Chapter 2: Polynomials
Exercise : 2.1(MCQ)

Choose the correct answer from the given four options in the following questions:

1. If one of the zeroes of the quadratic polynomial (k–1) x^2 + k x + 1 is –3, then the value of k is

(A) 4/3
(B) -4/3
(C) 2/3
(D) -2/3

Solution: (A) 4/3

According to the question, -3 is one of the zeros of quadratic polynomial
$(k-1)x^2 + kx + 1$.
Substituting -3 in the given polynomial,
$(k – 1)(- 3)^2 + k(-3) + 1 = 0$
or, $(k – 1)9 + k(-3) + 1 = 0$
or, $9k – 9 - 3k + 1 = 0$
or, $6k – 8 = 0$
or, $k = \frac{8}{6}$

Therefore, k = 4/3

2. A quadratic polynomial, whose zeroes are –3 and 4, is

(A) $x^2 – x + 12$
(B) $x^2 + x + 12$
(C) $\frac{x^2}{2} - \frac{x}{2} – 6$
(D) $2x^2 + 2x – 24$

Solution: (C) $\frac{x^2}{2} - \frac{x}{2}$ - 6

Sum of zeroes, $\alpha + \beta= -3 + 4 = 1$
Product of Zeroes, $\alpha\beta = -3 \times 4 = -12$

Therefore, the quadratic polynomial becomes,
x^2- (sum of zeroes)x+(product of zeroes)
$= x^2 - (\alpha + \beta)x + (\alpha\beta)$
$= x^2 – (1)x + (-12)$
$= x^2 – x – 12$

Divide by 2, we get

$= \frac{x^2}{2} - \frac{x}{2} - \frac{12}{2}$

$= \frac{x^2}{2} - \frac{x}{2} - 6$

Hence, option (C**)** is the correct answer.

3. If the zeroes of the quadratic polynomial x^2 + (a + 1) x + b are 2 and –3, then

(A) a = –7, b = –1
(B) a = 5, b = –1
(C) a = 2, b = – 6
(D) a = 0, b = – 6

Solution: (D) a = 2, b = – 6

According to the question,
$x^2 + (a + 1)x + b$

Given that, the zeroes of the polynomial = 2 and -3,
<u>When x = 2</u>
$2^2 + (a + 1)(2) + b = 0$
or, 4 + 2a + 2 + b = 0
or, 6 + 2a + b = 0
or, 2a + b = -6 ……………………………………………………….(1)

<u>When x = -3,</u>
$(-3)^2 + (a + 1)(-3) + b = 0$
or, 9 – 3a – 3 + b = 0
or, 6 – 3a + b = 0
or, -3a + b = -6………………………………………………………….(2)

Subtracting equation (2) from (1)
2a + b – (-3a + b) = -6 - (- 6)
or, 2a + b + 3a - b = -6 + 6
or, 5a = 0
or, a = 0

Substituting the value of 'a' in equation (1), we get,
2a + b = -6
2(0) + b = -6
b = -6

Hence, option (D) is the correct answer.

4. The number of polynomials having zeroes as –2 and 5 is
(A) 1
(B) 2
(C) 3
(D) more than 3

Solution: (D) more than 3

According to the question,
The zeroes of the polynomials = -2 and 5
We know that the polynomial is of the form, $p(x) = ax^2 + bx + c$.

Sum of the zeroes = – (coefficient of x) ÷ coefficient of x^2 i.e.
Sum of the zeroes = – b/a
– 2 + 5 = – b/a
or, 3 = – b/a

Hence, b = – 3 and a = 1

Product of the zeroes = constant term ÷ coefficient of x^2 i.e.
Product of zeroes = c/a
(- 2)5 = c/a
or, – 10 = c

Substituting the values of a, b and c in the polynomial p(x) = $ax^2 + bx + c$
We get, $x^2 – 3x – 10$
Therefore, we can conclude that x can take any value.

Hence, option (D) is the correct answer.

5. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

(A) (–c/a)
(B) c/a
(C) 0
(D) (–b/a)

Solution: (B) (c/a)

According to the question, we have the polynomial, $ax^3 + bx^2 + cx + d$

We know that, sum of product of roots of a cubic equation is given by c/a
It is given that one root = 0
Now, let the other roots be α, β

So, we get,
αβ + β(0) + (0)α = c/a
αβ = c/a
Hence the product of other two roots is c/a

Question 6:
If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is
(a) b – a +1 (b) b – a -1 (c) a – b +1 (d) a – b -1

Solution: (a) Let p(x) = $x^3 + ax^2 + bx + c$
Let a, p and y be the zeroes of the given cubic polynomial p(x).
∴ α = -1 ……[given]
and p(−1) = 0
or, $(-1)^3 + a(-1)^2 + b(-1) + c = 0$
or, -1 + a- b + c = 0
or, c = 1 -a + b…………………………………………………………………(1)
We know that,

Product of all zeroes = $(-1)^3 \frac{constant\ term}{coefficient\ of\ x^3} = -\frac{c}{1}$
αβγ = -c
or, (-1)βγ = −c…….[∴α = -1]
or, βγ = c
or, βγ = 1 -a + b……….[from Eq. (1)]
Hence, product of the other two roots is 1 -a + b.

Question 7: The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are
(a) both positive **(b) both negative**
(c) one positive and one negative **(d) both equal**

Solution: (b) Let given quadratic polynomial be $p(x) = x^2 + 99x + 127$.
On comparing p(x) with $ax^2 + bx + c$, we get, a = 1, b = 99 and c = 127

We know that,
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ ……[by quadratic formula]

$= \frac{-99 \pm \sqrt{99^2-4\times1\times127}}{2\times1}$

$= \frac{-99 \pm \sqrt{9801-508}}{2}$

$= \frac{-99 \pm \sqrt{9293}}{2}$

$= \frac{-99 \pm 96.4}{2}$

$= \frac{-99+96.4}{2}, \frac{-99-96.4}{2}$

$= \frac{-2.6}{2}, \frac{-195.4}{2}$

= -1.3 , -97.7

Hence, both zeroes of the given quadratic polynomial p(x) are negative.

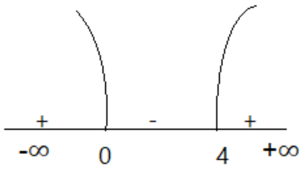
Question 8: The zeroes of the quadratic polynomial $x^2 + kx + k$ where $k \neq 0$,
(a) cannot both be positive **(b) cannot both be negative**
(c) are always unequal **(d) are always equal**

Solution: (a) Let $p(x) = x^2 + kx + k$, $k \neq 0$
On comparing p(x) with $ax^2 + bx + c$, we get,
a = 1, b = k and c = k

Now, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-k \pm \sqrt{k^2-4k}}{2\times1}$

$= \frac{-k \pm \sqrt{k(k-4)}}{2}$, $k \neq 0$



Here, we see that
$k(k - 4) > 0$
or, $k \in (-\infty, 0) \cup (4, \infty)$

Now, we know that, in quadratic polynomial $ax^2 + bx + c$
If $a > 0$, $b> 0$, $c> 0$ or $a< 0$, $b< 0$,$c< 0$, then the polynomial has always all negative zeroes, and if $a > 0$, $c < 0$ or $a < 0$, $c > 0$, then the polynomial has always zeroes of opposite sign

Case I
If $k\in (-\infty, 0)$ i.e., $k < 0$
or, $a = 1 > 0$, $b , c = k < 0$
So, both zeroes are of opposite sign.

Case II
If $k\in (4, \infty)$i.e., $k \geq 4$
or, $a = 1 > 0$, $b , c > 4$
So, both zeroes are negative.
Hence, in any case zeroes of the given quadratic polynomial cannot both be positive.

Question 9: If the zeroes of the quadratic polynomial $ax^2+ bx+ c$, where $c\neq 0$, are equal, then
(a) c and a have opposite signs **(b) c and b have opposite signs**
(c) c and a have same signs **(d) c and b have the same signs**

Solution:
(c) The zeroes of the given quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal. If coefficient of x^2 and constant term have the same sign i.e., c and a have the same sign. While b i.e., coefficient of x can be positive/negative but not zero.

