<u>**Chapter 1: Real Numbers**</u>

2016

Short Answer Type Questions I [2 Marks]

Question 1.
Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of either tanker in the exact number of times.
Solution:
Maximum capacity of a container, which can measure petrol in the exact number of times.

$= \text{HCF of } (850 \text{ and } 680)$

$$\begin{array}{r|r} 2 & 850 \\ \hline 5 & 425 \\ \hline 5 & 85 \\ \hline & 17 \end{array} \qquad \begin{array}{r|r} 2 & 680 \\ \hline 2 & 340 \\ \hline 2 & 170 \\ \hline 5 & 85 \\ \hline & 17 \end{array}$$

$850 = 2 \times 5 \times 5 \times 17$
$680 = 2 \times 2 \times 2 \times 5 \times 17$
$\text{HCF} = (850 \text{ and } 680) = 2 \times 5 \times 17 = 170$ litres.

Question 2.
Find the value of: $(-1) + (-1)^{2n} + (-l)^{2n+1} + (-l)^{4n+1}$, where n is any positive odd integer.
Solution:
To find $(-1)^n + (-1)^{2n} + (-1)^{2n+1} + (-1)^{4n+2}$
as 'n' is any positive odd integer
$\Rightarrow 2n$ and $4n + 2$ are even positive integers.

Now
$$(-1)^n = -1$$
$$(-1)^{2n} = +1$$
$$(-1)^{2n+1} = (-1)^{2n}(-1)^1 = 1(-1) = -1$$
$$(-1)^{4n+2} = (-1)^{4n}(-1)^2 = 1(1) = 1$$

Put all values in the above expression (i)
We have $-1 + 1 - 1 + 1 = 0$

Question 3.
Find whether a decimal expansion of 13/64 is a terminating or non-terminating decimal. If it terminates, find the number of decimal places its decimal expansion has.

Solution:

The given rational number is $\frac{13}{64}$

Now $\frac{13}{64} = \frac{13}{2^6} = \frac{13}{2^6 \times 5^0}$

The denominator of the given rational number is of the form $2^n \times 5^m$, i.e. $2^6 \times 5^0$

$\therefore$ The decimal expansion of $\frac{13}{64}$ is of the form of terminating.

The decimal expansion of $\frac{13}{64}$ terminates after 6 places of decimal.

$$
\begin{array}{r|l}
 & 0.203125 \\
64 & 13.000000 \\
 & \underline{128} \\
 & \;\;200 \\
 & \;\;\underline{192} \\
 & \;\;\;\;80 \\
 & \;\;\;\;\underline{64} \\
 & \;\;\;\;160 \\
 & \;\;\;\;\underline{128} \\
 & \;\;\;\;\;\;320 \\
 & \;\;\;\;\;\;\underline{320} \\
 & \;\;\;\;\;\;\;\;\underline{0}
\end{array}
$$

Short Answer Type Question II [3 Marks]

Question 4.
Explain whether the number 3 x 5 x 13 x 46 + 23 is a prime number or a composite number.
Solution:

We have $3 \times 5 \times 13 \times 46 + 23 = 23 \times (3 \times 5 \times 13 \times 2) + 23$

$= 23 \times (3 \times 5 \times 13 \times 2) + 23 = 23 \times 390 + 23$

It is clear that, the above number is a multiple of 23. Therefore, the number is $(3 \times 5 \times 13 \times 46 + 23)$ is not a prime number.

Long Answer Type Question [4 Marks]

Question 5.
Prove that the product of any three consecutive positive integers is divisible by 6.
Solution: Let three consecutive numbers are n, n + 1, n + 2
Solution:
1st Case: If n is even
This means n + 2 is also even.
Hence n and n + 2 are divisible by 2
Also, the product of n and (n + 2) is divisible by 2.
.’. n(n + 2) is divisible by 2.
This conclude n(n + 2) (n + 1) is divisible by 2 …(i)
As, n, n + 1, n + 2 are three consecutive numbers. n(n + 1) (n + 2) is a multiple of 3.
This shows n(n + 1) (n + 2) is divisible by 3. …(ii)
By equating (i) and (ii) we can say
n(n + 1) (n + 2) is divisible by 2 and 3 both.
Hence, n(n + 1) (n + 2) is divisible by 6.
2nd Case: When n is odd.
This show (n + 1) is even
Hence (n + 1) is divisible by 2. …(iii)
This conclude n(n + 1) (n + 2) is an even number and divisible by 2.

