Chapter 6 -Triangles
Exercise 6.1

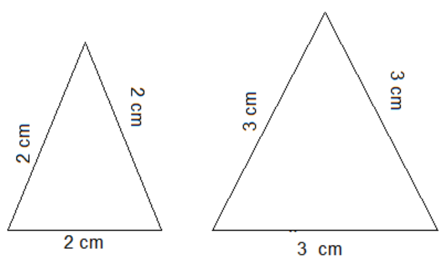
Question 1: Fill in the blanks using correct word given in the brackets:-

(i) All circles are __________. (congruent, similar)
(ii) All squares are __________. (similar, congruent)
(iii) All __________ triangles are similar. (isosceles, equilateral)
(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are __________ and (b) their corresponding sides are __________. (equal, proportional)

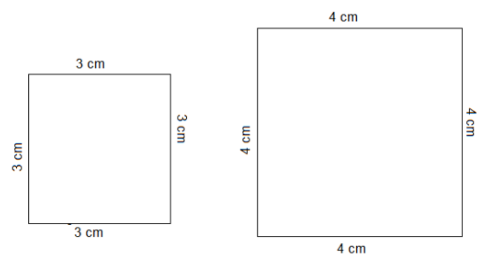
Answer:
(i) Similar
(ii) Similar
(iii) equilateral
(iv) (a) Equal ; (b) Proportional

Question 2: Give two different examples of pairs of
(i) similar figures.
(ii) non-similar figures.

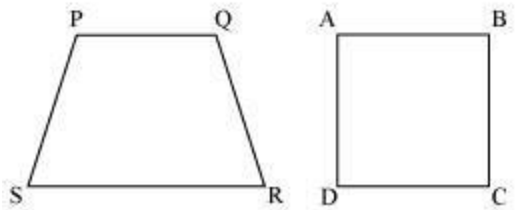
Answer: (i) Two equilateral triangles with sides 2 cm and 3 cm



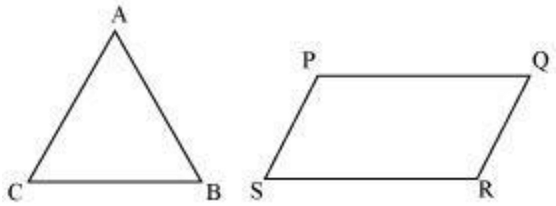
Two squares with sides 3 cm and 4 cm



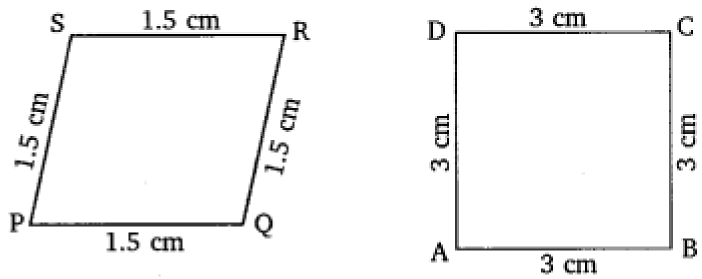
(ii) Trapezium and square



Triangle and parallelogram



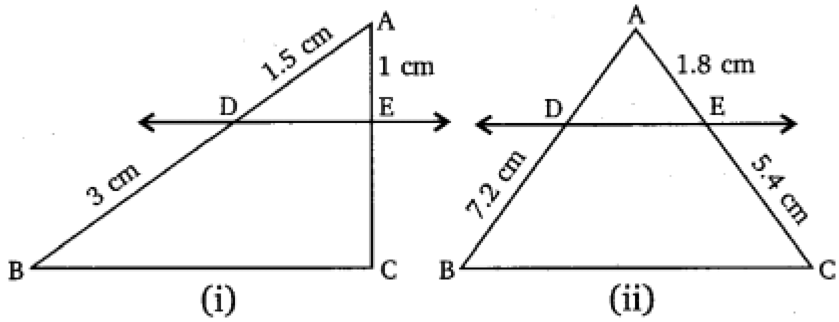
Question 3: State whether the following quadrilaterals are similar or not.



Answer: From the given above two figures, we can clearly see that, their corresponding angles are different or unequal. Therefore, they are not similar.

Exercise 6.2

Question 1: In the given figure (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).



Answer: (i) In ΔABC, DE ∥ BC

Hence, $\frac{AD}{DB} = \frac{AE}{EC}$ …………….[Using Basic proportionality theorem]

or, $\frac{1.5}{3} = \frac{1}{EC}$

or, EC = $\frac{3}{1.5}$

or, EC = 3 × $\frac{10}{15}$ = 2

Therefore , EC = 2 cm.

(ii) In ΔABC, DE ∥ BC

hence, $\frac{AD}{DB} = \frac{AE}{AC}$ …………………….[Using Basic proportionality theorem]

or, $\frac{AD}{7.2} = \frac{1.8}{5.4}$

or, AD = 1.8 × $\frac{7.2}{5.4} = \frac{18}{10} \times \frac{72}{10} \times \frac{10}{54}$

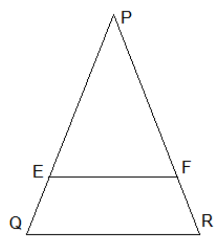
or, AD = $\frac{24}{10}$

or, AD = 2.4

Hence, AD = 2.4 cm.

Question 2: E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.
(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm
(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm
(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm

Answer:



(i) PE = 3.9 cm, [Given]
EQ = 3 cm, [Given]
PF = 3.6 cm [Given]
FR = 2.4 cm [Given]

Therefore,

$\frac{PE}{EQ} = \frac{3.9}{3}$ [using Basic proportionality theorem]

or, $\frac{39}{30} = \frac{13}{10}$ = 1.3

And $\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2}$ = 1.5

So, we get, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Hence, EF is not parallel to QR.

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8cm and RF = 9cm [Given]

$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9}$

and, $\frac{PF}{FR} = \frac{8}{9}$

So, $\frac{PE}{EQ} = \frac{PF}{RF}$

Hence, EF is parallel to QR.

(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm [Given]
From the above figure,
EQ = PQ – PE = (1.28 – 0.18) cm = 1.10 cm
And, FR = PR – PF = (2.56 – 0.36) cm = 2.20 cm

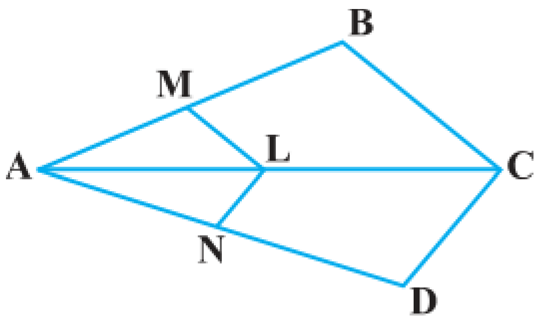
So, $\frac{PE}{EQ}$ = 0.18/1.10 $\frac{0.18}{1.10} = \frac{18}{110} = \frac{9}{55}$.. (1)

And, $\frac{PE}{FR} = \frac{0.36}{2.20} = \frac{36}{220} = \frac{9}{55}$ .... (2)

So, $\frac{PE}{EQ} = \frac{PF}{FR}$

Hence, EF is parallel to QR.

Question 3: In the figure, if LM || CB and LN || CD, prove that AM/AB = AN/AD



Answer: LM || CB [Given]

$\frac{AM}{AB} = \frac{AL}{AC}$................................(1) [Basic Proportionality theorem]

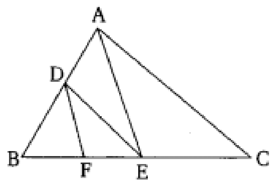
Again, LN || CD [Given]

$\frac{AN}{AD} = \frac{AL}{AC}$.....................................(2) [Basic Proportionality theorem]

From equation (1) and (2), we get,

$\frac{AM}{AB} = \frac{AN}{AD}$ [Proved]

Question 4: In the given figure, DE || AC and DF || AE.
Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.

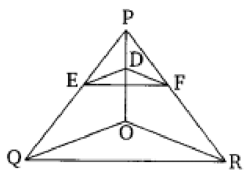


Answer: In ΔABC, DE || AC
Hence, $\frac{BD}{DA} = \frac{BE}{EC}$ ……………………………………………(1) [Basic Proportionality Theorem]

In ΔABC, DF || AE
Hence, $\frac{BD}{DA} = \frac{BF}{FE}$ ……………………………………………(2) [Basic Proportionality Theorem]

From equation (1) and (2), we get
$\frac{BE}{EC} = \frac{BF}{FE}$ [Given]

Question 5: In the given figure, DE || OQ and DF || OR. Show that EF || QR.



Answer: In ΔPQO, DE || OQ [Given]

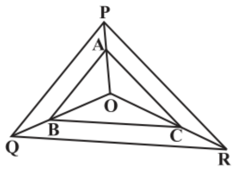
$\frac{PD}{DO} = \frac{PE}{EQ}$ ……………………………………………(1) [Basic Proportionality Theorem]

In ΔPQO, DE || OQ , [Given]

$\frac{PD}{DO} = \frac{PF}{FR}$…………………………………………..(2) [Basic Proportionality Theorem]

From the above two equations (1) and (2),
$\frac{PE}{EQ} = \frac{PF}{FE}$
Therefore, by using converse of Basic Proportionality Theorem, in ΔPQR, EF || QR.

Question 6: In the figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



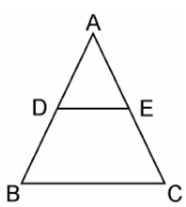
Answer: In ΔOPQ, AB || PQ, [Given]
$\frac{OA}{OP} = \frac{OB}{BQ}$ …………………………..(1) [Basic Proportionality Theorem]

In ΔOPR, AC || PR [Given]
$\frac{OA}{AP} = \frac{OC}{CR}$ …………………………(2) [Basic Proportionality Theorem]

From equations (1) and (2), we get,
$\frac{OB}{BQ} = \frac{OC}{CR}$

Therefore, by using converse of Basic Proportionality Theorem, in ΔOQR, BC || QR.

