Chapter 1: Number Systems
Exercise 1.1 – multiple choice questions (MCQ)

Question 1: Every rational number is
(a) **a natural number** (b) **an integer**
(c) **a real number** (d) **a whole number**

Answer: Option (c) is correct.
Since, real numbers are the combination of rational and irrational numbers, therefore, every rational number is a real number.

Question 2: Between two rational numbers
(a) **there is no rational number**
(b) **there is exactly one rational number**
(c) **there are infinitely many rational numbers**
(d) **there are only rational numbers and no irrational numbers**

Answer: Option (c) is correct.
We know that, between two rational numbers, there are infinitely many rational numbers.

Question 3: Decimal representation of a rational number cannot be
(a) **terminating** (b) **non-terminating**
(c) **non-terminating repeating** (d) **non-terminating non-repeating**

Answer: Option (d) is correct.
Decimal representation of a rational number cannot be non-terminating non-repeating because the decimal expansion of rational number is either terminating or non-terminating recurring.

Question 4: The product of any two irrational numbers is
(a) **always an irrational number** (b) **always a rational number**
(c) **always an integer** (d) **sometimes rational, sometimes irrational**

Answer: Option (d) is correct.
We know that, the product of any two irrational numbers is sometimes rational and sometimes irrational.
e.g., $\sqrt{2} \times \sqrt{2} = 2$ (rational)
and $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ (irrational)

Question 5: The decimal expansion of the number $\sqrt{2}$ is
(a) **a finite decimal** (b) **1.41421**
(c) **non-terminating recurring** (d) **non-terminating non-recurring**

Answer: Option (d) is correct.
As, √2 is an irrational number and also we know that an irrational number is non-terminating non-recurring.

Question 6: Which of the following is irrational?

(a) $\sqrt{\frac{4}{9}}$ **(b)** $\sqrt{\frac{12}{3}}$ **(c)** $\sqrt{7}$ **(d)** $\sqrt{81}$

Answer: Option (c) is the correct answer.

$\sqrt{\frac{4}{9}} = \frac{2}{3}$ (rational)

$\sqrt{\frac{12}{3}}$ = 2 (rational)

$\sqrt{81}$ = 9 (rational)
But $\sqrt{7}$ is an irrational number.

Question 7: Which of the following is irrational?
(a) 0.14 **(b)** $0.14\overline{16}$ **(c)** $0.\overline{1416}$ **(d) 0.4014001400014**

Answer: Option (d) is correct.
We have, 0.14, which is terminating. On the other hand 0.1416 and 0.1416 are non-terminating but recurring.
And 0.4014001400014… is non-terminating non-recurring and is an irrational number.

Question 8: A rational number between √2 and √3 is
(a) $\frac{\sqrt{2}+\sqrt{3}}{2}$ **(b)** $\frac{\sqrt{2}\times\sqrt{3}}{2}$ **(c) 1.5** **(d)1.8**

Answer: Option (c) is correct. (1.5)

Question 9: The value of 1.999… in the form of p/q, where p and q are integers and q≠0, is
(a) $\frac{19}{10}$ **(b)** $\frac{1999}{1000}$ **(c) 2** **(d)** $\frac{1}{9}$

Answer: Option (c) is correct
Let x = 1.999……………………………………………………………………….… (i)
Multiply (i) by 10, we get
10x = 19.999……………………………………………………………….………(ii)
On subtracting (i) from (ii), we get

10x – x = (19.999…) – (1.999…)
or, 9x = 18
or, x = 2

Question 10: 2√3 + √3 is equal to
(a) 2√6 (b) 6 (c) 3√3 (d) 4√6

Answer: Option (c) is correct.
We have 2√3 + √3 = √3(2+1) = 3√3

Question 11: 10 × √15 is equal to
(a) 6√5
(b) 5√6
(c) √25
(d) 10√5

Answer: Option (b) is correct.
We have √10 × √15 = √2.√5 × √3.√5 = (√2.√3) × (√5.√5) = 5√6

Question 12: The number obtained on rationalising the denominator of, $\frac{1}{\sqrt{7}-2}$ is,

(a) $\frac{\sqrt{7}+2}{3}$

(b) $\frac{\sqrt{7}-2}{3}$

(c) $\frac{\sqrt{7}+2}{5}$

(d) $\frac{\sqrt{7}+2}{45}$

Answer: Option (a) is correct.
We have, $\frac{1}{\sqrt{7}-2}$

= $\frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$ …………[By rationalising the denominator]

= $\frac{\sqrt{7}+2}{(\sqrt{7})^2-2^2}$

= $\frac{\sqrt{7}+2}{3}$

Question 13: $\frac{1}{\sqrt{9}-\sqrt{8}}$ **is equals to,**

(a) $\frac{1}{2}(3-2\sqrt{2})$

(b) $\frac{1}{3+2\sqrt{2}}$

(c) $(3-2\sqrt{2})$

(d) $(3+2\sqrt{2})$

Answer: Option (d) is correct.

We have $\frac{1}{\sqrt{9}-\sqrt{8}} = \frac{1}{3-2\sqrt{2}} = \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{3+2\sqrt{2}}{9-\left(2\sqrt{2}\right)^2} = 3+2\sqrt{2}$

Question 14: After rationalising the denominator of $\frac{7}{3\sqrt{3}-2\sqrt{2}}$, we get the denominator as

(a) 13 **(b) 19** **(c) 5** **(d) 35**

Answer: Option (b) is correct.