Also, the product of three consecutive number is a multiple of 3.
n(n + 1)(n + 2) is divisible by 3. …(iv)
Equating (iii) and (iv) we can say
n(n + 1) (n + 2) is divisible by both 2 and 3 Hence, n(n + 1)(n + 2) is divisible by 6.

2015

Short Answer Type Questions I [2 Marks]

Question 6.
Apply Euclid's division algorithm to find HCF of numbers 4052 and 420.
Solution:

Given numbers are 4052 and 420.

On applying Euclid's division algorithm, we have

$$4052 = 420 \times 9 + 272$$
$$420 = 272 \times 1 + 148$$
$$272 = 148 \times 1 + 124$$
$$148 = 124 \times 1 + 24$$
$$124 = 24 \times 5 + 4$$
$$24 = 4 \times 6 + 0$$

So, HCF of 4052 and 420 = 4.

420) 4052 (9
3780
272) 420 (1
272
148) 272 (1
148
124) 148 (1
124
24) 124 (5
120
4) 24 (6
24
0

Question 7.

Show that (√3+√5)² is an irrational number.
Solution:

We have to prove that $(\sqrt{3}+\sqrt{5})^2$ is an irrational number, i.e. to prove $(8+2\sqrt{15})$ is an irrational number.

Let us suppose $8+2\sqrt{15}$ is a rational number.

$\therefore$ There exists coprime integers (say) a and b, $b \neq 0$ such that

$$8 + 2\sqrt{15} = \frac{a}{b}$$

$$\Rightarrow \quad 2\sqrt{15} = \frac{a}{b} - 8 \quad \Rightarrow \quad \sqrt{15} = \frac{a-8b}{2b}$$

$\frac{a-8b}{2b}$ being rational number so, $\sqrt{15}$ becomes rational, which is contradiction with the fact that $\sqrt{15}$ is irrational. We led to contradiction due to wrong supposition. Hence, $(\sqrt{3}+\sqrt{5})^2$ is irrational.

Short Answer Type Question [3 Marks]

Question 8.
Three bells toll at intervals of 12 minutes, 15 minutes and 18 minutes respectively. If they start tolling together, after what time will they next toll together?
Solution:
LCM of 12, 15, 18 = 2^2x 3^2 x 5
=4x9x5 = 180
So, next time the bells will ring together after 180 minutes.

2	12, 15, 18
2	6, 15, 9
3	3, 15, 9
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

2014

Short Answer Type Questions I [2 Marks]

Question 9.
If HCF of 144 and 180 is expressed in form 13m – 3, find the value of m.
Solution:
On applying Euclid’s division algorithm,
180 = 144 x 1 + 36
144 = 36 x 4 + 0
At the last stage, the divisor is 36.
∴ HCF of 144 and 180 is 36.
∵ 36 = 13 x 3 – 3
So, m = 3
On applying Euclid’s division algorithm,
$180 = 144 \times 1 + 36$
$144 = 36 \times 4 + 0$
At the last stage, the divisor is 36.
∴ HCF of 144 and 180 is 36.
∵ $36 = 13 \times 3 - 3$
So, $m = 3$

144)180(1
144
36)144(4
144
0

Question 10.
Show that 9^n can not end with digit 0 for any natural number n.
Solution:
Since prime factorisation of 9^n is given by 9^n= $(3 \times 3)^n$ = 3271.
Prime factorisation of 9″ contains only prime number 3.
9 may end with the digit 0 for some natural number V if 5 must be in its prime factorisation, which is not present.
So, there is no natural number N for which 9^n ends with the digit zero.

Question 11.
Determine the values otp and q so that the prime factorisation of2520 is expressible as 23 X y X q x 7.
Solution:
Prime factorisation of 2520 is given by
2520 = 23 x 32 x 5 x 7
Given that 2520 = 23 x 3p x q x 7
On comparing both factorisation we get p = 2 and q = 5.