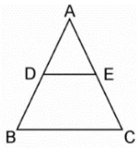
Question 7: Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).



Answer: In triangle ABC, D is the mid-point of AB and DE∥BC
In ΔABC, DE∥BC,
hence, $\frac{AD}{DB} = \frac{AE}{EC}$
But, AD = DB [As, D is the mid-point of AB]
or, $\frac{AD}{DB} = 1$
or, $\frac{AE}{EC} = 1$
Therefore, AE = EC
hence, DE bisects AC.

Question 8: Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer:



D is the midpoint of AB [given]
therefore, AD = DB
or, $\frac{AD}{BD}$ = 1……………………………………(1)

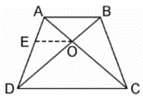
E is the mid-point of AC. [Given]
Therefore, AE = EC
or, $\frac{AE}{EC}$ = 1 ……………………………………(2)

From equations (1) and (2), we get,
$\frac{AD}{BD} = \frac{AE}{EC}$

Hence, by using converse of Basic Proportionality Theorem, DE || BC [Proved]

Question 9: ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$

Answer:



From point O, let draw a line EO touching AD at E, in such a way that,
EO || DC || AB [construction]

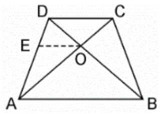
In ΔADC, we have OE || DC
or, $\frac{AE}{ED} = \frac{AO}{CO}$ …………………………………………..(1) [Basic Proportionality Theorem]

Now, In ΔABD, OE || AB
$\frac{DE}{EA} = \frac{DO}{BO}$ ……………………………………(2) [Basic Proportionality Theorem]

From equation (1) and (2) we get,
$\frac{AO}{CO} = \frac{DO}{BO}$
or, $\frac{AO}{BO} = \frac{CO}{DO}$ [Proved]

Question 10: The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Answer:



[Construction] From the point O, draw a line EO touching AD at E, in such a way that, EO || DC || AB

In ΔDAB, EO || AB

$\frac{DE}{EA} = \frac{DO}{OB}$ ……………………..(1) [Basic Proportionality Theorem]

$\frac{AO}{BO} = \frac{CO}{DO}$ [Given]

or, $\frac{AO}{CO} = \frac{BO}{DO}$

or, $\frac{CO}{AO} = \frac{DO}{BO}$

or, $\frac{DO}{BO} = \frac{CO}{AO}$ …………………………….(2)

From equations (1) and (2), we get

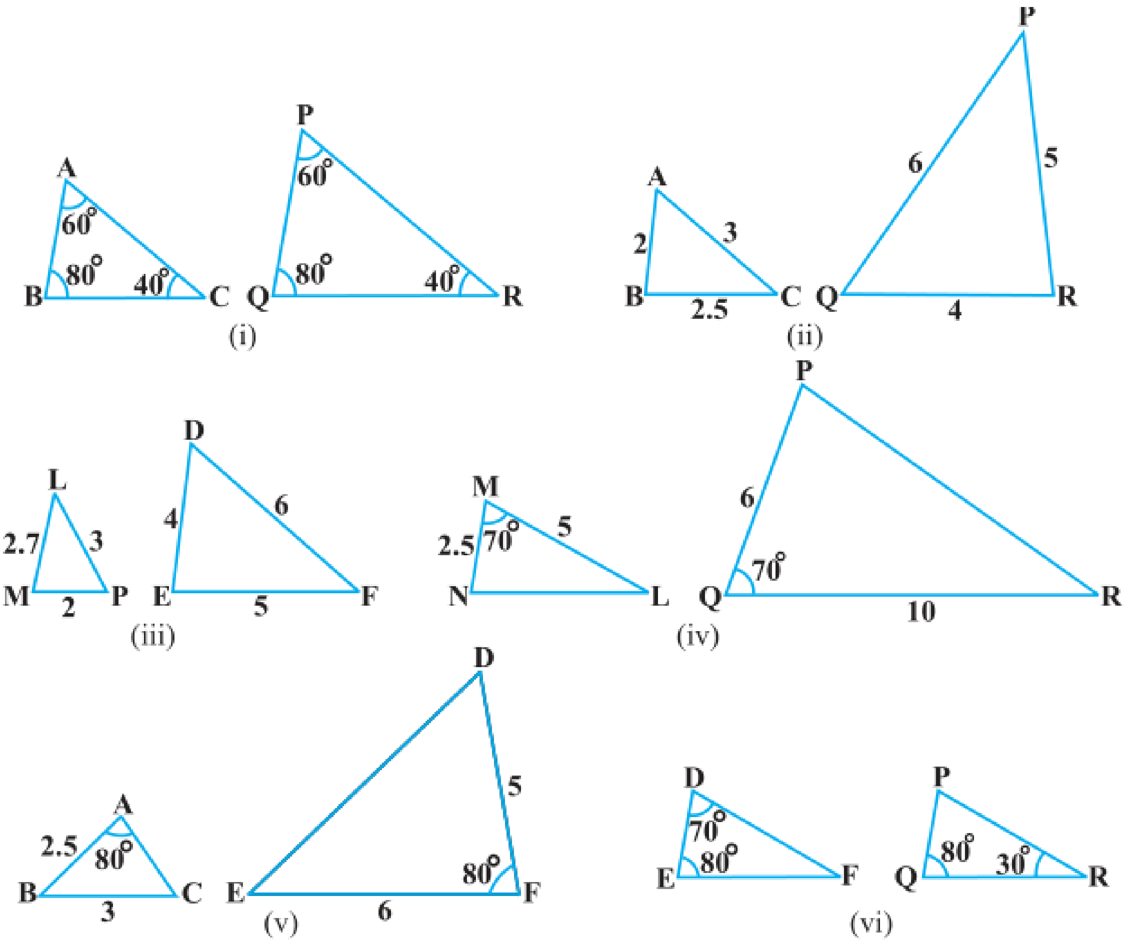
$\frac{DE}{EA} = \frac{CO}{AO}$

Therefore, By using converse of Basic Proportionality Theorem,
EO || DC also EO || AB, or, AB || DC.

Hence, quadrilateral ABCD is a trapezium with AB || CD.

Exercise 6.3

Question 1: State which pairs of triangles in Figure, are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



Answer : (i) In ΔABC and ΔPQR,
∠A = ∠P = 60°
∠B = ∠Q = 80°
∠C = ∠R = 40°

Therefore ΔABC ~ ΔPQR [AAA similarity criterion]

(ii) In ΔABC and ΔPQR,

$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$

By SSS similarity criterion,

Therefore, ΔABC ~ ΔQRP [SSS similarity criterion]

(iii) In ΔLMP and ΔDEF, LM = 2.7, MP = 2, LP = 3, EF = 5, DE = 4, DF = 6

$\frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}$

$\frac{PL}{DF} = \frac{3}{6} = \frac{1}{2}$

$\frac{LM}{EF} = \frac{2.7}{5} = \frac{27}{50}$

Here , $\frac{MP}{DE} = \frac{PL}{DF} \neq \frac{LM}{EF}$

Therefore, ΔLMP and ΔDEF are not similar.

(iv) In ΔMNL and ΔQPR,

$\frac{MN}{QP} = \frac{LM}{QR} = \frac{1}{2}$

∠M = ∠Q = 70° [Given]

Therefore, ΔMNL ~ ΔQPR [SAS similarity criterion]

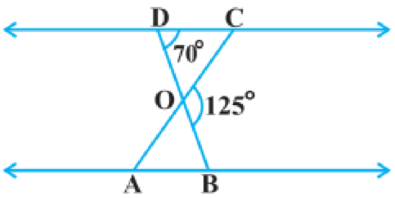
(v) In ΔABC and ΔDEF, AB = 2.5, BC = 3, ∠A = 80°, EF = 6, DF = 5, ∠F = 80° [Given]
Here, $\frac{AD}{DF} = \frac{2.5}{5} = \frac{1}{2}$
And, $\frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}$
or, ∠B ≠ ∠F
Hence, ΔABC and ΔDEF are not similar.

(vi) In ΔDEF, we know that,
∠D + ∠E + ∠F = 180° (sum of angles of triangles is 180°)
or, 70° + 80° + ∠F = 180°
or, ∠F = 180° – 70° – 80°
or, ∠F = 30°
Similarly, In ΔPQR,
∠P + ∠Q + ∠R = 180 (Sum of angles of Δ)
or, ∠P + 80° + 30° = 180°
or, ∠P = 180° – 80° -30°
or, ∠P = 70°

Now, comparing both the triangles, ΔDEF and ΔPQR, we have
∠D = ∠P = 70°
∠F = ∠Q = 80°
∠F = ∠R = 30°

Therefore, ΔDEF ~ ΔPQR. [AAA similarity criterion]

Question 2: In the figure, ΔODC ∝ ¼ ΔOBA, ∠ BOC = 125° and ∠ CDO = 70°. Find ∠ DOC, ∠ DCO and ∠ OAB.

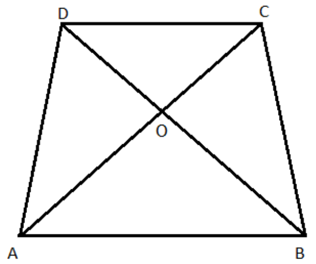


Answer: As we can see from the figure, DOB is a straight line.
Therefore, ∠DOC + ∠ COB = 180°
or, ∠DOC = 180° – 125° (Given, ∠ BOC = 125°)
= 55°

In ΔDOC,
∠DCO + ∠ CDO + ∠ DOC = 180° [sum of angles of Δ]
or, ∠DCO + 70º + 55º = 180°(Given, ∠ CDO = 70°)
or, ∠DCO = 55°
It is given that, ΔODC ∝ ¼ ΔOBA,
Therefore, ΔODC ~ ΔOBA.
Hence, are equal in similar triangles
∠OAB = ∠OCD
or, ∠ OAB = 55°

Question 3: Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that AO/OC = OB/OD

Answer:



In ΔDOC and ΔBOA,

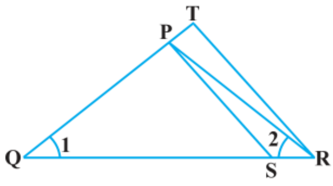
AB || CD,

Therefore, ∠CDO = ∠ABO [alternate interior are equal]

Similarly,
∠DCO = ∠BAO
Also, for the two triangles ΔDOC and ΔBOA,
∠DOC = ∠BOA [vertically opposite angles are equal]

Hence, by AAA similarity criterion,
ΔDOC ~ ΔBOA
Thus,
$\frac{DO}{BO} = \frac{OC}{OA}$ [corresponding sides are proportional]
or, $\frac{OA}{OC} = \frac{OB}{OD}$
Hence, proved.

