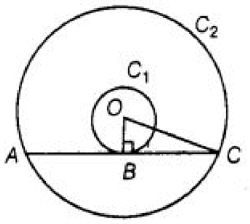
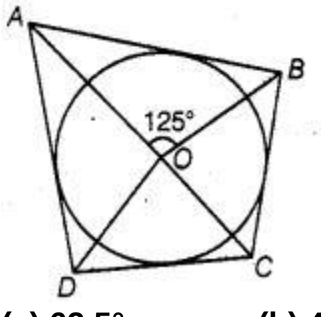
Chapter 9: Circles
Exercise 9.1

Question 1:
If the radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other circle is
(a) 3 cm (b) 6 cm (c) 9 cm (d) 1 cm
Solution:
(b) Let 0 be the centre of two concentric circles C_1 and C_2, whose radii are r_1 = 4 cm and r_2 = 5 cm. Now, we draw a chord AC of circle C_2, which touches the circle C_1 at B.
Also, join OB, which is perpendicular to AC. [Tangent at any point of a circle is perpendicular to the radius through the point of contact]



Now, in right-angled triangle OBC, by using Pythagoras theorem,
$OC^2 = BC^2 + BO^2$
or, $5^2 = BC^2 + 4^2$
or, $BC^2 = 25 – 46 = 9cm^2$
or, BC = 3cm
Length of the chord AC = 2BC = 2(3) = 6cm

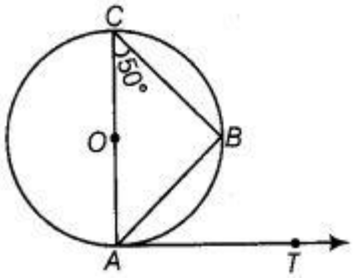
Question 2:
In the figure, if ∠AOB = 125°, then ∠COD is equal to



(a) 62.5° (b) 45° (c) 35° (d) 55°
Solution:
(d) We know that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.
$\angle AOB + \angle COD = 180^0$
$\angle COD = 180^0 - \angle AOB = 180^0 – 125^0 = 55^0$

Question 3:
In the figure, AB is a chord of the circle and AOC is its diameter such that

$\angle ACB = 50°$. If AT is the tangent to the circle at point A, then $\angle BAT$ is equal to



(a) 45° **(b) 60°** **(c) 50°** **(d) 55°**

Solution:

(c) In the figure, AOC is the diameter of the circle. We know that diameter subtends an angle of 90° at the circle.

$\angle ABC = 90^0$

In triangle ABC, $\angle A + \angle B + \angle C = 180^0$

$\angle A + 90^0 + 50^0 = 180^0$

$\angle A + 140^0 = 180^0$

$\angle A = 40^0$

$\angle A$ or $\angle OAB = 40^0$

now AT is the tangent to the circle at point A. So, OA is perpendicular to AT

$\angle OAT = 90^0$

$\angle OAB + \angle BAT = 90^0$

On putting $\angle OAB = 40^0$, we get

$\angle BAT = 90^0 - 40^0 = 50^0$

Hence, the value of $\angle BAT$ is 50^0

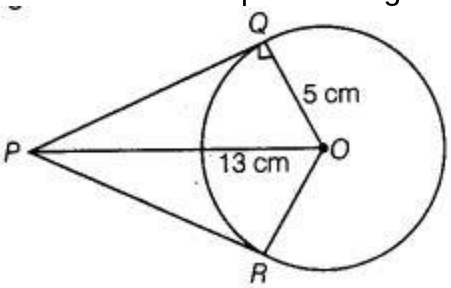
Question 4:

From a point P which is at a distance of 13 cm from the centre 0 of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle is drawn. Then, the area of the quadrilateral PQOR is

(a) 60 cm^2 **(b) 65 cm^2** **(c) 30 cm^2** **(d) 32.5 cm^2**

Solution:

(a) Firstly, draw a circle of radius 5 cm having centre O. P is a point at a distance of 13 cm from O. A pair of tangents PQ and PR are drawn.



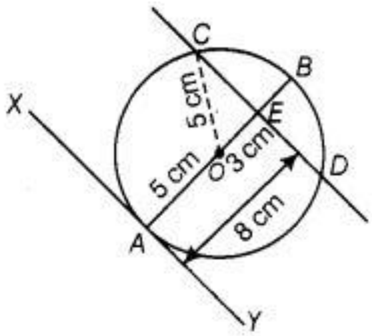
Thus, the quadrilateral POQR is formed.
Therefore, OQ ⊥ QP
In right-angled triangle PQO,
$OP^2 = 169 - 25 = 144$
QP = 12 cm

Now, area of triangle QOP = $\frac{1}{2} \times QP \times QO$
$= \frac{1}{2} \times 12 \times 5 = 30\text{cm}^2$
Area of quad QOPR = 2ΔOQP = 2(30) = 60cm^2

Question 5:
At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance of 8 cm from A, is

(a) 4 cm **(b) 5 cm**
(c) 6 cm **(d) 8 cm**

Solution:
(d) First, draw a circle of radius 5 cm having centre 0. A tangent XY is drawn at point A.



A chord CD is drawn which is parallel to XY and at a distance of 8 cm from A.

Now, $\angle OAY = 90°$

[Tangent and any point of a circle is perpendicular to the radius through the point of contact]

$\Delta OAY + \Delta OED = 180°$ [∵ sum of cointerior is 180°]

$\Rightarrow$ $\Delta OED = 180°$

Also, $AE = 8$ cm. Join OC

Now, in right angled ΔOEC, $OC^2 = OE^2 + EC^2$

$\Rightarrow$ $EC^2 = OC^2 - OE^2$ [by Pythagoras theorem]

$= 5^2 - 3^2$

[∵ OC = radius = 5 cm, $OE = AE - AO = 8 - 5 = 3$ cm]

$= 25 - 9 = 16$

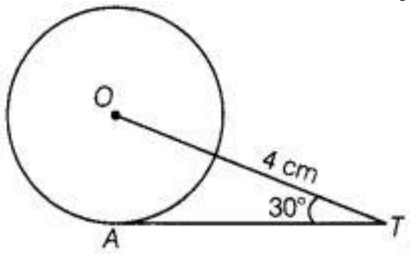
$\Rightarrow$ $EC = 4$ cm

Hence, length of chord $CD = 2\,CE = 2 \times 4 = 8$ cm

[since, perpendicular from centre to the chord bisects the chord]

Question 6:

In the figure, AT is a tangent to the circle with centre 0 such that OT = 4 cm and ∠OTA = 30°. Then, AT is equal to



(a) 4 cm **(b) 2 cm** **(c) 2√*3 cm*** **(d) 4√3 cm**

Solution:

(c) Join OA

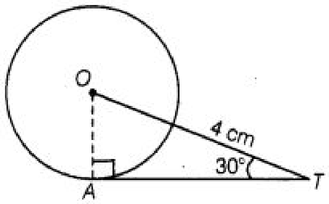
We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

therefore, ∠OAT = 90^0

In triangle OAT, Cos $30^0 = \frac{AT}{OT}$

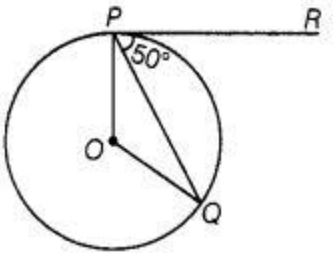
$$= \frac{\sqrt{3}}{2} = \frac{AT}{4}$$

$$= AT = 2\sqrt{3} \text{ cm}$$



Question 7:

In the figure, if 0 is the centre of a circle, PQ is a chord and the tangent PR at P, makes an angle of 50° with PQ, then ∠POQ is equal to



(a) 100° **(b) 80°** **(c) 90°** **(d) 75°**

Solution:

(a) Given, ∠QPR = 50°

We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OPR = 90°$

$\Rightarrow \quad \angle OPQ + \angle QPR = 90°$ [from figure]

$\Rightarrow \quad \angle OPQ = 90° - 50° = 40°$ [$\because \angle QPR = 50°$]

Now, $OP = OQ$ = Radius of circle

$\therefore \quad \angle OQP = \angle OPQ = 40°$

[since, angles opposite to equal sides are equal]

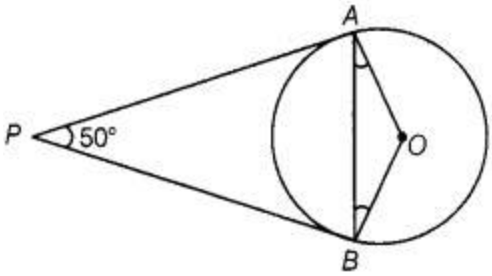
In ΔOPQ, $\angle O + \angle P + \angle Q = 180°$

[since, sum of angles of a triangle = 180°]

$\Rightarrow \quad \angle O = 180° - (40° + 40°)$ [$\because \angle P = 40° = \angle Q$]

$= 180° - 80° = 100°$

Question 8:
In the figure, if PA and PB are tangents to the circle with centre 0 such that ∠APB = 50°, then ∠OAB is equal to



(a) 25° **(b) 30°** **(c) 40°** **(d) 50°**

Solution:
(a) Given, PA and PB are tangent lines.

$\therefore \quad PA = PB$

[since, the length of tangents drawn from an external point to a circle is equal]

$\Rightarrow \quad \angle PBA = \angle PAB = \theta$ [say]

In ΔPAB, $\angle P + \angle A + \angle B = 180°$

[since, sum of angles of a triangle = 180°]

$\Rightarrow \quad 50° + \theta + \theta = 180°$

$\Rightarrow \quad 2\theta = 180° - 50° = 130°$

$\Rightarrow \quad \theta = 65°$

Also, $OA \perp PA$

[since, tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\therefore \quad \angle PAO = 90°$

$\Rightarrow \quad \angle PAB + \angle BAO = 90°$

$\Rightarrow \quad 65° + \angle BAO = 90°$

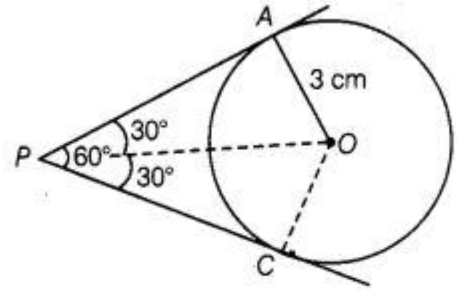
$\Rightarrow \quad \angle BAO = 90° - 65° = 25°$

Question 9:
If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then the length of each tangent is

(a) $\frac{3}{2}\sqrt{3}$ cm **(b) 6 cm** **(c) 3 cm** **(d) $3\sqrt{3}$ cm**

Solution:

(d) Let P be an external point and a pair of tangents are drawn from point P and the angle between these two tangents is 60°.



Join OA and OP.

Also, OP is a bisector line of $\angle APC$.

$\therefore$ $\angle APO = \angle CPO = 30°$

Also, $OA \perp AP$

A tangent at any point of a circle is perpendicular to the radius through the point of contact. In right-angled triangle OAP,

tan $30^0 = \frac{OP}{AP} = \frac{3}{AP}$

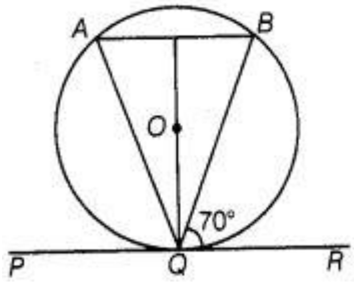
or, $\frac{1}{\sqrt{3}} = \frac{3}{AP}$

or, AP = $3\sqrt{3}$ cm

Hence, the length of each tangent is $3\sqrt{3}$ cm.

Question 10:

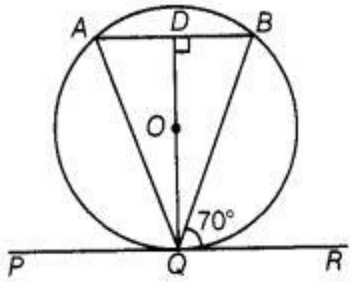
In the figure, if PQR is the tangent to a circle at Q whose centre is 0, AB is a chord parallel to PR and ∠BQR = 70°, then ∠ABQ is equal to



(a) 20° **(b) 40°** **(c) 35°** **(d) 45°**

Solution:

(b) Given, $AB \parallel PR$



$\therefore \quad \angle ABQ = \angle BQR = 70°$ [alternate angles]

Also, QD is perpendicular to AB and QD bisects AB.

In ΔQDA and ΔQDB, $\quad \angle QDA = \angle QDB$ [each 90°]

$AD = BD$

$QD = QD$ [common side]

$\therefore \quad \Delta ADQ \cong \Delta BDQ$ [by SAS similarity criterion]

Then $\quad \angle QAD = \angle QBD$ [CPCT] ...(i)

Also $\quad \angle ABQ = \angle BQR$ [alternate interior angle]

$\therefore \quad \angle ABQ = 70°$ [$\because \angle BQR = 70°$]

Hence, $\quad \angle QAB = 70°$ [from Eq. (i)]

Now, in ΔABQ, $\quad \angle A + \angle B + \angle Q = 180°$

$\Rightarrow \quad \angle Q = 180° - (70° + 70°) = 40°$

Exercise 9.2 Very Short Answer Type Questions

Question 1:
If a chord AB subtends an angle of 60° at the centre of a circle, then the angle between the tangents at A and B is also 60°.
Solution:
False
Since a chord, AB subtends an angle of 60° at the centre of a circle.

i.e., $\quad \angle AOB = 60°$

As $\quad OA = OB =$ Radius of the circle

$\therefore \quad \angle OAB = \angle OBA = 60°$

The tangent at points A and B is drawn, which intersect at C.

We know, $OA \perp AC$ and $OB \perp BC$.

$\therefore \quad \angle OAC = 90°, \angle OBC = 90°$

$\Rightarrow \quad \angle OAB + \angle BAC = 90°$

and $\quad \angle OBA + \angle ABC = 90°$

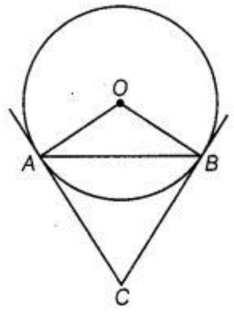
$\Rightarrow \quad \angle BAC = 90° - 60° = 30°$

and $\quad \angle ABC = 90° - 60° = 30°$

In ΔABC, $\quad \angle BAC + \angle CBA + \angle ACB = 180°$

[since, sum of all interior angles of a triangle is 180°]

$\Rightarrow \quad \angle ACB = 180° - (30° + 30°) = 120°$



Question 2:
The length of the tangent from an external point P on a circle is always greater than the radius of the circle.
Solution:

False
Because the length of the tangent from an external point P on a circle may or may not be greater than the radius of the circle.

