (APEA) = ar(AQFO).



Answer:
Given, ABCD and AEFD are two parallelograms.
To prove ar (APEA) = ar (AQFD)
Proof In quadrilateral PQDA,
AP || DQ [since, in parallelogram ABCD, AB || CD] and PQ || AD [since, in parallelogram AEFD, FE || AD]
Then, quadrilateral PQDA is a parallelogram.
Also, parallelogram PQDA and AEFD are on the same base AD and between the same parallels AD and EQ.
ar (parallelogram PQDA) = ar (parallelogram AEFD)

On subtracting ar (quadrilateral APFD) from both sides, we get
ar (parallelogram PQDA)- ar (quadrilateral APFD)
= ar (parallelogram AEFD) – ar (quadrilateral APFD) => ar (AQFD) = ar (APEA)
Hence proved.