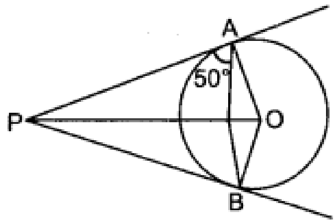
Chapter 10: Circles

2016

Very Short Answer Type Questions [1 Mark]

Question 1.
From an external point P, tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, then find ∠AOB.
Solution:



Given, $\angle PAB = 50°$

$\angle PAB + \angle OAB = 90°$

[∵ angle between radius OA and tangent PA is 90°]

$\Rightarrow 50° + \angle OAB = 90°$

$\Rightarrow \angle OAB = 90° - 50° = 40°$

Now, $PA = PB$ [∴ tangents from an external point are same]

$\Rightarrow \angle PBA = \angle PAB$

$\Rightarrow \angle PBA = 50°$

$\angle PBA + \angle OBA = 90°$ [∴ angle between radius OB and tangent PB is 180°]

$\Rightarrow 50° + \angle OBA = 90°$

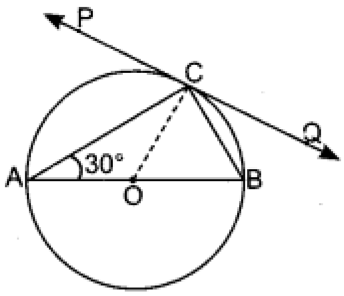
$\Rightarrow \angle OBA = 90° - 50° = 40°$

Now in ΔAOB we have

$\angle AOB + \angle ABO + \angle BAO = 180°$ [∵ sum of angles in triangle is 180°]

$\Rightarrow \angle AOB + 40° + 40° = 180° \Rightarrow \angle AOB = 180° - 80° = 100°$

Question 2.
In given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and ∠CAB = 30°, find ∠PCA



Solution:

Construction: Join AO.

Given: PQ is tangent. AB is diameter $\angle CAB = 30°$.

To Find: $\angle PCA$

Solution: In ΔAOC,

$$AO = CO \quad (\because \text{ Equal radii})$$

$$\angle CAO = \angle OCA \quad (\because \text{ Angles opposite to equal sides are equal})$$

or $$\angle CAB = \angle OCA$$

But, $$\angle CAB = 30° \quad \text{So,} \quad \angle OCA = 30° \quad (i)$$

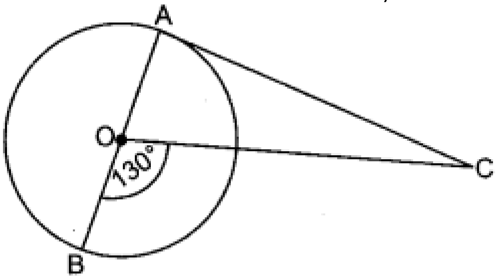
Since, $$OC \perp PQ \; (\because \text{ Tangent is perpendicular to radius at point of contact})$$

$$\Rightarrow \quad \angle PCO = 90° \Rightarrow \angle OCA + \angle PCA = 90° \Rightarrow 30° + \angle PCA = 90°$$

$$\therefore \quad \angle PCA = 60°$$

Question 3.

In figure given, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If ∠BOC = 130°, then find ∠ACO.



Solution:

$$\angle AOC + \angle BOC = 180°$$

$$[\because \text{ Linear Pair Axiom}]$$

$$\angle AOC + 130° = 180°$$

$$\angle AOC = 180° - 130°$$

$$\angle AOC = 50°$$

Now, $$\angle OAC = 90° \; [\text{angle between radius OA and tangent AC is } 90°]$$

Now, in ΔAOC,

$$\angle OAC + \angle AOC + \angle ACO = 180° \quad [\because \text{ sum of angles in triangle is } 180°]$$

$$90° + 50° + \angle ACO = 180°$$

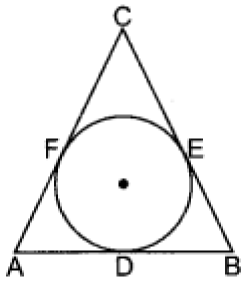
$$\angle ACO = 180° - 140°$$

$$\angle ACO = 40°$$

Short Answer Type Questions I [2 Marks]

Question 4.

In given figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF



Solution:
Given, AB = 12 cm, CA = 10 cm, BC = 8 cm

Let $\quad AD = AF = x \quad [\because$ Tangent drawn from external point to circle are equal]

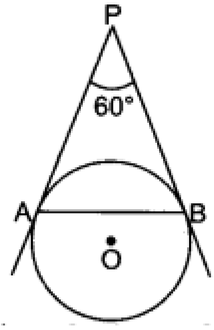
$\therefore \quad DB = BE = 12 - x \quad$ and $\quad CF = CE = 10 - x$

$$BC = BE + EC \Rightarrow 8 = 12 - x + 10 - x$$

$\Rightarrow \quad x = 7$

$\therefore$ AD = 7 cm, BE = 5 cm and CF = 3 cm

Question 5.
If given figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and ∠APB = 60°. Find the length of chord AB.



Solution:
In ΔAPB we have $\quad AP = BP$

$\Rightarrow \quad \angle PAB = \angle PBA$

[$\because$ Tangents from an external point are equally inclined to segment joining centre to point]

Let $\quad \angle PAB = x,$

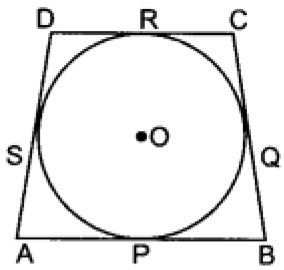
then in ΔAPB, $\quad x + x + 60° = 180°$

$$2x = 180° - 60° = 120°$$

$$x = 60°$$

As all three angles of ΔAPB are 60°. So ΔAPB is an equilateral triangle.
Hence AP = BP = AB = 5 cm

Question 6.
In figure, a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that AB + CD = BC + DA.



Solution:

We know that tangents drawn to a circle from an outer points are equal.

So, $AP = AS, BP = BQ,$
$CR = CQ$ and $DR = DS.$

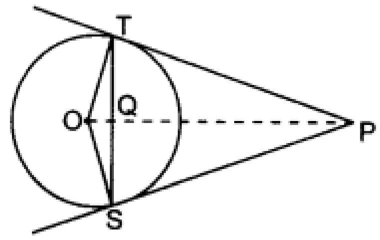
Now, consider

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$
$$\Rightarrow \quad AB + CD = AD + BC$$

Hence proved.

Question 7.

In given figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r.If PO = 2r, show that ∠OTS =∠OST = 30°.



Solution:

Let $\angle TOP = \theta$

In right triangle OTP we have

$$\therefore \quad \cos\theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2} = \cos 60° \Rightarrow \theta = 60°$$

Hence $\angle TOS = 2 \times 60 = 120°$ [$\because \angle TOP = \angle POS$ as angles opposite to equal tangent are equal]

In ΔOTS, we have $OT = OS$ [$\because$ Equal radii]

$\Rightarrow \quad \angle OTS = \angle OST$ [$\because$ Angle opposite to equal sides are equal]

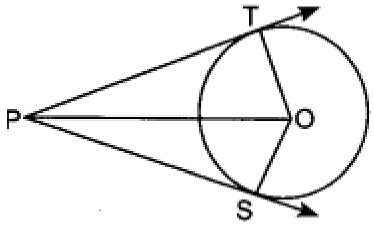
In ΔOTS,

$$\angle OTS + \angle OST + \angle TOS = 180°$$
$$2\angle OST = 60°$$
$$\therefore \quad \angle OST = \angle OTS = 30°$$

Hence proved.

Question 8.

In given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that ∠SPT = 120°, Prove that OP = 2PS



Solution:

Let $PT = x = PS$ [∵ Tangent drawn from external point to circle are equal]

$\angle SPT = 120°$

In ΔOTP and ΔOSP, $\angle OTP = \angle OSP$

[∵ each equal to 90°, since tangent perpendicular r radius]

$OT = OS$ [∵ Equal radii]

$OP = OP$ [common]

$\Rightarrow$ $\Delta OSP \cong \Delta OTP$ [∵ By SAS congruence rule]

$\therefore$ $\angle TPO = \angle SPO$ [∵ By CPCT]

$\Rightarrow$ $\angle TPO = \frac{1}{2}\angle SPT = \frac{1}{2} \times 120 = 60°$

In ΔOTP, $\frac{OP}{x} = \text{Sec } 60°$

$\Rightarrow$ $\frac{OP}{x} = 2 \Rightarrow OP = 2x \Rightarrow OP = 2PS$

Hence proved.

Question 9.

In given figure, there are two concentric circles of radii 6 cm and 4 cm with centre O. If AP is a tangent to the larger circle and BP to the smaller circle and length of AP is 8cm, find the length of BP

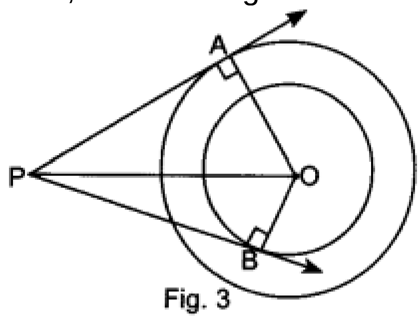


Fig. 3

Solution:

$OA = 6 \text{ cm}$ [∵ Given radius]

$OB = 4 \text{ cm}$ [∵ Given radius]

$AP = 8 \text{ cm}$

In ΔOAP, $OP^2 = OA^2 + AP^2 = 36 + 64 = 100$ [∵ Pythagoras theorem]

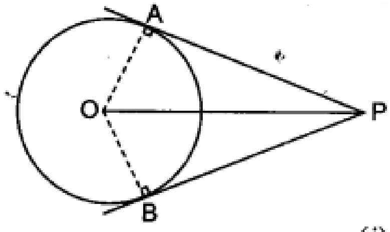
$\Rightarrow$ $OP = 10 \text{ cm}$

In ΔOBP, $BP^2 = OP^2 - OB^2 = 100 - 16 = 84$ [∵ Pythagoras theorem]

$BP = 2\sqrt{21} \text{ cm}$

Long Answer Type Questions [4 Marks]

Question 10.
Prove that the lengths of tangents drawn from an external point to a circle are equal
Solution:



Given: A circle $C(O, r)$, P is a point outside the circle and PA and PB are tangents to a circle.

To Prove: PA = PB

Construction: Draw OA, OB and OP.

Proof: Consider triangles OAP and OBP.

$$\angle OAP = \angle OBP = 90^\circ \quad ...(i)$$

[Radius is perpendicular to the tangent at the point of contact]

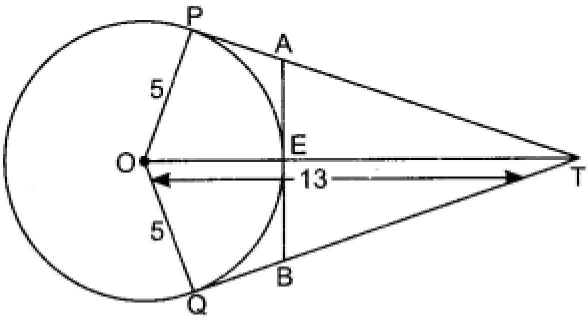
$$OA = OB \text{ (radii)} \quad ...(ii)$$

OP is common ...(*iii*)

$\therefore$ $\Delta OAP \cong \Delta OBP$ (RHS) [from (*i*), (*ii*) and (*iii*)]

Hence, $AP = BP$ (CPCT)

Question 11.
In given figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Solution:

In ΔOPT, $OP^2 + PT^2 = OT^2$ [$\because$ Pythagoras theorem]

$$PT = \sqrt{OT^2 - OP^2}$$

$$= \sqrt{169 - 25} = 12 \text{ cm}$$

and $TE = OT - OE = 13 - 5 = 8$ cm

Let $PA = AE = x$ [tangent from outer point A]

In ΔTEA, $TE^2 + EA^2 = TA^2$ [$\because$ Pythagoras theorem]

$$(8)^2 + (x)^2 = (12 - x)^2$$

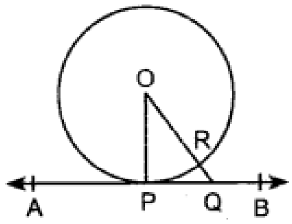
$$64 + x^2 = (12 - x)^2$$

$$\Rightarrow \quad 64 + x^2 = 144 + x^2 - 24x$$

$$\Rightarrow \quad 80 = 24x \quad \Rightarrow x = 3.3 \text{ cm}$$

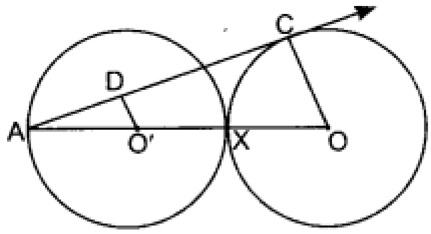
Thus $AB = 2 \times 3.3$ cm $= 6.6$ cm [$\because$ AE = EB, as AB is tangent to circle at E]

Question 12.
Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact
Solution:
Given: A circle $C(O, r)$ and a tangent AB at a point P.
To prove: $OP \perp AB$
Construction: Take any point Q other than P on the tangent AB.
Join OQ, intersecting circle at R.