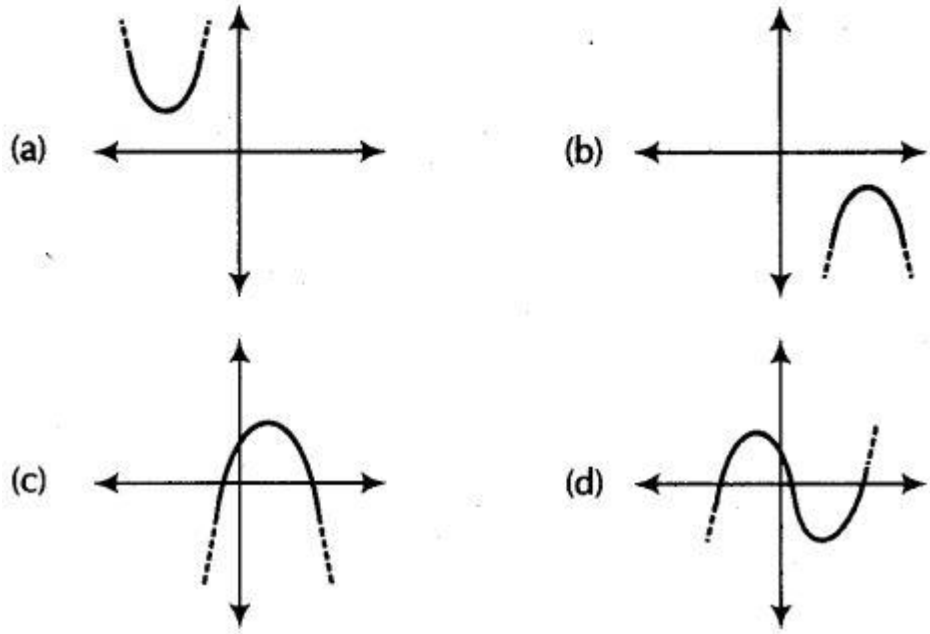
e.g., (i) $x^2 + 4x + 4 = 0$
or, $(x + 2)^2 = 0$
or, $x = -2, -2$

(ii) $x^2 – 4x + 4 = 0$
or, $(x – 2)^2 = 0$
or, $x = 2, 2$
which is only possible when a and c have the same signs.

Question 10: If one of the zeroes of a quadratic polynomial of the form x^2 + ax + b is the negative of the other, then it
(a) has no linear term and the constant term is negative
(b) has no linear term and the constant term is positive
(c) can have a linear term but the constant term is negative
(d) can have a linear term but the constant term is positive

Solution: (a) Let, p(x) = x^2 + ax + b.
Put a = 0, then, p(x) = x^2 + b = 0
or, x^2 = -b
or, x = $\pm\sqrt{-b}$ [b < 0]
Hence, if one of the zeroes of quadratic polynomial p(x) is the negative of the other, then it has no linear term i.e., a = O and the constant term is negative i.e., b< 0.

Question 11: Which of the following is not the graph of a quadratic polynomial?



Solution: (d) For any quadratic polynomial ax^2 + bx + c, a≠0, the graph of the Corresponding equation y = ax^2 + bx + c has one of the two shapes either open upwards like u or open downwards like ∩ depending on whether a > 0 or a < 0. These curves are called parabolas. So, option (d) cannot be possible.
Also, the curve of a quadratic polynomial crosses the X-axis on at most two points but in option (d) the curve crosses the X-axis on the three points, so it does not represent the quadratic polynomial.

Exercise 2.2 (Very Short Answer Type Questions)

Question 1: Answer the following and justify.
(i) Can x^2 -1 be the quotient on division of x^6 +2x^3 +x-l by a polynomial in x of degree 5?

(ii) What will the quotient and remainder be on division of $ox^2 + bx + c$ by $px^3 + qx^2 + rx + s$, $p \neq 0$?
(iii) If on division of a polynomial p(x) by a polynomial g(x),the quotient is zero, what is the relation between the degree of p(x)and g(x)l
(vi) If on division of a non-zero polynomial p(x)by a polynomial g(x),the remainder is zero, what is the relation between the degrees of p(x) and g(x)?
(v) Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer k > 1?

Solution: (i) No. because whenever we divide a polynomial $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5, then we get quotient always as in linear form i.e., polynomial in x of degree 1. Let divisor = a polynomial in x of degree 5
$= ax^5 + bx^4 + cx^3 + dx^2 + ex + f$
quotient = $x^2 - 1$ and dividend = $x^6 + 2x^3 + x - 1$
By division algorithm for polynomials, Dividend = Divisor x Quotient + Remainder
$= (ax^5 + bx^4 + cx^3 + dx^2 + ex + f)x(x^2 - 1)$ + Remainder
= (a polynomial of degree 7) + Remainder
= (a polynomial of degree 7)
But, dividend = a polynomial of degree 6
So, division algorithm is not satisfied.
Hence, $x^2 - 1$ is not a required quotient.