Question 12.
Show that 2√2 is an irrational number.
Solution:
Let us assume that $2\sqrt{2}$ is rational.
$\because$ There exists coprime integers a and b $(b \neq 0)$ such that

$$2\sqrt{2} = \frac{a}{b} \Rightarrow \sqrt{2} = \frac{a}{2b}$$

Since a and b are integers, we get $\frac{a}{2b}$ is rational and so $\sqrt{2}$ is rational.
But this contradicts the fact that $\sqrt{2}$ is irrational.
This contradiction has arisen because of our incorrect assumption that $2\sqrt{2}$ is rational.
Hence, we conclude that $2\sqrt{2}$ is irrational.

Question 13.
Show that any positive odd integer is of form 4m + 1 or 4m + 3, where m is some integer.
Solution:
Let 'a' be any positive integer and $b = 4$, then by Euclid's division algorithm, we have
$a = 4q + r$, $0 \leq r < 4$ where $q \geq 0$ and r = 0, 1, 2, 3.
Now 'a' may be of the form of $4q$, $4q + 1$, $4q + 2$, $4q + 3$.

When	When	When	When
$a = 4q$	$a = 4q + 1$	$a = 4q + 2$	$a = 4q + 3$
$= 2.2q$	$= 2.(2q) + 1$	$= 2.(2q) + 2$	$= 2.2q + 2 + 1$
$= 2m$	$= 2m + 1$	$= 2(2q + 1)$	$= 2(2q + 1) + 1$
where $m = 2q$	= odd number	$= 2m$	$= 2m + 1$
= even number		= even number	= odd number

Clearly, it is seen that any positive odd integer is of the form $4m + 1$ or $4m + 3$ for some integer m.

Short Answer Type Questions II [3 Marks]

Question 14.
By using, Euclid's algorithm, find the largest number which divides 650 and 1170.
Solution:
Given numbers are 650 and 1170.
On applying Euclid's division algorithm,

we get 1170 = 650 x 1 + 520
650 = 520 x 1 + 130
520 = 130 x 4 + 0
∵ At the last stage, the divisor is 130.
∴ The HCF of 650 and 1170 is 130.

$$
\begin{array}{l}
650\overline{)1170}(1 \\
\quad 650 \\
\quad \overline{520)650}(1 \\
\qquad\quad 520 \\
\qquad\quad \overline{130)520}(4 \\
\qquad\qquad\quad 520 \\
\qquad\qquad\quad \overline{\ \ 0\ \ }
\end{array}
$$

Question 15.
Show that reciprocal of 3+2√2 is an irrational number
Solution:
We have to prove that $\frac{1}{3+2\sqrt{2}} = 3 - 2\sqrt{2}$ is an irrational number.

Let us assume that $3 - 2\sqrt{2}$ is rational.
∴ There exists coprime integers a and b ($b \neq 0$) such that

$$3 - 2\sqrt{2} = \frac{a}{b} \Rightarrow 2\sqrt{2} = 3 - \frac{a}{b}$$

$$\Rightarrow \quad 2\sqrt{2} = \frac{3b-a}{b}$$

$$\Rightarrow \quad \sqrt{2} = \frac{3b-a}{2b} = \frac{3}{2} - \frac{a}{2b}$$

Since a and b are integers, we get $\frac{3}{2} - \frac{a}{2b}$ is rational and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.
This contradiction has arisen because of our incorrect assumption that $3 - 2\sqrt{2}$ is rational.
Hence, we conclude that $3 - 2\sqrt{2}$ is irrational.

Long Answer Type Question [4 Marks]

Question 16.
Find HCF of 378,180 and 420 by prime factorisation method. Is HCF x LCM of three numbers equal to the product of the three numbers?
Solution:
378 = 2 x 33 x 7
180 = 22 x 32 x 5
420 = 22 x 3 x 5 x 7

∴ HCF (378, 180, 420) = 2 x 3 = 6.
No. HCF (p, q, r) x LCM (p, q, r) ≠ p x q x r. where p, q, r are positive integers.