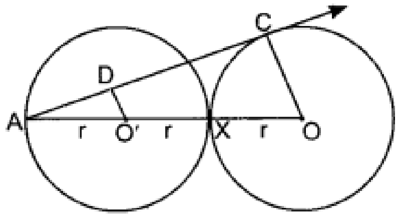


Proof: We have, $OP = OR$ [Radii]
$OQ = OR + RQ$
$\therefore$ $OQ > OR \Rightarrow OQ > OP$ [$\because$ OR = OP = radius]
Thus, $OP < OQ$, i.e. OP is shorter than any other segment joining O to any point of AB.
But among all line segments, joining point O to point on AB, shortest one is perpendicular from O on AB.
Hence, $OP \perp AB$

Question 13.
In given figure, two equal circles, with centres O and O', touch each other at X. OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. O'D is perpendicular to AC. Find the value of DO'/CO.

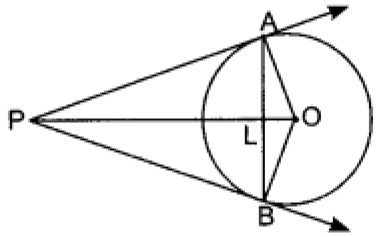


Solution:
AC is tangent to the circle with centre O.
In $\Delta ADO'$ and ΔACO, $\angle ADO' = \angle ACO$ (each 90°)
$\angle DAO = \angle CAO$ (common)



$\therefore$ By AA criterion, $\frac{AO'}{AO} = \frac{DO'}{CO}$ [$\because$ corresponding parts of similar triangle]
$AO = AO' + O'X + XO = r + r + r = 3r$
$\frac{DO'}{CO} = \frac{r}{3r}$ [$\therefore$ AO = AO' + O'X + XO = 3AO]
$\Rightarrow$ $\frac{DO'}{CO} = \frac{1}{3}$

Question 14.
In given figure, AB is a chord of a circle, with centre O, such that AB = 16 cm and radius of circle is 10 cm. Tangents at A and B intersect each other at P. Find the length of PA



Solution:

Let $PL = x$

As OP is perpendicular bisector of AB. Then

$$AL = BL = 8 \text{ cm}$$

In ΔALO, $OL^2 = OA^2 - AL^2 = 10^2 - 8^2 = 36 \Rightarrow OL = 6 \text{ cm}$

$AP^2 = OP^2 - OA^2$ [$\because$ Pythagoras theorem]

In ΔOAP, $AP^2 = (x+6)^2 - 10^2$

$AP^2 = AL^2 + PL^2$ [$\because$ Pythagoras theorem]

In ΔALP, $AP^2 = x^2 + 64$

Now, $(x+6)^2 - 10^2 = x^2 + 64$

$$x^2 + 12x + 36 - 100 = x^2 + 64$$

$$\Rightarrow \quad 12x = 128$$

$$\Rightarrow \quad x = \frac{128}{12} = \frac{32}{3} \text{ cm}$$

From ΔALP,

$$AP^2 = \left(\frac{32}{3}\right)^2 + 64 = \frac{1024}{9} + 64 = \frac{1024 + 576}{9} \text{ cm}$$

$$AP^2 = \frac{1600}{9} \text{ cm}$$

$$AP = \frac{40}{3} \text{ cm} = 13.3 \text{ cm}$$

2015

Very Short Answer Type Questions [1 Mark

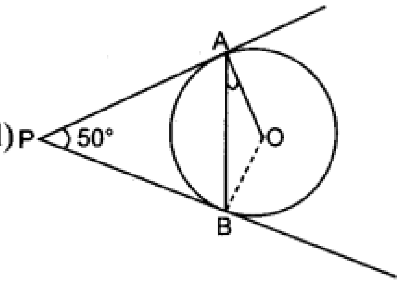
Question 15.

In figure, PA and PB are tangents to the circle with centre O such that ∠APB = 50°.
Write the measure of ∠OAB

Solution:

Join OB.

$\because$ PA and PB are tangents to the circle drawn from an external point P. We know that, tangent is perpendicular r to radius.



$$\angle OAP = \angle OBP = 90°$$

Then, $\angle OAP + \angle APB + \angle OBP + \angle AOB = 360°$

(ASP of quadrilateral)

$\therefore \quad \angle APB + \angle AOB = 180°$

$\Rightarrow \quad 50° + \angle AOB = 180°$

$\Rightarrow \quad \angle AOB = 130°$

In ΔOAB, $\quad OA = OB$ $\quad$ ($\because$ equal radii)

$\Rightarrow \quad \angle A = \angle B = x$ (say)($\because$ angles opposite to equal sides are equal)

$\angle A + \angle B + \angle AOB = 180°$ $\quad$ ($\because$ ASP of triangles)

$\Rightarrow \quad x + x + 130° = 180°$

$\Rightarrow \quad 2x = 50°$

$\Rightarrow \quad x = 25°$

$\therefore \quad \angle OAB = 25°$

Question 16.

Find the relation between x and y if the points A(x, y), B(-5, 7) and C(-4, 5) are collinear

Solution:

$\because$ A, B and C are collinear. Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

So, $\quad ar(\Delta ABC) = 0$

$\therefore \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\frac{1}{2}[x(7-5) + (-5)(5-y) + (-4)(y-7)] = 0$

$2x - 25 + 5y - 4y + 28 = 0$

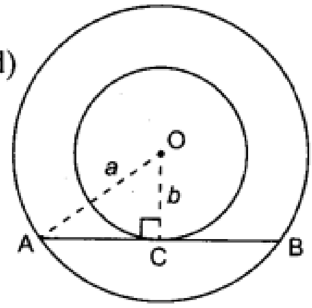
$\Rightarrow \quad 2x + y + 3 = 0$. Required relation between x and y.

Question 17.

Two concentric circles of radii a and b(a > b) are given. Find the length of the chord of the larger circle which touches the smaller circle.

Solution:

AB is tangent at C to circle C(O, b)

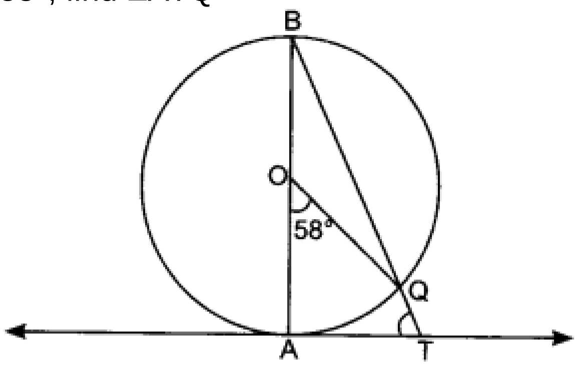


$$\therefore \quad OC \perp AB$$
$$\therefore \quad \angle OCB \perp 90°$$
$$\Rightarrow \quad AC = BC \Rightarrow AB = 2AC$$
($\because$ perpendicular from centre to chord bisects chord)

Now, in ΔOCA, $AO^2 = OC^2 + AC^2$
$$\Rightarrow \quad a^2 = b^2 + AC^2$$
$$\Rightarrow \quad AC = \sqrt{a^2 - b^2}$$
$$\therefore \quad AB = 2\sqrt{a^2 - b^2} = 2AC$$
$$\text{length of chord} = 2\sqrt{a^2 - b^2}$$

Short Answer Type Questions I [2 Marks]

Question 18.
In figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ$ = 58°, find $\angle ATQ$



Solution:
$\because$ AT is a tangent and BA is a diameter.

So, $OA \perp AT$

[radius is perpendicular to the tangent at point of contact]

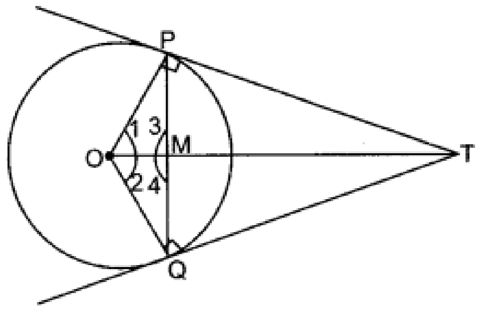
$$\Rightarrow \quad \angle OAT = 90° \text{ or } \angle BAT = 90°$$

Arc AQ subtends an angle of 58° at the circle.
$$\angle AOQ = 2\angle ABQ$$
So, $\angle ABQ = 29°$ [angle subtended by the arc at the centre is double the angle subtended by the same arc on the circle]

In ΔABT,
$$\angle A + \angle ABT + \angle ATB = 180°$$
$$\Rightarrow \quad 90° + 29° + \angle ATB = 180°$$
$$\Rightarrow \quad \angle ATB = 61°$$
Hence, $\angle ATQ = 61°$

Question 19.
From a point T outside a circle of centre O, tangents TP and TQ are drawn to the

circle. Prove that OT is the right bisector of the line segment PQ.
Solution:
Given: TP and TQ are tangents to the circle of centre O.
To Prove: $\angle OMP = 90°$ and PM = MQ.
Proof: $\because$ TP and TQ are tangents at P and Q respectively.



So, $OP \perp PT$ and $OQ \perp QT$
($\because$ radius is perpendicular to the tangent at point of contact)

$\therefore \quad \angle OPT = \angle OQR = 90°$

In ΔOPT and ΔOQT

$OP = OQ$ (radius)
$\angle P = \angle Q$ (each 90°)
$OT = OT$ (common)

So, $\Delta OPT \cong \Delta OQT$ (By RHS)
$\Rightarrow \angle 1 = \angle 2$ (By CPCT)

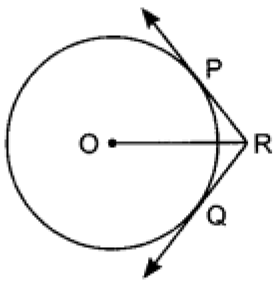
Now, In ΔOMP and ΔOMQ,

$OP = OQ$ (radius)
$\angle 1 = \angle 2$ (Proved above)
$OM = OM$ (common)

So, $\Delta OMP \cong \Delta OMQ$ (By SAS)
$\Rightarrow PM = MQ$ and $\angle 3 = \angle 4$ (By CPCT)
Now $\angle 3 + \angle 4 = 180°$ ($\because$ Linear Pair Axiom)
$\Rightarrow 2\angle 3 = 180° \Rightarrow \angle 3 = 90° \Rightarrow \angle OMP = 90°$

Hence, OT is the right bisector of the line segment PQ.

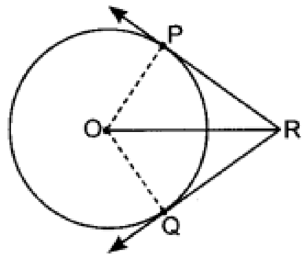
Question 20.
In figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If ∠PRQ = 120°, then prove that OR = PR + RQ.



Solution:

We know that, tangent is perpendicular r to radius. Perpendicular from centre bisects angle.

OR bisects $\angle PRQ$



$$\therefore \quad \angle PRO = \angle QRO = 60^\circ$$

$$[\because \angle PRQ = \angle ORP + \angle ORQ = 120^\circ]$$

In right ΔOPR ($\because OP \perp PR$) [$\because$ radius is perpendicular to the targent at point of contact]

$$\therefore \quad \cos \angle ORP = \frac{PR}{OR} = \cos 60^\circ$$

$$\Rightarrow \quad OR = 2PR \quad ...(i)$$

Similarly, in right ΔOQR, $\frac{QR}{OR} = \frac{1}{2} = \cos 60^\circ$

$$\Rightarrow \quad OR = 2QR \quad ...(ii)$$

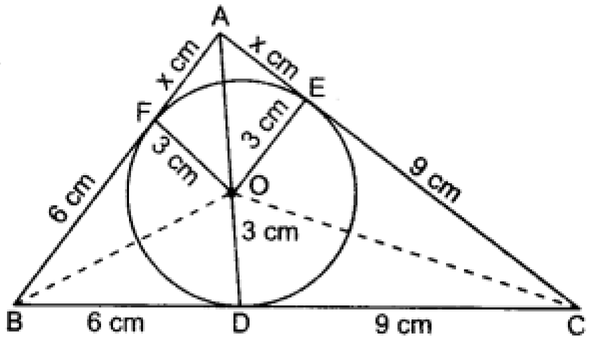
Adding (i) and (ii), we get

$$2OR = 2PR + 2QR$$

$$\Rightarrow \quad OR = PR + RQ$$

Question 21.