(ii) Given that, Divisor $px^3 + gx^2 + rx + s$, $p \neq 0$ and dividend = $ax^2 + bx + c$
We know that, Degree of divisor > Degree of dividend
So, by division algorithm, quotient = 0 and remainder = $ax^2 + bx + c$
If degree of dividend < degree of divisor, then quotient will be zero and remainder as same as dividend.

(iii) If division of a polynomial p(x) by a polynomial g(x), the quotient is zero, then relation between the degrees of p(x) and g(x) is degree of p(x) < degree of g(x).
(iv) If division of a non-zero polynomial p(x) by a polynomial g(x), the remainder is zero, then g(x) is a factor of p(x) and has degree less than or equal to the degree of p(x). e., degree of g(x) < degree of p(x).
(v) No, let $p(x) = x^2 + kx + k$
If p(x) has equal zeroes, then its discriminant should be zero.
$D = b^2 – 4ac = 0$……………………………………………………………(1)
On comparing p(x) with $Ax^2 + Bx + C$, we get
A =1 B = k and C = k
$\therefore$ $(k)^2 - 4(1)(k) = 0$ [from Eq. (1)]
or, k(k- 4) = 0
or, k = 0, 4
So, the quadratic polynomial p(x) have equal zeroes only at k = 0, 4.

Question 2: Are the following statements True' or 'False'? Justify your answer.
(i) If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a, b and c all have the same sign.
(ii) If the graph of a polynomial intersects the X-axis at only one point, it cannot

be a quadratic polynomial.
(iii) If the graph of a polynomial intersects the X-axis at exactly two points, it need not be a quadratic polynomial.
(iv) If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.
(v) If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of the polynomial have the same sign.
(vi) If all three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then atleast one of a, b and c is non-negative.
(vii) The only value of k for which the quadratic polynomial $kx^z + x + k$ has equal zeroes is $\frac{1}{2}$

Solution: (i) False, if the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then $\alpha + \beta = -\frac{b}{a}$ and, $\alpha.\beta = \frac{c}{a}$
where, α and β are the zeroes of quadratic polynomial.
$c < 0$, $a < 0$ and $b > 0$
$c > 0$, $a > 0$ and $b < 0$

(ii) True, if the graph of a polynomial intersects the X-axis at only one point, then it cannot be a quadratic polynomial because a quadratic polynomial may touch the X-axis at exactly one point or intersects X-axis at exactly two points or do not touch the X-axis.

(iii) True, if the graph of a polynomial intersects the X-axis at exactly two points, then it may or may not be a quadratic polynomial. As, a polynomial of degree more than z is possible which intersects the X-axis at exactly two points when it has two real roots and other imaginary roots.

(iv) True, let a, p and y be the zeroes of the cubic polynomial and given that two of the zeroes have value 0.
Let, $\alpha = \beta = 0$ and $f(x) = (x- \alpha)(x-\beta)(x-\gamma)$
$= (x - a)(x - 0)(x - 0)$
$= x^3 - ax^{2,}$ which does not have linear and constant terms.

(v) True, if $f(x) = ax^3 + bx^2 + cx + d$. Then, for all negative roots, a, b, c and d must have same sign.

(vi) False, let α, β, and γ be the three zeroes of cubic polynomial $x^3 + ax^2 - bx + c$
Then, product of zeroes = $(-1)^3 \frac{\text{constant term}}{\text{coefficient of } x^3}$
or, $\alpha \beta \gamma = -\frac{+c}{1}$
or, $\alpha \beta \gamma = -c$ ……………………………………………(1)

Given, that, all three zeroes are positive. So, the product of all three zeroes is also positive i.e., $\alpha \beta \gamma > 0$
or, $-c > 0$ ………..[From eq. (1)]
or, $c < 0$

Now, sum of the zeroes = $\alpha + \beta + \gamma$ = (-1) $\frac{\text{constant term } x^2}{\text{coefficient of } x^3}$
or, $\alpha + \beta + \gamma = -\frac{a}{1}$ = -a
But, α β γ are all positive.
Thus, its sum is also positive.
So, $\alpha + \beta + \gamma > 0$
or, - a > 0
or, a < 0
and sum of the product of two zeroes at a time = $(-1)^2 \cdot \frac{\text{constant term}}{\text{coefficient of } x^3} = -\frac{b}{1}$
or, $\alpha\beta + \beta\gamma + \gamma\alpha$ = -b
or, $\alpha\beta + \beta\gamma + \gamma\alpha > 0$
or, -b > 0
or, b < 0

