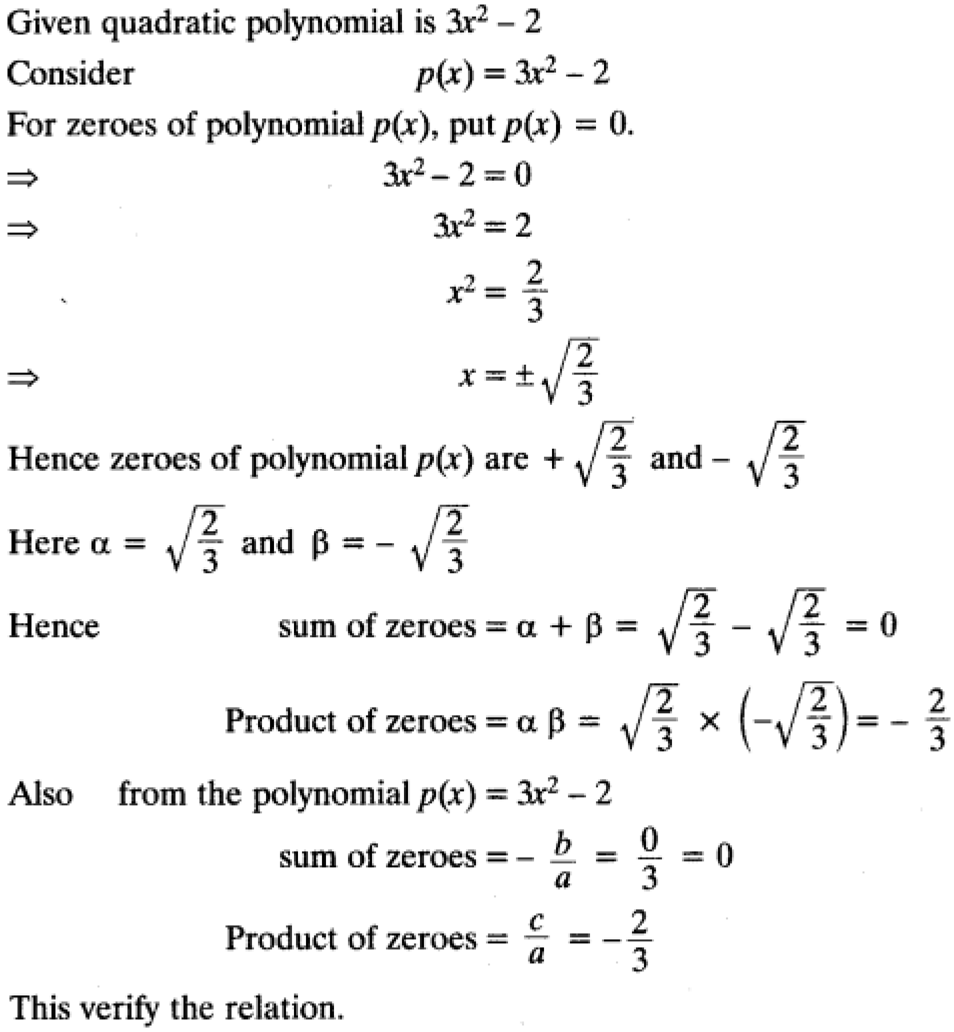
Chapter 2: Polynomials

Short Answer Type Questions II [3 Marks]

Question 1
Find the zeroes of the quadratic polynomial $3x^2 - 2$ and verify the relationship between the zeroes and the coefficients.
Answer:



Given quadratic polynomial is $3x^2 - 2$

Consider $\quad p(x) = 3x^2 - 2$

For zeroes of polynomial $p(x)$, put $p(x) = 0$.

$$\Rightarrow \quad 3x^2 - 2 = 0$$
$$\Rightarrow \quad 3x^2 = 2$$
$$x^2 = \frac{2}{3}$$
$$\Rightarrow \quad x = \pm\sqrt{\frac{2}{3}}$$

Hence zeroes of polynomial $p(x)$ are $+\sqrt{\frac{2}{3}}$ and $-\sqrt{\frac{2}{3}}$

Here $\alpha = \sqrt{\frac{2}{3}}$ and $\beta = -\sqrt{\frac{2}{3}}$

Hence $\quad$ sum of zeroes $= \alpha + \beta = \sqrt{\frac{2}{3}} - \sqrt{\frac{2}{3}} = 0$

$$\text{Product of zeroes} = \alpha\,\beta = \sqrt{\frac{2}{3}} \times \left(-\sqrt{\frac{2}{3}}\right) = -\frac{2}{3}$$

Also $\quad$ from the polynomial $p(x) = 3x^2 - 2$

$$\text{sum of zeroes} = -\frac{b}{a} = \frac{0}{3} = 0$$
$$\text{Product of zeroes} = \frac{c}{a} = -\frac{2}{3}$$

This verify the relation.

Question 2
On dividing $x^3 - 8x^2 + 20x - 10$ by a polynomial g(x), the quotient and the remainder were x – 4 and 6 respectively. Find g(x).

Answer:

Consider Divided: $f(x) = x^3 - 8x^2 + 20x - 10$

Quotient $q(x) = x - 4$

remainder $r(x) = 6$ and divisor is $g(x)$

Applying division algorithm, we get

$$f(x) = g(x).\, q(x) + r(x)$$

$$\Rightarrow \qquad g(x) = \frac{f(x) - r(x)}{q(x)} = \frac{(x^3 - 8x^2 + 20x - 10) - (6)}{x - 4}$$

$$= \frac{x^3 - 8x^2 + 20x - 16}{x - 4}$$

So, $\qquad g(x) = x^2 - 4x + 4$

$$
\begin{array}{rr}
 & x^2 - 4x + 4 \\ \cline{2-2}
x - 4 \big) & x^3 - 8x^2 + 20x - 16 \\
 & \underset{-}{x^3} \underset{+}{-} 4x^2 \qquad\qquad \\ \cline{2-2}
 & -4x^2 + 20x \\
 & \underset{+}{-} 4x^2 \underset{-}{+} 16x \\ \cline{2-2}
 & 4x - 16 \\
 & 4x - 16 \\
 & \underset{}{-} \quad + \\ \cline{2-2}
 & 0 \\ \cline{2-2}
\end{array}
$$

Question 3

Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

Answer:

α and β are the zeroes of the polynomial $2x^2 - 3x + 1$

$$\Rightarrow \qquad \alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$$

$$\alpha\beta = \frac{c}{a} = \frac{1}{2}$$

Now, zeroes of the required polynomial are 3α and 3β

$$\Rightarrow \qquad S = 3\alpha + 3\beta = 3(\alpha + \beta) = 3\left(\frac{3}{2}\right) = \frac{9}{2}$$

$$\Rightarrow \qquad P = (3\alpha)(3\beta) = 9(\alpha\beta) = 9 \times \frac{1}{2} = \frac{9}{2}$$