In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm², then find the lengths of sides AB and AC



Solution:

Let AF = x cm, BC = (6 + 9) = 15 cm

$\because \quad AF = AE$

[tangents drawn from an external point are equal]

$\therefore \quad AE = x$ cm

Also $\quad BD = BF = 6$ cm

and $\quad CD = CE = 9$ cm

$\therefore \quad AB = (x + 6)$ cm

In ΔABC, $\quad AC = (x + 9)$ cm

$$\text{Area } \Delta ABC = \text{Area } \Delta BOC + \text{Area } \Delta COA + \text{Area } \Delta AOB$$

$$\Rightarrow \quad 54 = \frac{1}{2} BC \times OD + \frac{1}{2} AC \times OE + \frac{1}{2} AB \times OF$$

$$\Rightarrow \quad 54 \times 2 = 15 \times 3 + (9 + x) \times 3 + (6 + x) \times 3$$

$$108 = 45 + 18 + 3x + 27 + 3x$$

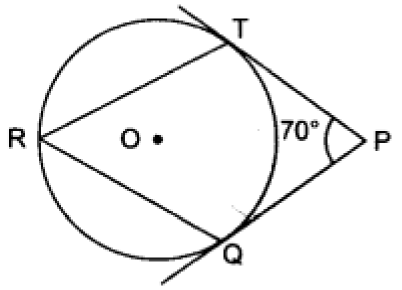
$$6x = 18 \Rightarrow x = 3$$

$$\Rightarrow \quad AB = 6 + x = 6 + 3 = 9 \text{ cm}$$

$$AC = 9 + x = 9 + 3 = 12 \text{ cm}$$

Question 22.

In figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If ∠TPQ = 70°, find ∠TRQ



Solution:

We know that tangent is perpendicular to radius. Hence,

$$\angle OTP = \angle OQP = 90^\circ$$

In quadrilateral PQOT,

$$\angle QOT + \angle OTP + \angle TPQ + \angle OQP = 360^\circ \quad [\because \text{ ASP of quadrilateral}]$$

$$\angle TOQ + \angle TPQ = 180^\circ$$

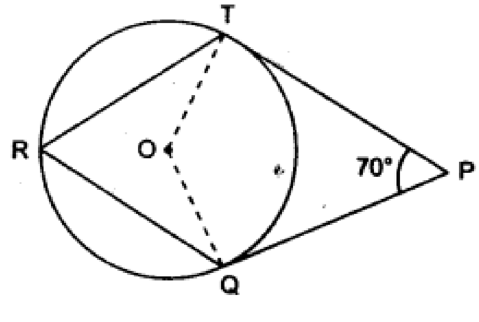
$$\Rightarrow \quad \angle TOQ = 110^\circ$$

Also $\quad \angle TOQ = 2\angle TRQ$

[angle subtended by an arc at centre of the circle is twice the angle subtended by it in alternate segment]

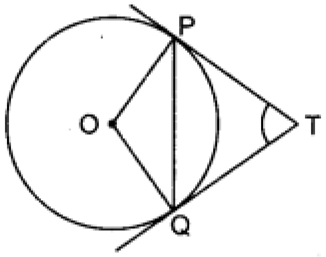
$$\Rightarrow \quad 110^\circ = 2\angle TRQ$$

$$\Rightarrow \quad \angle TRQ = 55^\circ$$



Question 23.

In figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the lengths of TP and TQ



Solution:

Join OT intersecting PQ at R.

OT bisects $\angle PTQ$

$\therefore \quad \angle PTO = \angle QTO$

$\therefore \quad \angle PTR = \angle QTR \quad \ldots(i)$

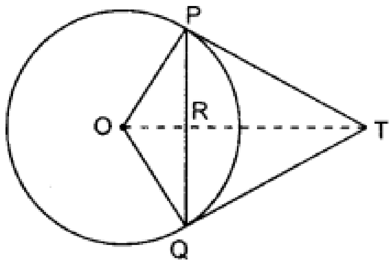
In ΔPTR and ΔQTR, $\quad PT = QT$ [length of tangents drawn from common external point are equal]

$RT = RT$ [common]

$\angle PTR = \angle QTR$ [$\because$ from (i)]

$\therefore \quad \Delta PTR \cong \Delta QTR$ [By SAS]

$\Rightarrow \quad PR = RQ$ [$\because$ By CPCT]



$\Rightarrow$ R is mid-point of PQ

$\therefore \quad OR \perp PQ$

In right triangle ORP

$OP^2 = PR^2 + OR^2$ [$\because$ Given, OP = 5 cm, PQ = 8 cm $\therefore$ PR = QR = 4 cm]

$\Rightarrow \quad 25 = 16 + OR^2$

$OR = 3$ cm

In ΔORQ and ΔOQT

$\angle ORQ = \angle OQT$ (Each 90°)

$\angle ROQ = \angle ROQ$ (Common)

$\therefore \quad \Delta ORQ \sim \Delta OQT$ (By AA criterion)

$\Rightarrow \quad \dfrac{OR}{OQ} = \dfrac{RQ}{QT}$ (By C.P. of similar triangles)

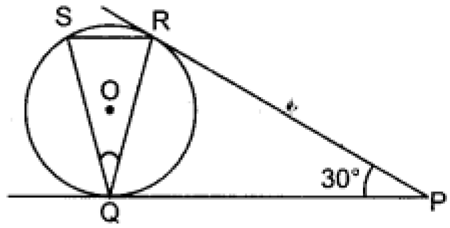
$\Rightarrow \quad \dfrac{3}{5} = \dfrac{4}{QT} \quad \Rightarrow \quad QT = \dfrac{20}{3}$ cm

Also $\quad PT = QT \quad \Rightarrow \quad PT = \dfrac{20}{3}$ cm

Long Answer Type Questions [4 Marks]

Question 24.

In figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find ∠RQS



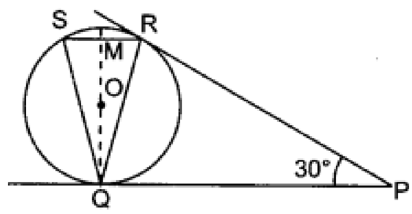
Solution:

Construction: Draw a line through Q and perpendicular to PQ.

Proof: As $MQ \perp PQ$

So, MQ passes through the centre O. [If a line is perpendicular to the tangent, then it must be passed through centre of the circle]

$\Rightarrow \quad \angle OQP = 90°$



Also $\quad PQ = PR$

[lengths of tangents drawn from an external point to a circle are equal]

In $\Delta PQR \quad PQ = PR$

$\Rightarrow \quad \angle Q = \angle R$ [$\because$ angles opposite to equal sides are equal]

Now, $\quad \angle P + \angle Q + \angle R = 180°$ [$\because$ ASP of triangle]

$\Rightarrow \quad 30° + 2\angle Q = 180°$

$\Rightarrow \quad 2\angle Q = 150°$

$\Rightarrow \quad \angle Q = 75°$

$\because$ Given, $\quad SR \parallel PQ$

$\Rightarrow \quad \angle QRS = \angle RQP$ [if 2 lines are parallel, alternate angles are equal]

$\Rightarrow \quad \angle QRS = 75°$

Also, $\quad \angle QSR = \angle RQP$ [$\because$ Corresponding angles equal]

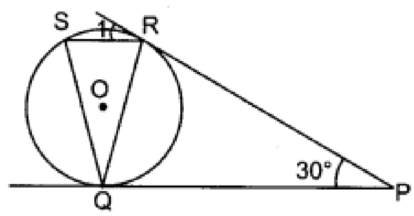
$\Rightarrow \quad \angle QSR = 75°$

Now, In ΔRQS,

$\angle RQS + \angle RSQ + \angle SRQ = 180°$ [$\because$ ASP of triangle]

$\Rightarrow \quad \angle RQS + 75° + 75° = 180°$

$\Rightarrow \quad \angle RQS = 180 - 150 = 30°$



Alternate method:

Given: PQ and PR are tangents and $\angle RPQ = 30°$

To find: $\angle RQS$

$\because \quad SR \parallel QP$

$\Rightarrow \quad \angle 1 = \angle P$ [Corresponding angles are equal]

$\Rightarrow \quad \angle 1 = 30°$

$\because$ PR is tangent at P and $\angle 1 = 30°$

So, $\quad \angle RQS = \angle 1$ [By alternate segment theorem]

$\Rightarrow \quad \angle RQS = 30°$

Question 25.

Prove that the tangent at any point of a circle is perpendicular to the radius throug the point of contact

Solution:

Refer to Ans. 12

Question 26.

Prove that the lengths of the tangents drawn from an external point to a circle are equal

Solution:

Refer to Ans. 10.

Question 27.

Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

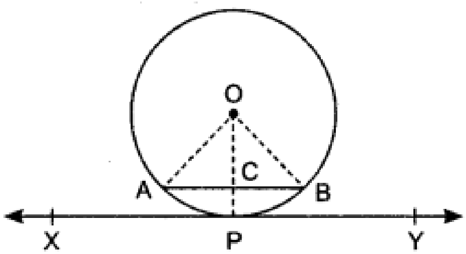
Solution:

Given: APB is arc of the circle $C(O, r)$, P is mid-point of arc APB and XY is tangent to the circle at P.

To Prove: $AB || XY$

Join OA and OB

Here



$\angle AOP = \angle BOP$ (angle subtended by equal arc)

$OA = OB$ ($\because$ equal radii)

$OC = OC$ (common)

$\therefore \quad \Delta ACO \cong \Delta BCO$ (SAS)

So, $\quad AC = BC$ (CPCT)

$\Rightarrow \quad OC \perp AB$

$\therefore \quad \angle OCB = \angle OCA = 90°$ (line joining mid-point of chord with centre of the circle is perpendicular to the chord)

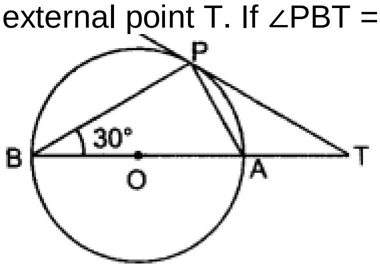
Also $\quad \angle OPY = 90°$

$\Rightarrow \quad \angle OCB = \angle OPY$ (these are corresponding angles)

$\therefore \quad AB || XY$ ($\because$ if corresponding angles are equal, 2 lines are parallel)

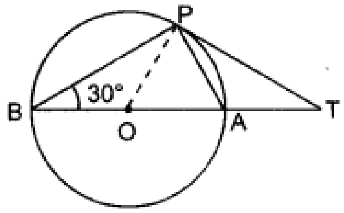
Question 28.

In figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If $\angle PBT = 30°$, prove that BA: AT = 2:1



Solution:

We have, $\angle BPA = 90°$ [$\because$ PT is tangent to circle, tangent perpendicular radius]



In ΔBPA, $\angle ABP + \angle BPA + \angle PAB = 180°$ [$\because$ ASP of triangle]

$\Rightarrow \quad 30° + 90° + \angle PAB = 180°$ [$\because \angle PBT = \angle ABP = 30°$]

$\Rightarrow \quad \angle PAB = 60°$

Also $\quad \angle POA = 2\angle PBA$

[$\because$ Angle subtended by an arc at centre is twice angle subtended by arc on circle]

$\Rightarrow \quad \angle POA = 2 \times 30° = 60°$

$\therefore \quad \angle PAO = \angle POA$

$\Rightarrow \quad OP = AP$ (sides opposite to equal angles) ...(*i*)

In ΔOPT, $\angle OPT = 90°$ (radius is perpendicular to tangent)

$\angle POT = 60°$

$\therefore \quad \angle PTO = 30°$ [angle sum property of a triangle)

Also, $\angle APT + \angle ATP = \angle PAO$ (exterior angle property)

$\therefore \quad \angle APT + 30° = 60° \Rightarrow \angle APT = 30°$

$\angle PTA = \angle APT$ ($\because$ from above)

$\therefore \quad AP = AT$ (sides opposite to equal angles) ... (*ii*)

From (*i*) and (*ii*)

$\Rightarrow \quad AT = OP =$ radius of the circle $= r$ [$\because$ AP is radius of circle]

Now $\quad AB = 2r$

$$\Rightarrow \quad AB = 2AT \Rightarrow \frac{AB}{AT} = 2 \Rightarrow AB : AT = 2 : 1$$

2014

Short Answer Type Questions I [2 Marks]

Question 29.