Question 3:
The length of the tangent from an external point P on a circle with centre 0 is always less than OP.
Solution:
True

PT is a tangent drawn from external point P. Join OT.

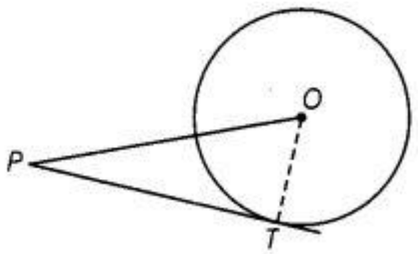
$\because \quad OT \perp PT$

So, OPT is a right angled triangle formed.

In right angled triangle, hypotenuse is always greater than any of the two sides of the triangle.

$\therefore \quad OP > PT$

or $\quad PT < OP$



Question 4:
The angle between two tangents to a circle may be 0°.
Solution:
True
'This may be possible only when both tangent lines coincide or are parallel to each other.

Question 5:
If the angle between two tangents drawn from a point P to a circle of radius a and centre 0 is 90°, then OP = a √2.
Solution:
True

From point P, two tangents are drawn.

Given, $\quad OT = a$

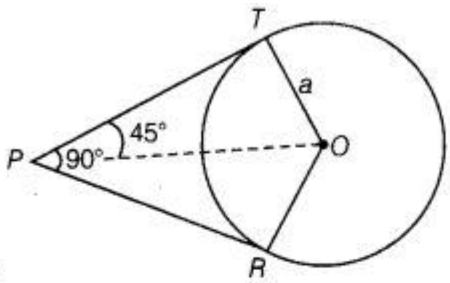
Also, line OP bisects the $\angle RPT$.

$\therefore \quad \angle TPO = \angle RPO = 45°$

Also, $\quad OT \perp PT$

In right angled ΔOTP, $\quad \sin 45° = \frac{OT}{OP}$

$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{a}{OP} \Rightarrow OP = a\sqrt{2}$



Question 6:
If the angle between two tangents drawn from a point P to a circle of radius a and centre 0 is 60°, then OP = a√3.
Solution:

True

From point P, two tangents are drawn.

Given, $OT = a$

Also, line OP bisects the $\angle RPT$.

$\therefore \quad \angle TPO = \angle RPO = 30°$

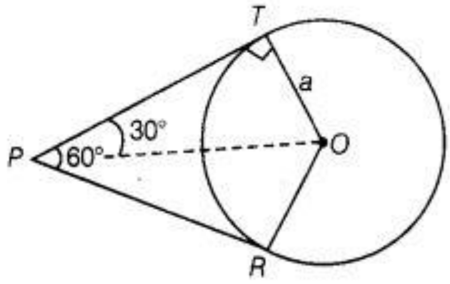
Also, $OT \perp PT$

In right angled ΔOTP,

$$\sin 30° = \frac{OT}{OP}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{a}{OP}$$

$$\Rightarrow \quad OP = 2a$$



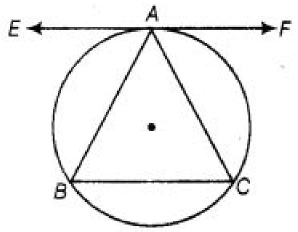
Question 7:

The tangent to the circumcircle of an isosceles ΔABC at A, in which AB = AC, is parallel to BC.

Solution:

True

Let EAF be tangent to the circumcircle of ΔABC.



To prove EAF ‖BC

∠EAB = ∠ABC

here, AB = AC

or, ∠ABC = ∠ACB ……………………………………(i) [angle between a tangent and is chord equal to the angle made by a chord in the alternate segment]

Also, ∠EAB = ∠BCA

From eq(i) and eq(ii), we get,

∠EAB = ∠ABC

or, EAF ‖BC

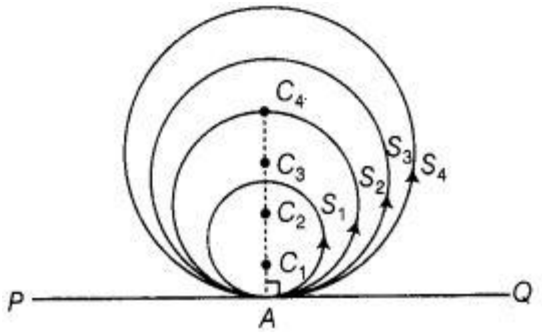
Question 8:

If several circles touch a given line segment PQ at a point A, then their centres lie on the perpendicular bisector of PQ.

Solution:

False

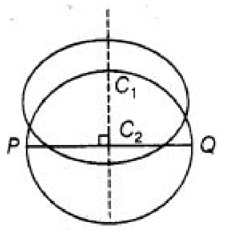
Given that PQ is any line segment and S_1, S_2, S_3, S_4,… circles are touches a line segment PQ at a point A. Let the centres of the circlesS_1, S_2, S_3, S_4,… be C_1 C_2, C_3, C_4,… respectively.



To prove centres of these circles lie on the perpendicular bisector PQ
Now, joining each centre of the circles to point A on the line segment PQ by a line segment i.e., C_1A, C_2A, C_3A, C_4A… so on.
We know that, if we draw a line from the centre of a circle to its tangent line, then the line is always perpendicular to the tangent line. But it not bisect the line segment PQ.
So, $C_1A \perp PQ$ …………..[for S_1]
$C_2A \perp PQ$…………….[for S_2]
$C_3A \perp PQ$ ……………[for S_3]
$C_4A \perp PQ$ ………………[For S_4]
……………………………………………..so on.

Since each circle is passing through a point A. Therefore, all the line segments C_1A, C_2A, C_3A, C_4A…. so on are coincident.
So, the centre of each circle lies on the perpendicular line of PQ but they do not lie on the perpendicular bisector of PQ.
Hence, several circles touch a given line segment PQ at a point A, then their centres lie

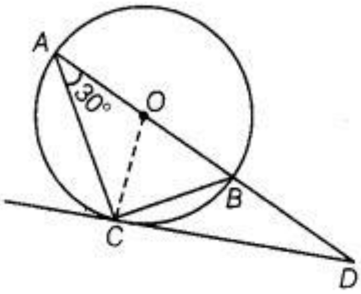
Question 9:
If several circles pass through the endpoints P and Q of a line segment PQ, then their centres lie on the perpendicular bisector of PQ.
Solution: true



We draw two circles with centres C_1 and C_2 passing through the endpoints P and Q of a line segment PQ. We know that perpendicular bisectors of a chord of a circle always passes through the centre of the circle
Thus, the perpendicular bisector of PQ passes through C_1 and C_2. Similarly, all the circle passing through PQ will haVe their centre on perpendiculars bisectors of PQ

Question 10:
AB is a diameter of a circle and AC is its chord such that ∠BAC – 30°. If the tangent at C intersects AB extended at D, then BC = BD.

Solution:
True
To Prove, BC = BD



Join BC and OC.