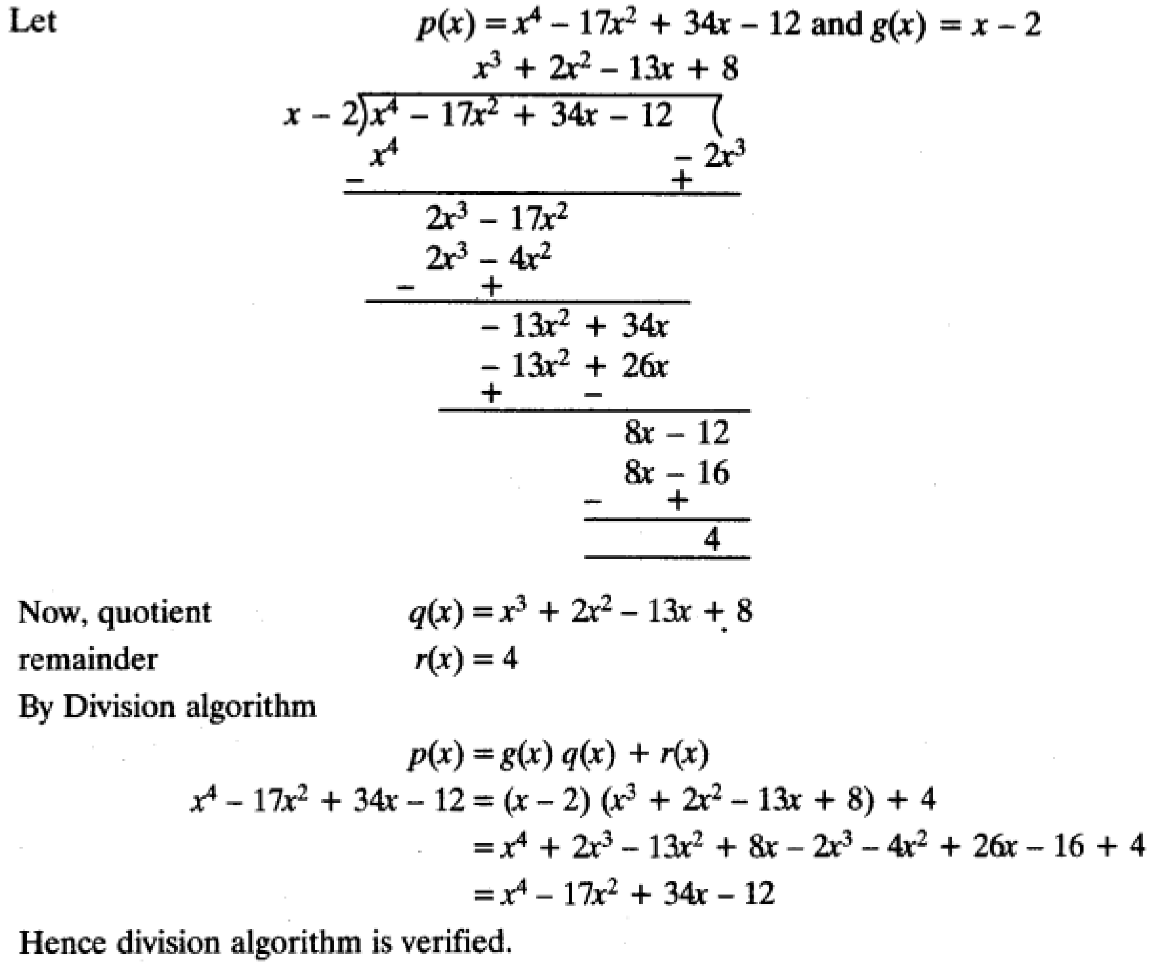
Now, required polynomial is $x^2 - Sx + p$

$$= x^2 - \frac{9}{2}x + \frac{9}{2} = \frac{k}{2}(2x^2 - 9x + 9), \text{ where } k \text{ be any constant.}$$

Question 4.

Divide the polynomial $x^4 - 11x^2 + 34x - 12$ by $x - 2$ and find the quotient and the remainder. Also, verify the division algorithm.

Answer:



Let $p(x) = x^4 - 17x^2 + 34x - 12$ and $g(x) = x - 2$

$$\begin{array}{r l}
 & x^3 + 2x^2 - 13x + 8 \\
x - 2 \big) & x^4 - 17x^2 + 34x - 12 \quad (\\
 & x^4 \qquad\qquad\qquad\quad - 2x^3 \\
 & - \qquad\qquad\qquad\quad\; + \\ \hline
 & \qquad 2x^3 - 17x^2 \\
 & \qquad 2x^3 - 4x^2 \\
 & \quad - \qquad + \\ \hline
 & \qquad\quad - 13x^2 + 34x \\
 & \qquad\quad - 13x^2 + 26x \\
 & \qquad\quad + \qquad - \\ \hline
 & \qquad\qquad\qquad 8x - 12 \\
 & \qquad\qquad\qquad 8x - 16 \\
 & \qquad\qquad\quad - \qquad + \\ \hline
 & \qquad\qquad\qquad\qquad 4 \\ \hline
\end{array}$$

Now, quotient $q(x) = x^3 + 2x^2 - 13x + 8$

remainder $r(x) = 4$

By Division algorithm

$$p(x) = g(x)\, q(x) + r(x)$$

$$x^4 - 17x^2 + 34x - 12 = (x - 2)\,(x^3 + 2x^2 - 13x + 8) + 4$$

$$= x^4 + 2x^3 - 13x^2 + 8x - 2x^3 - 4x^2 + 26x - 16 + 4$$

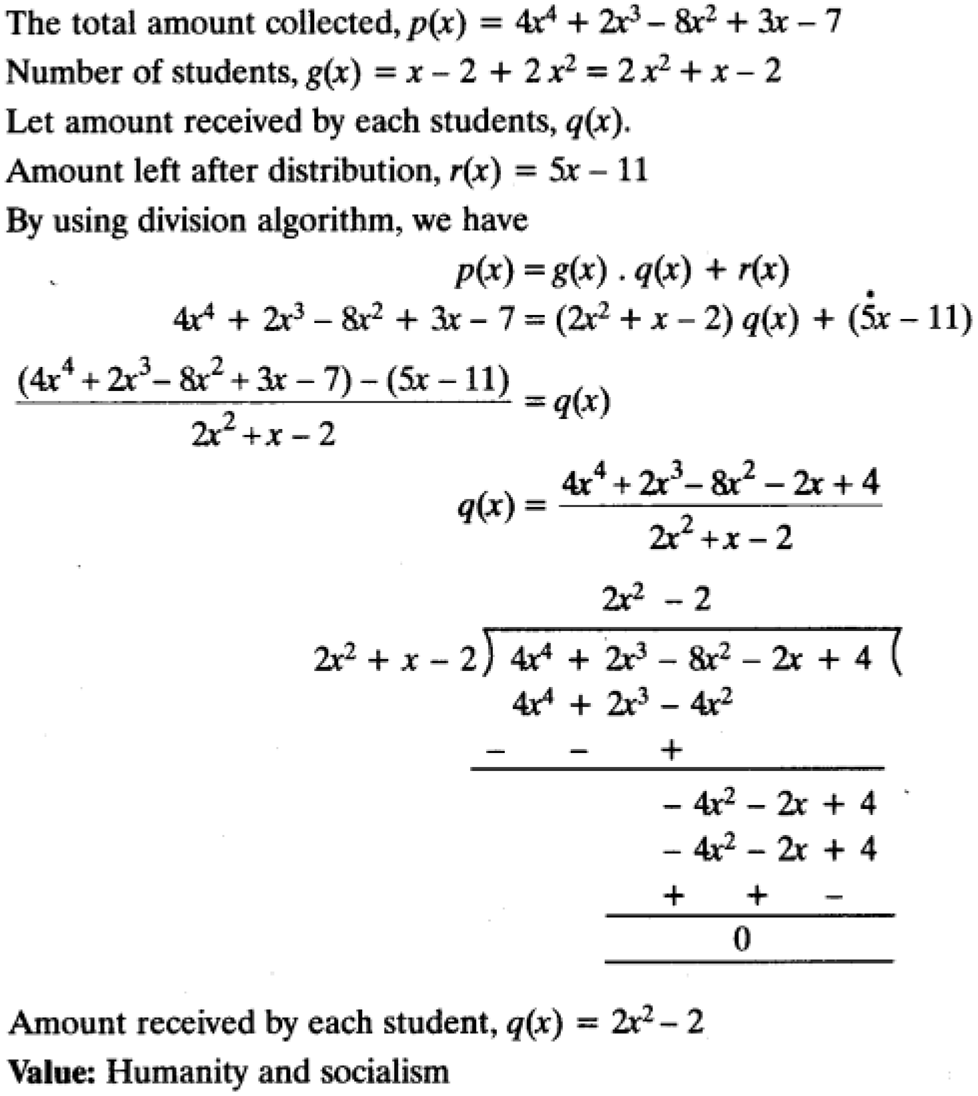
$$= x^4 - 17x^2 + 34x - 12$$

Hence division algorithm is verified.

Question 5.

An NGO decided to distribute books and pencils to the students of a school running by some other NGO. For this, they collected some amount from different people. The total amount collected is represented by $4x^4 + 2x^3 – 8x^2 + 3x – 7$. From this fund, each student received an equal amount. The number of students, who received the amount, is represented by $x – 2 + 2x^2$. After distribution, $5x - 11$, the amount is left with the NGO which they donated to the school for their infrastructure. Find the amount received by each student from the NGO. What value has been depicted here?

Answer:



The total amount collected, $p(x) = 4x^4 + 2x^3 - 8x^2 + 3x - 7$

Number of students, $g(x) = x - 2 + 2x^2 = 2x^2 + x - 2$

Let amount received by each students, $q(x)$.