So, the cubic polynomial $x^3 + ax^2 - bx + c$ has all three zeroes which are positive only when all constants a, b and c are negative

(vii) False, let $f(x) = kx^2 + x + k$
For equal roots. Its discriminant should be zero i.e., $D = b^2 - 4ac = 0$
or, 1 - 4k.k = 0
or, $k = \pm\frac{1}{2}$
So, for two values of k, given quadratic polynomial has equal zeroes

Exercise 2.3

Question 1: Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and the coefficients of the polynomials
(i) $4x^2 - 3x - 1$.
Solution: Let, f(x) = $4x^2 - 3x - 1$
= $4x^2 - 4x + x - 1$
= 4x(x – 1) +1(x – 1)
= (4x + 1)(x – 1)
So the value of $4x^2 - 3x - 1$ is zero when x – 1 = 0, or 4x + 1 = 0
or, x = 1 , $-\frac{1}{4}$
Hence the zeros are 1 and $-\frac{1}{4}$
Therefore, sum of zeros = $1 - \frac{1}{4} = \frac{3}{4} = \frac{-(-3)}{4}$
= $(-1)\left(\frac{Coefficient\ of\ x}{Coefficient\ of\ x^2}\right)$
And, Product of zeros = $(1)(-\frac{1}{4}) = -\frac{1}{4}$
= $(-1)^2\left(\frac{Constant\ Term}{Coefficient\ of\ x^2}\right)$
Hence the relations between the zeros and coefficients of the polynomials are

verified.

(ii) $3x^2 + 4x - 4$.

Solution: The zeros of the polynomial are -2 and $\frac{2}{3}$.

Sum of zeros = $-\frac{4}{3}$ and product of zeros = $-\frac{4}{3}$

(iii) $51^2 + 12t + 7$.

Solution: The zeros of the polynomial are -1 and $-\frac{7}{5}$.

Sum of zeros = $-\frac{12}{5}$ and product of zeros = $\frac{7}{5}$

(iv) $t^3 - 2t^2 - 15t$.

Solution: The zeros of the polynomial are -3 , 0 and $-\frac{7}{5}$.

Sum of zeros = 2, product of zeros two at a time = -15 and product of zeros = 0

(v) $2x^2 + \frac{7}{2}x + \frac{3}{4}$

Solution: The zeros of the polynomial are $-\frac{1}{4}$ and $-\frac{3}{2}$.

Sum of zeros = $-\frac{7}{4}$ and product of zeros = $\frac{3}{8}$

(vi) 4×2 +5√2x – 3.

Solution: The zeros of the polynomial are $-\frac{3}{\sqrt{2}}$ and $\frac{1}{2\sqrt{2}}$.

Sum of zeros = $-\frac{5\sqrt{2}}{4}$ and product of zeros = $-\frac{3}{4}$

(vii) $2s^2 -(1+2\sqrt{2})s +\sqrt{2}$

Solution: The zeros of the polynomial are $\sqrt{2}$ and $\frac{1}{2}$.

Sum of zeros = $\frac{1}{2} + \sqrt{2}$ and product of zeros = $\frac{1}{\sqrt{2}}$

(viii) $v^2 + 4\sqrt{3}v - 15$.

Solution: The zeros of the polynomial are $\sqrt{3}$ and $-5\sqrt{3}$.

Sum of zeros = $-4\sqrt{3}$ and product of zeros = -15

(ix) $y^2 + \frac{3}{2}\sqrt{5}y - 5$.

Solution: The zeros of the polynomial are $-2\sqrt{5}$ and $\frac{\sqrt{5}}{2}$.

Sum of zeros = $-\frac{3\sqrt{5}}{2}$ and product of zeros = -5

(x) $7y^2 - \frac{11}{3}y - \frac{2}{3}$

Solution: The zeros of the polynomial are $-\frac{1}{7}$ and $\frac{2}{3}$.
Sum of zeros = $\frac{11}{21}$ and product of zeros = $-\frac{2}{21}$

Exercise 2.4

1. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomials by factorisation.