Question 4: In the fig.6.36, QR/QS = QT/PR and ∠1 = ∠2. Show that ΔPQS ~ ΔTQR.



Answer:

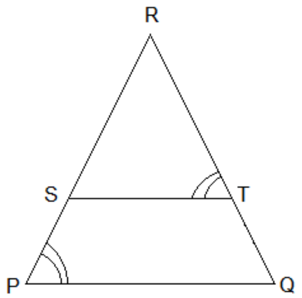
In ΔPQR,
∠PQR = ∠PRQ
Therefore, PQ = PR ……………………….**(i)**

Given,
$\frac{QS}{QR} = \frac{QT}{PR}$
Using equation **(i)**, we get
$\frac{QS}{QR} = \frac{QT}{QP}$……………….**(ii)**
In ΔPQS and ΔTQR, by equation (ii),
$\frac{QS}{QR} = \frac{QT}{QP}$
∠Q = ∠Q

Therefore, ΔPQS ~ ΔTQR [By SAS similarity criterion]

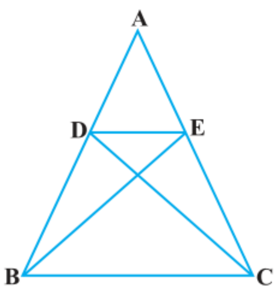
Question 5: S and T are point on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Answer: Given, S and T are point on sides PR and QR of ΔPQR
And ∠P = ∠RTS.



In ΔRPQ and ΔRTS,
∠RTS = ∠QPS (Given)
∠R = ∠R (Common angle)
Therefore ΔRPQ ~ ΔRTS (AA similarity criterion)

Question 6: In the figure, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.

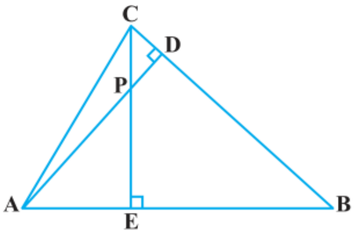


Answer: Given, ΔABE ≅ ΔACD.
Therefore, AB = AC [By CPCT] ………………………………..**(i)**
And, AD = AE [By CPCT] …………………………….**(ii)**
In ΔADE and ΔABC, dividing eq.(ii) by eq(i),

$\frac{AD}{AB} = \frac{AE}{AC}$
∠A = ∠A [Common angle]
Therefore, ΔADE ~ ΔABC [SAS similarity criterion]

Question 7: In the figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:

(i) ΔAEP ~ ΔCDP
(ii) ΔABD ~ ΔCBE
(iii) ΔAEP ~ ΔADB
(iv) ΔPDC ~ ΔBEC



Answer: Given, altitudes AD and CE of ΔABC intersect each other at the point P.

(i) In ΔAEP and ΔCDP,
∠AEP = ∠CDP (90° each)
∠APE = ∠CPD (Vertically opposite angles)
Hence, by AA similarity criterion,
ΔAEP ~ ΔCDP

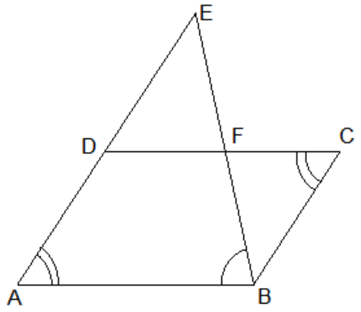
(ii) In ΔABD and ΔCBE,
∠ADB = ∠CEB (90° each)
∠ABD = ∠CBE (Common Angles)
Hence, by AA similarity criterion,
ΔABD ~ ΔCBE

(iii) In ΔAEP and ΔADB,
∠AEP = ∠ADB (90° each)
∠PAE = ∠DAB (Common Angles)
Hence, by AA similarity criterion,
ΔAEP ~ ΔADB

(iv) In ΔPDC and ΔBEC,
∠PDC = ∠BEC (90° each)
∠PCD = ∠BCE (Common angles)
Hence, by AA similarity criterion,
ΔPDC ~ ΔBEC

Question 8: E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.

Answer: Given, E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Consider the figure below,

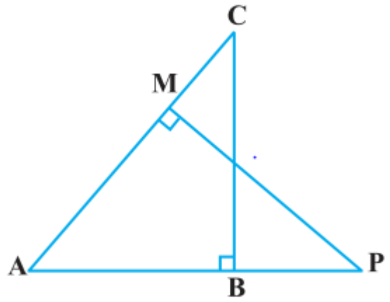


In ΔABE and ΔCFB,
∠A = ∠C (Opposite angles of a parallelogram)
∠AEB = ∠CBF (Alternate interior angles as AE || BC)
Therefore ΔABE ~ ΔCFB (AA similarity criterion)

Question 9: In the figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:

(i) ΔABC ~ ΔAMP

(ii) CA/PA = BC/MP



Answer: Given, ABC and AMP are two right triangles, right angled at B and M respectively.

(i) In ΔABC and ΔAMP, we have,
∠CAB = ∠MAP (common angles)
∠ABC = ∠AMP = 90° (each 90°)
Therefore ΔABC ~ ΔAMP (AA similarity criterion) [Proved]

(ii) As, ΔABC ~ ΔAMP (AA similarity criterion)
If two triangles are similar then the corresponding sides are always equal,
Hence, $\frac{CA}{PA} = \frac{BC}{MP}$ [Proved]

Question 10: CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:

(i) CD/GH = AC/FG
(ii) ΔDCB ~ ΔHGE
(iii) ΔDCA ~ ΔHGF

Answer: Given, CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively.

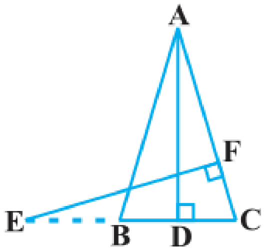
(i) From the given condition,
ΔABC ~ ΔFEG.
Therefore ∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE
Since, ∠ACB = ∠FGE
Therefore ∠ACD = ∠FGH (Angle bisector)

And, ∠DCB = ∠HGE (Angle bisector)
In ΔACD and ΔFGH,
∠A = ∠F
∠ACD = ∠FGH
Therefore ΔACD ~ ΔFGH (AA similarity criterion)
or, $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) In ΔDCB and ΔHGE,
∠DCB = ∠HGE (Already proved)
∠B = ∠E (Already proved)
Therefore ΔDCB ~ ΔHGE (AA similarity criterion)

(iii) In ΔDCA and ΔHGF,
∠ACD = ∠FGH (Already proved)
∠A = ∠F (Already proved)
Therefore ΔDCA ~ ΔHGF (AA similarity criterion)

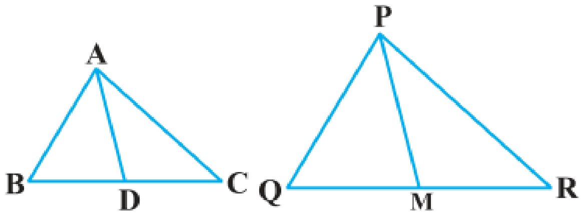
Question 11: In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF.



Answer: Given, ABC is an isosceles triangle.
Therefore AB = AC
or, ∠ABD = ∠ECF

In ΔABD and ΔECF,
∠ADB = ∠EFC (Each 90°)
∠BAD = ∠CEF (Already proved)
Therefore ΔABD ~ ΔECF (using AA similarity criterion)

Question 12: Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see Fig 6.41).
Show that ΔABC ~ ΔPQR.



Answer: Given, ΔABC and ΔPQR, AB, BC and median AD of ΔABC are proportional to sides PQ, QR and median PM of ΔPQR

i.e. AB/PQ = BC/QR = AD/PM

We have to prove: ΔABC ~ ΔPQR

As we know here,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM} \text{ ....(i)}$$

or, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ (D is the midpoint of BC. M is the midpoint of QR)

or, ΔABD ~ ΔPQM [SSS similarity criterion]

Therefore ∠ABD = ∠PQM [Corresponding angles of two similar triangles are equal]
or, ∠ABC = ∠PQR

In ΔABC and ΔPQR

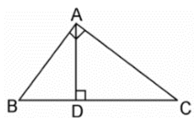
$\frac{AB}{PQ} = \frac{BC}{QR}$ ....(i)

∠ABC = ∠PQR ....(ii)

From equation (i) and (ii), we get, ΔABC ~ ΔPQR [SAS similarity criterion]

Question 13: D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that CA^2 = CB.CD

Answer: D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. [Given]



In ΔADC and ΔBAC,
∠ADC = ∠BAC [given]
∠ACD = ∠BCA [Common angles]

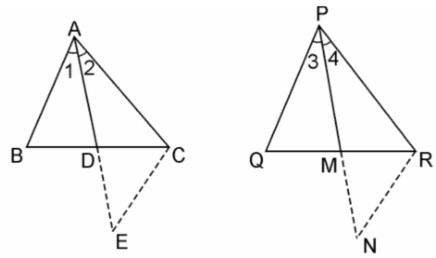
Therefore, ΔADC ~ ΔBAC [AA similarity criterion]
As, we know that corresponding sides of similar triangles are in proportion.
therefore, $\frac{CA}{CB} = \frac{CD}{CA}$
or, CA^2 = CB.CD. [Proved]

Question 14: Sides AB and AC and median AD of a triangle ABC are respectively proportional to PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

Answer:

ΔABC and ΔPQR in which AD and PM are medians [Given]
Therefore, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

Construction: We need to produce AD to E so that AD = DE.
Join CE, Similarly produce PM to N such that PM = MN, also Join RN.