Amount left after distribution, $r(x) = 5x - 11$

By using division algorithm, we have

$$p(x) = g(x) \cdot q(x) + r(x)$$

$$4x^4 + 2x^3 - 8x^2 + 3x - 7 = (2x^2 + x - 2)\, q(x) + (5x - 11)$$

$$\frac{(4x^4 + 2x^3 - 8x^2 + 3x - 7) - (5x - 11)}{2x^2 + x - 2} = q(x)$$

$$q(x) = \frac{4x^4 + 2x^3 - 8x^2 - 2x + 4}{2x^2 + x - 2}$$

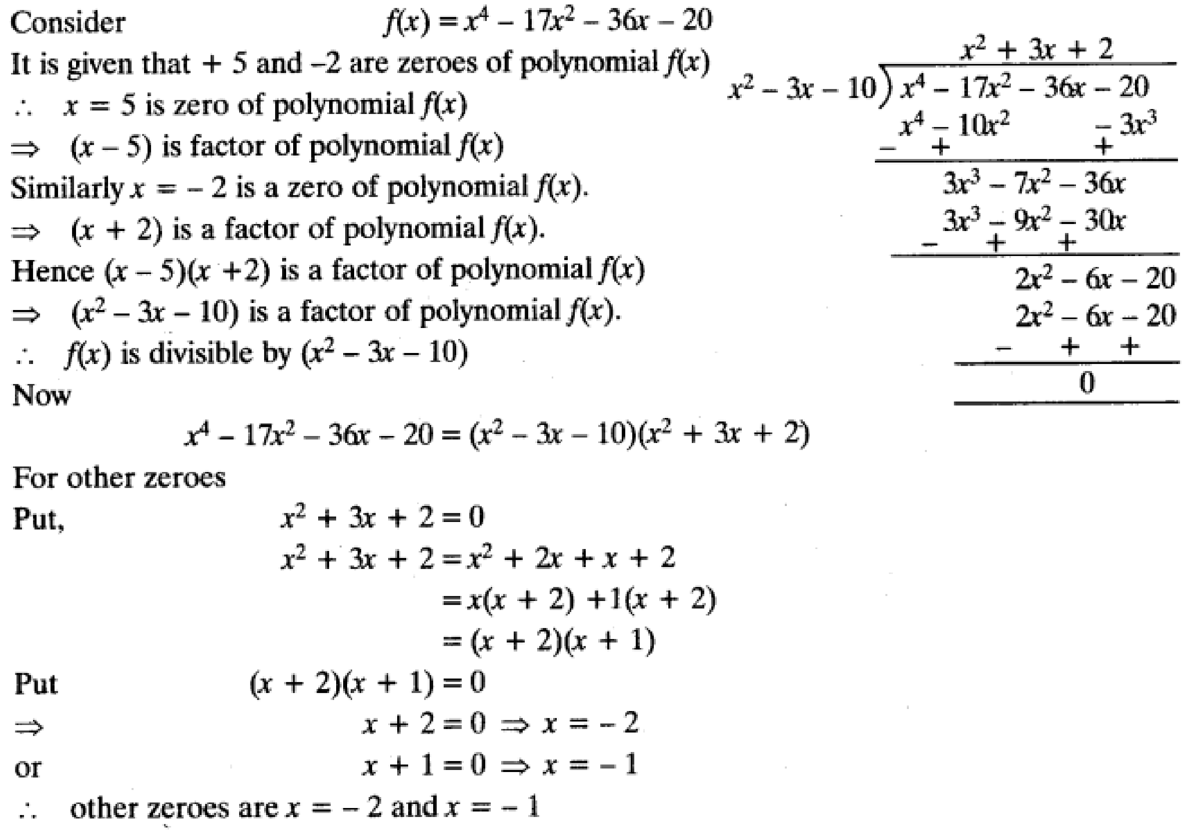
$$\begin{array}{r|l}
 & 2x^2 \quad - 2 \\
\hline
2x^2 + x - 2 & 4x^4 + 2x^3 - 8x^2 - 2x + 4 \;(\\
 & 4x^4 + 2x^3 - 4x^2 \\
 & - \quad - \quad + \\
\hline
 & \qquad\quad - 4x^2 - 2x + 4 \\
 & \qquad\quad - 4x^2 - 2x + 4 \\
 & \qquad\quad + \quad + \quad - \\
\hline
 & \qquad\qquad 0 \\
\hline
\end{array}$$

Amount received by each student, $q(x) = 2x^2 - 2$

Value: Humanity and socialism

Question 6.
Obtain all other zeroes of the polynomial $x^4 - 17x^2 - 36x - 20$, if two of its zeroes are + 5 and – 2.

Answer:



Consider $f(x) = x^4 - 17x^2 - 36x - 20$

It is given that + 5 and –2 are zeroes of polynomial $f(x)$

∴ $x = 5$ is zero of polynomial $f(x)$

⇒ $(x - 5)$ is factor of polynomial $f(x)$

Similarly $x = -2$ is a zero of polynomial $f(x)$.

⇒ $(x + 2)$ is a factor of polynomial $f(x)$.

Hence $(x - 5)(x + 2)$ is a factor of polynomial $f(x)$

⇒ $(x^2 - 3x - 10)$ is a factor of polynomial $f(x)$.

∴ $f(x)$ is divisible by $(x^2 - 3x - 10)$

$$\begin{array}{r|l}
 & x^2 + 3x + 2 \\
\hline
x^2 - 3x - 10 & x^4 - 17x^2 - 36x - 20 \\
 & x^4 \mp 10x^2 \qquad \mp 3x^3 \\
 & - \quad + \qquad\quad + \\
\hline
 & 3x^3 - 7x^2 - 36x \\
 & 3x^3 \mp 9x^2 \mp 30x \\
 & - \quad + \quad + \\
\hline
 & 2x^2 - 6x - 20 \\
 & 2x^2 - 6x - 20 \\
 & - \quad + \quad + \\
\hline
 & 0 \\
\hline
\end{array}$$

Now

$$x^4 - 17x^2 - 36x - 20 = (x^2 - 3x - 10)(x^2 + 3x + 2)$$

For other zeroes

Put, $x^2 + 3x + 2 = 0$

$$\begin{aligned} x^2 + 3x + 2 &= x^2 + 2x + x + 2 \\ &= x(x + 2) + 1(x + 2) \\ &= (x + 2)(x + 1) \end{aligned}$$

Put $(x + 2)(x + 1) = 0$

⇒ $x + 2 = 0 \Rightarrow x = -2$

or $x + 1 = 0 \Rightarrow x = -1$

∴ other zeroes are $x = -2$ and $x = -1$

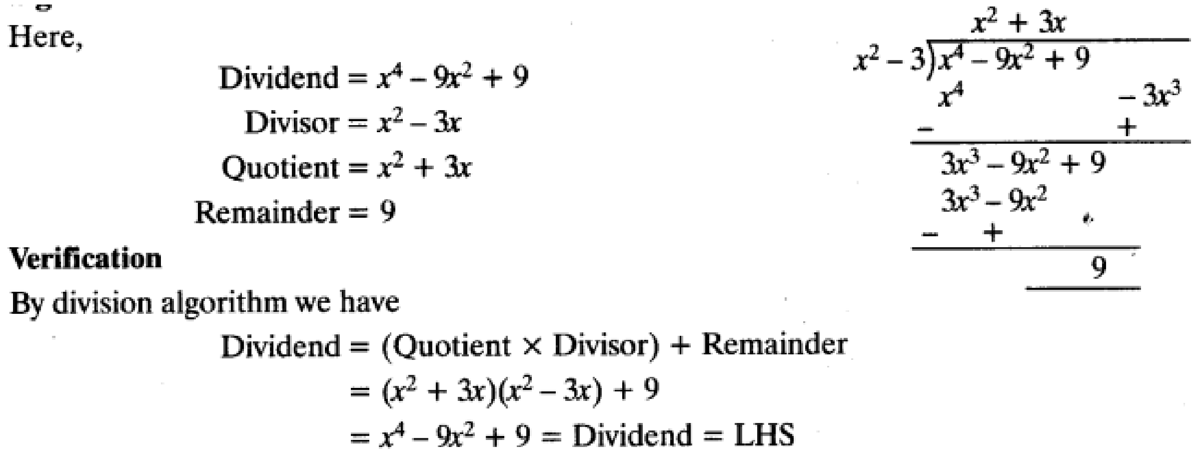
2015

Short Answer Type Questions II [3 Marks]

Question 7.

Divide the polynomial $x^4 – 9x^2 + 9$ by the polynomial x^2 -3x and verify the division algorithm.