2013

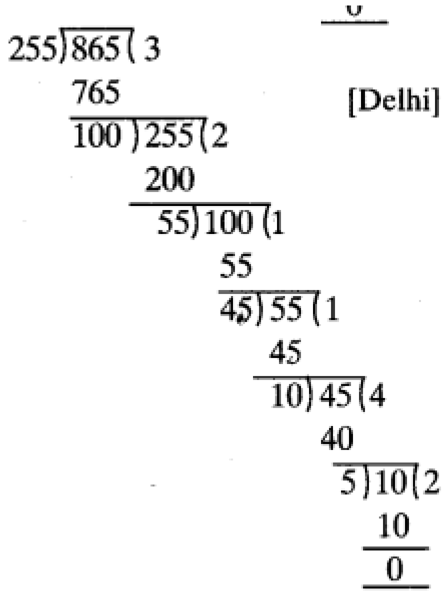
Short Answer Type Questions I [2 Marks]

Question 17.
Find the HCF of 255 and 867 by Euclid’s division algorithm
Solution:
Given numbers are 255 and 867.
On applying Euclid’s division algorithm, we have
867 = 255 x 3 + 102
255 = 102 x 2 + 51
102 = 51 x 2 + 0
∵ At the last stage, the divisor is 51
∴ The HCF of 255 and 867 is 51.

255)867(3
765
102)255(2
204
51)102(2
102
0

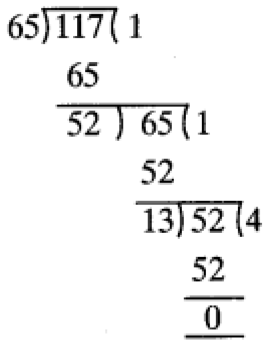
Question 18.
Find the HCF (865, 255) using Euclid’s division lemma.
Solution:
Given numbers are 255 and 865.
On applying Euclid’s division algorithm, we have
865 = 255 x 3 + 100
255 = 100 x 2 + 55
100 = 55 x 1 + 45
55 = 45 x 1 + 10
45 = 10 x 4 + 5
10 = 5 x 2 + 0
∵At the last stage, the divisor is 5

∴ The HCF of 255 and 865 is 5.



Short Answer Type Questions II [3 Marks]

Question 19.
Find HCF of 65 and 117 and find a pair of integral values of m and n such that HCF = 65m + 117n.
Solution:
Given numbers are 65 and 117.
On applying Euclid's division algorithm, we get
117 = 65 x 1 + 52
65 = 52 x 1 + 13
52 = 13 x 4 + 0
∵At the last stage, the divisor is 13.
∴ The HCF of 65 and 117 is 13.
The required pair of integral values of m and n is
(2,-1) which satisfies the given relation HCF = 65m + 117n.



Question 20.
By using Euclid's algorithm, find the largest number which divides 650 and 1170
Solution:

Since prime factorisation of 9^n is given by $9^n = (3 \times 3)^n = 3271$.
Prime factorisation of 9″ contains only prime number 3.
9 may end with the digit 0 for some natural number V if 5 must be in its prime factorisation, which is not present.
So, there is no natural number N for which 9^n ends with the digit zero.

2012

Short Answer Type Question I [2 Marks]

Question 21.
If $\frac{241}{4000} = \frac{241}{2^m.5^n}$ find the values of m and n where m and n are non-negative integers. Hence write its decimal expansion without actual division.
Solution:

$$\frac{241}{4000} = \frac{241}{2^m \cdot 5^n} \quad \Rightarrow 4000 = 2^m.5^n$$

$$2^5 \times 5^3 = 2^m.5^n \quad \Rightarrow m = 5, n = 3$$

$$\frac{241}{4\times1000} = \frac{1}{4}\times0.241 = 0.06025 \text{ (Terminating).}$$

Short Answer Type Questions II [3 Marks]

Question 22.
Express the number 0.3178 in the form of rational number a/b.
Solution:

Let $x = 0.3\overline{178}$

$$\Rightarrow \quad 10x = 3.\overline{178} \quad ...(i)$$

$$\Rightarrow \quad 10000x = 3178.\overline{178} \quad ...(ii)$$

Subtracting (i) from (ii)

$$\Rightarrow \quad 9990x = 3175$$

$$x = \frac{635}{1998}$$

Question 23.
Using Euclid's division algorithm, find whether the pair of numbers 847,2160 are or not.