Given, $\angle BAC = 30°$

$\Rightarrow$ $\angle BCD = 30°$

[angle between tangent and chord is equal to angle made by chord in the alternate segment]

$\therefore$ $\angle ACD = \angle ACO + \angle OCD = 30° + 90° = 120°$

[$\because OC \perp CD$ and $OA = OC$ = radius $\Rightarrow \angle OAC = \angle OCA = 30°$]

In ΔACD, $\angle CAD + \angle ACD + \angle ADC = 180°$

[since, sum of all interior angles of a triangle is 180°]

$\Rightarrow$ $30° + 120° + \angle ADC = 180°$

$\Rightarrow$ $\angle ADC = 180° - (30° + 120°) = 30°$

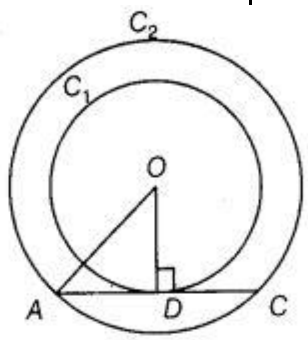
Now, in ΔBCD $\angle BCD = \angle BDC = 30°$

$\Rightarrow$ $BC = BD$

[since, sides opposite to equal angles are equal]

Exercise 9.3 Short Answer Type Questions

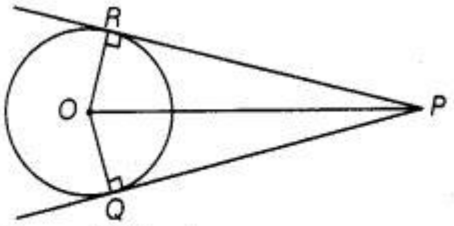
Question 1:
Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.
Solution:
Let C_1 and C_2 be the two circles having the same centre O. AC is a chord that touches the C_1 at point D.



Join OD, OD ⊥ AC
Thus, AD = DC = 4cm
In right-angled triangle AOD,
$DO^2 = 5^2 - 4^2$
$= 25 - 16 = 9$

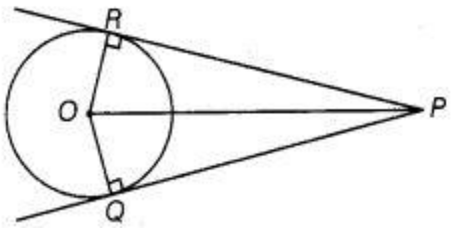
DO = 3 cm
The radius of the inner circle OD = 3 cm

Question 2:
Two tangents PQ and PR are drawn from an external point to a circle with centre 0. Prove that QORP is a cyclic quadrilateral.
Solution:
Given Two tangents PQ and PR are drawn from an external point to a circle with centre 0.



To prove *QORP* is a cyclic quadrilateral.
proof Since, *PR* and *PQ* are tangents.
So, $OR \perp PR$ and $OQ \perp PQ$
[since, if we drawn a line from centre of a circle to its tangent line. Then, the line always perpendicular to the tangent line]
$\therefore$ $\angle ORP = \angle OQP = 90°$
Hence, $\angle ORP + \angle OQP = 180°$
So, *QOPR* is cyclic quadrilateral.
[If sum of opposite angles is quadrilateral in 180°, then the quadrilateral is cyclic]
Hence proved.

Question 3:
Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.
Solution:
Given Two tangents PQ and PR are drawn from an external point P to a circle with centre 0.

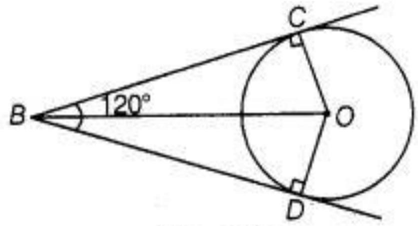


To prove the Centre of a circle touching two intersecting lines lies on the angle bisector of the lines.
In ∠RPQ.
Construction Join OR, and OQ.
In ΔPOP and ΔPOO
∠PRP = ∠PQO = 90^0 [tangent at any point of a circle is perpendicular to the radius through the point of contact]
OR = OQ [radii of some circle]

Since OP is common
$\Delta PRP \cong \Delta PQO$ [RHS

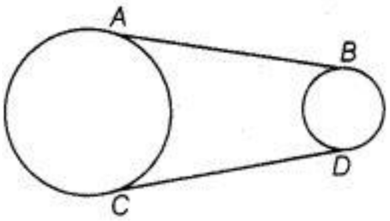
Hence, ∠RPO = ∠QPO [CPCT]
Thus, O lies on the angle bisector of PR and PQ. Hence proved.

Question 4:
If from an external point B of a circle with centre 0, two tangents BC and BD are drawn such that ∠DBC = 120°, prove that BC + BD = B0 i.e., BO = 2 BC.
Solution:
Two tangents BD and BC are drawn from an external point B.



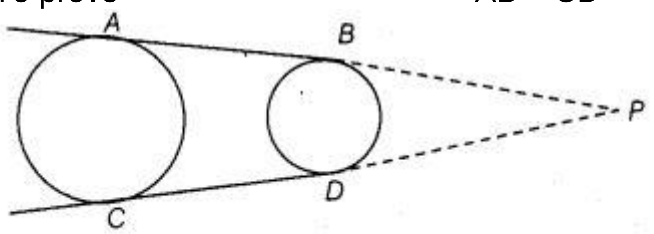
To prove $BO = 2BC$
Given, $\angle DBC = 120°$
Join OC, OD and BO.
Since, BC and BD are tangents.
∴ $OC \perp BC$ and $OD \perp BD$
We know, OB is a angle bisector of $\angle DBC$.
∴ $\angle OBC = \angle DBO = 60°$
In right angled ΔOBC, $\cos 60° = \dfrac{BC}{OB}$
⇒ $\dfrac{1}{2} = \dfrac{BC}{OB}$
⇒ $OB = 2\ BC$
Also, $BC = BD$
[tangent drawn from internal point to circle are equal]
∴ $OB = BC + BC$
⇒ $OB = BC + BD$

Question 5:
In the figure, AB and CD are common tangents to two circles of unequal radii. Prove that AB = CD



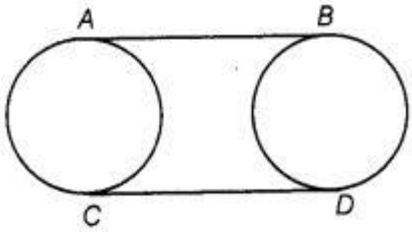
Solution:
Given AS and CD are common tangent to two circles of unequal radius

To prove AB = CD

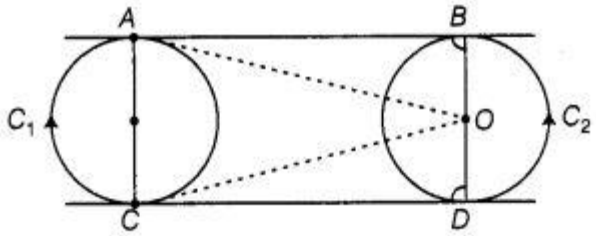


Construction: Produce AB and CD, to intersect at P
Proof: PA = PC [the length of tangents drawn from an internal point to a circle are equal]
PB = PD [The lengths of tangents drawn from an internal point to a circle are equal]
PA – PB = PC = PD
AB = CD

Question 6:
In the figure, AB and CD are common tangents to two circles of equal radii. Prove that AB = CD.



Solution:
Given AB and CD are tangents to two circles of equal radii?
To prove AB = CD



Construction Join OA, OC, $O'B$ and $O'D$

Proof Now, $\angle OAB = 90^\circ$

[tangent at any point of a circle is perpendicular to radius through the point of contact]

Thus, AC is a straight line.