(i) (–8/3), 4/3
(ii) 21/8, 5/16
(iii) -2√3, -9
(iv) (-3/(2√5)), -½

Solution: (i) Sum of the zeroes = – 8/3
Product of the zeroes = 4/3
P(x) = x^2 – (sum of the zeroes) + (product of the zeroes)
Then, P(x)= x^2 – (-8x)/3 + 4/3

P(x)= $3x^2 + 8x + 4$
Using splitting the middle term method,
$3x^2 + 8x + 4 = 0$
or, $3x^2 + (6x + 2x) + 4 = 0$
or, $3x^2 + 6x + 2x + 4 = 0$
or, 3x(x + 2) + 2(x + 2) = 0
or, (x + 2)(3x + 2) = 0
or, x = -2, -2/3

(ii) Sum of the zeroes = 21/8
Product of the zeroes = 5/16
P(x) = x^2 – (sum of the zeroes) + (product of the zeroes)

Then, P(x)= $x^2 - \frac{21}{8}x + \frac{5}{16}$
P(x)= $16x^2 - 42x + 5$

Using splitting the middle term method,
$16x^2 - 42x + 5 = 0$
or, $16x^2 - (2x + 40x) + 5 = 0$
or, $16x^2 - 2x - 40x + 5 = 0$
or, 2x (8x – 1) – 5(8x – 1) = 0
or, (8x – 1)(2x – 5) = 0
or, x = $\frac{1}{8}, \frac{5}{2}$

(iii) Sum of the zeroes = – 2√3
Product of the zeroes = – 9

$P(x) = x^2$ – (sum of the zeroes) + (product of the zeroes)
Then, $P(x) = x^2 - (-2\sqrt{3}x) - 9$

Using splitting the middle term method,
$x^2 + 2\sqrt{3}x - 9 = 0$
or, $x^2 + (3\sqrt{3}x - \sqrt{3}x) - 9 = 0$
or, $x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3}) = 0$
or, $(x - \sqrt{3})(x + 3\sqrt{3}) = 0$
or, $x = \sqrt{3}, -3\sqrt{3}$

(iv) Sum of the zeroes = $-\frac{3}{2\sqrt{5}}x$
Product of the zeroes = – ½

$P(x) = x^2$ – (sum of the zeroes) + (product of the zeroes)
Then, $P(x)= x^2 -(-\frac{3}{2\sqrt{5}}x) - ½$
$P(x)= 2\sqrt{5}x^2 + 3x - \sqrt{5}$

Using splitting the middle term method,
$2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$
or, $2\sqrt{5}x^2 + (5x - 2x) - \sqrt{5} = 0$
or, $2\sqrt{5}x^2 - 5x + 2x - \sqrt{5} = 0$
or, $\sqrt{5}x (2x + \sqrt{5}) - (2x + \sqrt{5}) = 0$
or, $(2x + \sqrt{5})(\sqrt{5}x - 1) = 0$
or, $x = \frac{1}{\sqrt{5}}, -\frac{\sqrt{5}}{2}$

2. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, a + b, a + 2b for some real numbers a and b, find the values of *a* and *b* as well as the zeroes of the given polynomial.

Solution: Given that a, a + b, a + 2b are roots of given polynomial $x^3 - 6x^2 + 3x + 10$
Sum of the roots ⇒ a + 2b + a + a + b = -coefficient of x^2/ coefficient of x^3
or, 3a + 3b = -(-6)/1 = 6
or, 3(a + b) = 6
or, a + b = 2…………………………………………………………(1)
or, b = 2 – a

Product of roots, (a + 2b)(a + b)a = -constant/coefficient of x^3
or, $(a + b + b)(a + b)a = \frac{-10}{1}$

Substituting the value of a + b = 2 in it,
or, (2 + b)(2)a = -10
or, (2 + b)2a = -10
or, (2 + 2 – a)2a = -10
or, (4 – a)2a = -10
or, $4a - a^2 = -5$
or, $a^2 - 4a - 5 = 0$
or, $a^2 - 5a + a - 5 = 0$
or, (a – 5)(a + 1) = 0

a – 5 = 0 or a + 1 = 0
a = 5 a = -1
a = 5, -1 in (1) a + b = 2

When $a = 5$, $5 + b = 2$, or, $b = (-3)$
$a = -1$, $-1 + b = 2$, or, $b = 3$

∴ If $a = 5$ then $b = (-3)$
or, If $a = -1$ then $b = 3$