Solution:

$$
\begin{aligned}
2160 &= 847 \times 2 + 466 \\
847 &= 466 \times 1 + 381 \\
466 &= 381 \times 1 + 85 \\
381 &= 85 \times 4 + 41 \\
85 &= 41 \times 2 + 3 \\
41 &= 3 \times 13 + 2 \\
3 &= 2 \times 1 + 1 \\
1 &= 1 + 0
\end{aligned}
$$

$\because$ HCF $= 1$

$\therefore$ Numbers 847 and 2160 are coprimes.

Question 24.
The LCM of the two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number.
Solution:
Let HCF $= x$

$\therefore$ LCM $= 14x$

A.T.Q. $x + 14x = 600 \Rightarrow x = 40$

Now, $280 \times$ other number $=$ HCF $\times$ LCM $= 40 \times 560$

Other number $= 80$

2011

Short Answer Type Questions I [2 Marks]

Question 25.
Prove that 15 + 17√3 is an irrational number.
Solution:
Let $15 + 17\sqrt{3}$ is a rational number.

$\therefore 15 + 17\sqrt{3} = \frac{a}{b}$, where a and b are coprime, $b \neq 0$

$\Rightarrow \quad 17\sqrt{3} = \frac{a}{b} - 15 \quad \Rightarrow \sqrt{3} = \frac{a - 15b}{17b}$

$\because$ a and b are integers

So, $\frac{a - 15b}{17b}$ is a rational number and so, $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational. This contradiction has arisen because of our incorrect assumption that $15 + 17\sqrt{3}$ is rational.

Hence, $15 + 17\sqrt{3}$ is irrational.

Question 26.
Find the LCM and HCF of 120 and 144 by using Fundamethe mental Theorem of

Arithmetic.
Solution:
120 = 23 x 3 x 5
144 = 24 x 32
∴ HCF = 23 x 3 = 24
LCM = 24 x 5 x 32 = 720

Short Answer Type Questions II [3 Marks]

Question 27.
An army contingent of 1000 members is to march behind an army band of 56 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?
Solution:
1000 =2x2x2x5x5x5
56 = 2x2x2x7
HCF of 1000 and 56 = 8
A maximum number of columns = 8.

Question 28.
Show that any positive odd integer is of the form 4q + 1 or 4q + 3 where q is a positive integer.
Solution:
Let N be any positive integer and $b = 4$
Then by Euclid's division lemma, $N = 4q + r, 0 \le r < 4; q > 0$
$\therefore N = 4q$ or $4q + 1$ or $4q + 2$ or $4q + 3$
(*i*) when $N = 4q = 2(2q) =$ even
(*ii*) when $N = 4q + 1 = 2(2q) + 1 =$ even $+ 1 =$ odd
(*iii*) when $N = 4q + 2 = 2(2q + 1) =$ even
(*iv*) when $N = 4q + 3 = 4q + 2 + 1 = 2(2q + 1) + 1 =$ Even $+ 1 =$ odd
$\therefore$ When $N = 4q + 1$ or $4q + 3$, then it is odd
$\Rightarrow$ Any positive odd integer is of the form $4q + 1$ or $4q + 3$.

Question 29.
Prove that 2√3/5 is irrational

Solution:

Let $\frac{2\sqrt{3}}{5}$ is rational.

$\therefore$ There exists coprime integers a and $b (b \neq 0)$

Such that $\frac{2\sqrt{3}}{5} = \frac{a}{b} \Rightarrow \sqrt{3} = \frac{5a}{2b}$

$\because$ a and b are integers we get $\frac{5a}{2b}$ is rational and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $\frac{2\sqrt{3}}{5}$ is rational.

So, we conclude that $\frac{2\sqrt{3}}{5}$ is irrational.

2010

Very Short Answer Type Questions [1 Mark]

Question 30.
Has the rational number a terminating or a non-terminating decimal representation
Solution:

$\frac{441}{2^2 \cdot 5^7 \cdot 7^2}$ is non-terminating decimal.

Since $q = 2^2 \times 5^7 \times 7^2$ is not of the form $2^m \times 5^n$.