Answer:



Here,

$$\begin{aligned} \text{Dividend} &= x^4 - 9x^2 + 9 \\ \text{Divisor} &= x^2 - 3x \\ \text{Quotient} &= x^2 + 3x \\ \text{Remainder} &= 9 \end{aligned}$$

$$\begin{array}{r|l}
 & x^2 + 3x \\
\hline
x^2 - 3 & x^4 - 9x^2 + 9 \\
 & x^4 \qquad\qquad - 3x^3 \\
 & - \qquad\qquad\quad + \\
\hline
 & 3x^3 - 9x^2 + 9 \\
 & 3x^3 - 9x^2 \\
 & - \quad + \\
\hline
 & 9 \\
\hline
\end{array}$$

Verification

By division algorithm we have

$$\begin{aligned} \text{Dividend} &= (\text{Quotient} \times \text{Divisor}) + \text{Remainder} \\ &= (x^2 + 3x)(x^2 - 3x) + 9 \\ &= x^4 - 9x^2 + 9 = \text{Dividend} = \text{LHS} \end{aligned}$$

Question 8.

If one zero of the quadratic polynomial f(x) = $4x^2 – 8kx + 8x – 9$ is negative of the other, then find the zeroes of $kx^2 + 3kx + 2$.

Answer:

$$f(x) = 4x^2 - 8kx + 8x - 9 = 4x^2 - (8k - 8)x - 9$$

Let one zero of $f(x)$ be α then other zero be $-\alpha$.

So, sum of zeroes = 0

$$\Rightarrow \quad \frac{8k - 8}{4} = 0 \Rightarrow 8k - 8 = 0 \Rightarrow k = 1$$

Now, other given polynomial is $p(x) = kx^2 + 3kx + 2$

$$= x^2 + 3x + 2$$

$$= (x + 2)(x + 1)$$

So, zeroes of $p(x)$ are -1 and -2.

Question 9.

An NGO decided to distribute books and pencils to the students of a school running by some other NGO. For this, they collected some amount from a different number of people. The total amount collected is represented by $4\ x^4 + 2x^3 - 8x^2 + 3x - 7$. . The amount is equally divided between each of the students. The number of students, who received the amount is represented by $x - 2 + 2x^2$. After distribution, 5x- 11, the amount is left with the NGO which they donated to the school for their infrastructure. Find the amount received by each student from the NGO.

What value has been depicted here?

Answer:

$$
\begin{array}{r l}
 & \quad 2x^2 - 2 \\
2x^2 + x - 2 \big) & 4x^4 + 2x^3 - 8x^2 + 3x - 7 \\
 & 4x^4 + 2x^3 - 4x^2 \\
 & - \quad\; - \quad\; + \\
\hline
 & \qquad\qquad -4x^2 + 3x - 7 \\
 & \qquad\qquad -4x^2 - 2x + 4 \\
 & \qquad\qquad + \quad\; + \quad\; - \\
\hline
 & \qquad\qquad\qquad 5x - 11 \\
\hline
\end{array}
$$

So, the amount received by each student = ₹$(2x^2 - 2)$

The distribution of books and pencils by NGO to school students shows the helping nature of NGO. These activities boost the students.

2014

Short Answer Type Questions I [2 Marks]

Question 10.

If the product of zeroes of the polynomial $ax^2 - 6x - 6$ is 4, find the value of a. Find the sum of zeroes of the polynomial.

Answer:

Let α, β be the zeroes of given polynomial $p(x) = ax^2 - 6x - 6$.

Then,

$$\alpha\beta = \frac{-6}{a} \Rightarrow 4 = \frac{-6}{a} \Rightarrow a = \frac{-6}{4} = \frac{-3}{2}$$

Thus, $a = \frac{-3}{2}$.

Now, sum of zeroes $= \frac{6}{a} = \frac{6}{\left(\frac{-3}{2}\right)} = \frac{2 \times 6}{-3} = -4$

Question 11.

Find the zeroes of the quadratic polynomial $9t^2$-6t + 1 and verify the relationship between the zeroes and the coefficients.

Answer:

$f(t) = 9t^2 - 6t + 1 = (3t - 1)^2$

So, $\frac{1}{3}, \frac{1}{3}$ are the zeroes of $f(t)$.

Now, sum of zeroes $= \frac{1}{3} + \frac{1}{3} = \frac{2}{3} = -\frac{(\text{coefficient of } t)}{(\text{coefficient of } t^2)}$

and product of zeroes $= \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{9} = \frac{\text{constant term}}{\text{coefficient of } t^2}$

Hence, verified the relationship between the zeroes and the coefficients.

Question 12.

When a polynomial $6x^4 + 8x^3 + 290x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$ the remainder is in the form ax + b. Find a and b.

Answer:

$$\begin{array}{r}
2x^2 + 9 \\
3x^2 + 4x + 1 \overline{)\, 6x^4 + 8x^3 + 29x^2 + 21x + 7} \\
6x^4 + 8x^3 + 2x^2 \\
- \qquad - \qquad - \\
\hline
27x^2 + 21x + 7 \\
27x^2 + 36x + 9 \\
- \qquad - \qquad - \\
\hline
-15x - 2 \\
\hline
\end{array}$$

$\because ax + b$ is given as remainder.

So, $ax + b = -15x - 2 \Rightarrow a = -15$ and $b = -2$

Short Answer Type Questions II [3 Marks]

Question 13.
Obtain all other zeroes of the polynomial $x^4 + 4x^3 - 2x^2 - 20x - 15$ if two of its zeroes are √5 and -√5

Answer:

$\because \sqrt{5}$ and $-\sqrt{5}$ are the zeroes of $p(x) = x^4 + 4x^3 - 2x^2 - 20x - 15$.

$\therefore (x - \sqrt{5})(x + \sqrt{5})$ *i.e.* $(x^2 - 5)$ is the factor of $p(x)$.

To find other factor,

$$
\begin{array}{rl}
 & \qquad\quad x^2 + 4x + 3 \\ \cline{2-2}
x^2 - 5 \big) & x^4 + 4x^3 - 2x^2 - 20x - 15 \\
 & x^4 \qquad\quad - 5x^2 \\
 & - \qquad\quad + \\ \cline{2-2}
 & \qquad 4x^3 + 3x^2 - 20x - 15 \\
 & \qquad 4x^3 \qquad\quad - 20x \\
 & \qquad - \qquad\quad + \\ \cline{2-2}
 & \qquad\qquad 3x^2 \qquad - 15 \\
 & \qquad\qquad 3x^2 \qquad - 15 \\
 & \qquad\qquad - \qquad\quad + \\ \cline{2-2}
 & \qquad\qquad\qquad 0 \\ \cline{2-2}
\end{array}
$$

So, $\quad x^4 + 4x^3 - 2x^2 - 20x - 15 = (x^2 - 5)(x^2 + 4x + 3)$
$\qquad\qquad\qquad\qquad\qquad\qquad\quad = (x^2 - 5)(x + 3)(x + 1)$

$\therefore$ Other zeroes of $p(x)$ are -1 and -3.

Question 14.
If α and β are zeroes of a polynomial $x^2 + 6x + 9$, then form a polynomial whose zeroes are $-\alpha$ and $-\beta$.

Answer:
Let $p(x) = x^2 + 6x + 9$
$\because \alpha, \beta$ are the zeroes of $p(x)$
So, sum of zeroes, $\alpha + \beta = -6$ and product of zeroes $\alpha\beta = 9$.
Now, $-\alpha$ and $-\beta$ are the zeroes of required quadratic polynomial.
So, sum of zeroes of required polynomial $= -\alpha - \beta = -(\alpha + \beta) = +6$
and product of zeroes of required polynomial $= (-\alpha)(-\beta) = \alpha\beta = 9$.
$\therefore$ Required quadratic polynomial is given by
$q(x) = x^2 -$ (sum of zeroes)x + product of zeroes
$\qquad = x^2 - 6x + 9$

Question 15.
Find the zeroes of the quadratic polynomial $3x^2 - 2$ and verify the relationship between the zeroes and the coefficients.

Answer:

$$p(x) = 3x^2 - 2 = (\sqrt{3}x)^2 - (\sqrt{2})^2 = (\sqrt{3}x - \sqrt{2})(\sqrt{3}x + \sqrt{2})$$

So, zeroes of $p(x)$ are $\frac{\sqrt{2}}{\sqrt{3}}$ and $\frac{-\sqrt{2}}{\sqrt{3}}$.

Now,

$$\text{Sum of zeroes} = \frac{\sqrt{2}}{\sqrt{3}} - \frac{\sqrt{2}}{\sqrt{3}} = 0 = -\frac{(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

$$\text{and product of zeroes} = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)\left(\frac{-\sqrt{2}}{\sqrt{3}}\right) = \frac{-2}{3} = \frac{\text{constant term}}{(\text{coefficient of } x^2)}$$

Hence, verified the relationship between the zeroes and the coefficients.