In ΔABD and ΔCDE,
AD = DE [By Construction.]
BD = DC [AP is the median]
and, ∠ADB = ∠CDE [Vertically opposite angles]

Hence, ΔABD ≅ ΔCDE [SAS criterion of congruence]
Thus, AB = CE [By CPCT……………………………………………..(1)

Also, in ΔPQM and ΔMNR,
PM = MN [By Construction.]
QM = MR [PM is the median]
and, ∠PMQ = ∠NMR [Vertically opposite angles]

Therefore, ΔPQM = ΔMNR [SAS criterion of congruence]
Thus, PQ = RN [CPCT]…………………………………………………….(2)

Now, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

From equations (1) and (2),

or, $\frac{CE}{RN} = \frac{AC}{PR} = \frac{AD}{PM}$

or, $\frac{CE}{RN} = \frac{AC}{PR} = \frac{2AD}{2PM}$

or, $\frac{CE}{RN} = \frac{AC}{PR} = \frac{AE}{PN}$ [Since 2AD = AE and 2PM = PN] therefore, ΔACE ~ ΔPRN [SSS similarity criterion]

Therefore, ∠2 = ∠4
And, similarly, ∠1 = ∠3

Thus, ∠1 + ∠2 = ∠3 + ∠4
or, ∠A = ∠P …………………………………………………………..(3)

Now, in ΔABC and ΔPQR,

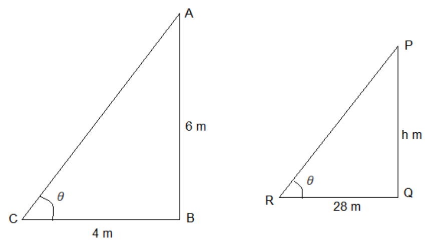
$\frac{AB}{PQ} = \frac{AC}{PR}$ [Given]

From equation (3), ∠A = ∠P

Therefore, ΔABC ~ ΔPQR [SAS similarity criterion]

Question 15: A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer:



In ΔABC, AB is the pole and BC its shadow.
Also, ΔPQR, PQ be the tower of height h meters and QR be its shadow.

When Q is the altitude of the sun.
ΔABC ~ ΔPQR [By AA similarity]

Or, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

Or, $\frac{BC}{RQ} = \frac{AC}{PR}$

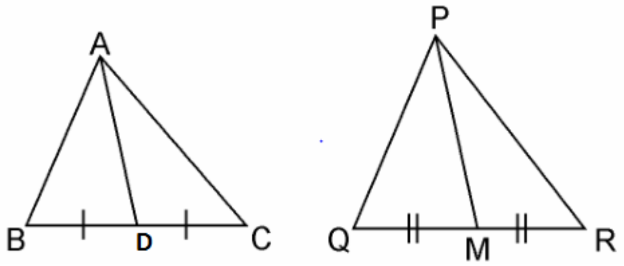
Or, $\frac{4}{28} = \frac{6}{h}$

Or, h = $\frac{6 \times 28}{4}$ = 42

Hence, the height of the tower is 42 m.

Question 16: If AD and PM are medians of triangles ABC and PQR, respectively where ΔABC ~ ΔPQR prove that AB/PQ = AD/PM.

Answer:



ΔABC ~ ΔPQR [Given]
or, ∠ABC = ∠PQR

$\frac{AB}{PQ} = \frac{BC}{QR}$

Or, $\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$

Or, $\frac{AB}{PQ} = \frac{BD}{QM}$

In ΔABD and ΔPQM

$\frac{AB}{PQ} = \frac{BD}{QM}$ [Proved]

∠B = ∠Q

Hence, ΔABD ~ ΔPQM

$\frac{AB}{PQ} = \frac{AD}{PM}$ [Corresponding sides of similar trinagles]

Exercise 6.4

Question 1: Let ΔABC ~ ΔDEF and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Answer: ΔABC ~ ΔDEF,
ar (ΔABC) = 64 cm^2
ar (ΔDEF) = 121 cm^2

EF = 15.4 cm

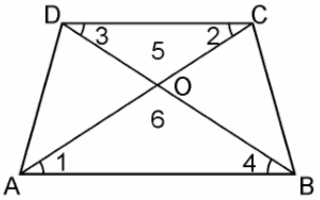
Therefore, $\frac{\text{ar }(\Delta\text{ABC})}{\text{ar }(\Delta\text{DEF})} = \frac{AB^2}{DE^2}$

As we know, if two triangles are similar, the ratio of their areas is equal to the square of the ratio of their corresponding sides,
$= \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$
Hence, $\frac{64}{121} = \frac{BC^2}{EF^2}$
or, $\left(\frac{8}{11}\right)^2 = \left(\frac{BC}{15.4}\right)^2$

or, $\left(\frac{8}{11}\right)^2 = \frac{BC}{15.4}$
or, BC = $\frac{8 \times 15.4}{11}$
or, BC = 8 × 1.4
or, BC = 11.2 cm

Question 2: Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

Answer:



In ΔAOB and ΔCOD, we have

∠1 = ∠2 (Alternate angles)

∠3 = ∠4 (Alternate angles)

∠5 = ∠6 (Vertically opposite angle)

Therefore ΔAOB ~ ΔCOD [AAA similarity criterion]

As we know, If two triangles are similar, then the ratio of their areas is equal to the square of the ratio of their corresponding sides. Therefore,

Area of (ΔAOB)/Area of (ΔCOD) = AB^2/CD^2

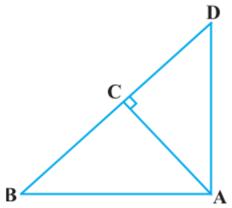
$= \frac{(2CD)^2}{CD^2}$ [because AB = 2CD]

Therefore Area of (ΔAOB)/Area of (ΔCOD)

$= \frac{4CD^2}{CD^2} = \frac{4}{1}$

Hence, the required ratio of the area of ΔAOB and ΔCOD = 4:1

3. In the figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that area (ΔABC)/area (ΔDBC) = AO/DO.

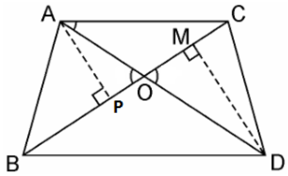


Solution:

Given, ABC and DBC are two triangles on the same base BC. AD intersects BC at O.

We have to prove: Area (ΔABC)/Area (ΔDBC) = $\frac{AO}{DO}$

Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle = 1/2 × Base × Height

$\frac{\Delta \text{ABC}}{\Delta \text{DEF}} = \frac{\frac{1}{2} BC \times AP}{\frac{1}{2} BC \times DM} = \frac{AP}{DM}$

In ΔAPO and ΔDMO,

∠APO = ∠DMO (Each 90°)

∠AOP = ∠DOM (Vertically opposite angles)

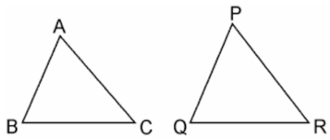
Therefore ΔAPO ~ ΔDMO (AA similarity criterion)

Therefore $\frac{AP}{DM} = \frac{AO}{DO}$

or, $\frac{\text{ar}\,(\Delta \text{ABC})}{\text{ar}(\Delta \text{DBC})} = \frac{\text{AO}}{\text{BO}}$

4. If the areas of two similar triangles are equal, prove that they are congruent.

Answer: Let ΔABC and ΔPQR are two similar triangles and equal in area



Now let us prove ΔABC ≅ ΔPQR.

Since ΔABC ~ ΔPQR

Therefore Area of (ΔABC)/Area of (ΔPQR) = BC^2/QR^2

or, $\frac{BC^2}{QR^2}$ =1 [Since, Area(ΔABC) = (ΔPQR)

or, $\frac{BC^2}{QR^2}$

or, BC = QR

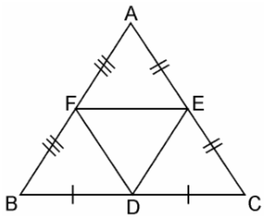
Similarly, we can prove that

AB = PQ and AC = PR

Thus, ΔABC ≅ ΔPQR [SSS criterion of congruence]

Question 5. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the area of ΔDEF and ΔABC.

Answer: D, E and F are the mid-points of sides AB, BC and CA of ΔABC.[Given]



In ΔABC,

F is the mid-point of AB (Already given)
E is the mid-point of AC (Already given)
So, by the mid-point theorem, we have,
FE || BC and FE = $\frac{1}{2}$BCor, FE || BC and FE || BD [BD = $\frac{1}{2}$BC]
Since opposite sides of a parallelogram are equal and parallel
Therefore BDEF is a parallelogram.

Similarly, in ΔFBD and ΔDEF, we have
FB = DE (Opposite sides of parallelogram BDEF)
FD = FD (Common sides)
BD = FE (Opposite sides of parallelogram BDEF)

Therefore ΔFBD ≅ ΔDEF

Similarly, we can prove that
ΔAFE ≅ ΔDEF
ΔEDC ≅ ΔDEF

As we know, if triangles are congruent, then they are equal in area.
So, Area(ΔFBD) = Area(ΔDEF) ……………………………**(i)**
Area(ΔAFE) = Area(ΔDEF) ……………………………….**(ii)**
Area(ΔEDC) = Area(ΔDEF) ………………………….**(iii)**

Now,
Area(ΔABC) = Area(ΔFBD) + Area(ΔDEF) + Area(ΔAFE) + Area(ΔEDC) ………**(iv)**

Area(ΔABC) = Area(ΔDEF) + Area(ΔDEF) + Area(ΔDEF) + Area(ΔDEF)

From equation **(i)**, **(ii)** and **(iii)**,

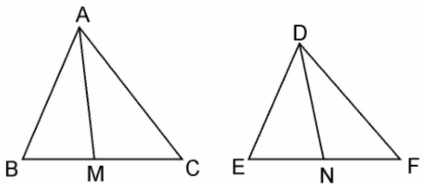
or, Area(ΔDEF) = ($\frac{1}{4}$)Area(ΔABC)

or, $\frac{\text{Area}(\Delta\text{DEF})}{\text{Area}(\Delta\text{ABC})} = \frac{1}{4}$

Hence, Area(ΔDEF): Area(ΔABC) = 1:4

Question 6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer: AM and DN are the medians of triangles ABC and DEF respectively and ΔABC ~ ΔDEF.