Question 31.
Write whether $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplification gives a rational or an irrational number.
Solution:

$$\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}} = \frac{2\sqrt{9\times 5}+3\sqrt{4\times 5}}{2\sqrt{5}} = \frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}} = \frac{12\sqrt{5}}{2\sqrt{5}}$$

$= 6$ which is rational number.

Question 32.
The HCF of 45 and 105 is 15. Write their LCM.
Solution:

HCF (45, 105) $= 15$

$\therefore \quad \text{LCM} = \frac{45\times 105}{15} = 315$

Short Answer Type Questions II [3 Marks]

Question 33.
Prove that 2-3√5 is an irrational number.
Solution:

Let us assume, to contrary that $2 - 3\sqrt{5}$ is rational

Let $2 - 3\sqrt{5} = \frac{a}{b}$ where a and b are coprime numbers, $b \neq 0$

$$2 - \frac{a}{b} = 3\sqrt{5}$$

$$\frac{2b-a}{3b} = \sqrt{5}$$

Since a and b are integers, we get $\frac{2b-a}{3b}$ is rational, and so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational. This contradiction has arisen because of our incorrect assumption that $2 - 3\sqrt{5}$ is rational.

So, we conclude that $2 - 3\sqrt{5}$ is irrational.

Question 34.
Prove that 2√3 – 1 is an irrational number.
Solution:

Let 2√3 – 1 is an irrational number.

$\therefore\ 2\sqrt{3} - 1 = \frac{a}{b}$, where a and b are coprimes and $b \neq 0$

$\Rightarrow \quad 2\sqrt{3} = \frac{a}{b} + 1 \Rightarrow 2\sqrt{3} = \frac{a+b}{b}$

$\Rightarrow \quad \sqrt{3} = \frac{a+b}{2b} \qquad ...(i)$

From (i), we notice

LHS is an irrational number and RHS is rational number, which is not possible. Hence, our supposition is wrong. Hence, $2\sqrt{3} - 1$ is an irrational number.

Question 35.
Prove that √2 is irrational.

Solution:

Let us assume that $\sqrt{2}$ is rational.

$\therefore$ There exists coprime integers a and b ($b \neq 0$)

Such that

$$\sqrt{2} = \frac{a}{b} \Rightarrow \sqrt{2}b = a$$

Squaring on both sides, we get

$$2b^2 = a^2 \qquad ...(i)$$

$\Rightarrow$ 2 divides a^2 $\Rightarrow$ 2 divides a

So, we can write

$$a = 2c \text{ for some integer } c \qquad ...(ii)$$

From (i) and (ii),

$$2b^2 = 4c^2 \Rightarrow b^2 = 2c^2$$

$\Rightarrow$ 2 divides b^2 $\Rightarrow$ 2 divides b

$\therefore$ 2 is a common factor of a and b.

But this contradicts the fact that a and b are coprimes.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

Hence, $\sqrt{2}$ is irrational.

Question 36.

Prove that 7 – 2√3 is an irrational number.

Solution:

Let, $7 - 2\sqrt{3}$ be a rational number.

Let $7 - 2\sqrt{3} = \frac{a}{b}$, where a and b are coprimes and $b \neq 0$

$$\Rightarrow 7 - \frac{a}{b} = 2\sqrt{3} \Rightarrow \frac{7b - a}{b} = 2\sqrt{3} \Rightarrow \frac{7b - a}{2b} = \sqrt{3}$$

We notice that LHS is a rational number, whereas RHS is an irrational number, which is a contradiction. Hence, our supposition is wrong.

Hence, $7 - 2\sqrt{3}$ is an irrational number.

Question 37.

Show that 5 + 3√2 is an irrational number.

Solution:

Let us assume 5 + 3√2 is an irrational number.

There coprime integers a and b (b≠0)

Such that

$$5 + 3\sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \quad 3\sqrt{2} = \frac{a}{b} - 5 \Rightarrow \sqrt{2} = \frac{a-5b}{3b}$$

$\because$ a and b are integers we get $\frac{a-5b}{3b}$ is rational number and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5+3\sqrt{2}$ is rational.

So, we conclude that $5+3\sqrt{2}$ is irrational.

2009

Very Short Answer Type Questions [1 Mark]

Question 38.

The decimal expansion of the rational number $\frac{43}{2^4 . 5^3}$, will terminate after how many places of decimals.