Also, $\angle OAB + \angle OCD = 180^\circ$

∴ $AB || CD$

Similarly, BD is a straight line

and $\angle O'BA = \angle O'DC = 90^\circ$

Also, $AC = BD$ [radii of two circles are equal]

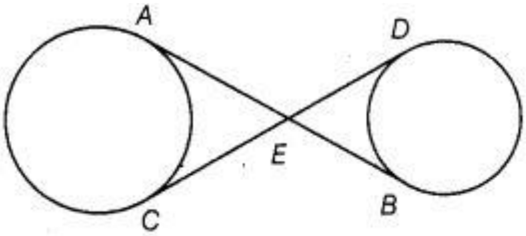
In quadrilateral $ABCD$, $\angle A = \angle B = \angle C = \angle D = 90^\circ$

and $AC = BD$

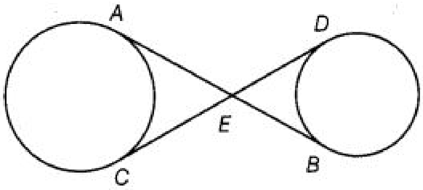
$ABCD$ is a rectangle

Hence, $AB = CD$ [opposite sides of rectangle are equal]

Question 7:
In the figure, common tangents AB and CD to two circles intersect at E. Prove that AB = CD.

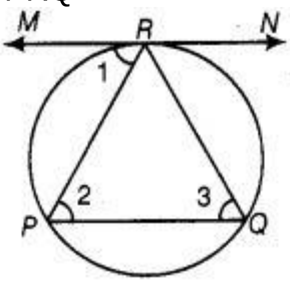


Solution:
Given Common tangents AB and CD to two circles intersecting at E.
To prove   AB = CD



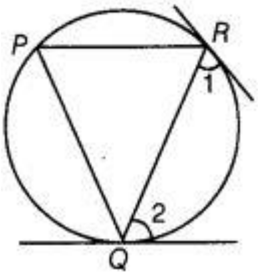
Proof: EA = EC ………………………..(i)[The lengths of tangents drawn from an internal point to a circle are equal]
EB = ED ………………………(ii)
On adding eq(i) and (ii),
EA + EB = EC + ED
AB = CD

Question 8:
A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.
Solution:
Given that Chord, PQ is parallel to the tangent at R.
To prove R bisects the arc PRQ



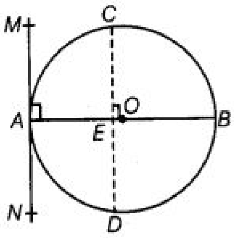
Proof $\angle 1 = \angle 2$ [alternate interior angles]
$\angle 1 = \angle 3$
[angle between tangent and chord is equal to angle made by chord in alternate segment]
$\therefore$ $\angle 2 = \angle 3$
$\Rightarrow$ $PR = QR$ [sides opposite to equal angles are equal]
$\Rightarrow$ $PR = QR$
So, R bisects PQ.

Question 9:
Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.
Solution:
To prove ∠1 = ∠2, let PQ be a chord of the circle. Tangents are drawn at the points R and Q.



Let P be another point on the circle, then, join PQ and PR.
Since, at point Q, there is a tangent.
∠2 = ∠P [angles in alternate segments are equal]
Since at point R, there is a tangent
∠1 = ∠P [angles in alternate segments are equal]
Thus, ∠1 = ∠2 = ∠P

Question 10:
Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at point A.
Solution:
Given, AB is the diameter of the circle.
A tangent is drawn from point A. Draw a chord CD parallel to the tangent MAN.

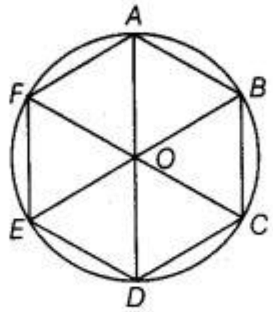


So, the CD is a chord of the circle and OA is a radius of the circle.
∠MAO = 90^0 [tangent at any point of a circle is perpendicular to the radius through the point of contact]
∠CEO = ∠MAO [Corresponding angles]
∠CEO = 90^0

Thus, OE bisects CD, [perpendicular from the centre of the circle to a chord bisects the chord] Similarly, the diameter AB bisects all. Chords that are parallel to the tangent at point A.

Exercise 9.4 Long Answer Type Questions

Question 1:
If a hexagon ABCDEF circumscribe a circle, prove that
AB + CD + EF =BC + DE + FA
Solution:
Given A hexagon ABCDEF circumscribes a circle.



To prove $AB + CD + EF = BC + DE + FA$

Proof $AB + CD + EF = (AQ + QB) + (CS + SD) + (EU + UF)$

$= AP + BR + CR + DT + ET + FP$

$= (AP + FP) + (BR + CR) + (DT + ET)$

$AB + CD + EF = AF + BC + DE$

$\because$ $AQ = AP$

$QB = BR$

$CS = CR$

$DS = DT$

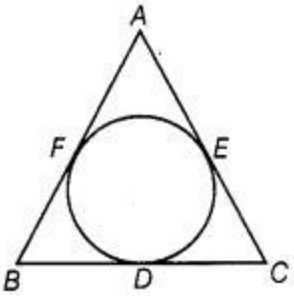
$EU = ET$

[tangents drawn from an external point to a circle are equal]

Hence proved.

Question 2:
Let s denotes the semi-perimeter of a Δ ABC in which BC = a, CA = b and AB = c. If a circle touches the sides BC, CA, AB at D, E, F, respectively. Prove that BD = s – b.
Solution:

A circle is inscribed in the A ABC, which touches the BC, CA and AB.



Given, $BC = a, CA = b$ and $AB = c$

By using the property, tangents are drawn from an external point to the circle are equal in length.

$\therefore \quad BD = BF = x$ [say]

$DC = CE = y$ [say]

and $AE = AF = z$ [say]

Now, $BC + CA + AB = a + b + c$

$\Rightarrow (BD + DC) + (CE + EA) + (AF + FB) = a + b + c$

$\Rightarrow (x + y) + (y + z) + (z + x) = a + b + c$

$\Rightarrow 2(x + y + z) = 2s$

[$\because 2s = a + b + c$ = perimeter of ΔABC]

$\Rightarrow s = x + y + z$

$\Rightarrow x = s - (y + z)$

$\Rightarrow BD = s - b$ [$\because b = AE + EC = z + y$]

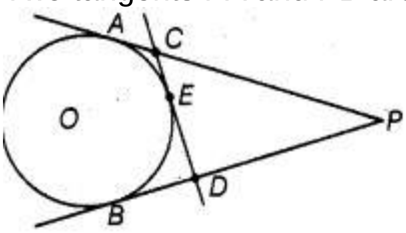
Hence proved.

Question 3:

From an external point P, two tangents, PA and PB are drawn to a circle with centre 0. At one point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of the triangle PCD.

Solution:

Two tangents PA and PB are drawn to a circle with centre 0 from an external point P



Perimeter of $\Delta PCD = PC + CD + PD$

$= PC + CE + ED + PD$

$= PC + CA + DB + PD$

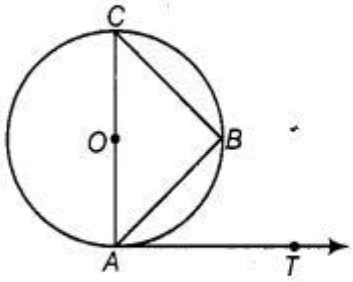
$= PA + PB$

$= 2PA = 2(10)$

$= 20$ cm

[$\because CE = CA, DE = DB, PA = PB$ tangents from internal point to a circle are equal]

Question 4:
If AB is a chord of a circle with centre 0, AOC is diameter and AT is the tangent at A as shown in the figure. Prove that ∠BAT = ∠ACB.