We have to prove: Area(ΔABC)/Area(ΔDEF) = $\frac{AM^2}{DN^2}$

Since ΔABC ~ ΔDEF (Given)

Therefore $\frac{\text{Area}(\Delta\text{ABC})}{\text{Area}(\Delta\text{DEF})} = (\frac{AB^2}{DE^2})$ ……………………………**(i)**

and, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ ………………………………………**(ii)**

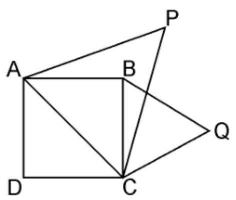
or, $\frac{AB}{DE} = \frac{\frac{1}{2}AB}{\frac{1}{2}DE} = \frac{CD}{FD}$

In ΔABM and ΔDEN,
Since ΔABC ~ ΔDEF
Therefore ∠B = ∠E
$\frac{AB}{DE} = \frac{BM}{EN}$ [Already Proved in equation **(i)**]
Therefore ΔABC ~ ΔDEF [SAS similarity criterion]
or, $\frac{AB}{DE} = \frac{AM}{DN}$ …………………………………………………..**(iii)**

Therefore ΔABM ~ ΔDEN
As the areas of two similar triangles are proportional to the squares of the corresponding sides.
Therefore area(ΔABC)/area(ΔDEF) = $AB^2/DE^2 = AM^2/DN^2$
Hence, proved.

7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the equilateral triangle area described on one of its diagonals.

Solution:



Given, ABCD is a square whose one diagonal is AC. ΔAPC and ΔBQC are two equilateral triangles described on the diagonals AC and side BC of the square ABCD.

Area(ΔBQC) = ½ Area(ΔAPC)

Since, ΔAPC and ΔBQC are both equilateral triangles,
hence, ΔAPC ~ ΔBQC [AAA similarity criterion]

$\frac{\text{ar}(\Delta\text{APC})}{\text{ar}(\Delta\text{BQC})} = \frac{AC^2}{BC^2}$

Since, Diagonal = √2 side = √2 BC = AC

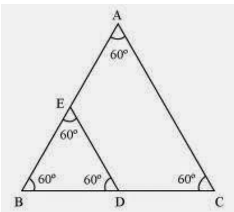
$\left(\frac{\sqrt{2}\,BC}{BC}\right)^2$ = 2

area(ΔAPC) = 2 × area(ΔBQC)
Or, area(ΔBQC) = 1/2area(ΔAPC) [proved]

Tick the correct answer and justify:

8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. The ratio of the area of triangles ABC and BDE is
(A) 2: 1
(B) 1 : 2
(C) 4: 1
(D) 1 : 4

Answer: ΔABC and ΔBDE are two equilateral triangles. D is the midpoint of BC. [Given]



Hence, BD = DC = 1/2BC
Let each side of the triangle be 2*a*.

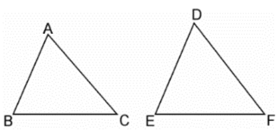
As ΔABC ~ ΔBDE

Hence, Area(ΔABC)/Area(ΔBDE)
= $\frac{AB^2}{BD^2}$ = $\frac{(2a)^2}{a^2}$ = $\frac{4a^2}{a^2}$ = $\frac{4}{1}$= 4 : 1

The correct answer is (C).

9. Sides of two similar triangles are in the ratio 4: 9. Areas of these triangles are in the ratio
(A) 2 : 3
(B) 4: 9
(C) 81 : 16
(D) 16 : 81

Answer: Sides of two similar triangles are in the ratio 4: 9. [Given]



Let ABC and DEF are two similar triangles, such that,

ΔABC ~ ΔDEF

And AB/DE = AC/DF = BC/EF = 4/9

As the ratio of the areas of these triangles will be equal to the square of the balance of the corresponding sides,

Hence, Area(ΔABC)/Area(ΔDEF) = $\frac{AB^2}{DE^2}$

Area(ΔABC)/Area(ΔDEF) = $\left(\frac{4}{9}\right)^2 = \frac{16}{18}$ = 16:81

Hence, the correct answer is (D).

Exercise 6.5

Question 1: Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.
(i) 7 cm, 24 cm, 25 cm
(ii) 3 cm, 8 cm, 6 cm
(iii) 50 cm, 80 cm, 100 cm
(iv) 13 cm, 12 cm, 5 cm

Answer: (i) $7^2 + 24^2 = 49 + 576 = 625 = 25^2$

Hence, the given triangle makes a right-angled triangle with hypotenuse 25cm.

(ii) $3^2 + 6^2 = 9 + 36 = 45 \neq 8^2$

The given triangle is not right-angled.

(iii) $50^2 + 80^2 = 2500 + 6400 = 8900 \neq 100^2$

Hence, the given triangle is not right-angled.

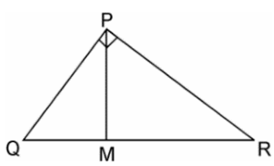
(iv) $12^2 + 5^2 = 144 + 25 = 169 = 13^2$

Hence, the given triangle makes a right-angled triangle with hypotenuse 13cm.

Question 2: QPR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that PM^2 = QM × MR.

Solution:

Given, ΔPQR is right-angled at P is a point on QR such that PM ⊥QR



We have to prove, PM^2 = QM × MR

In ΔPQM,
$PQ^2 = PM^2 + QM^2$ [Pythagoras theorem]
Or, $PM^2 = PQ^2 – QM^2$ ……………………………….(1)

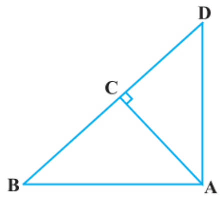
In ΔPMR,
$PR^2 = PM^2 + MR^2$ [Pythagoras theorem]
Or, $PM^2 = PR^2 – MR^2$ ……………………………………….(2)

Adding equation, (1) and (2) we get,

$2PM^2 = (PQ^2 + PM^2) – (QM^2 + MR^2)$
$= QR^2 – QM^2 – MR^2$ [$QR^2 = PQ^2 + PR^2$]
$= (QM + MR)^2 – QM^2 – MR^2$
$= 2QM \times MR$

Hence, $PM^2 = QM \times MR$ [Proved]

3. In Figure, ABD is a triangle right angled at A and AC ⊥ BD. Show that
(i) $AB^2 = BC \times BD$
(ii) $AC^2 = BC \times DC$
(iii) $AD^2 = BD \times CD$



Answer:

(i) In ΔADB and ΔCAB,

∠DAB = ∠ACB [Each 90°]

∠ABD = ∠CBA [Common angles]

Hence, ΔADB ~ ΔCAB [AA similarity criterion]
or, $\frac{AB}{CB} = \frac{BD}{AB}$
or, $AB^2 = CB \times BD$

(ii) Let ∠CAB = x

In ΔCBA, ∠CBA = 180° – 90° – x
or, ∠CBA = 90° – x

Similarly, in ΔCAD ∠CAD = 90° – ∠CBA = 90° – x
or, ∠CDA = 180° – 90° – (90° – x)
or, ∠CDA = x

In ΔCBA and ΔCAD, we have

∠CBA = ∠CAD
∠CAB = ∠CDA
∠ACB = ∠DCA (Each 90°)

Hence, ΔCBA ~ ΔCAD [AAA similarity criterion]

Or, $\frac{AC}{DC} = \frac{BC}{AC}$
or, AC^2 = DC × BC

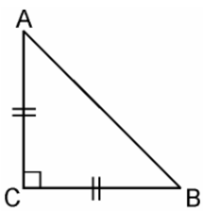
(iii) In ΔDCA and ΔDAB,
∠DCA = ∠DAB (Each 90°)
∠CDA = ∠ADB (common angles)

Hence, ΔDCA ~ ΔDAB [AA similarity criterion]
or, $\frac{DC}{DA} = \frac{AD}{BD}$

Or, AD^2 = BD × CD

Question 4. ABC is an isosceles triangle right angled at C. Prove that AB^2 = $2AC^2$.

Answer: ΔABC is an isosceles triangle right angled at C.



In ΔACB, ∠C = 90° [Given]

AC = BC [isosceles triangle property]

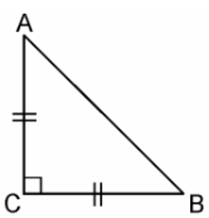
$AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]

$= AC^2 + AC^2$ [Since, AC = BC]

$AB^2 = 2AC^2$ [proved]

Question 5. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Answer: ΔABC is an isosceles triangle having AC = BC and $AB^2 = 2AC^2$ [Given]



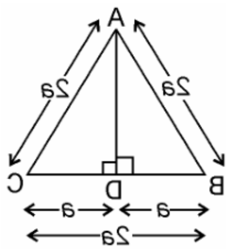
In ΔACB,

AC = BC
$AB^2 = 2AC^2$
$AB^2 = AC^2 + AC^2 = AC^2 + BC^2$ [AC = BC]

Hence, by Pythagoras theorem, ΔABC is a right-angle triangle.

Question 6. ABC is an equilateral triangle of side 2a. Find each of its altitudes.

Answer: ABC is an equilateral triangle of side 2a.



Draw, AD ⊥ BC

In ΔADB and ΔADC,
AB = AC
AD = AD
∠ADB = ∠ADC [90°]

Therefore, ΔADB ≅ ΔADC [RHS congruence.]
Hence, BD = DC [by CPCT]

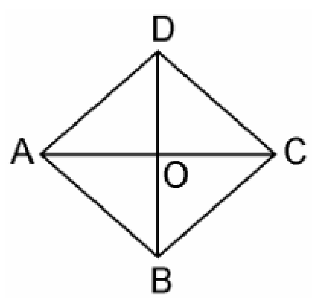
In right-angled ΔADB,

$AB^2 = AD^2 + BD^2$
or, $(2a)^2 = AD^2 + a^2$
or, $AD^2 = 4a^2 - a^2$
or, $AD^2 = 3a^2$
or, $AD = \sqrt{3}a$

7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Solution:

Given, ABCD is a rhombus whose diagonals AC and BD intersect at O.