Question 16.

If a polynomial $x^4 – 3x^3 – 6x^2 + kx – 16$ is exactly divisible by $x^2 – 3x + 2$, then find the value of A.

Answer:

Let $p(x) = x^4 - 3x^3 - 6x^2 + kx - 16$ and $g(x) = x^2 - 3x + 2 = (x - 1)(x - 2)$

$\because$ $p(x)$ is divisible by $g(x)$

So, 1 and 2 are zeroes of $p(x)$.

$\therefore p(1) = 0$

$$\begin{aligned}
\Rightarrow \quad (1)^4 - 3(1)^3 - 6(1)^2 + k(1) - 16 &= 0 \\
\Rightarrow \quad 1 - 3 - 6 + k - 16 &= 0 \\
\Rightarrow \quad k - 24 &= 0 \\
\Rightarrow \quad k &= 24
\end{aligned}$$

Thus, the value of k is 24.

Question 17.

Obtain all other zeroes of the polynomial $x^4 – 17x^2 – 36x – 20$, if two of its zeroes are 5 and – 2.

Answer:

$\because$ 5 and – 2 are the zeroes of $p(x) = x^4 - 17x^2 - 36x - 20$

$\therefore$ $(x - 5)(x + 2)$ or $(x^2 - 3x - 10)$ is the factor of $p(x)$.

$$\begin{array}{r|l}
 & x^2 + 3x + 2 \\
\hline
x^2 - 3x - 10 & x^4 - 17x^2 - 36x - 20 \\
 & x^4 - 10x^2 \qquad - 3x^3 \\
 & - \quad + \qquad\qquad + \\
\cline{2-2}
 & 3x^3 - 7x^2 \quad - 36x - 20 \\
 & 3x^3 - 9x^2 \quad - 30x \\
 & - \quad + \quad + \\
\cline{2-2}
 & \quad 2x^2 \quad - \quad 6x - 20 \\
 & \quad 2x^2 \quad - \quad 6x - 20 \\
 & \quad - \quad + \quad + \\
\cline{2-2}
 & \qquad 0 \\
\cline{2-2}
\end{array}$$

So, $\qquad x^4 - 17x^2 - 36x - 20 = (x^2 - 3x - 10)(x^2 + 3x + 2)$

$\qquad\qquad = (x^2 - 3x - 10)(x + 1)(x + 2)$

$\therefore$ Other zeroes of $p(x)$ are – 2 and – 1.

Question 18.

Obtain all other zeroes of the polynomial $x^4 – 3\sqrt{2}x^3 – 3x^2 + 3\sqrt{2}x – 4$, if two of its zeroes are $\sqrt{2}$ and $2\sqrt{2}$.

Answer:

$\because$ $\sqrt{2}$ and $2\sqrt{2}$ are the zeroes of $p(x) = x^4 - 3\sqrt{2}x^3 + 3x^2 + 3\sqrt{2}x - 4$

$\therefore$ $(x - \sqrt{2})(x - 2\sqrt{2})$ or $(x^2 - 3\sqrt{2}x + 4)$ is the factor of $p(x)$

$$\begin{array}{r|l}
 & x^2 - 1 \\
\hline
x^2 - 3\sqrt{2}x + 4 & x^4 - 3\sqrt{2}x^3 + 3x^2 + 3\sqrt{2}x - \sqrt{4} \\
 & x^4 - 3\sqrt{2}x^3 + 4x^2 \\
 & - \quad + \qquad - \\
\cline{2-2}
 & \qquad - x^2 + 3\sqrt{2}x - 4 \\
 & \qquad - x^2 + 3\sqrt{2}x - 4 \\
 & \qquad + \quad - \quad + \\
\cline{2-2}
 & \qquad\qquad 0 \\
\cline{2-2}
\end{array}$$

So, $x^4 - 3\sqrt{2}x^3 + 3x^2 + 3\sqrt{2}x - 4 = (x^2 - 3\sqrt{2}x + 4)(x^2 - 1) = (x^2 - 3\sqrt{2}x + 4)(x - 1)(x + 1)$

$\therefore$ Other zeroes of $p(x)$ are – 1 and 1.

Question 19.

Divide the polynomial $3x^3– 2x^2 + 5x – 5$ by $3x + 1$ and verify the division algorithm.

Answer:

Here, Dividend = $3x^3 - 2x^2 + 5x - 5$

Divisor = $3x + 1$

Quotient = $x^2 - x + 2$

Remainder = -7

Division algorithm is

$$\text{Dividend} = (\text{Divisor}) \times (\text{Quotient}) + \text{Remainder}$$

Now, $\text{RHS} = (\text{Divisor}) \times (\text{Quotient}) + (\text{Remainder})$

$$= (3x + 1)(x^2 - x + 2) + (-7)$$
$$= 3x^3 - 3x^2 + 6x + x^2 - x + 2 - 7$$
$$= 3x^3 - 2x^2 + 5x - 5 = \text{Dividend} = \text{LHS}$$

Hence, division algorithm is verified.

$$\begin{array}{r|l} & x^2 - x + 2 \\ \hline 3x + 1 & 3x^3 - 2x^2 + 5x - 5 \\ & 3x^3 + x^2 \\ & - \quad\; - \\ \hline & -3x^2 + 5x - 5 \\ & -3x^2 - x \\ & + \quad\;\; + \\ \hline & 6x - 5 \\ & 6x + 2 \\ & - \quad\; - \\ \hline & -7 \\ \hline \end{array}$$

2013

Short Answer Type Questions I [2 Marks]

Question 20.
Find a quadratic polynomial whose zeroes are 3 + √2 and 3 – √2.
Answer:

Sum of zeroes = $(3+\sqrt{2})+(3-\sqrt{2}) = 6$

Product of zeroes = $(3+\sqrt{2})(3-\sqrt{2}) = 7$

∴ Required quadratic polynomial is x^2 – (sum of zeroes)x + product of zeroes, i.e. $x^2 - 6x + 7$.

Question 21.
Find a quadratic polynomial whose zeroes are $\frac{3+\sqrt{5}}{5}$ **and** $\frac{3-\sqrt{5}}{5}$.
Answer:

Sum of zeroes = $\frac{3+\sqrt{5}}{5} + \frac{3-\sqrt{5}}{5} = \frac{6}{5}$

Product of zeroes = $\left(\frac{3+\sqrt{5}}{5}\right)\left(\frac{3-\sqrt{5}}{5}\right) = \frac{9-5}{25} = \frac{4}{25}$

∴ Required quadratic polynomial is x^2 – (sum of zeroes)x + product of zeroes,

i.e. $x^2 - \frac{6}{5}x + \frac{4}{25}$ or $25x^2 - 30x + 4$.

Question 22.
Verify whether 2,3 and 1/2 are the zeroes of the polynomial

Answer:

First get the factors of $p(x)$.

$$p(x) = 2x^3 - 11x^2 + 17x - 6 = (x-2)(x-3)(2x-1)$$

So, zeroes of $p(x)$ are 2, 3 and $\frac{1}{2}$.

Alternative Method:

$$p(x) = 2x^3 - 11x^2 + 17x - 6$$

Now, $$p(2) = 2(2)^3 - 11(2)^2 + 17(2) - 6 = 16 - 44 + 34 - 6 = 0$$

$$p(3) = 2(3)^3 - 11(3)^2 + 17(3) - 6 = 54 - 99 + 51 - 6 = 0$$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 11\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6 = \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6 = 0$$

$\because$ $$p(2) = p(3) = p\left(\frac{1}{2}\right) = 0 \; \therefore 2, 3 \text{ and } \frac{1}{2} \text{ are the zeroes of } p(x).$$

Question 23.

Obtain all other zeroes of the polynomial $x^4 + 4x^3 - 2x^2 - 20x - 15$ if two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

Answer:

Here,

Dividend $= x^4 - 9x^2 + 9$

Divisor $= x^2 - 3x$

Quotient $= x^2 + 3x$

Remainder $= 9$

$$\begin{array}{r|l} & x^2 + 3x \\ \hline x^2 - 3 & x^4 - 9x^2 + 9 \\ & x^4 \quad\quad - 3x^3 \\ & - \quad\quad\quad + \\ \hline & 3x^3 - 9x^2 + 9 \\ & 3x^3 - 9x^2 \\ & - \quad + \\ \hline & \quad\quad 9 \end{array}$$

Verification

By division algorithm we have

$$\text{Dividend} = (\text{Quotient} \times \text{Divisor}) + \text{Remainder}$$
$$= (x^2 + 3x)(x^2 - 3x) + 9$$
$$= x^4 - 9x^2 + 9 = \text{Dividend} = \text{LHS}$$

2012

Short Answer Type Questions I [2 Marks]

Question 24.