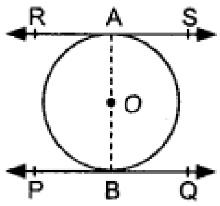
Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.

Solution:

Given: PQ and RS are two parallel tangents to a circle at B and A respectively. O is the centre of the circle. AOB be a line segment.

To prove: AB passes through O.

Construction: Join OA and OB.



Proof: As we know, OB is perpendicular to PQ.
[Tangent is perpendicular to radius at the point of contact.]

Now, given, PQ || RS ⇒ BO (Produced to RS) is perpendicular to RS. ....(*i*)
[A line perpendicular to one of the two parallel lines is perpendicular to other line also]
Also, OA is perpendicular to RS [∵ Tangent perpendicular to radius] ...(*ii*)
From (*i*) and (*ii*), OA and OB must coincide as only one line can be drawn perpendicular from a point outside the line to the line.

∴ AOB is straight line.

∴ A, O, B are collinear.

⇒ AB Passes through O, the centre of the circle.

Question 30.
If from an external point P of a circle with centre O, two tangents PQ and PR are drawn, such that ∠QPR = 120°, prove that 2PQ = PO.

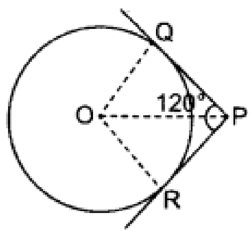
Solution:

Given: PQ and PR are tangents from point P to circle with centre O.

Also, $\angle QPR = 120°$

To Prove: $2PQ = OP$

Construction: Join OQ, OP and OR



Proof: In triangles OQP and ORP,

$OQ = OR = r$ (say) [∵ equal radii]

$OP = OP$ (common)

$PQ = PR$

[The lengths of the tangents drawn from an external point to a circle are equal]

∴ $\Delta OQP \cong \Delta ORP$ (by SSS)

∴ $\angle OPQ = \angle OPR$ (By CPCT)

Now, given $\angle QPR = 120°$

⇒ $\angle OPQ + \angle OPR = 120°$

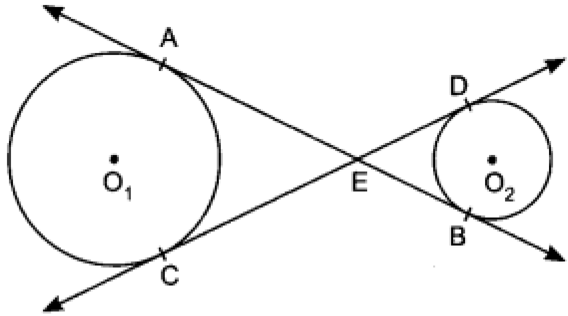
⇒ $2\angle OPQ = 120°$

⇒ $\angle OPQ = 60° = \angle OPR$

Now, In ΔOQP, $\angle Q = 90°$ [∵ Tangent perpendicular to radius]

Then, $\frac{PQ}{OP} = \cos 60° = \frac{1}{2} \Rightarrow OP = 2PQ$

Question 31.
In figure, common tangents AB and CD to the two circles with Centres O1 and O2 intersect at E. Prove that AB = CD.



Solution:

In the given figure, AB and CD are common tangents to the two given circles with centres O_1 and O_2 respectively.

We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal in length.

$\therefore$ AE = EC and EB = ED

$\Rightarrow$ AE + EB = CE + ED

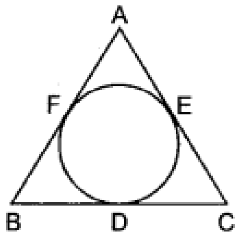
$\Rightarrow$ AB = CD.

Question 32.

The incircle of an isosceles triangle ABC, in which AB = AC, touches the sides BC, CA and AB at D, E and F respectively. Prove that BD = DC

Solution:

We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal in length.



$\therefore$ AF = AE, BF = BD and CD = CE ...(i)

Given AB = AC

$\Rightarrow$ AF + FB = AE + EC

$\Rightarrow$ FB = EC [using (i), AF = AE]

$\Rightarrow$ BD = CD [using (i), BF = BD and CD = CE]

Question 33.

In figure, XP and XQ are two tangents to the circle with centre O, drawn from an external point X. ARB is another tangent, touching the circle at R. Prove that XA+AR=XB+BR.

Solution:

Lengths of the tangents drawn from a point outside the circle to the circle are equal.

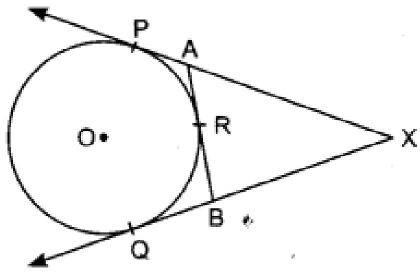
$\therefore$ XP = XQ, AP = AR and BR = BQ ...(i)

Now, XP = XQ [$\because$ equal tangents]

$\Rightarrow$ XA + AP = XB + BQ

$\Rightarrow$ XA + AR = XB + BR [using (i)]

Hence proved.

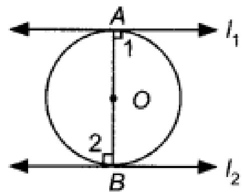


Question 34.

Prove that the tangents drawn at the ends of any diameter of a circle are parallel.

Solution:

AB is diameter of a circle with centre O and l_1, l_2 are the tangents to the circle at A and B.
We know that radius is perpendicular to the tangent at the point of contact or diameter is perpendicular to the tangent at the point of contact.



$\therefore$ $\angle 1 = 90°$ and $\angle 2 = 90°$ [$\because$ See from figure]

$\Rightarrow$ $\angle 1 = \angle 2$

But these are alternate angles.

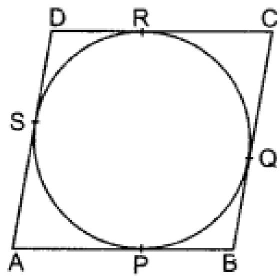
$\therefore$ l_1 is parallel to l_2.

[$\because$ If alternate angles are equal, so 2 lines are parallel]

Long Answer Type Questions [4 Marks]

Question 35.
Prove that the length of the tangents drawn from an external point to a circle are equal.
Solution:
Refer to Ans. 10.

Question 36.
Prove that a parallelogram circumscribing a circle is a rhombus
Solution:
Given: ABCD is parallelogram circumscribing a circle.
To prove: ABCD is a rhombus
Proof: We have, $DR = DS$ ...(*i*)
[Lengths of tangents drawn from an external point to a circle are equal]



Also, $AP = AS$ ...(*ii*)
$BP = BQ$ ...(*iii*)
$CR = CQ$ ...(*iv*)

Adding (*i*), (*ii*), (*iii*) and (*iv*),

$$(DR + CR) + (AP + BP) = (DS + AS) + (BQ + CQ)$$

$\Rightarrow$ $CD + AB = AD + BC$

$\Rightarrow$ $2AB = 2AD$ [$\because$ In parallelogram, opposite sides are equal $AB = CD$ and $AD = BC$]

$\Rightarrow$ $AB = AD$

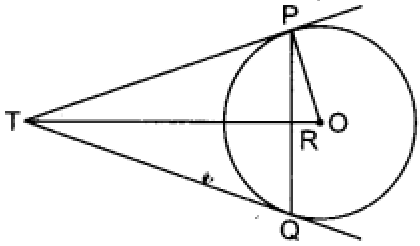
$\therefore$ $AB = AD = BC = CD$

Hence, ABCD is a rhombus as all sides are equal in rhombus.

Question 37.
Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact
Solution:
Refer to Ans. 12.

Question 38.
In figure, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at

P and Q intersect at a point T. Find the length of TP.

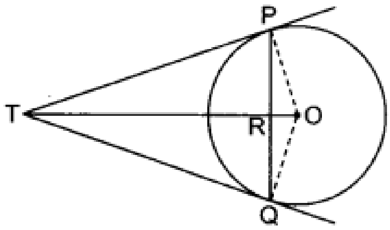


Solution:

Given: PQ is chord of length 16 cm, TP and TQ are the tangents to a circle with centre O, radius 10 cm.

To find: TP.

Solution: Join OP and OQ.



In triangles OTP and OTQ,

OT is common

$$OP = OQ \quad \text{(radii)}$$

$$TP = TQ$$

[length of the tangents drawn from a point outside the circle to the circle are equal]

$\therefore \quad \Delta OPT \cong \Delta OQT$ (SSS congruence rule)

$\therefore \quad \angle POT = \angle QOT$ ...(*i*) (By CPCT)

Consider, triangles OPR and OQR

$OP = OQ$ (radii)

OR is common $\quad \angle POR = \angle QOR$ [from (*i*)]

$\therefore \quad \Delta OPR \cong \Delta OQR$ (SAS congruence rule)

So, $\quad PR = RQ = \frac{1}{2} \times 16 = 8$ cm ...(*ii*) (By CPCT)

$\angle ORP = \angle ORQ = 90°$ ...(*iii*) (By CPCT)

In right-angled triangle TRP,

$TR^2 = TP^2 - (8)^2 = TP^2 - 64$ ...(*iv*) [From (*iii*)]

Also, in ΔTOP, $\quad OT^2 = TP^2 + (10)^2 = TP^2 + PO^2$ ($\because$ Pythagoras theorem)

$(TR + OR)^2 = TP + 100$

$(TR + 6)^2 = TP^2 + 100$ $\quad [\because OR = \sqrt{100 - 64} = 6]$

$TR^2 + 12TR + 36 = TP^2 + 100$

$TP^2 - 64 + 12TR + 36 = TP^2 + 100$ [From (*iv*)]

$12TR = 128 \Rightarrow TR = \frac{32}{3}$ cm

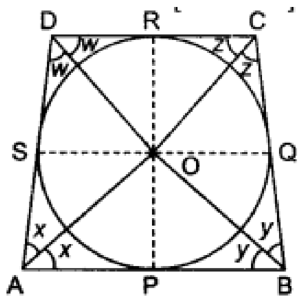
From (*iv*), $\quad \left(\frac{32}{3}\right)^2 = TP^2 - 64$

$\Rightarrow \quad TP^2 = \frac{1024}{9} + 64 = \frac{1024 + 576}{9} = \frac{1600}{9} \Rightarrow TP = \frac{40}{3}$ cm.

Question 39.

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:



Given: ABCD is a quadrilateral, circumscribing a circle with centre O and touches the quadrilateral at P, Q, R and S respectively.

To Prove: (*i*) $\angle AOB + \angle COD = 180°$

(*ii*) $\angle BOC + \angle AOD = 180°$

Construction: Join OP, OQ, OR and OS.

Proof: Consider, triangles APO and ASO,

$$AP = AS$$

[Lengths of the tangents drawn from a point outside the circle to the circle are equal]

$$OS = OP \text{ (radii)}$$

OA is common

$\therefore \quad \Delta APO \cong \angle ASO$ **(SSS congruency rule)**

$\therefore \quad \angle OAP = \angle OAS = x$ (say) **(CPCT)**

Similarly, $\angle OBP = \angle OBQ = y$ (say)

$\angle OCQ = \angle OCR = z$ (say)

and $\angle ODR = \angle ODS = w$ (say)

We have, $\angle DAB + \angle ABC + \angle BCD + \angle CDA = 360°$

[$\because$ Angle sum property of quadrilateral]

$\Rightarrow \quad 2x + 2y + 2z + 2w = 360°$

$\Rightarrow \quad x + y + z + w = 180°$ ...(*i*)

Consider, $\angle AOB + \angle COD = [180° - x - y] + [180° - w - z]$

[Sum of angles of a triangle is 180°]

$= 360° - (x + y + z + w)$

$= 360° - 180°$ [using (*i*)]

$\therefore \quad \angle AOB + \angle COD = 180°$

Again consider, $\angle BOC + \angle AOD$

$= [180° - y - z] + [180° - x - w]$

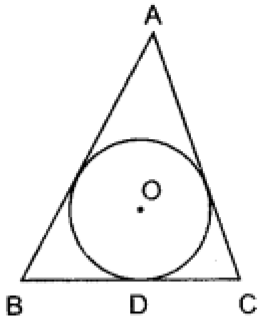
[Sum of angles of a triangle is 180°]

$= 360° - (x + y + z + w)$

$= 360° - 180° = 180°$ [using (*i*)]

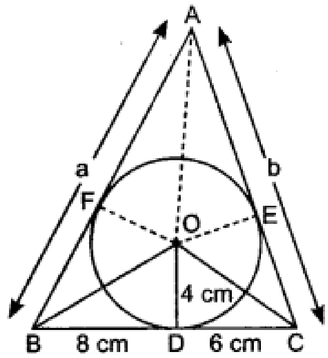
Hence proved.