We have to prove, as per the question,

$AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

Since the diagonals of a rhombus bisect each other at right angles.

Therefore, AO = CO and BO = DO

In ΔAOB,

∠AOB = 90°

$AB^2 = AO^2 + BO^2$ …………………….. **(i)** [By Pythagoras theorem]

Similarly,

$AD^2 = AO^2 + DO^2$ …………………….. **(ii)**

$DC^2 = DO^2 + CO^2$ …………………….. **(iii)**

$BC^2 = CO^2 + BO^2$ …………………….. **(iv)**

Adding equations **(i) + (ii) + (iii) + (iv)**, we get,

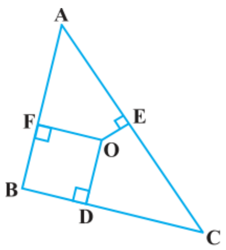
$AB^2 + AD^2 + DC^2 + BC^2 = 2(AO^2 + BO^2 + DO^2 + CO^2)$

$= 4AO^2 + 4BO^2$ [Since, AO = CO and BO =DO]

$= (2AO)^2 + (2BO)^2 = AC^2 + BD^2$

$AB^2 + AD^2 + DC^2 + BC^2 = AC^2 + BD^2$ [Proved]

8. In Fig. 6.54, O is a point in the interior of a triangle.



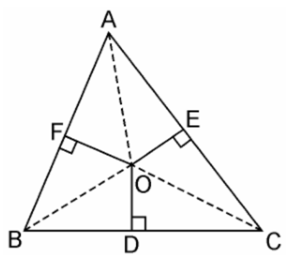
ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that:
(i) $OA^2 + OB^2 + OC^2 – OD^2 – OE^2 – OF^2 = AF^2 + BD^2 + CE^2$,
(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.

Solution:

Given, in ΔABC, O is a point in the interior of a triangle.

And OD ⊥ BC, OE ⊥ AC and OF ⊥ AB.

Join OA, OB and OC



(i) By Pythagoras theorem in ΔAOF, we have

$OA^2 = OF^2 + AF^2$

Similarly, in ΔBOD

$OB^2 = OD^2 + BD^2$

Similarly, in ΔCOE

$OC^2 = OE^2 + EC^2$

Adding these equations,

$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$

$OA^2 + OB^2 + OC^2 – OD^2 – OE^2 – OF^2 = AF^2 + BD^2 + CE^2$.

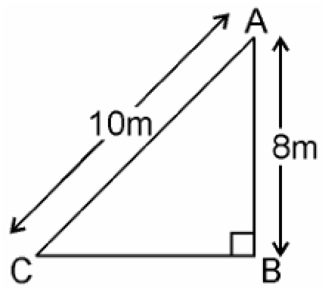
(ii) $AF^2 + BD^2 + EC^2 = (OA^2 – OE^2) + (OC^2 – OD^2) + (OB^2 – OF^2)$

Hence, $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.

9. A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from the base of the wall.

Solution:

Given, a ladder 10 m long reaches a window 8 m above the ground.



Let BA be the wall and AC be the ladder,

Therefore, by Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$10^2 = 8^2 + BC^2$

$BC^2 = 100 – 64$

$BC^2 = 36$

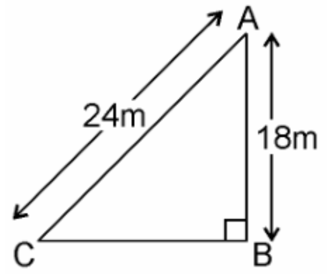
BC = 6m

Therefore, the distance of the foot of the ladder from the base of the wall is 6 m.

10. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Solution:

Given, a guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end.



Let AB be the pole and AC be the wire.

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$24^2 = 18^2 + BC^2$

$BC^2 = 576 – 324$

$BC^2 = 252$

BC = $6\sqrt{7}$m

Therefore, the distance from the base is $6\sqrt{7}$m.

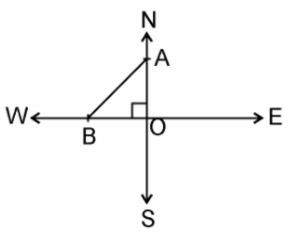
11. An aeroplane leaves an airport and flies due north at 1,000 km per hour. Simultaneously, another aeroplane leaves the same airport and flies due west at a speed of 1,200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer: Speed of first aeroplane = 1000 km/hr [Given]

Distance covered by a first aeroplane flying due north in $1\frac{1}{2}$ hours (OA) = 100 × $\frac{3}{2}$ km = 1500 km

Speed of second aeroplane = 1200 km/hr

Distance covered by a second aeroplane flying due west in $1\frac{1}{2}$ hours (OB) = 1200 × $\frac{3}{2}$km = 1800 km



In right-angle ΔAOB, by Pythagoras Theorem,

$AB^2 = AO^2 + OB^2$
or, $AB^2 = (1500)^2 + (1800)^2$
or, AB = $\sqrt{(2250000 + 3240000)}$ = $\sqrt{5490000}$

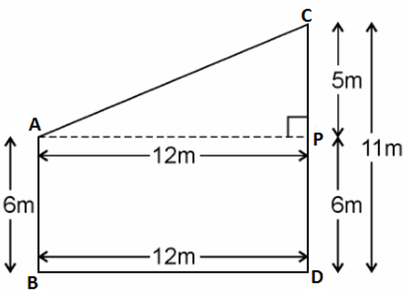
AB = 300√61 km

Hence, the distance between the two aeroplanes will be 300√61 km.

12. Two poles of heights 6 m and 11 m stand on bare ground. If the distance between the bars' feet is 12 m, find the distance between their tops.

answer: Two poles of heights 6 m and 11 m stand on a plain ground. [Given]

And the distance between the feet of the poles is 12 m.



Let AB and CD be the poles of height 6m and 11m.

Therefore, CP = 11 – 6 = 5m

From the figure, it can be observed that AP = 12m

By Pythagoras theorem for ΔAPC, we get,

$AP^2 = PC^2 + AC^2$

$(12m)^2 + (5m)^2 = (AC)^2$

$AC^2 = (144+25)\ m^2 = 169\ m^2$

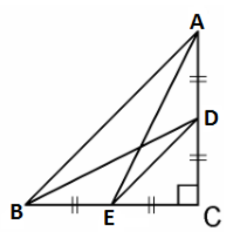
AC = 13m

Therefore, the distance between their tops is 13 m.

13. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Solution:

Given, D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C.



By Pythagoras theorem in ΔACE, we get

$AC^2 + CE^2 = AE^2$ ………………………………………….**(i)**

In ΔBCD, by Pythagoras theorem, we get

$BC^2 + CD^2 = BD^2$ ………………………………..**(ii)**

From equations **(i)** and **(ii)**, we get,

$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2$ …………..**(iii)**

In ΔCDE, by Pythagoras theorem, we get

$DE^2 = CD^2 + CE^2$

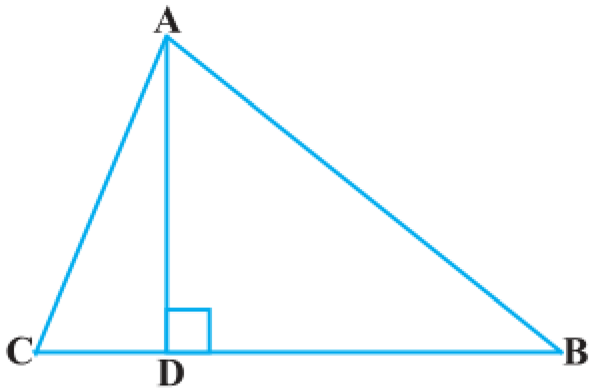
In ΔABC, by Pythagoras theorem, we get

$AB^2 = AC^2 + CB^2$

Putting the above two values in equation **(iii)**, we get

$DE^2 + AB^2 = AE^2 + BD^2$.

14. The perpendicular from A on side BC of a Δ ABC intersects BC at D such that DB = 3CD (see Figure). Prove that $2AB^2 = 2AC^2 + BC^2$.



Solution:

Given, the perpendicular from A on side BC of a Δ ABC intersects BC at D such that;

DB = 3CD.

In Δ ABC,

AD ⊥BC and BD = 3CD

In a right-angled triangle, ADB and ADC, by Pythagoras theorem,

$AB^2 = AD^2 + BD^2$ ……………………….**(i)**

$AC^2 = AD^2 + DC^2$ ……………………………..**(ii)**

Subtracting equation **(ii)** from equation **(i)**, we get

$AB^2 – AC^2 = BD^2 – DC^2$

$= 9CD^2 – CD^2$ [As, BD = 3CD]

$= 8CD^2$

$= 8(BC/4)^2$ [As, BC = DB + CD = 3CD + CD = 4CD]

Therefore, $AB^2 – AC^2 = BC^2/2$

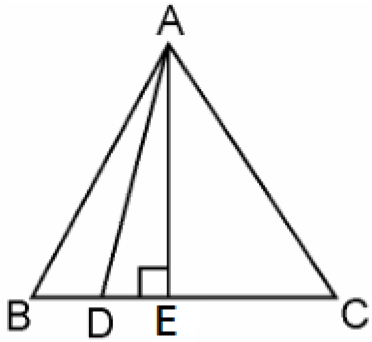
Or, $2(AB^2 – AC^2) = BC^2$

Or,$2AB^2 - 2AC^2 = BC^2$

Hence, $2AB^2 = 2AC^2 + BC^2$.

15. In an equilateral triangle ABC, D is a point on side BC such that BD = 1/3BC. Prove that $9AD^2 = 7AB^2$.

Answer: ABC is an equilateral triangle and D is a point on side BC such that BD = $\frac{1}{3}$BC [Given]



Let the side of the equilateral triangle be *a*, and AE be the altitude of ΔABC.