If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the value of p and q.

Answer:

Let α, β are zeroes of $2x^2 - 5x - 3$.

$$\therefore \alpha + \beta = \frac{5}{2}, \alpha\beta = \frac{-3}{2}$$

A.T.Q.,

It is given that 2α and 2β are zeroes of $x^2 + px + q$

$$\therefore \quad 2\alpha + 2\beta = -p \Rightarrow 2 \times (\alpha + \beta) = -p$$

$$2 \times \frac{5}{2} = -p \Rightarrow p = -5$$

$$\text{and} \quad 2\alpha \times 2\beta = q \Rightarrow 4\,\alpha\beta = q$$

$$\Rightarrow \quad 4 \times \left(\frac{-3}{2}\right) = q \Rightarrow q = -6$$

Question 25.

Show that 1/2 and -3/2 are the zeroes of the polynomial $4x^2 + 4x- 3$ and verify the relationship between zeroes and coefficients of the polynomial.

Answer:

$$\text{Here} \quad p(x) = 4x^2 + 4x - 3 = (2x + 3)(2x - 1)$$

$\therefore$ Zeroes of $p(x)$ are $\frac{1}{2}$ and $\frac{-3}{2}$.

Now,

$$\text{Sum of zeroes} = \frac{1}{2} - \frac{3}{2} = -1 = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$$

$$\text{and product of zeroes} = \left(\frac{1}{2}\right)\left(\frac{-3}{2}\right) = \frac{-3}{4} = \frac{(\text{constant term})}{(\text{coefficient of } x^2)}$$

Hence, verified the relationships between the zeroes and the coefficients.

Question 26.

Find the value of b for which (2x + 3) is a factor of $2x^3 + 9x^2 – x-b$.

Answer:

$p(x) = 2x^3 + 9x^2 - x - b$

$2x + 3$ is a factor of $p(x)$.

$$\therefore \quad p\left(\frac{-3}{2}\right) = 0 \Rightarrow 2\left(\frac{-3}{2}\right)^3 + 9\left(\frac{-3}{2}\right)^2 - \left(\frac{-3}{2}\right) - b = 0$$

$$\Rightarrow \quad 2 \times \left(\frac{-27}{8}\right) + \frac{81}{4} + \frac{3}{2} - b = 0 \Rightarrow b = \frac{-27}{4} + \frac{81}{4} + \frac{3}{2} = 15$$

Question 27.

Given that $x-\sqrt{5}$ is a factor of the polynomial $x^3 – 3 -\sqrt{5} x^2 – 5x + 15\sqrt{5}$, find all the zeroes of the polynomial.

Answer:

$$
\begin{array}{r|l}
 & x^2 - 2\sqrt{5}x - 15 \\
\hline
x - \sqrt{5} & x^3 - 3\sqrt{5}x^2 - 5x + 15\sqrt{5} \\
 & x^3 - \sqrt{5}x^2 \\
 & - \quad + \\
\hline
 & -2\sqrt{5}x^2 - 5x + 15\sqrt{5} \\
 & -2\sqrt{5}x^2 + 10x \\
 & + \quad - \\
\hline
 & -15x + 15\sqrt{5} \\
 & -15x + 15\sqrt{5} \\
 & + \quad - \\
\hline
 & \times \\
\hline
\end{array}
$$

Question 28.
If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by ($x^2 - 2x + k$) the remainder comes out to be x+ a, find k and a.
Answer:

$P(x) = x^4 - 6x^3 + 16x^2 - 25x + 10,$

$g(x) = x^2 - 2x + k$

$$
\begin{array}{r|l}
 & x^2 - 4x + (8 - k) \\
\hline
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + (16 - k)x^2 - 25x + 10 \\
 & -4x^3 + 8x^2 \qquad - 4kx \\
 & + \quad - \qquad + \\
\hline
 & (8 - k)x^2 + (4k - 25)x + 10 \\
 & (8 - k)x^2 - (16 - 2k)x + 8k - k^2 \\
 & - \qquad + \qquad - \quad + \\
\hline
 & (2k - 9)x + (10 - 8k - k^2) \\
\hline
\end{array}
$$

$\because$ Remainder is given as $x + a$

$$
\begin{array}{lrcl}
\therefore & x + a & = & (2k - 9)x + (10 - 8k + k^2) \\
\Rightarrow & 2k - 9 & = & 1 \\
\text{and} & k^2 - 8k + 10 & = & a \\
\Rightarrow & 2k & = & 10 \\
\Rightarrow & k & = & 5 \\
\text{and} & 5^2 - 8(5) + 10 & = & a \\
\Rightarrow & a & = & 25 - 40 + 10 \\
\Rightarrow & a & = & -5
\end{array}
$$

Thus, $a = -5$ and $k = 5$

Question 29.
What must be subtracted or added to $p(x) = 8x^4 + 14x^3 - 2x^2 + 8x - 12$ so that $4x^2 + 3x - 2$ is a factor of $p(x)$?
Answer:

$$
\begin{array}{r|l}
 & 2x^2+2x-1 \\ \cline{2-2}
4x^2+3x-2 & 8x^4+14x^3-2x^2+8x-12 \\
 & 8x^4+6x^3-4x^2 \\
 & - \quad - \quad + \\ \cline{2-2}
 & 8x^3+2x^2+8x-12 \\
 & 8x^3+6x^2-4x \\
 & - \quad - \quad + \\ \cline{2-2}
 & -4x^2+12x-12 \\
 & -4x^2-3x+2 \\
 & + \quad + \quad - \\ \cline{2-2}
 & 15x-14 \\ \cline{2-2}
\end{array}
$$

Remainder $= 15x - 14$

$\therefore$ If we subtract $15x - 14$ or add $-15x + 14$ then remainder will be 0.

Then $4x^2 + 3x - 2$ will be a factor of given polynomial.

2011

Short Answer Type Questions I [2 Marks]

Question 30.
Divide $x^4 - 3x^2 + 4x + 5$ by $x^2 - x + 1$, find quotient and remainder.
Answer:

$$
\begin{array}{r|l}
 & x^2+x-3 \\ \cline{2-2}
x^2-x+1 & x^4-3x^2+4x+5 \\
 & x^4+x^2 \quad -x^3 \\
 & - \quad - \quad + \\ \cline{2-2}
 & x^3-4x^2+4x+5 \\
 & x^3-x^2+x \\
 & - \quad + \quad - \\ \cline{2-2}
 & -3x^2+3x+5 \\
 & -3x^2+3x-3 \\
 & + \quad - \quad + \\ \cline{2-2}
 & 8 \\ \cline{2-2}
\end{array}
$$

Quotient $= x^2 + x - 3$, Remainder $= 8$

Question 31.
If 2 and -3 are the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$; then find the values of a and b.

Answer:

$p(x) = x^2 + (a + 1)x + b$

$\because$ 2 is a zero of $p(x)$

$$\begin{aligned} \Rightarrow \quad & & p(2) &= 0 \\ \Rightarrow \quad & & 2^2 + (a + 1)\,2 + b &= 0 \\ \Rightarrow \quad & & 2a + b &= -6 \\ \text{Also,} \quad & & p(-3) &= 0 \\ \Rightarrow \quad & & (-3)^2 + (a + 1)(-3) + b &= 0 \\ \Rightarrow \quad & & -3a + b &= -6 \end{aligned}$$

Solving equation (*i*) and (*ii*), we get $a = 0, b = -6$

Question 32.
It is given that 1 is one of the zeroes of the polynomial $7x – x^3 – 6$. Find its other zeroes.

Answer:

$$\begin{array}{r|l} & -x^2 - x + 6 \\ \cline{2-2} x - 1 & -x^3 + 7x - 6 \\ & -x^3 \qquad\quad +x^2 \\ & + \qquad\qquad\; - \\ \cline{2-2} & -x^2 + 7x - 6 \\ & -x^2 + x \\ & + \quad\; - \\ \cline{2-2} & \qquad 6x - 6 \\ & \qquad 6x - 6 \\ & \qquad - \quad + \\ \cline{2-2} & \qquad\quad 0 \\ \cline{2-2} \end{array}$$

$p(x) = 7x - x^3 - 6$

$\because$ 1 is a zero of $p(x) \Rightarrow (x - 1)$ is a factor of $p(x)$

For other zeroes,

$$\begin{aligned} \therefore \qquad 7x - x^3 - 6 &= (x - 1)(-x^2 - x + 6) \\ &= (1 - x)(x^2 + x - 6) \\ &= (1 - x)(x + 3)(x - 2) \end{aligned}$$

$\therefore$ Other two zeroes of $p(x)$ are 2 and -3.