Question 40.
In figure, a triangle ABC is drawn to circumscribe a circle of radius 4 cm, such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.



Solution:

We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal in length.



$\therefore \quad AF = AE = x$, (say)

$BF = BD = 8$ cm

$CE = CD = 6$ cm

$\therefore \quad AB = (x + 8)$ cm $= a$ (say)

$AC = (x + 6)$ cm $= b$ (say)

and $\quad c$ (say) $= BC = 14$ cm $= (8 + 6)$ cm

Now, perimeter of triangle using Heron's formula

$$2S = x + 8 + x + 6 + 14$$
$$= 28 + 2x$$
$$\Rightarrow \quad S = 14 + x$$

$\therefore$ **Area of ΔABC using Heron's formula,**

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{(14+x)(14+x-x-8)(14+x-x-6)(14+x-14)}$$
$$= \sqrt{(14+x)\times 6\times 8\times x} \quad \ldots(i)$$

Also, $\quad$ area of $\Delta ABC = ar(BOC) + ar(BOA) + ar(AOC)$

$$= \frac{1}{2}\times 14\times 4+\frac{1}{2}\times (8+x)\times 4+\frac{1}{2}(6+x)\times 4$$
$$= 28 + 16 + 2x + 12 + 2x = 56 + 4x \quad \ldots(ii)$$

From (i) and (ii), we get

$$\sqrt{48x(14+x)} = 56 + 4x$$

Squaring both sides, we get

$$48x(14 + x) = (56 + 4x)^2$$
$$\Rightarrow \quad 48x(14 + x) = 16(14 + x)^2$$
$$\Rightarrow \quad 3x(14 + x) - (14 + x)^2 = 0$$
$$\Rightarrow \quad (14 + x)(3x - 14 - x) = 0$$
$$\Rightarrow \quad (14 + x)(2x - 14) = 0$$
$$\Rightarrow \quad 14 + x = 0 \text{ or } 2x - 14 = 0$$
$$\Rightarrow \quad x = -14 \text{ (rejected) or } x = 7$$

$\therefore \quad AB = (7 + 8)$ cm $= 15$ cm,

and $\quad AC = (7 + 6)$ cm $= 13$ cm.

Question 41.

Prove that the lengths of the tangents drawn from an external point to a circle are equal.

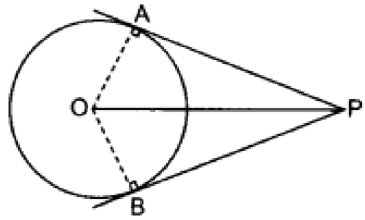
Solution:

Given: A circle $C(o, r)$. P is a point outside the circle and PA and PB are tangents to a circle.

To Prove: PA = PB

Construct: Draw OA, OB and OP.

Proof: Consider triangle OAP and OBP.



$\angle OAP = \angle OBP = 90°$ ...(*i*)

[Radius is perpendicular to the tangent at the point of contact]

OA = OB (radii) ...(*ii*)

OP is common ...(*iii*)

$\therefore$ $\Delta OAP \cong \Delta OBP$ (RHS) [from (*i*), (*ii*), (*iii*)]

$\Rightarrow$ AP = BP (CPCT)

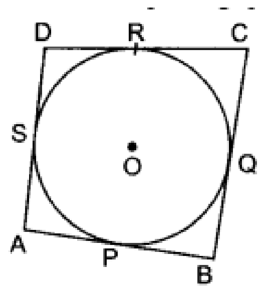
Question 42.
A quadrilateral is drawn to circumscribe a circle. Prove that the sums of opposite sides are equal

Solution:

Given: A quadrilateral ABCD which circumscribes a circle.

Let it touches the circle at P, Q, R and S as shown in figure.

To Prove: AB + CD = AD + BC



Proof: We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal.

$\therefore$ AP = AS; BP = BQ; CQ = CR and DR = DS ...(*i*)

Consider, AB + CD = AP + PB + CR + RD

= AS + BQ + CQ + DS [using (*i*)]

= (AS + DS) + (BQ + CQ) = AD + BC

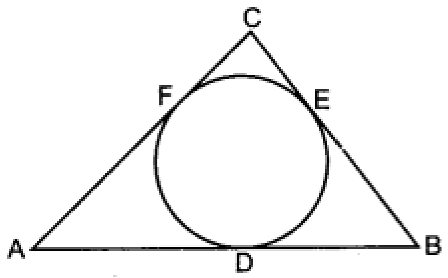
2013

Short Answer Type Questions I [2 Marks]

Question 43.
Prove that the parallelogram circumscribing a circle is a rhombus

Solution:
Refer to Ans. 36.

Question 44.
In the given figure, a circle inscribed in ΔABC touches its sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, then find the lengths of AD, BE and CF



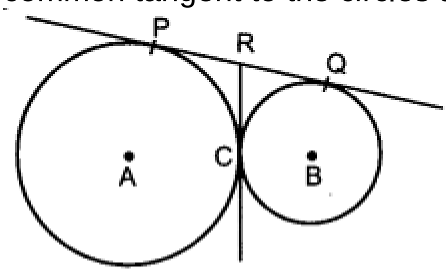
Solution:

Given, AB = 12 cm, BC = 8 cm, AC = 10 cm

Let	$AD = x$ cm	
$\therefore$	$BD = AB - AD = (12 - x)$ cm	
$\because$	$AD = AF$	[tangents from point A]
$\therefore$	$AF = x$ cm	
Now,	$CF = AC - AF = (10 - x)$ cm	
Also,	$CE = CF \Rightarrow CE = (10 - x)$ cm	
And	$BD = BE$	[$\because$ tangents from B]
$\Rightarrow$	$BE = (12 - x)$ cm	[From (*i*)]
Now,	$BC = CE + BE$	
$\Rightarrow$	$8 = (10 - x) + (12 - x)$	
$\Rightarrow$	$8 = 22 - 2x \Rightarrow 2x = 14$	
$\Rightarrow$	$x = 7$ cm	
$\Rightarrow$	$AD = 7$ cm	
	$BE = 12 - x = 12 - 7 = 5$ cm	
and	$CF = 10 - x = 10 - 7 = 3$ cm	

Question 45.

In the given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.



Solution:

PR and RC are tangents to circle with centre A.

$\therefore$ $PR = RC$ [tangent from common point R] ...(*i*)

Similarly RQ and RC are tangents to circle with centre B

$\therefore$ $RQ = RC$ ...(*ii*)

From (*i*) and (*ii*), $PR = RQ$

$\therefore$ CR bisects PQ.

Question 46.

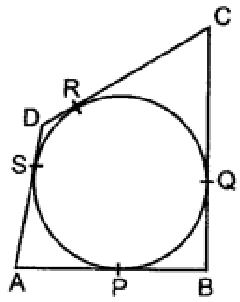
In the given figure, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that

AB + CD = AD + BC

Solution:

Quadrilateral ABCD circumscribing a circle.

$\therefore$ AP = AS [tangents drawn from common external point to a circle are equal in length.]



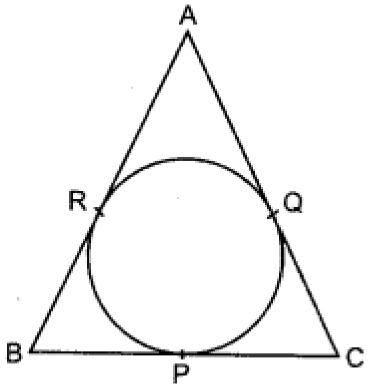
$$BP = BQ$$
$$DR = DS$$
$$CR = CQ$$

On adding,

$$AP + BP + DR + CR = AS + BQ + DS + CQ$$
$$(AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ)$$
$$AB + CD = AD + BC$$

Question 47.

In the given figure, a circle inscribed in ΔABC, touches its sides BC, CA and AB at the points P, Q and R respectively. If AB = AC, then prove that BP = CP.



Solution:

$$AB = AC$$

$\therefore$ $AR + BR = AQ + CQ$

$$AR + BR = AR + CQ$$

[AQ = AR, euqal tangents]

$\Rightarrow$ $BR = CQ$

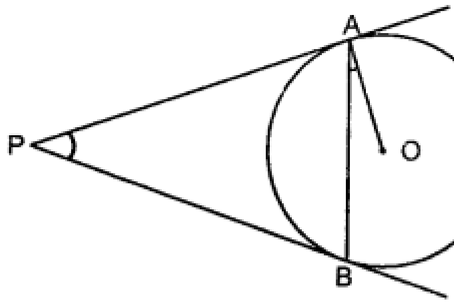
Now, BR = BP [Length of equal tangents]

$$CQ = CP$$

$\Rightarrow$ $BP = CP$

Question 48.

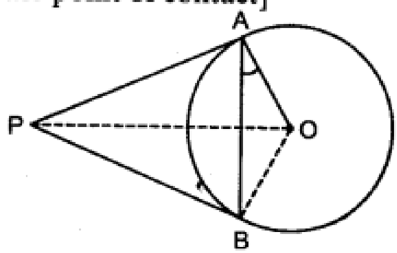
In the given figure, two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that ∠APB = 2 ∠OAB



Solution:

Construction: Join OP and OB.

Proof: Now, $OA \perp AP$ [Radius is perpendicular to tangent at the point of contact]



$\Rightarrow \quad \angle OAP = 90°$

Similarly, $\quad OB \perp BP$

$\Rightarrow \quad \angle OBP = 90°$

In quadrilateral OAPB

$$\angle OAP + \angle OBP = 90° + 90° = 180°$$

$\therefore$ Quadrilateral OAPB is a cyclic quadrilateral.

$\Rightarrow \quad \angle OAB = \angle OPB$ [Angles in same segment] ...(*i*)

Also, $\quad \angle OPB = \angle OPA$ [OP bisects $\angle APB$]

$\Rightarrow \quad \angle OPB + \angle OPA = 2\angle OPB$ [From (*i*)]

$\Rightarrow \quad \angle APB = 2\angle OPB$ ...(*ii*)

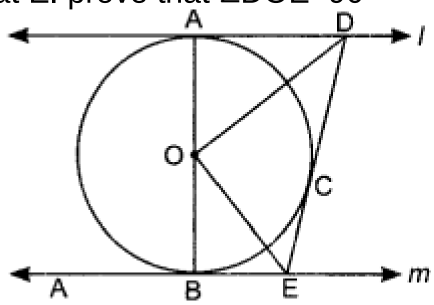
From (*i*) and (*ii*)

$$\angle APB = 2\angle OAB \qquad [\because \angle OAB = \angle OPB]$$

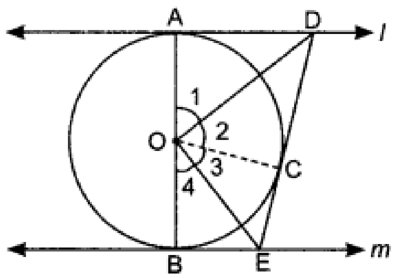
Long Answer Type Questions [4 Marks]

Question 49.
Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact
Solution:
Refer to ANS.12

Question 50.
In the given figure l,m are two parallel tangents to the circle with center O,touching the circle at A and B respectively.Another tangent at C intersect the line l at D and m at E. prove that ∠DOE=90



Solution:



Given: Line $l \parallel m$ and both are tangents to a circle at points A and B respectively.