Solution:
Since AC is a diameter line, so angle in a semi-circle makes an angle 90°.

$\therefore \quad \angle ABC = 90^\circ$ [by property]

In ΔABC, $\quad \angle CAB + \angle ABC + \angle ACB = 180^\circ$

[$\because$ sum of all interior angles of any triangle is 180°]

$\Rightarrow \quad \angle CAB + \angle ACB = 180^\circ - 90^\circ = 90^\circ$ ...(i)

Since, diameter of a circle is perpendicular to the tangent.

i.e. $\quad CA \perp AT$

$\therefore \quad \angle CAT = 90^\circ$

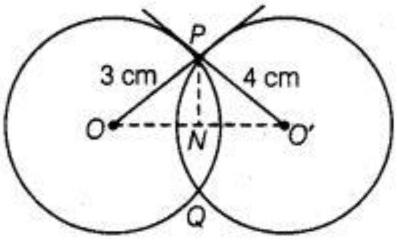
$\Rightarrow \quad \angle CAB + \angle BAT = 90^\circ$ ...(ii)

From Eqs. (i) and (ii),

$$\angle CAB + \angle ACB = \angle CAB + \angle BAT$$

$\Rightarrow \quad \angle ACB = \angle BAT$ **Hence proved.**

Question 5:
Two circles with centres 0 and 0′ of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and 0'P are tangents to the two circles. Find the length of the common chord PQ.
Solution:
Here, two circles are of radii OP = 3 cm and PO' = 4 cm
These two circles intersect at P and Q.



Here, OP and PO' are two tangents drawn at point P.

$$\angle OPO' = 90^\circ$$

[tangent at any point of circle is perpendicular to radius through the point of contact]

Join OO' and PN.

In right angled $\Delta OPO'$,

$$(OO')^2 = (OP)^2 + (PO')^2 \quad \text{[by Pythagoras theorem]}$$

i.e., $$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$= (3)^2 + (4)^2 = 25$$

$\Rightarrow$ $$OO' = 5\,\text{cm}$$

Also, $$PN \perp OO'$$

Let $ON = x$, then $NO' = 5 - x$

In right angled ΔOPN,

$$(OP)^2 = (ON)^2 + (NP)^2 \quad \text{[by Pythagoras theorem]}$$

$\Rightarrow$ $$(NP)^2 = 3^2 - x^2 = 9 - x^2 \quad \ldots\text{(i)}$$

and in right angled $\Delta PNO'$,

$$(PO')^2 = (PN)^2 + (NO')^2 \quad \text{[by Pythagoras theorem]}$$

$\Rightarrow$ $$(4)^2 = (PN)^2 + (5 - x)^2$$

$\Rightarrow$ $$(PN)^2 = 16 - (5 - x)^2 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii),

$$9 - x^2 = 16 - (5 - x)^2$$

$\Rightarrow$ $$7 + x^2 - (25 + x^2 - 10x) = 0$$

$\Rightarrow$ $$10x = 18$$

$\therefore$ $$x = 1.8$$

Again, in right angled ΔOPN,

$$OP^2 = (ON)^2 + (NP)^2 \quad \text{[by Pythagoras theorem]}$$

$\Rightarrow$ $$3^2 = (1.8)^2 + (NP)^2$$

$\Rightarrow$ $$(NP)^2 = 9 - 3.24 = 5.76$$

$\therefore$ $$(NP) = 2.4$$

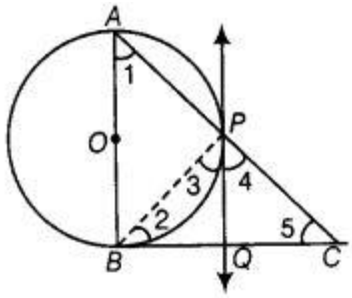
$\therefore$ Length of common chord, $$PQ = 2\,PN = 2 \times 2.4 = 4.8\,\text{cm}$$

Question 6:

In a right angle, ΔABC is which ∠B = 90°, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Prove that the tangent to the circle at PQ bisects BC.

Solution:

Let O be the centre of the given circle. Suppose, the tangent at P meets BC at 0.
Join BP.



To prove $BQ = QC$ [angles in alternate segment]

Proof $\angle ABC = 90°$

[tangent at any point of circle is perpendicular to radius through the point of contact]

$\therefore$ In ΔABC, $\angle 1 + \angle 5 = 90°$ [angle sum property, $\angle ABC = 90°$]

$\angle 3 = \angle 1$

[angle between tangent and the chord equals angle made by the chord in alternate segment]

$\therefore$ $\angle 3 + \angle 5 = 90°$ ...(i)

Also, $\angle APB = 90°$ [angle in semi-circle]

$\Rightarrow$ $\angle 3 + \angle 4 = 90°$ [$\angle APB + \angle BPC = 180°$, linear pair]

From Eqs. (i) and (ii), we get

$$\angle 3 + \angle 5 = \angle 3 + \angle 4$$

$\Rightarrow$ $\angle 5 = \angle 4$

$\Rightarrow$ $PQ = QC$ [sides opposite to equal angles are equal]

Also, $QP = QB$

[tangents drawn from an internal point to a circle are equal]

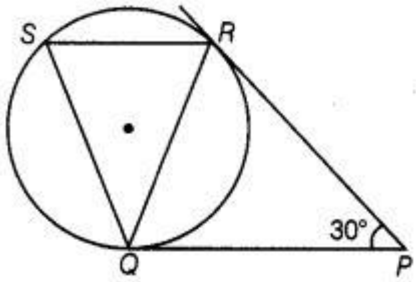
$\Rightarrow$ $QB = QC$ **Hence proved.**

Question 7:

In the figure, tangents PQ and PR are drawn to a circle such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find ∠RQS.

Solution:

PQ and PR are two tangents drawn from an external point P.



$\therefore \quad PQ = PR$

[the lengths of tangents drawn from an external point to a circle are equal]

$\Rightarrow \quad \angle PQR = \angle QRP$

[angles opposite to equal sides are equal]

Now, in $\Delta PQR \quad \angle PQR + \angle QRP + \angle RPQ = 180°$

[sum of all interior angles of any triangle is 180°]

$\Rightarrow \quad \angle PQR + \angle PQR + 30° = 180°$

$\Rightarrow \quad 2\angle PQR = 180° - 30°$

$\Rightarrow \quad \angle PQR = \dfrac{180° - 30°}{2} = 75°$

Since, $\quad SR \parallel QP$

$\therefore \quad \angle SRQ = \angle RQP = 75°$ [alternate interior angles]

Also, $\quad \angle PQR = \angle QSR = 75°$ [by alternate segment theorem]

In ΔQRS, $\quad \angle Q + \angle R + \angle S = 180°$

[sum of all interior angles of any triangle is 180°]

$\Rightarrow \quad \angle Q = 180° - (75° + 75°)$

$= 30°$

$\therefore \quad \angle RQS = 30°$

Question 8:
AB is diameter and AC is a chord of a circle with centre 0 such that ∠BAC = 30°. The tangent at C intersects extended AB at a point D. Prove that BC = BD.
Solution:
A circle is drawn with centre O and AB is a diameter.
AC is a chord such that ∠BAC = 30°.
Given AB is diameter and AC is a chord of a circle with centre O, ∠BAC = 30°.