Short Answer Type Questions II [3 Marks]

Question 33.
If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, then what will be the quotient and remainder?

Answer:

$$\begin{array}{r|l}
 & \quad 2x^2 + 5 \\ \hline
3x^2 + 4x + 1 & 6x^4 + 8x^3 + 17x^2 + 21x + 7 \\
 & 6x^4 + 8x^3 + 2x^2 \\
 & - \quad\; - \quad\;\; - \\ \hline
 & \qquad\qquad 15x^2 + 21x + 7 \\
 & \qquad\qquad 15x^2 + 20x + 5 \\
 & \qquad\quad - \quad\;\; - \quad\;\; - \\ \hline
 & \qquad\qquad\qquad\quad x + 2 \\ \hline
\end{array}$$

Quotient $= 2x^2 + 5$, remainder $= x + 2$

Question 34

On dividing the polynomial $4x^4 – 5x^3 – 39x^2 – 46x – 2$ by the polynomial g(x), the quotient and remainder were $x^2 – 3x – 5$ and $-5x + 8$ respectively. Find g(x).

Answer:

$p(x) = 4x^4 - 5x^3 - 39x^2 - 46x - 2$

$q(x) = x^2 - 3x - 5, r(x) = -5x + 8$

According to division algorithm,

$$p(x) = g(x) \cdot q(x) + r(x) \Rightarrow p(x) - r(x) = g(x) \cdot q(x)$$

$$\Rightarrow \qquad \frac{p(x) - r(x)}{q(x)} = g(x)$$

$$\Rightarrow \qquad g(x) = \frac{(4x^4 - 5x^3 - 39x^2 - 46x - 2) - (-5x + 8)}{x^2 - 3x - 5} = \frac{4x^4 - 5x^3 - 39x^2 - 41x - 10}{x^2 - 3x - 5}$$

$$= 4x^2 + 7x + 2$$

Long Answer Type Questions [4 Marks]

Question 35.

Find other zeroes of the polynomial $x^4 – 7x^2 + 12$ if it is given that two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

Answer:

$p(x) = x^4 - 7x^2 + 12$

$\because \sqrt{3}$ and $-\sqrt{3}$ are the zeroes of $p(x)$,

$\therefore \quad (x - \sqrt{3})(x + \sqrt{3})$ is a factor of $p(x)$

$\Rightarrow x^2 - 3$ is a factor of $p(x)$.

For other factors of $p(x)$,

$\therefore \quad x^4 - 7x^2 + 12 = (x^2 - 3)(x^2 - 4) = (x^2 - 3)(x - 2)(x + 2)$

$\therefore$ Other two zeroes are 2 and -2.

$$\begin{array}{r|l}
 & \quad x^2 - 4 \\ \hline
x^2 - 3 & x^4 - 7x^2 + 12 \\
 & x^4 - 3x^2 \\
 & - \quad + \\ \hline
 & \quad -4x^2 + 12 \\
 & \quad -4x^2 + 12 \\
 & \quad + \qquad - \\ \hline
 & \qquad\quad \times \\ \hline
\end{array}$$

Question 36.

Obtain all other zeroes of the polynomial $x^4 – 3x^3 - x^2 + 9x – 6$, if two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

Answer:

$p(x) = x^4 - 3x^3 - x^2 + 9x - 6$

$\because \quad \sqrt{3}$ and $-\sqrt{3}$ are the zeroes of $p(x)$

$\therefore \quad (x - \sqrt{3})\,(x + \sqrt{3})$ is a factor of $p(x)$

$\Rightarrow \quad x^2 - 3$ is a factor of $p(x)$

For other factor

$$
\begin{aligned}
\therefore \qquad p(x) &= (x^2 - 3)(x^2 - 3x + 2) \\
&= (x^2 - 3)(x - 2)(x - 1)
\end{aligned}
$$

$\therefore$ Other two zeroes are 1 and 2.

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
\hline
x^2 - 3 & x^4 - 3x^3 - x^2 + 9x - 6 \\
 & x^4 \qquad\quad - 3x^2 \\
 & - \qquad\qquad + \\
\hline
 & -3x^3 + 2x^2 + 9x - 6 \\
 & -3x^3 \qquad\quad + 9x \\
 & + \qquad\qquad - \\
\hline
 & \qquad 2x^2 \qquad - 6 \\
 & \qquad 2x^2 \qquad - 6 \\
 & \qquad - \qquad\quad + \\
\hline
 & \qquad\qquad \times \\
\hline
\end{array}
$$

Question 37.
Divide $2x^4 – 9x^3 + 5x^2 + 3x – 8$ bv $x^2 – 4x + 1$ and verify the division algorithm.
Answer:

Checking

$p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 8$

$q(x) = 2x^2 - x - 1, r(x) = -7$

$g(x) = x^2 - 4x + 1$

Now, $g(x) \times q(x) + r(x)$

$$
\begin{aligned}
&= (x^2 - 4x + 1)\,(2x^2 - x - 1) + (-7) \\
&= 2x^4 - x^3 - x^2 - 8x^3 + 4x^2 + 4x + 2x^2 - x - 1 - 7 \\
&= 2x^4 - 9x^3 + 5x^2 + 3x - 8 = p(x)
\end{aligned}
$$

$\therefore \quad p(x) = g(x)\,.\,q(x) + r(x)$

$$
\begin{array}{r|l}
 & 2x^2 - x - 1 \\
\hline
x^2 - 4x + 1 & 2x^4 - 9x^3 + 5x^2 + 3x - 8 \\
 & 2x^4 - 8x^3 + 2x^2 \\
 & - \quad + \quad - \\
\hline
 & \quad -x^3 + 3x^2 + 3x - 8 \\
 & \quad -x^3 + 4x^2 - x \\
 & \quad + \quad - \quad + \\
\hline
 & \qquad -x^2 + 4x - 8 \\
 & \qquad -x^2 + 4x - 1 \\
 & \qquad + \quad - \quad + \\
\hline
 & \qquad\qquad\quad -7 \\
\hline
\end{array}
$$

2010

Very Short Answer Type Questions [1 Mark]

Question 38.
If α, β are the zeroes of a polynomial, such that $\alpha + \beta = 6$ and $\alpha\beta = 4$, then write the polynomial.
Answer:

α, β are the zeroes of a polynomial $\alpha + \beta = 6, \alpha\beta = 4$

The required polynomial $g(x)$ is given by

$$
\begin{aligned}
g(x) &= k(x^2 - Sx + P) \\
g(x) &= k(x^2 - 6x + 4)
\end{aligned}
$$

Where k is any non zero real number.

Question 39.
If α, β are the zeroes of the polynomial $2y^2 + 7y + 5$, write the value of $\alpha + \beta + \alpha\beta$.

Answer:

$P(y) = 2y^2 + 7y + 5$

$\because$ α, β are zeroes of P(y)

$$\therefore \quad \alpha + \beta = \frac{-7}{2}$$

$$\alpha\beta = \frac{5}{2}$$

$$\therefore \quad \alpha + \beta + \alpha\beta = \frac{-7}{2} + \frac{5}{2} = \frac{-2}{2} = -1.$$

Question 40.
If one zero of the polynomial x ^{2} – 4x + 1 is 2 + √3, write the other zero
Answer:

Let other zero be α,

$$\therefore \quad (2+\sqrt{3}) + \alpha = -\left(\frac{-4}{1}\right)$$

$$\Rightarrow \quad \alpha = 4 - 2 - \sqrt{3} = 2 - \sqrt{3}.$$

Short Answer Type Questions I [2 Marks]

Question 41.
If two zeroes of the polynomial x ^{3} – 4x ^{2} – 3x + 12 are √3 and -√3, then find its third zero.
Answer:

As $\sqrt{3}$ and $-\sqrt{3}$ are zeroes of polynomial $x^3 - 4x^2 - 3x + 12$.

$\therefore (x + \sqrt{3})(x - \sqrt{3})$ are factors of $x^3 - 4x^2 - 3x + 12$

$$\begin{aligned} \text{Now,} \quad x^3 - 4x^2 - 3x + 12 &= x^2(x-4) - 3(x-4) \\ &= (x^2 - 3)(x - 4) \\ &= (x - \sqrt{3})(x + \sqrt{3})(x - 4) \end{aligned}$$

$\therefore$ Third zero of $x^3 - 4x^2 - 3x + 12$ is 4.