BE = EC = $\frac{BC}{2} = \frac{a}{2}$ and AE = $\frac{\sqrt{3}a}{2}$

BD = $\frac{1}{3}$BC [Given] and BD = $\frac{a}{3}$

DE = BE – BD = $\frac{a}{2} - \frac{a}{3} = \frac{a}{6}$

In ΔADE, by Pythagoras theorem,

$AD^2 = AE^2 + DE^2$

$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$

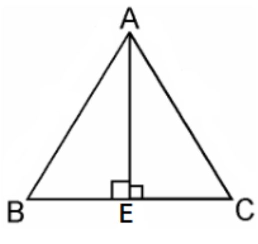
$= \frac{3a^2}{4} + \frac{a^2}{36}$

$= \frac{7}{9} AB^2$

Or, $9\ AD^2 = 7\ AB^2$

16. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer: An equilateral triangle ABC,



Let the sides of the equilateral triangle be of length a, and AE be the altitude of ΔABC. Hence, BE = EC = $\frac{BC}{2} = \frac{a}{2}$

In ΔABE, by Pythagoras Theorem, we get

$AB^2 = AE^2 + BE^2$

$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$

$AE^2 = a^2 - \frac{a^2}{4}$

$AE^2 = \frac{3a^2}{4}$

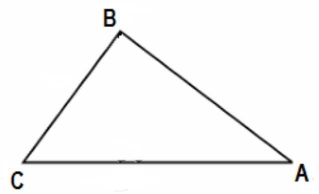
$4AE^2 = 3a^2$
or, 4 × (Square of altitude) = 3 × (Square of one side) [Proved]

Question **17. Tick the correct answer and justify: In ΔABC, AB = 6√3 cm, AC = 12 cm and BC = 6 cm.**
The angle B is:
(A) 120°

(B) 60°
(C) 90°

(D) 45°

Answer: In ΔABC, AB = 6√3 cm, AC = 12 cm and BC = 6 cm. [Given]



We can observe that,

$AB^2 = 108$

$AC^2 = 144$

And, $BC^2 = 36$

$AB^2 + BC^2 = AC^2$

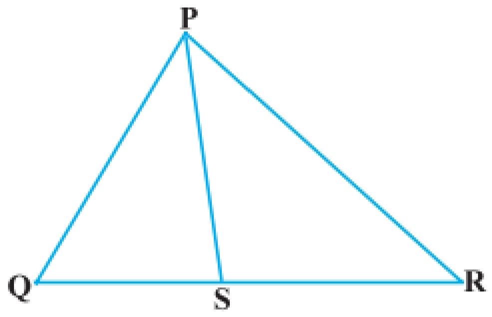
Hence, ΔABC is satisfying Pythagoras theorem.

Therefore, the triangle is a right triangle, right-angled at B.
Hence, ∠B = 90°

Hence, the correct answer is (C).

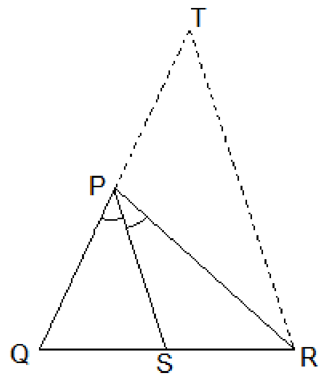
Exercise 6.6

1. In Figure, PS is the bisector of ∠ QPR of Δ QPR. Prove that QS/PQ = SR/PR



Solution:

Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.
Given, PS is the angle bisector of ∠QPR. Therefore,
∠QPS = ∠SPR……………………………….(i)

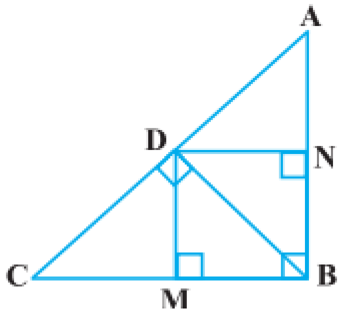


As per the constructed figure,
∠SPR=∠PRT(Since, PS||TR)……………(ii)
∠QPS = ∠QRT(Since, PS||TR) …………..(iii)
From the above equations, we get,
∠PRT=∠QTR

Therefore,
PT=PR
In △QTR, by basic proportionality theorem,
$\frac{QS}{SR} = \frac{QP}{PT}$

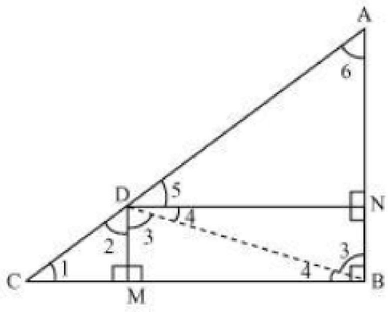
Since PT=TR
Therefore,
$\frac{QS}{SR} = \frac{PQ}{PR}$
Hence, proved.

2. In Fig. 6.57, D is a point on hypotenuse AC of △ABC, such that BD ⊥AC, DM ⊥ BC and DN ⊥ AB. Prove that: (i) DM^2 = DN. MC (ii) DN^2 = DM . AN.



Solution:

1. Let us join Point D and B.



Given,
BD ⊥AC, DM ⊥ BC and DN ⊥ AB

Now from the figure we have,
DN || CB, DM || AB and ∠B = 90 °
Therefore, DMBN is a rectangle.
So, DN = MB and DM = NB

The given condition which we have to prove is when D is the foot of the perpendicular drawn from B to AC.

Therefore ∠CDB = 90° ⇒ ∠2 + ∠3 = 90° …………………….. (i)

In ∆CDM, ∠1 + ∠2 + ∠DMC = 180°
or, ∠1 + ∠2 = 90° …………………………………….. (ii)
In ∆DMB, ∠3 + ∠DMB + ∠4 = 180°
or, ∠3 + ∠4 = 90° …………………………………….. (iii)
From equation (i) and (ii), we get
∠1 = ∠3
From equation (i) and (iii), we get
∠2 = ∠4
In ∆DCM and ∆BDM,
∠1 = ∠3 (Already Proved)
∠2 = ∠4 (Already Proved)

Therefore ∆DCM ~ ∆BDM (AA similarity criterion)
$\frac{BM}{DM} = \frac{DM}{MC}$
DN/DM = DM/MC (BM = DN)

or, $DM^2 = DN \times MC$
Hence, proved.

(ii) In right triangle DBN,
$\angle 5 + \angle 7 = 90°$ ……………….. (iv)

In right triangle DAN,
$\angle 6 + \angle 8 = 90°$ ………………… (v)

D is the point in the triangle, which is the perpendicular foot drawn from B to AC.
Therefore $\angle ADB = 90° \Rightarrow \angle 5 + \angle 6 = 90°$ ………….. (vi)

From equation (iv) and (vi), we get,
$\angle 6 = \angle 7$

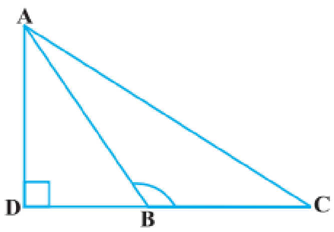
From equation (v) and (vi), we get,
$\angle 8 = \angle 5$

In ΔDNA and ΔBND,
$\angle 6 = \angle 7$ [Proved earlier]
$\angle 8 = \angle 5$ [Proved earlier]

Therefore $\Delta DNA \sim \Delta BND$ (AA similarity criterion)
$\frac{AN}{DN} = \frac{DN}{NB}$
or, $DN^2 = AN \times NB$
or, $DN^2 = AN \times DM$ (NB = DM) [Proved]

3. In Figure, ABC is a triangle in which $\angle ABC > 90°$ and AD ⊥ CB produced. Prove that

$AC^2 = AB^2 + BC^2 + 2\ BC.BD$.



Solution:

By applying Pythagoras Theorem in ΔADB, we get,

$AB^2 = AD^2 + DB^2$ ……………………… (i)

Again, by applying Pythagoras Theorem in ΔACD, we get,

$AC^2 = AD^2 + DC^2$

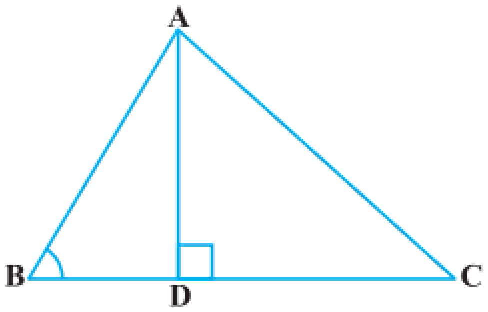
$AC^2 = AD^2 + (DB + BC)^2$

$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$

From equation (i),

$AC^2 = AB^2 + BC^2 + 2DB \times BC$ [Proved]

4. In Figure, ABC is a triangle in which ∠ ABC < 90° and AD ⊥ BC. Prove that $AC^2 = AB^2 + BC^2 - 2\ BC.BD$.



Answer: By applying Pythagoras Theorem in ΔADB, we get,

$AB^2 = AD^2 + DB^2$

or, $AD^2 = AB^2 - DB^2$ ……………….. (i)

By applying Pythagoras Theorem in ΔADC, we get,

$AD^2 + DC^2 = AC^2$

From equation (i),

$AB^2 - BD^2 + DC^2 = AC^2$

or, $AB^2 - BD^2 + (BC - BD)^2 = AC^2$

or, $AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$

or, $AC^2 = AB^2 + BC^2 - 2BC \times BD$

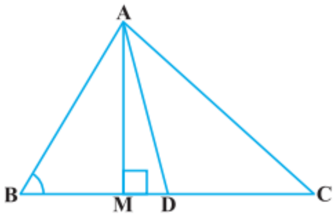
Hence, proved.