Question 9:
Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the endpoints of the arc.
Solution:
Let mid-point of an arc AMB be M and TMT’ be the tangent to the circle.
Join AB, AM and MB.

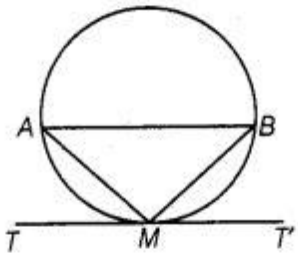
Since, arc AM = arc MB

$\Rightarrow$ Chord AM = Chord MB

In ΔAMB, $AM = MB$

$\Rightarrow$ $\angle MAB = \angle MBA$

[equal sides corresponding to the equal angle] ...(i)



Since, TMT' is a tangent line.

$\therefore$ $\angle AMT = \angle MBA$

[angles in alternate segments are equal]

$= \angle MAB$ [from Eq. (i)]

But ∠AMT and ∠MAB are alternate angles, which is possible only when

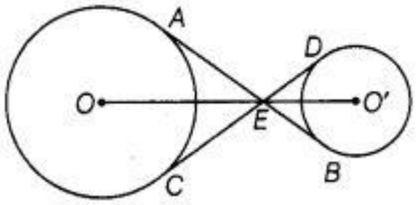
AB || TMT'

Hence, the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the endpoints of the arc

Hence proved.

Question 10:

In a figure the common tangents, AB and CD to two circles with centres 0 and O' intersect at E. Prove that the points 0, E and O' are collinear.

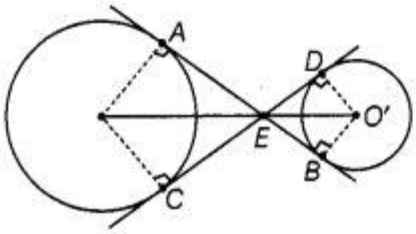


Solution:

Joint AO, OC and $O'D, O'B$.
Now, in $\Delta EO'D$ and $\Delta EO'B$,

$$O'D = O'B \quad \text{[radius]}$$
$$O'E = O'E \quad \text{[common side]}$$
$$ED = EB$$

[since, tangents drawn from an external point to the circle are equal in length]



$\therefore \quad \Delta EO'D \cong \Delta EO'B$ [by SSS congruence rule]

$\Rightarrow \quad \angle O'ED = \angle O'EB$

$O'E$ is the angle bisector of $\angle DEB$. ...(i)

Similarly, OE is the angle bisector of $\angle AEC$.

Now, in quadrilateral $DEBO'$,

$$\angle O'DE = \angle O'BE = 90^\circ$$

[since, CED is a tangent to the circle and $O'D$ is the radius, *i.e.*, $O'D \perp CED$]

$\Rightarrow \quad \angle O'DE + \angle O'BE = 180^\circ$

$\therefore \quad \angle DEB + \angle DO'B = 180^\circ$ [since, $DEBO'$ is cyclic quadrilateral] ...(ii)

Since, AB is a straight line.

$\therefore \quad \angle AED + \angle DEB = 180^\circ$

$\Rightarrow \quad \angle AED + 180^\circ - \angle DO'B = 180^\circ$ [from Eq. (ii)]

$\Rightarrow \quad \angle AED = \angle DO'B$ ...(iii)

Similarly, $\quad \angle AED = \angle AOC$ ...(iv)

Again from Eq. (ii), $\quad \angle DEB = 180^\circ - \angle DO'B$

Divided by 2 on both sides, we get

$$\frac{1}{2}\angle DEB = 90^\circ - \frac{1}{2}\angle DO'B$$

$\Rightarrow \quad \angle DEO' = 90^\circ - \frac{1}{2}\angle DO'B$ ...(v)

[since, $O'E$ is the angle bisector of $\angle DEB$ *i.e.*, $\frac{1}{2}\angle DEB = \angle DEO'$]

Similarly, $\quad \angle AEC = 180^\circ - \angle AOC$

Divided by 2 on both sides, we get

$$\frac{1}{2}\angle AEC = 90° - \frac{1}{2}\angle AOC$$

$$\Rightarrow \quad \angle AEO = 90° - \frac{1}{2}\angle AOC \qquad \ldots(vi)$$

[since, OE is the angle bisector of $\angle AEC$ i.e., $\frac{1}{2}\angle AEC = \angle AEO$]

Now, $\angle AED + \angle DEO' + \angle AEO = \angle AED + \left(90° - \frac{1}{2}\angle DO'B\right) + \left(90° - \frac{1}{2}\angle AOC\right)$

$$= \angle AED + 180° - \frac{1}{2}(\angle DO'B + \angle AOC)$$

$$= \angle AED + 180° - \frac{1}{2}(\angle AED + \angle AED) \quad \text{[from Eqs. (iii) and (iv)]}$$

$$= \angle AED + 180° - \frac{1}{2}(2 \times \angle AED)$$

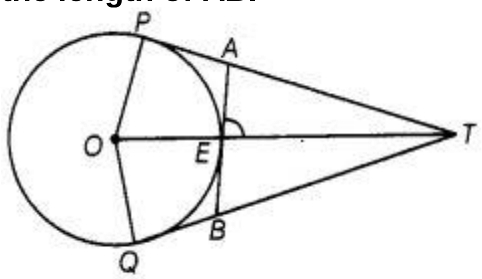
$$= \angle AED + 180° - \angle AED = 180°$$

$$\therefore \quad \angle AEO + \angle AED + \angle DEO' = 180°$$

So, OEO' is straight line.

Hence, O, E and O' are collinear. **Hence proved.**

Question 11:
In the figure, 0 is the centre of a circle of radius 5 cm, T is a point such that OT = 13 and 0T intersects the circle at E, if AB is the tangent to the circle at E, find the length of AB.



Solution:
Given, OT = 13 cm and OP = 5 cm
Since, if we drew a line from the centre to the tangent of the circle. It is always perpendicular to the tangent i.e., OP⊥PT.

In right angled ΔOPT, $OT^2 = OP^2 + PT^2$

[by Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$]

$\Rightarrow$ $PT^2 = (13)^2 - (5)^2 = 169 - 25 = 144$

$\Rightarrow$ $PT = 12$ cm

Since, the length of pair of tangents from an external point T is equal.

$\therefore$ $QT = 12$cm

Now, $TA = PT - PA$

$\Rightarrow$ $TA = 12 - PA$ ...(i)

and $TB = QT - QB$

$\Rightarrow$ $TB = 12 - QB$ ...(ii)

Again, using the property,length of pair of tangents from an external point is equal.

$\therefore$ $PA = AE$ and $QB = EB$ ...(iii)

$\therefore$ $OT = 13$cm

$\therefore$ $ET = OT - OE$ [$\because OE = 5$cm = radius]

$\Rightarrow$ $ET = 13 - 5$

$\Rightarrow$ $ET = 8$cm

Since, AB is a tangent and OE is the radius.