To prove: $\angle DOE = 90^\circ$

Construction: Join OC

Proof: In ΔADO and ΔCDO,

$AD = DC$ [tangents from an external point are equal]

$OD = OD$ [common]

$\angle OAD = \angle OCD = 90^\circ$ [radius is perpendicular to tangent]

$\therefore \Delta ADO \cong \Delta COD$ [By RHS]

$\therefore \angle 1 = \angle 2$ ...(*i*) [By CPCT]

Similarly, in ΔBOE and ΔCOE,

$\angle 3 = \angle 4$ ...(*ii*)

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$ [angles on a straight line]

$\Rightarrow 2\angle 2 + 2\angle 3 = 180^\circ$ [$\because \angle 1 = \angle 2$ and $\angle 3 = \angle 4$]

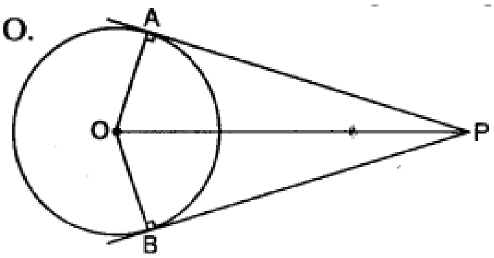
$\Rightarrow 2(\angle 2 + \angle 3) = 180^\circ$

$\Rightarrow \angle 2 + \angle 3 = 90^\circ \Rightarrow \angle DOE = 90^\circ$

Question 51.

Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution:



Given: PA and PB are tangents to a circle with centre O.

To Prove: $PA = PB$

Construction: Join OP, OA, OB.

Proof: In ΔAOP and ΔBOP

$OP = OP$ (common)

$OA = OB$ (radii of circle)

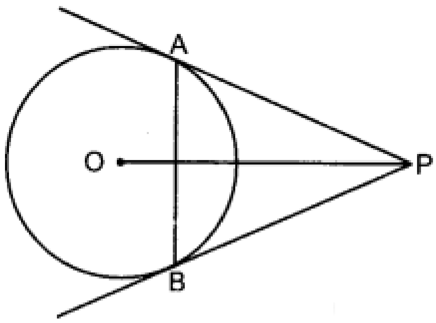
$\angle OAP = \angle OBP = 90^\circ$ (radius is perpendicular to tangent)

$\Delta AOP \cong \Delta BOP$ (RHS)

$\therefore PA = PB$ (CPCT)

Question 52.

In the given figure, PA and PB are two tangents drawn from an external point P to a circle with centre O. Prove that OP is the right bisector of line segment AB.



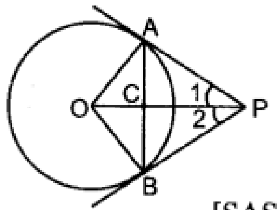
Solution:

Join OA and OB.

In ΔPAO and ΔPBO

	$OA = OB$	[Radii]
	$OP = OP$	[Common]
and	$AP = BP$	[Tangents from P]
$\therefore$	$\Delta PAO \cong \Delta PBO$	(SSS congruence rule)
$\Rightarrow$	$\angle 1 = \angle 2$	

In ΔAPC and ΔBPC



	$\angle 1 = \angle 2$ [Proved]	
	$AP = BP$	
and	$PC = PC$	
$\therefore$	$\Delta APC \cong \Delta BPC$	[SAS congruence rule]
$\Rightarrow$	$AC = BC$	[CPCT]
and	$\angle ACP = \angle BCP$	
Also,	$\angle ACP + \angle BCP = 180°$	
$\Rightarrow$	$\angle ACP = \angle BCP = 90°$	

$\therefore$ OP is the right bisector of AB.

Question 53.

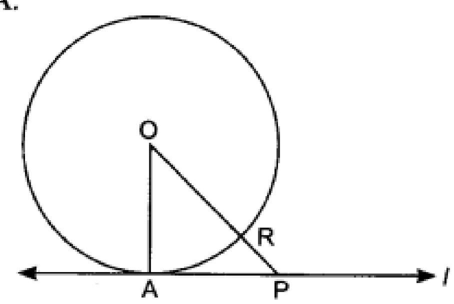
Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

Solution:

Given: A circle with centre O, line l is tangent to the circle at A.

To Prove: Radius OA is perpendicular to the tangent at A.

Construction: Take a point P, other than A, on tangent l. Join OP, meeting the circle at R.



Proof: We know that tangent to the circle touches, the circle at one point and all other points on the tangent lie in the exterior of a circle.

$\therefore$ $OP > OR$ (radius of circle)

$\Rightarrow$ $OP > OA$ ($\because$ $OR = OA$, radius of circle)

$\Rightarrow$ $OA < OP$

$\Rightarrow$ OA is the smallest segment, from O to a point on the tangent.

We know that smallest line segment from a point outside the circle to the line is perpendicular segment.

Hence, $OA \perp$ tangent l.

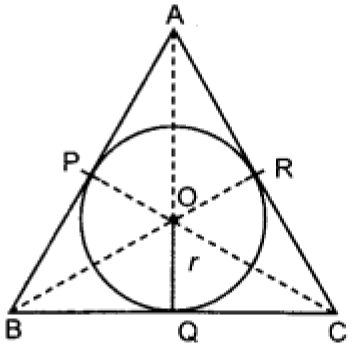
$\Rightarrow$ tangent at any point of a circle is perpendicular to the radius through the point of contact.

Question 54.

In the given figure, the sides AB, BC and CA of ΔABC touch a circle with centre O and radius r at P, Q and R respectively.

Prove that:

1. AB + CQ = AC + BQ
2. Area (ΔABC) = 1/2 (perimeter of ΔABC) X r



Solution:

(i) We have, $AP = AR$ [Tangents from A] ...(i)

Similarly, $BP = BQ$ [Tangents from B] ...(ii)

$CR = CQ$ [Tangents from C] ...(iii)

Now, we have

$\because \quad AP = AR$

$\Rightarrow \quad (AB - BP) = (AC - CR)$

$\Rightarrow \quad AB + CR = AC + BP$

$\Rightarrow \quad AB + CQ = AC + BQ$ [Using eq. (ii) and (iii)]

(ii) $\text{Area}(\Delta ABC) = \text{Area}(\Delta ABO + \Delta OBC + OAC)$

$= \frac{1}{2}(AB + BC + AC) \times r$ [$\because$ Area (Δ) $= \frac{1}{2} \times$ base $\times$ height]

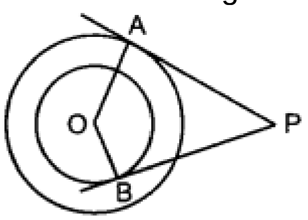
$= \frac{1}{2}(\text{perimeter of } \Delta ABC) \times r$

2012

Short Answer Type Questions I [2 Marks]

Question 55.

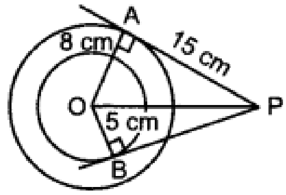
Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radii 8 cm and 5 cm respectively, as shown in Fig. If AP = 15 cm, then find the length of BP



Solution:

Join OP.

In ΔPAO and ΔPBO



$$\angle PAO = 90°, \angle PBO = 90°$$

($\because$ tangent is perpendicular to radius at the point of contact)

In right angled ΔPAO

$$PA^2 + OA^2 = OP^2 \quad [\because \text{ Pythagoras theorem}]$$
$$15^2 + 8^2 = OP^2$$
$$225 + 64 = OP^2$$
$$OP^2 = 289$$
$$OP = \sqrt{289} = 17 \text{ cm}$$

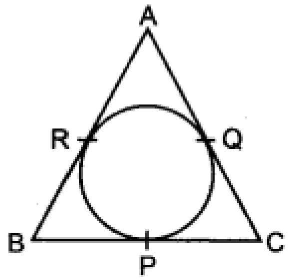
Now, In right angled ΔPBO

$$PB^2 + BO^2 = PO^2 \quad [\because \text{ Pythagoras theorem}]$$

$$PB^2 + 5^2 = (17)^2$$
$$PB^2 + 25 = 289$$
$$PB^2 = 289 - 25$$
$$PB^2 = 264$$
$$PB = \sqrt{264} = 2\sqrt{66} \text{ cm.}$$

Question 56.

In figure, an isosceles triangle ABC, with AB = AC, circumscribes a circle. Prove that the point of contact P bisects the base BC



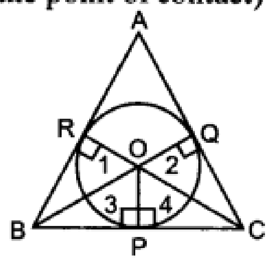
Solution:

Let the centre of circle be O.

Join OR, OQ, OB, OP, OC.

$$\angle 1 = \angle 2 = \angle 3 = \angle 4 = 90°$$

($\because$ Radius is perpendicular to tangent at the point of contact)



In ΔORB and ΔOQC

$OR = OQ$ (Radii of same circle)

$\angle 1 = \angle 2$ (each 90°)

$RB = QC$ $\left(\begin{array}{l}\because AB = AC \text{ and } AR = AQ \\ \text{So, } AB - AR = AC - AQ\end{array}\right)$

By SAS congruence rule,

$\Delta ORB \cong \angle OQC$

$\therefore$ $OB = OC$ (By CPCT)

In ΔOPB and ΔOPC

$OP = OP$ (common)

$\angle 3 = \angle 4$ (each 90°)

$OB = OC$ (Proved above)

By RHS congruence rule,

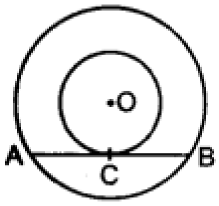
$\Delta OPB \cong \Delta OPC$

$BP = PC$ (By CPCT)

Hence, P bisects the base BC.

Question 57.

In figure, the chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C. Prove that AC = CB.



Solution:

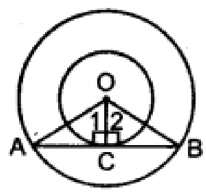
Given: Two concentric circles with **centre O.**

AB is chord of bigger circle **which touches the smaller circle to C.**

To Prove: AC = CB

Construction: Join OA, OC, OB

Proof: In ΔOCA and ΔOCB



$OC = OC$ (common)

$\angle 1 = \angle 2$ (Radius perpendicular tangent)

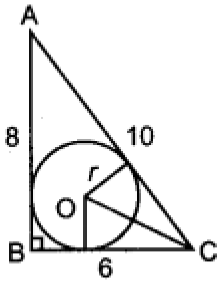
$OA = OB$ (radii of same circle).

By RHS congruence rule,

$\Delta OCA \cong \Delta OCB$

$\therefore$ $AC = BC$

Question 58.
In figure, a right triangle ABC, circumscribes a circle of radius r. If AB and BC are of lengths 8 cm and 6 cm respectively, find the value of r.



Solution:
$\because$ ABC is right angle Δ, right $\angle d$ at B.
So, By Pythagoras theorem

$$AC^2 = AB^2 + BC^2 = 8^2 + 6^2 = 100$$
$$AC = 10 \text{ cm}$$

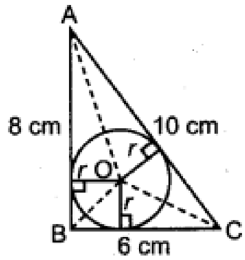
So, $\quad ar(\Delta ABC) = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$

Also, $\quad ar(\Delta ABC) = ar(\Delta OBC) + ar(\Delta OAC) + ar(\Delta OAB)$

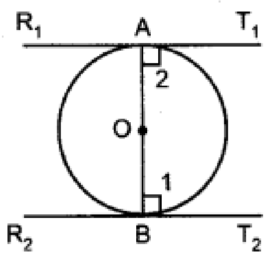
$\Rightarrow \quad 24 = \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r + \frac{1}{2} \times 8 \times r$

$\Rightarrow \quad 24 = 3r + 5r + 4r \Rightarrow 12r = 24$

$\Rightarrow \quad r = 2 \text{ cm}$



Question 59.
Prove that the tangents drawn at the ends of a diameter of a circle are parallel
Solution:
AB is the diameter.
R_1T_1 and R_2T_2 are the tangents at point A and B respectively.