Question 42.
If √5 and -√5 are two zeroes of the polynomial x ^{3} + 3 x ^{2} – 5x – 15, find its third zero.

Answer:

As $\sqrt{5}$ and $-\sqrt{5}$ are zeroes of the polynomial $x^3 + 3x^2 - 5x - 15$ therefore $(x - \sqrt{5})$ and $(x + \sqrt{5})$ are its factors.

$$
\begin{aligned}
\text{Consider } x^3 + 3x^2 - 5x - 15 &= x^2(x + 3) - 5(x + 3) \\
&= (x^2 - 5)(x + 3) \\
&= (x - \sqrt{5})(x + \sqrt{5})(x + 3)
\end{aligned}
$$

∴ Third zero of the polynomial is – 3.

Question 43.
For what value of k, is 3 a zero of the polynomial 2x ^{2} + x + kx

Answer:

Since 3 is a zero of the polynomial $p(x) = 2x^2 + x + k$

$\therefore \quad P(3) = 0$

$\Rightarrow \quad P(3) = 2(3)^2 + 3 + k$

$\Rightarrow \quad 0 = 18 + 3 + k$

$\Rightarrow \quad k = -21$

Question 44.
If -1 and 2 are two zeroes of the polynomial 2x ^{3} – x ^{2} – 5x – 2, find its third zero.
Answer:

-1 and 2 are zeroes of $p(x) = 2x^3 - x^2 - 5x - 2$.

$\therefore$ $(x + 1)$ and $(x - 2)$ are factors of $p(x)$

$\Rightarrow (x^2 - x - 2)$ is a factor of $p(x)$.

Dividing $p(x)$ by $x^2 - x - 2$

$\therefore$ $(x^2 - x - 2)(2x + 1)$ are factor of $p(x)$.

$\therefore$ $2x^3 - x^2 - 5x - 2 = (x^2 - x - 2)(2x + 1)$

$\therefore$ Third zero of given polynomial is $\frac{-1}{2}$.

$$
\begin{array}{r|l}
 & 2x + 1 \\
\hline
x^2 - x - 2 & 2x^3 - x^2 - 5x - 2 \\
 & 2x^3 - 2x^2 - 4x \\
 & - \quad + \quad + \\
\hline
 & x^2 - x - 2 \\
 & x^2 - x - 2 \\
 & - \quad + \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

2009

Very Short Answer Type Questions [1 Mark]

Question 45.
For what value of k, (- 4) is a zero of the polynomial x ^{2}-x- (2k + 2)

Answer:

$\because -4$ is the zero of polynomial $p(x) = x^2 - x - (2k + 2)$

$$\begin{aligned} \text{So,} \quad & p(-4) = 0 \\ \Rightarrow \quad & (-4)^2 - (-4) - (2k + 2) = 0 \\ \Rightarrow \quad & 16 + 4 - 2k - 2 = 0 \\ \Rightarrow \quad & 18 = 2k \quad \Rightarrow \quad k = 9 \end{aligned}$$

Question 46.
For what value of p, (-4) is a zero of the polynomial x ^{2} – 2x – (7p + 3)
Answer:

$\because (-4)$ is a zero of polynomial $p(x) = x^2 - 2x - (7p + 3)$

$$\begin{aligned} \therefore \quad & p(-4) = 0 \\ \text{Hence,} \quad & (-4)^2 - 2(-4) - (7p + 3) = 0 \\ & 16 + 8 - 7p - 3 = 0 \\ & 21 - 7p = 0 \quad \Rightarrow \quad p = \frac{21}{7} = 3 \end{aligned}$$

Question 47.
If 1 is a zero of polynomial p(x) = ax ^{2} – 3 (a – 1) – 1, then find the value of a.
Answer:

$\because$ 1 is a zero of $p(x)$

$$\begin{aligned} \Rightarrow \quad & p(1) = 0 \quad \Rightarrow a(1)^2 - 3(a - 1) - 1 = 0 \\ \Rightarrow \quad & a - 3a + 3 - 1 = 0 \quad \Rightarrow \quad a = 1 \end{aligned}$$

Question 48.
Write the polynomial, the product and sum of whose zeroes are -9/2 and -3/2 respectively.
Answer:

Required polynomial is given by
$p(x) = x^2 -$ (sum of zeroes)x + product of zeroes

Here, sum of zeroes $= \frac{-3}{2}$ and product of zeroes $= \frac{-9}{2}$

$$\therefore \quad p(x) = x^2 - \left(\frac{-3}{2}\right)x + \left(\frac{-9}{2}\right) = x^2 + \frac{3}{2}x - \frac{9}{2}$$

Short Answer Type Questions I [2 marks]

Question 49.
If the polynomial 6x ^{4}+ 8x ^{3} + 17x ^{2} + 21x + 7 is divided by another polynomial 3x ^{2} + 4x + 1, the remainder comes out to be (ax + b), find a and b.

Answer:

The given remainder is $ax + b$.

then $ax + b = x + 2$

So, $a = 1, b = 2$.

$$
\begin{array}{rl}
 & 2x^2 + 5 \\
3x^2 + 4x + 1 \,\big) & 6x^4 + 8x^3 + 17x^2 + 21x + 7 \\
 & 6x^4 + 8x^3 + 2x^2 \\
 & - \quad\; - \quad\; - \\
\hline
 & 15x^2 + 21x + 7 \\
 & 15x^2 + 20x + 5 \\
 & - \quad\; - \quad\; - \\
\hline
 & x + 2 \\
\hline
\end{array}
$$

Question 50.

If the polynomial x ^{4} + 2x ^{3} + 8x ^{2} + 12x + 18 is divided by another polynomial x ^{2} + 5, the remainder comes out to be px + q, find values of p and q.

Answer:

Polynomial $p(x) = x^4 + 2x^3 + 8x^2 + 12x + 18$

is divided by $x^2 + 5$.

Remainder is $2x + 3$

$\therefore\ 2x + 3 = px + q$

Hence, $p = 2$ and $q = 3$

$$
\begin{array}{rl}
 & x^2 + 2x + 3 \\
x^2 + 5 \,\big) & x^4 + 2x^3 + 8x^2 + 12x + 18 \\
 & x^4 \qquad\; + 5x^2 \\
 & - \qquad\quad - \\
\hline
 & 2x^3 + 3x^2 + 12x + 18 \\
 & 2x^3 \qquad\; + 10x \\
 & - \qquad\quad - \\
\hline
 & 3x^2 + 2x \;\; + 18 \\
 & 3x^2 \qquad\; + 15 \\
 & - \qquad\quad - \\
\hline
 & 2x + 3 \\
\hline
\end{array}
$$

Question 51.

Find all the zeroes of the polynomial x ^{3} + 3x ^{2} – 2x – 6, if two of its zeroes are -√2 and √2.

Answer:

$p(x) = x^3 + 3x^2 - 2x - 6$

$\because\ -\sqrt{2}$ and $\sqrt{2}$ are zeroes of $p(x)$.

$\therefore\ \{x - (-\sqrt{2})\}\,(x - \sqrt{2})$ is a factor of $p(x)$.

$\Rightarrow\ x^2 - 2$ is a factor of $p(x)$.

$\therefore\ x^3 + 3x^2 - 2x - 6 = (x^2 - 2)(x + 3)$

$\therefore$ Other zero is -3,

$\therefore$ All the zeroes of $p(x)$ are $-\sqrt{2}$, $\sqrt{2}$ and -3.

$$
\begin{array}{rl}
 & x + 3 \\
x^2 - 2 \,\big) & x^3 + 3x^2 - 2x - 6 \\
 & x^3 \qquad\; - 2x \\
 & - \qquad\quad + \\
\hline
 & 3x^2 \qquad - 6 \\
 & 3x^2 \qquad - 6 \\
 & - \qquad\quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

Question 52.

Find all the zeroes of the polynomial 2 x ^{3} +x ^{2} – 6 x – 3, if two of its zeroes are -√3 and √3.

Answer:

Let $p(x) = 2x^3 + x^2 - 6x - 3$

$$= x^2(2x + 1) - 3(2x + 1) = (x^2 - 3)(2x + 1) = (x + \sqrt{3})\ (x - \sqrt{3})\ (2x + 1)$$

$-\sqrt{3}$ and $\sqrt{3}$ are two zeroes of $p(x)$ (given)

$\therefore$ All the zeroes of given polynomial are $-\sqrt{3}$, $\sqrt{3}$ and $-\frac{1}{2}$.