$\therefore$ $OE \perp AB$

$\Rightarrow$ $\angle OEA = 90°$

$\therefore$ $\angle AET = 180° - \angle OEA$ [linear pair]

$\Rightarrow$ $\angle AET = 90°$

Now, in right angled ΔAET,

$(AT)^2 = (AE)^2 + (ET)^2$ [by Pythagoras theorem]

$\Rightarrow$ $(PT - PA)^2 = (AE)^2 + (8)^2$

$\Rightarrow$ $(12 - PA)^2 = (PA)^2 + (8)^2$ [from Eq. (iii)]

$\Rightarrow$ $144 + (PA)^2 - 24 \cdot PA = (PA)^2 + 64$

$\Rightarrow$ $24 \cdot PA = 80$

$\Rightarrow$ $PA = \frac{10}{3}$cm

$\therefore$ $AE = \frac{10}{3}$ cm [from Eq. (iii)]

Join OQ.

Similarly $BE = \frac{10}{3}$cm

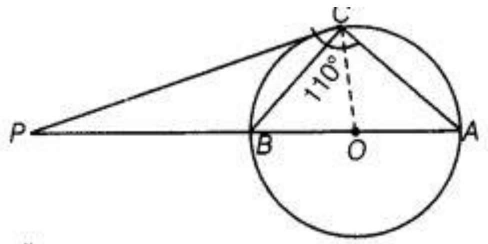
Hence, $AB = AE + EB$

$= \frac{10}{3} + \frac{10}{3}$

$= \frac{20}{3}$cm

Hence,the required length AB is $\frac{20}{3}$cm.

Question 12:
The tangent at a point C of a circle and a diameter AB when extended intersect at P. If ∠PCA = 110°, find ∠CBA.
Solution:
Here, AB is the diameter of the circle from point C and a tangent is drawn which meets at a point P.



Join OC. Here, OC is radius.
Since, tangent at any point of a circle is perpendicular to the radius through point of contact circle.

$\therefore$ $OC \perp PC$

Now, $\angle PCA = 110°$ [given]

$\Rightarrow$ $\angle PCO + \angle OCA = 110°$

$\Rightarrow$ $90° + \angle OCA = 110°$

$\Rightarrow$ $\angle OCA = 20°$

$\therefore$ $OC = OA =$ Radius of circle

$\Rightarrow$ $\angle OCA = \angle OAC = 20°$

[since, two sides are equal, then their opposite angles are equal]

Since, PC is a tangent, so $\angle BCP = \angle CAB = 20°$

[angles in a alternate segment are equal]

In ΔPBC, $\angle P + \angle C + \angle A = 180°$

$\angle P = 180° - (\angle C + \angle A)$

$= 180° - (110° + 20°)$

$= 180° - 130° = 50°$

In ΔPBC, $\angle BPC + \angle PCB + \angle PBC = 180°$

[sum of all interior angles of any triangle is 180°]

$\Rightarrow$ $50° + 20° + \angle PBC = 180°$

$\Rightarrow$ $\angle PBC = 180° - 70°$

$\Rightarrow$ $\angle PBC = 110°$

Since, APB is a straight line.

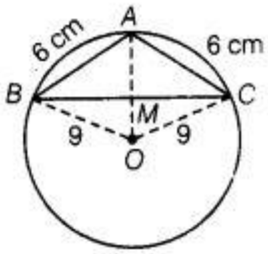
$\therefore$ $\angle PBC + \angle CBA = 180°$

$\Rightarrow$ $\angle CBA = 180° - 110° = 70°$

Question 13:
If an isosceles ΔABC in which AB = AC = 6 cm, is inscribed in a circle of radius 9 cm, find the area of the triangle.
Solution:
In a circle, ΔABC is inscribed.
Join OB, OC and OA.

Conside ΔABO and ΔACO

$$AB = AC \quad \text{[given]}$$
$$BO = CO \quad \text{[radii of same circle]}$$



AO is common.

$\therefore \quad \Delta ABO \cong \Delta ACO$ [by SSS congruence rule]

$\Rightarrow \quad \angle 1 = \angle 2$ [CPOT]

Now, in ΔABM and ΔACM,

$$AB = AC \quad \text{[given]}$$
$$\angle 1 = \angle 2 \quad \text{[proved above]}$$

AM is common.

$\therefore \quad \Delta AMB \cong \Delta AMC$ [by SAS congruence rule]

$\Rightarrow \quad \angle AMB = \angle AMC$ [CPCT]

Also, $\quad \angle AMB + \angle AMC = 180^\circ$ [linear pair]

$\Rightarrow \quad \angle AMB = 90^\circ$

We know that a perpendicular from centre of circle bisects the chord.

So, OA is perpendicular bisector of BC.

Let $AM = x$, then $OM = 9 - x$ [$\because OA$ = radius = 9 cm]

In right angled ΔAMC, $\quad AC^2 = AM^2 + MC^2$ [by Pythagoras theorem]

i.e., $\quad (\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$\Rightarrow \quad MC^2 = 6^2 - x^2 \quad \text{...(i)}$

and in right ΔOMC, $\quad OC^2 = OM^2 + MC^2$ [by Pythagoras theorem]

$\Rightarrow \quad MC^2 = 9^2 - (9 - x)^2 \quad \text{...(ii)}$

From Eqs. (i) and (ii), $\quad 6^2 - x^2 = 9^2 - (9 - x)^2$

$\Rightarrow \quad 36 - x^2 = 81 - (81 + x^2 - 18x)$

$\Rightarrow \quad 36 = 18x \Rightarrow x = 2$

$\therefore \quad AM = x = 2$

In right angled ΔABM, $\quad AB^2 = BM^2 + AM^2$ [by Pythagoras theorem]

$$6^2 = BM^2 + 2^2$$

$\Rightarrow \quad BM^2 = 36 - 4 = 32$

$\Rightarrow \quad BM = 4\sqrt{2}$

$\therefore \quad BC = 2\,BM = 8\sqrt{2}$ cm

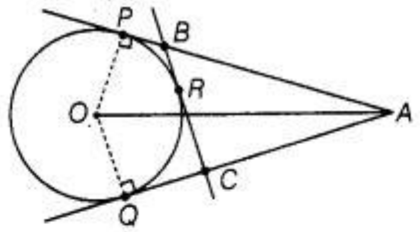
$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times BC \times AM$$

$$= \frac{1}{2} \times 8\sqrt{2} \times 2 = 8\sqrt{2} \text{ cm}^2$$

Hence, the required area of ΔABC is $8\sqrt{2}$ cm^2.

Question 14:
A is a point at a distance of 13 cm from the centre 0 of a circle of radius 5 cm. AP and AQ are the tangents to the circle at P and Q. If a tangent BC is drawn at a point R lying on the minor arc PQ to intersect AP at B and AQ at C, find the perimeter of the ΔABC.
Solution:
Given Two tangents are drawn from an external point A to the circle with centre 0,



$$OA = 13\text{ cm}$$

Tangent BC is drawn at a point R. radius of circle equals 5cm.

To find perimeter of ΔABC.

Proof $\angle OPA = 90°$

[tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\therefore$ $OA^2 = OP^2 + PA^2$ [by Pythagoras theorm]

$(13)^2 = 5^2 + PA^2$

$\Rightarrow$ $PA^2 = 144 = 12^2$

$\Rightarrow$ $PA = 12\text{cm}$

Now, perimeter of $\Delta ABC = AB + BC + CA$

$= (AB + BR) + (RC + CA)$

$= AB + BP + CQ + CA$

[$\because BR = BP, RC = CQ$ tangents from internal point to a circle are equal]

$= AP + AQ$

$= 2\,AP$

$= 2(12)$

$= 24\text{cm}$

[$AP = AQ$ tangent from internal point to a circle are equal]

Hence, the perimeter of $\Delta ABC = 24\text{ cm}$.