Now, $\quad OB \perp R_2T_2$
[radius perpendicular the tangent at point of contact]
$\Rightarrow \quad \angle 1 = 90°$
Also, $\quad OA \perp R_1T_1$
[radius perpendicular the tangent at point of contact]
$\Rightarrow \quad \angle 2 = 90°$
Now, $\angle 1 + \angle 2 = 90° + 90° = 180°$
$\Rightarrow \quad R_1T_1 \parallel R_2T_2$ $\quad$ [$\because$ if interior angles on same side is supplementary, 2 lines are parallel]

Question 60.
The incircle of an isosceles triangle ABC, with AB = AC, touches the sides AB, BC and CA at D, E and F respectively. Prove that E bisects BC
Solution:

We have, $AB = AC$ [Given] ...(*i*)

$AD = AF$

[tangents drawn from an external point are equal] ...(*ii*)

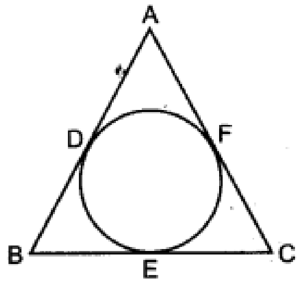
On subtracting eq (*ii*) from eq (*i*), we get

$AB - AD = AC - AF \Rightarrow BD = CF$ ...(*iii*)

$\because$ Also, $BD = BE$ ...(*iv*)

$\therefore$ from (*iii*) and (*iv*), we have

$BE = CF$ ...(*v*)



Also, $CF = CE$ ...(*vi*)

$\therefore$ from (*v*) and (*vi*)

$BE = CE$

$\therefore$ E bisects BC.

Question 61.
Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact

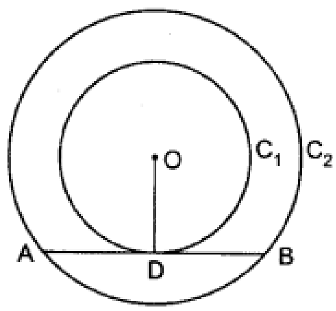
Solution:

Given: Let O be the centre of two concentric circles C_1 and C_2. Let AB be the chord of larger circle C_2 which is a tangent to the smaller circle C_1 at D.

To prove: Now we have to prove that the chord AB is bisected at D that is AD = BD.

Construction: Join OD.

Proof: Now since OD is the radius of the circle C_1 and AB is the tangent to the circle C_1 at D.



So, $OD \perp AB$ [radius of the circle is perpendicular to tangent at any point of contact]

Since AB is the chord of the circle C_2 and $OD \perp AB$.

$\therefore$ $AD = DB$ [perpendicular drawn from the centre to the chord always bisects the chord]

Short Answer Type Questions II [3 Marks]

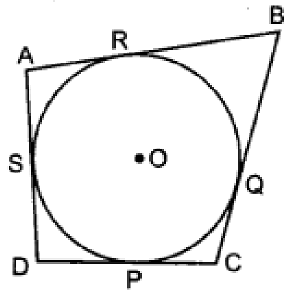
Question 62.
Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:

Given: A circle with centre O.

ABCD is a parallelogram circumscribing the circle, touching it at P, Q, R, S

To Prove: ABCD is a Rhombus.



Proof:

$$AR = AS \quad ...(i)$$

[$\because$ tangents from an external point are equal]

$$RB = BQ \quad ...(ii)$$

$$DP = DS \quad ...(iii)$$

$$PC = CQ \quad ...(iv)$$

Consider $AB + DC = AR + RB + DP + PC$

$= AS + BQ + DS + QC$ [By (*i*), (*ii*), (*iii*), (*iv*)]

Now, $AB + DC = AD + BC$

$AB + AB = AD + AD$

[$\because$ ABCD is a parallelogram, so opposite sides are equal, i.e. AB = CD. AD = BC]

$2AB = 2AD$

$AB = AD$

So, $AB = BC = CD = AD$

ABCD is a parallelogram with all sides equal.

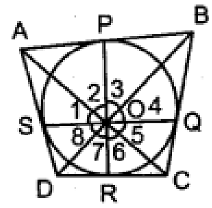
Hence, ABCD is a Rhombus.

Question 63.

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:

Given: ABCD is a quadrilateral circumscribing the circle with centre O touching it at P, Q, R, S.



To Prove: $\angle AOB + \angle DOC = 180°$

$\angle AOD + \angle BOC = 180°$

Construction: Join AO, PO, BO, QO, CO, RO, DO, SO,

Proof: In ΔAOS and ΔAOP

$AO = AO$ (common)

$AS = AP$ (tangents from external point)

$OS = OP$ (radii of same circle)

By SSS congruence

$\Delta AOS \cong \Delta AOP$

$\angle 1 = \angle 2$ (By CPCT) ...(*i*)

Similarly, $\angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$ ...(*ii*)

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ [$\because$ ASP of quadrilateral]

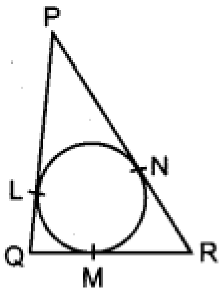
$\angle 2 + \angle 2 + \angle 3 + \angle 3 + \angle 6 + \angle 6 + \angle 7 + \angle 7 = 360°$ [By (*i*), (*ii*)]

$2[\angle 2 + \angle 3 + \angle 6 + \angle 7] = 360°$

$\angle AOB + \angle COD = 180°$

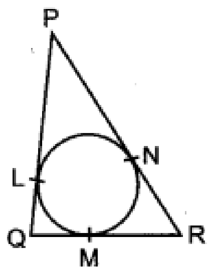
Similarly, $\angle AOD + \angle BOC = 180°$

Question 64.
In figure, a circle is inscribed in a triangle PQR with PQ = 10 cm, QR = 8 cm and PR = 12 cm. Find the lengths QM, RN and PL.



Solution:
We know that the tangents drawn from an external point to a circle are equal.

Therefore



Let $\quad QM = x = QL$

$\quad MR = y = RN$

and $\quad PL = z = PN$

Now $\quad PQ = 10 \text{ cm}, QR = 8 \text{ cm}, PR = 12 \text{ cm}$

$\Rightarrow \quad x + y = 8, y + z = 12, z + x = 10$

$\Rightarrow \quad 2x + 2y + 2z = 8 + 12 + 10 = 30$

$\Rightarrow \quad x + y + z = 15 \Rightarrow 8 + z = 15 \Rightarrow z = 7$

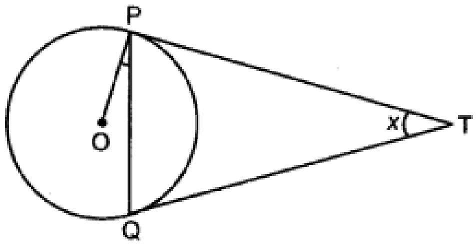
$\Rightarrow \quad x + 12 = 15 \Rightarrow x = 3$

$\Rightarrow \quad y + 10 = 15 \Rightarrow y = 5$

Hence, QM = 3 cm, RN = 5 cm and PL = 7 cm.

Question 65.
Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2∠OPQ

Solution:



Given: A circle with centre O. External point T and two tangents TP and TQ to the circle, where P, Q are the points of contact.

To Prove: $\angle PTQ = 2\angle OPQ$

Proof: Let $\angle PTQ = x$

In ΔPTQ, $PT = PQ$ [The lengths of tangents drawn from an external point to a circle are equal]

$\angle TPQ = \angle TQP$ [angles opposite to equal sides are equal]

$\angle TPQ = \angle TQP = \frac{1}{2}(180° - x) = 90° = \frac{x}{2}$ [$\because$ ASP of triangle]

$\angle OPT = 90°$ [The tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\therefore$ From figure, $\angle OPQ = \angle OPT - \angle TPQ$

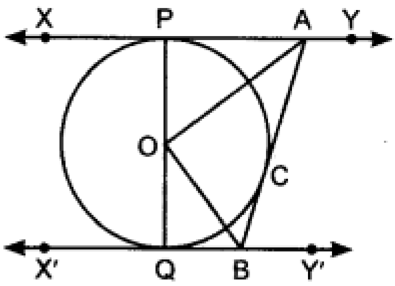
$$= 90° - \left(90° - \frac{x}{2}\right) = 90° - 90° + \frac{x}{2}$$

$$\angle OPQ = \frac{1}{2}\angle PTQ$$

This gives $\angle PTQ = 2\angle OPQ$ Hence, Proved.

Question 66.

In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X'Y' at B. Prove that ∠AOB = 90°.



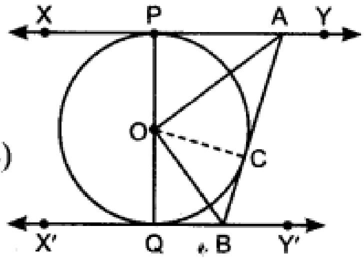
Solution:

Given: XY and X′Y′ are are two parallel tangents to circle with centre O. Tangent AB with point of contact C intersects XY at A and X′Y′ at B.

To Prove: $\angle AOB = 90°$

Construction: Join OC.

Proof: In ΔOPA and ΔOCA,



$$OP = OC \text{ (Radii of the same circle)}$$
$$AP = AC \text{ (Tangents from point A)}$$
$$AO = AO \text{ (common side)}$$
$$\Delta OPA \cong \Delta OCA \quad \text{(SSS congruence rule)}$$

Therefore, $P \to e, A \to A, O \to o$,

$$\angle POA = \angle COA \quad ...(i) \text{ (CPCT)}$$

Similarly, we prove: $\Delta OQB \cong \Delta OCB$

Then: $\angle QOB = \angle COB \quad ...(ii) \text{ (CPCT)}$

Since, POQ is the diameter of the circle, it is a straight line.

$\therefore \angle POA + \angle COA + \angle COB + \angle QOB = 180°$

from equation (i) and (ii),

$$2\angle COA + 2\angle COB = 180°$$
$$2(\angle COA + \angle COB) = 180°$$
$$\angle COA + \angle COB = \frac{180°}{2}$$
$$\angle COA + \angle COB = 90°$$
$$\angle AOB = 90°$$

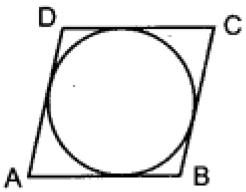
Hence, Proved.

Long Answer Type Questions [4 Marks]

Question 67.
Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact
Solution:
Refer to Ans. 12.

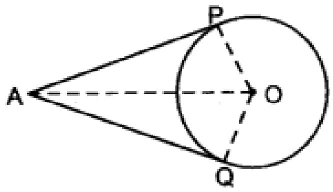
Question 68.
A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.
Solution:
Refer to Ans. 46.

Question 69.
Prove that the lengths of tangents drawn from an external point to a circle are equal. Using it, prove: quadrilateral ABCD is drawn to circumscribe a circle. Such’that AB + CD = AD + BC



Solution:

Given: A circle with centre O. Through the external point A, tangents AP and AQ are drawn.



To prove: AP = AQ

Construction: Join OA, OP and OQ

Proof: In ΔOAP and ΔOAQ,

$OP = OQ$ [Radii of the same circle]

$OA = OA$ [Common]

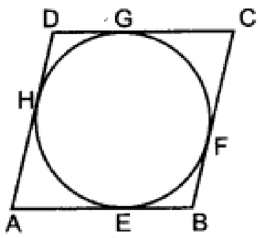
$\angle OPA = \angle OQA = 90^\circ$ [radius is perpendicular to the tangent at point of contact]

$\therefore$ $\Delta OAP \cong \Delta OAQ$ [By RHS]

$\Rightarrow$ $AP = AQ$ [CPCT]

Hence proved.

Second Part:



In the given figure, AE = AH ...(*i*)

[Tangents drawn from an external point are equal]

BE = BF ...(*ii*)

[Tangents drawn from an external point are equal]

DG = DH ...(*iii*)

[Tangents drawn from an external point are equal]

CG = CF ...(*iv*) [Tangents drawn from an external point are equal]

Adding equation (*i*), (*ii*), (*iii*) and (*iv*), we get

$AE + BE + DG + CG = AH + BF + DH + CF$

$\Rightarrow$ $(AE + BE) + (DG + CG) = (AH + DH) + (BF + CF)$

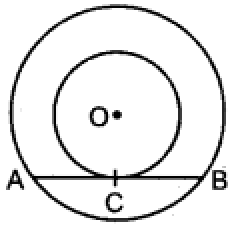
$\Rightarrow$ $AB + CD = AD + BC$.

Hence proved.

Question 70.

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Solution:



Refer to Ans. 12.