Solution:

The decimal expansion of the rational number $\frac{43}{2^4 . 5^3}$, will terminate after 4 places of decimal.

$$\frac{43}{2^4 . 5^3} = \frac{43}{16\times125} = \frac{43}{2000} = 0.0215$$

Question 39.

Find the [HCF X LCM] for the numbers 100 and 190.

Solution:

HCF x LCM = one number x another number

= 100 x 190 = 19000

Question 40.

Find the [HCF and LCM] for the numbers 105 and 120. [All India]

Solution:

105 = 5 x 7 x 3

120 = 2x2x2x3x5

HCF = 3 X 5 = 15

LCM = 5x7x3x2x2x2 = 840

Question 41.

Write whether the rational number $\frac{51}{1500}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

Solution: $\frac{51}{1500} = \frac{17}{500}$
Prime factorization of 500 = (2)(2)(5)(5)(5) = $2^2 \times 5^3$
Its denominator has prime factors of form $2^m \times 5^n$
So, it has a terminating decimal expansion.

Question 42.
The HCF and LCM of two numbers are 9 and 360 respectively. If one number is 45, write the other number.
Solution:
Let another number = x

$$\begin{aligned} \text{HCF }(45, x) &= 9 \\ \text{LCM }(45, x) &= 360 \\ \text{HCF} \times \text{LCM} &= 45 \times x \\ 9 \times 360 &= 45 \times x \\ x &= \frac{360}{5} = 72 \end{aligned}$$

Short Answer Type Questions II [3 Marks]

Question 43.
Show that 5 – 2√3 is an irrational number.
Solution:
Let us assume that $5 - 2\sqrt{3}$ is rational number.
$\therefore$ There exists coprime integers a and b $(b \neq 0)$ such that

$5 - 2\sqrt{3} = \frac{a}{b}$

$\Rightarrow 2\sqrt{3} = 5 - \frac{a}{b} \Rightarrow \sqrt{3} = \frac{5b - a}{2b}$

$\because$ a and b are integers, we get $\frac{5b-a}{2b}$ is rational and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.
This contradiction has arisen because of our incorrect assumption that $5 - 2\sqrt{3}$ is rational.
So, we conclude that $5 - 2\sqrt{3}$ is irrational.

Question 44.
Show that 3 + 5√2 is an irrational number.
Solution:

Let us assume that $3+5\sqrt{2}$ is rational number.

$\therefore$ There exists coprime integers a and b $(b \neq 0)$

Such that $3+5\sqrt{2} = \frac{a}{b}$

$\Rightarrow \quad 5\sqrt{2} = \frac{a}{b} - 3 \quad \Rightarrow \sqrt{2} = \frac{a-3b}{5b}$

$\because$ a and b are integers we get $\frac{a-3b}{5b}$ is rational number and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $3+5\sqrt{2}$ is rational.

So, we conclude that $3+5\sqrt{2}$ is irrational.

Question 45.

Show that the square of any positive odd integer is of form 8m + 1, for some integer m.

Solution:

Let $a = 2q + 1$ be any positive odd integer.

Now,

$$\begin{aligned} a^2 &= (2q+1)^2 \\ &= 4q^2 + 4q + 1 \\ &= 4q(q+1) + 1 \\ &= 4(2m) + 1 \\ &= 8m + 1 \end{aligned}$$

[$\because$ q and $(q+1)$ are consecutive numbers, so one of them must be even and of the form $2m$.]

$\therefore$ Square of any positive odd integer is of the form $8m + 1$, for some integer m.

Question 46.

Prove that 7 + 3√2 is not a rational number.

Solution:

Let $7 + 3\sqrt{2}$ be rational

$\Rightarrow \quad 7 + 3\sqrt{2} = \frac{p}{q}$, where $q \neq 0$ and p and q are coprimes

$\Rightarrow \quad 3\sqrt{2} = \frac{p}{q} - 7$

$\Rightarrow \quad \sqrt{2} = \frac{p-7q}{3q}$

Here, on RHS $\frac{p-7q}{3q}$ is rational, whereas $\sqrt{2}$ is irrational.

Therefore, our assumption is wrong.

Hence, $7 + 3\sqrt{2}$ is irrational.