5. In Figure, AD is a median of a triangle ABC and AM ⊥ BC. Prove that :

(i) $AC^2 = AD^2 + BC.DM + 2\ (BC/2)^2$

(ii) $AB^2 = AD^2 - BC.DM + 2\ (BC/2)^2$

(iii) $AC^2 + AB^2 = 2\ AD^2 + \frac{1}{2}\ BC^2$



Answer:

(i) By applying Pythagoras Theorem in ΔAMD, we get,
$AM^2 + MD^2 = AD^2$ ………………………………………………. (i)
Again, by applying Pythagoras Theorem in ΔAMC, we get,

$AM^2 + MC^2 = AC^2$
or, $AM^2 + (MD + DC)^2 = AC^2$
or, $(AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$

From equation(i), we get,
$AD^2 + DC^2 + 2MD.DC = AC^2$
Since, DC=BC/2, thus, we get,
$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\left(\frac{BC}{2}\right)^2 = AC^2$
or, $AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD \times BC = AC^2$
Hence, proved.

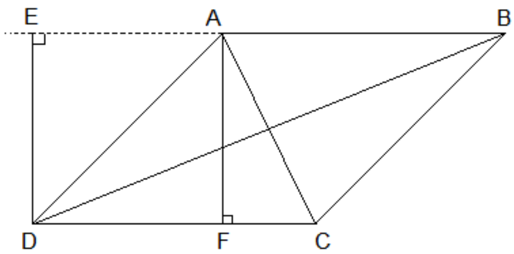
(ii) By applying Pythagoras Theorem in ∆ABM, we get;
$AB^2 = AM^2 + MB^2$
$= (AD^2 - DM^2) + MB^2$
$= (AD^2 - DM^2) + (BD - MD)^2$
$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$
$= AD^2 + BD^2 - 2BD \times MD$
$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) MD$
$= AD^2 + \left(\frac{BC}{2}\right)^2 {}^2 - BC.MD$
Hence, proved.

(iii) By applying Pythagoras Theorem in ∆ABM, we get,
$AM^2 + MB^2 = AB^2$ …………………..… (i)
By applying Pythagoras Theorem in ∆AMC, we get,
$AM^2 + MC^2 = AC^2$ …………………..… (ii)
Adding both the equations (i) and (ii), we get,
$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$
$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$
$2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$
$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) = AB^2 + AC^2$
$2(AM^2 + MD^2) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right)^2 = AB^2 + AC^2$
$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$

6. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Solution:

Let, ABCD be a parallelogram. Now, we need to draw perpendicular DE on extended side of AB and draw a vertical AF meeting DC at point F.



In ΔDEA, we get,
$DE^2 + EA^2 = DA^2$ ……………………………………(1) [Pythagoras Theorem]
In ΔDEB, we get,
$DE^2 + EB^2 = DB^2$ [[Pythagoras Theorem]
$DE^2 + (EA + AB)^2 = DB^2$
$(DE^2 + EA^2) + AB^2 + 2EA \times AB = DB^2$
$DA^2 + AB^2 + 2EA \times AB = DB^2$ ……………………………………….(2)
By applying Pythagoras Theorem in ΔADF, we get,
$AD^2 = AF^2 + FD^2$

Again, in ΔAFC,
$AC^2 = AF^2 + FC^2$ [By Pythagoras Theorem]
$= AF^2 + (DC - FD)^2$
$= AF^2 + DC^2 + FD^2 - 2DC \times FD$
$= (AF^2 + FD^2) + DC^2 - 2DC \times FD\ AC^2$
$AC^2 = AD^2 + DC^2 - 2DC \times FD$ ………………………………………….(3)

Since ABCD is a parallelogram,
$AB = CD$ …………………………………………………….(4)
And $BC = AD$ ……………………………………….(5)

In ΔDEA and ΔADF,
$\angle DEA = \angle AFD$ (Each 90°)
$\angle EAD = \angle ADF$ ($EA \parallel DF$)
$AD = AD$ (Common Angles)
Therefore $\Delta EAD \cong \Delta FDA$ (AAS congruence criterion)
or, $EA = DF$ ………………………………………………………..(6)

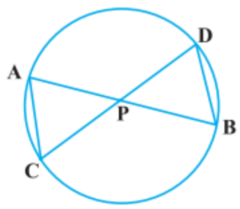
Adding equations (1) and (3), we get,
$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$
$DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$

From equation (4) and (6),
$BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2$
$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

7. In Figure, two chords AB and CD intersect each other at the point P. Prove that :

(i) $\Delta APC \sim \Delta DPB$

(ii) AP. PB = CP . DP



Solution:

Firstly, let us join CB in the given figure.

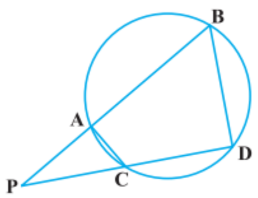
(i) In ΔAPC and ΔDPB,
∠APC = ∠DPB   [Vertically opposite angles]
∠CAP = ∠BDP   [Angles in the same segment for chord CB]
Therefore,
ΔAPC ~ ΔDPB   [AA similarity criterion]

(ii) In the above, we have proved that ΔAPC ~ ΔDPB
We know that the corresponding sides of similar triangles are proportional.
Therefore $\frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$
or, $\frac{AP}{DP} = \frac{PC}{PB}$
Therefore AP. PB = PC. DP [Proved]

8. In Fig. 6.62, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that:

(i) Δ PAC ~ Δ PDB

(ii) PA. PB = PC . PD.



Solution:

(i) In ΔPAC and ΔPDB,
∠P = ∠P [Common Angles]
As we know, the exterior angle of a cyclic quadrilateral is ∠PCA and ∠PBD is the opposite interior angle, which is both equal.
∠PAC = ∠PDB
Thus, ΔPAC ~ ΔPDB   [AA similarity criterion]

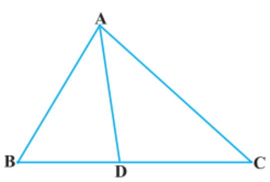
(ii) We have already proved above,
ΔAPC ~ ΔDPB
We know that the corresponding sides of similar triangles are proportional.
Therefore,

$\frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD}$

$\frac{AP}{DP} = \frac{PC}{PB}$

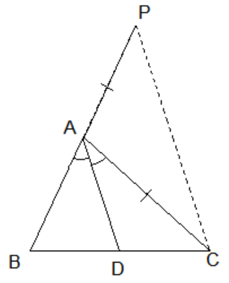
Therefore AP. PB = PC. DP

9. In Figure, D is a point on side BC of Δ ABC such that BD/CD = AB/AC. Prove that AD is the bisector of ∠ BAC.



Solution:

In the given figure, let us extend BA to P such that;
AP = AC.
Now join PC.

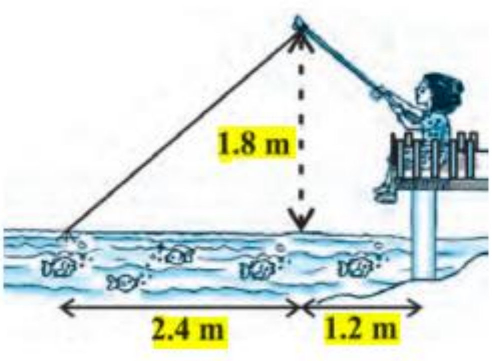


Given, $\frac{BD}{CD} = \frac{AB}{BC}$
or, $\frac{BD}{CD} = \frac{AP}{AC}$
By using the converse of basic proportionality theorem, we get,
AD || PC
∠BAD = ∠APC [Corresponding angles] …………………………………….. (i)
And, ∠DAC = ∠ACP [Alternate interior angles] ……………………………..… (ii)
From the new figure,

AP = AC
or, ∠APC = ∠ACP …………………… (iii)

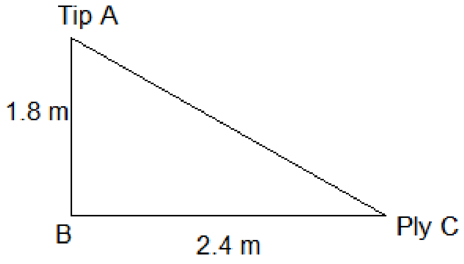
On comparing equations (i), (ii), and (iii), we get,
∠BAD = ∠APC
Therefore, AD is the bisector of the angle BAC.
Hence, proved.

10. Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water, and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Figure)? If she pulls in the line at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Solution:

Let us consider; AB is the height of the tip of the fishing rod from the water surface, and BC is the
The horizontal distance of the fly from the tip of the fishing rod. Therefore, AC is now the length of the string.

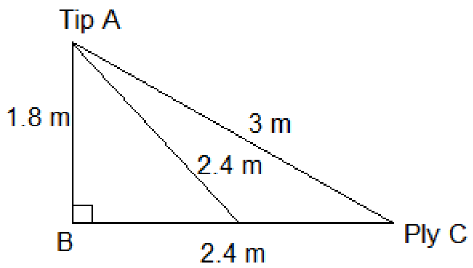


To find AC, we have to use Pythagoras theorem in ∆ABC, is such a way;
$AC^2 = AB^2 + BC^2$
or, $AB^2 = (1.8\text{ m})^2 + (2.4\text{ m})^2$
or, $AB^2 = (3.24 + 5.76)\text{ m}^2$
or, $AB^2 = 9.00\text{ m}^2$
or, $AB = \sqrt{9}\text{ m} = 3\text{m}$

Thus, the length of the string out is 3 m.

As its given, she pulls the string at the rate of 5 cm per second.

Therefore, string pulled in 12 seconds = 12 × 5 = 60 cm = 0.6 m



Let us consider; the fly is at point D after 12 seconds.
Length of string out after 12 seconds is AD.
AD = AC − String pulled by Nazima in 12 seconds
= (3.00 − 0.6) m
= 2.4 m

In ∆ADB,
$AB^2 + BD^2 = AD^2$ [Pythagoras Theorem]
or, $(1.8\text{ m})^2 + BD^2 = (2.4\text{ m})^2$
or, $BD^2 = (5.76 - 3.24)\text{ m}^2 = 2.52\text{ m}^2$
or, BD = 1.587 m

Horizontal distance of fly = BD + 1.2 m = (1.587 + 1.2) m = 2.787 m = 2.79 m