2011

Short Answer Type Questions I [2 Marks]

Question 71.
Two concentric circles are of radii 7 cm and r cm respectively, where r >7 .A chord of the larger circle, of length 48 cm, touches the smaller circle. Find the value of r.
Solution:

Given: $OP = 7$ cm; $OA = r$ cm
$AB = 48$ cm

Now, $OP \perp AB$ (as radius makes an angle of 90° with the tangent at point of contact)

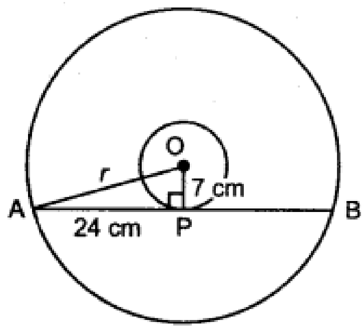
Also, $AP = PB$ (perpendicular drawn from centre to the chord bisects the chord)

So, $AP = 24$ cm

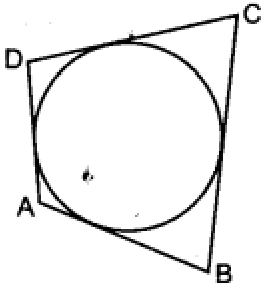
In ΔOPA, $\angle P = 90°$

By Pythagoras theorem in ΔOPA,

$$OA^2 = AP^2 + OP^2$$
$$r^2 = 24^2 + 7^2 = 576 + 49 = 625$$
$$\Rightarrow \quad r = 25 \text{ cm}$$



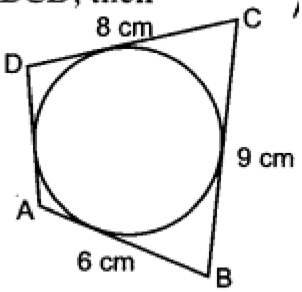
Question 72.
In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.



Solution:

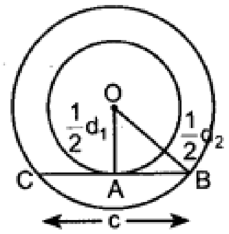
If a circle touches all the four sides of quadrilateral ABCD, then we know that

$$AD + BC = AB + CD$$
$$\therefore \quad AD + 9 = 6 + 8$$
$$\Rightarrow \quad AD = 5 \text{ cm}$$



Question 73.
If d1, d2 (d2 > d1) be the diameters of two concentric circles and c be the length of a chord of a circle which is tangent to the other circle, prove that $d^2 2 = c^2 + d^2$.

Solution:



$\because$ Diameter of bigger circle $= d_2$

So, Radius of bigger circle $= \frac{1}{2}d_2 = OB$

and Diameter of smaller circle $= d_1$

So, Radius of smaller circle $= \frac{1}{2}d_1 = OA$

$AB = \frac{c}{2}$ [$\because$ Diameter of CB = 'C']

In right ΔOAB, $\angle A = 90°$

[$\because$ radius is perpendicular the tangent at point of contact]

By pythagoras theorem $OB^2 = AB^2 + OA^2$

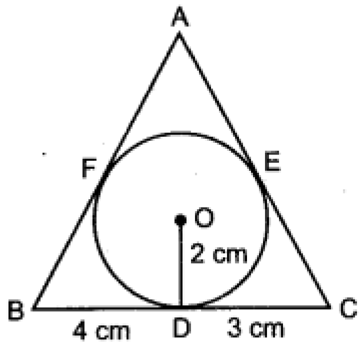
$\Rightarrow \left(\frac{1}{2}d_2\right)^2 = \left(\frac{1}{2}c\right)^2 + \left(\frac{1}{2}d_1\right)^2 \Rightarrow \frac{1}{4}d_2^2 = \frac{1}{4}c^2 + \frac{1}{4}d_1^2$

$\Rightarrow d_2^2 = c^2 + d_1^2$ Hence proved.

Short Answer Type Questions II [3 Marks]

Question 74.

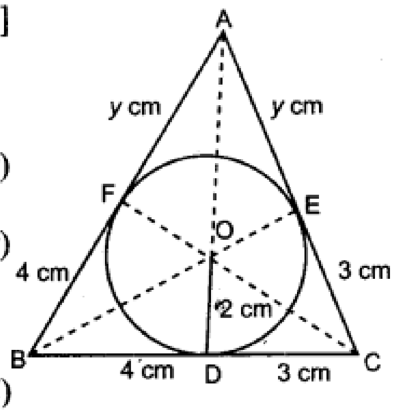
In figure, a triangle ABC is drawn to circumscribe a circle of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D are the lengths 4 cm and 3 cm respectively. If area of $\Delta ABC = 21$ cm^2, then find the lengths of sides AB and AC.



Solution:

Let AE = AF = y (say)

[Tangents drawn from an external point are equal]



$$\text{ar } \Delta BOC = \frac{1}{2} \times 7 \times 2 = 7 \text{ cm}^2 = b \text{ (say)}$$

$$\text{ar } \Delta AOB = \frac{1}{2} \times (4 + y) \times 2 = (4 + y) \text{ cm}^2 = a \text{ (say)}$$

$$\text{ar } \Delta AOC = \frac{1}{2} \times (3 + y) \times 2 = (3 + y) \text{ cm}^2 = c \text{ (say)}$$

Now, ar ΔABC = ar ΔAOB + ar ΔBOC + ar ΔAOC

$$= 4 + y + 7 + 3 + y$$

$$\text{ar } \Delta ABC = 14 + 2y \qquad ...(i)$$

For ΔABC,

Semi-perimeter, $$s = \frac{a+b+c}{2} = \frac{4+y+7+3+y}{2} = \frac{14+2y}{2} = 7 + y$$

$\therefore$ $$\text{ar } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} \qquad [\because \text{ By Heron's formula}]$$

$$= \sqrt{(7+y)(7+y-4-y)[7+y-7)(7+y-3-y)}$$

$$= \sqrt{(7+y) \times 3 \times y \times 4}$$

$$\text{ar } \Delta ABC = 2\sqrt{3y(7+y)} \qquad ...(ii)$$

From (i) and (ii)

$$\Rightarrow \quad 2\sqrt{3y(7+y)} = 14 + 2y$$

$$\Rightarrow \quad \sqrt{3y(7+y)} = 7 + y$$

Squaring both sides, we get

$$\Rightarrow \quad 3y(7+y) = (7+y)^2 \Rightarrow 21y + 3y^2 = 49 + y^2 + 14y$$

$$\Rightarrow \quad 2y^2 + 7y - 49 = 0 \Rightarrow 2y^2 + 14y - 7y - 49 = 0$$

$$\Rightarrow \quad 2y(y+7) - 7(y+7) = 0 \Rightarrow (2y-7)(y+7) = 0$$

$$\Rightarrow \quad y = \frac{7}{2}, y = -7 \qquad \text{[Rejected]}$$

Hence, length of side AB = 4 + 3.5 = 7.5 cm and AC = 3 + 3.5 = 6.5 cm.

Long Answer Type Questions [4 Marks]

Question 75.
Prove that the lengths of tangents drawn from an external point to a circle are equal.

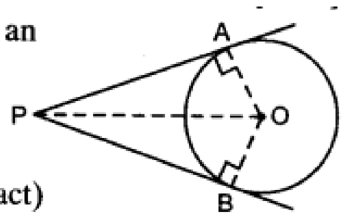
Solution:

Given: PA and PB are two tangents to a given circle drawn from an external point P.

To prove: PA = PB

Proof: OA ⊥ PA and OB ⊥ PB

(radius perpendicular to tangent at point of contact)



Join OP.

Now, In ΔOAP and ΔOBP,

	OA = OB	(radii)
	∠A = ∠B	(each 90°)
	OP = OP	(common)
So,	ΔOAP ≅ ΔOBP	(By RHS)
So,	PA = PB	(By CPCT)

Question 76.

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

Solution:

Refer to Ans. 12.

Question 77.

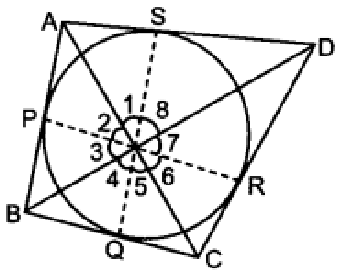
Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:

The given quadrilateral ABCD is circumscribing the circle having its centre at O.

The sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively.

Join OA, OB, OC, OD; OP, OQ, OR, OS.



We observe that OA bisects ∠POS [∵ By CPCT, applied to ΔPOA and ΔSOA]

⇒ $\angle 1 = \angle 2$ ...(*i*)

similarly $\angle 3 = \angle 4$ ...(*ii*)

$\angle 5 = \angle 6$ ...(*iii*)

and $\angle 7 = \angle 8$ ...(*iv*)

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ [∵ ASP of quadrilateral]

⇒ $2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360°$

⇒ $(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$ ⇒ $\angle AOD + \angle BOC = 180°$

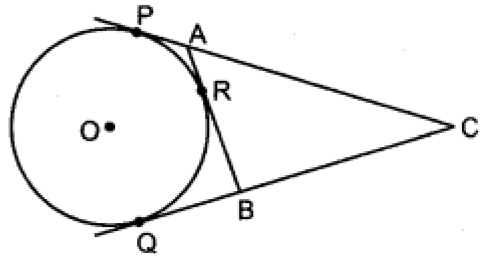
Similarly, $\angle AOB + \angle COD = 180°$

Hence, opposite sides of the quadrilateral ABCD subtend supplementary angles at the centre

2010

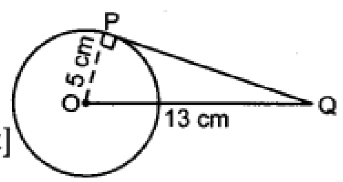
Very Short Answer Type Questions [1 Mark]

Question 78.
In figure, CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If CP = 11 cm and BR = 4 cm, find the length of BC.
Solution:



In the given figure, $CP = CQ$
[tangents drawn from an external point are equal]
So, $CP = CQ = 11$ cm
Also, $BR = BQ$
[tangents drawn from an external point are equal]
So, $BR = BQ = 4$ cm
$\therefore$ Now, $BC = CQ - BQ = (11 - 4)$ cm $= 7$ cm

Question 79.
A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 13 cm. Find the length PQ.
Solution:



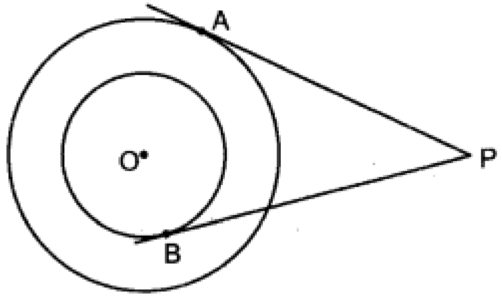
Given, $OP = 5$ cm [radius]
$OQ = 13$ cm
Now, In ΔOPQ, $\angle P = 90°$ [radius is perpendicular to tangent at point of to contact]
$$(OQ)^2 = (OP)^2 + (PQ)^2$$
$\therefore$ $PQ = \sqrt{(13)^2 - (5)^2} = 12$ cm [By pythagoras theorem]

Short Answer Type Questions I [2 Marks]

Question 80.
Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above prove the following: In Fig., PA and PB are tangents from an external point P , to a circle with centre O. LN touches the circle at M. Prove that PL + LM = PN + MN.
Solution:
Refer to Ans. 10 and 33.

Question 81.
In figure, there are two concentric circles, with centre O and of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12

cm, find the length of BP.



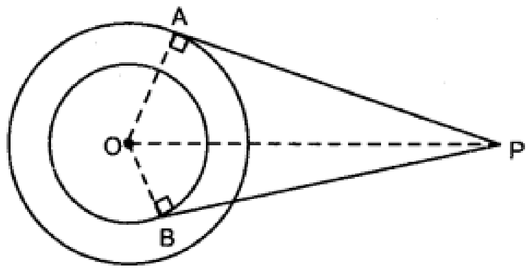
Solution:

Construction: Join OA, OB and OP.

AP = 12 cm, OA = 5 cm, OB = 3 cm

In ΔAOP, $\angle A = 90°$ [radius is perpendicular to the tangent at point of contact]

ΔBOP, $\angle B = 90°$ [radius is perpendicular to the tangent at point of contact]



So, $OP^2 = OA^2 + AP^2$

and $OP^2 = OB^2 + BP^2$

Using Pythagoras theorem for ΔAOP and ΔBOP.

$\therefore \quad OA^2 + AP^2 = OB^2 + BP^2$

$5^2 + 12^2 = 3^2 + BP^2 \Rightarrow 25 + 144 = 9 + BP^2 \Rightarrow 169 - 9 = BP^2$

$\Rightarrow \quad BP = \sqrt{160}$ cm = 12.65 cm