We have, $\frac{7}{3\sqrt{3}-2\sqrt{2}} = \frac{7}{3\sqrt{3}-2\sqrt{2}} \cdot \frac{3\sqrt{3}+2\sqrt{2}}{3\sqrt{3}+2\sqrt{2}}$

$= \frac{7(3\sqrt{3}+2\sqrt{2})}{(3\sqrt{3})^2-(2\sqrt{2})^2} = \frac{7(3\sqrt{3}+2\sqrt{2})}{27-8} = \frac{7(3\sqrt{3}+2\sqrt{2})}{19}$

Question 15: The value of $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$ is

(a) $\sqrt{2}$ **(b) 2** **(c) 4** **(d) 8**

Answer: Option (b) is correct.

We have, $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}} = \frac{\sqrt{16\times3}+\sqrt{16\times3}}{\sqrt{4\times2}+\sqrt{4\times3}} = \frac{4\sqrt{2}+4\sqrt{3}}{2\sqrt{2}+2\sqrt{3}} = \frac{4(\sqrt{2}+\sqrt{3})}{2(\sqrt{2}+\sqrt{3})} = 2$

Question 16: If $\sqrt{2} = 1.4142$ ..., then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to

(a) 2.4142… **(b) 5.8282…** **(c) 0.4142…** **(d) 0.1718…**

Answer: Option (c) is correct.

We have, $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1}} = \sqrt{\frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-(1)^2}} = \sqrt{\frac{(\sqrt{2}-1)^2}{2-1}} = \sqrt{2}-1 = 0.4142$ ...

Question 17: $\sqrt[4]{\sqrt[3]{2^2}}$ equals to

(a) $2^{-\frac{1}{6}}$ **(b)** 2^{-6} **(c)** $2^{\frac{1}{6}}$ **(d)** 2^{6}

Answer: Option (c) is correct.

We have, $\sqrt[4]{\sqrt[3]{2^2}} = \left[\sqrt[3]{2^2}\right]^{\frac{1}{4}} = \left[[2^2]^{\frac{1}{3}}\right]^{\frac{1}{4}} = \left[[2]^{\frac{2}{3}}\right]^{\frac{1}{4}} = [2]^{\frac{1}{6}}$

Question 18: The product $\sqrt[3]{2} \times \sqrt[4]{2} \times \sqrt[12]{32}$ equals to

(a) $\sqrt{2}$ **(b) 2** **(c)** $\sqrt[12]{2}$ **(d)** $\sqrt[12]{32}$

Answer: Option (b) is correct.

We have , LCM of the three irrational numbers 3, 4 and 12 = 12

Therefore, $\sqrt[3]{2} = \sqrt[12]{2^4}$

$\sqrt[4]{2} = \sqrt[12]{2^3}$

$\sqrt[12]{32} = \sqrt[12]{2^5}$

Therefore the product = $\sqrt[3]{2} \times \sqrt[4]{2} \times \sqrt[12]{32} = \sqrt[12]{2^4} \times \sqrt[12]{2^3} \times \sqrt[12]{2^5}$

$= \sqrt[12]{2^{4+3+5}} = \sqrt[12]{2^{12}} = 2^{12 \times \frac{1}{12}} = 2$

Question 19: The value of $\sqrt[4]{81^{-2}}$ is

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$ (c) 9 (d) $\frac{1}{81}$

Answer: Option (a) is correct.

We have $\sqrt[4]{\frac{1}{(81)^2}} = \frac{1}{(81)^{\frac{2}{4}}} = \frac{1}{(81)^{\frac{1}{2}}} = \frac{1}{(9^2)^{\frac{1}{2}}} = \frac{1}{9^{\frac{2}{2}}} = \frac{1}{9}$

Question 20: The value of $(256)^{0.16} \times (256)^{0.09}$ is

(a) 4 (b) 16 (c) 64 (d) 256.25

Answer: Option (a) is correct.

We have $(256)^{0.16} \times (256)^{0.09} = (256)^{\frac{16}{100}} \times (256)^{\frac{9}{100}} = (256)^{\frac{16}{100}+\frac{9}{100}} = (256)^{\frac{25}{100}} = (256)^{\frac{1}{4}}$

$= [(4)^4]^{\frac{1}{4}} = 4$

Question 21: Which of the following is equal to x?

(a) $x^{\frac{12}{7}} - x^{\frac{5}{7}}$ (b) $\sqrt[12]{(x^4)^{\frac{1}{3}}}$ (c) $\left(\sqrt{x^3}\right)^{\frac{2}{3}}$ (d) $x^{\frac{12}{7}} \times x^{\frac{7}{12}}$

Answer: (a) $x^{\frac{12}{7}} - x^{\frac{5}{7}} = x^{\frac{5}{7}+1} - x^{\frac{5}{7}} = x^{\frac{5}{7}}.x - x^{\frac{5}{7}} \neq x$

(b) $\sqrt[12]{(x^4)^{\frac{1}{3}}} = ((x^4)^{\frac{1}{3}})^{\frac{1}{12}} = x^{4 \times \frac{1}{3} \times \frac{1}{12}} = x^{\frac{1}{9}} \neq x$

(c) $\left(\sqrt{x^3}\right)^{\frac{2}{3}} = \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = x^{\frac{3}{2} \times \frac{2}{3}} = x$

(d) $x^{\frac{12}{7}} \times x^{\frac{7}{12}} = x^{\frac{12}{7}+\frac{7}{12}} = x^{\frac{144+49}{84}} = x^{\frac{193}{84}} \neq x$

Hence (c) is the correct option.

Exercise 1.2 Short Answer Type Questions

Question 1: Let x and y be rational and irrational numbers, respectively. Is x+y necessarily an irrational number? Give an example in support of your answer.

Answer: Yes, (x + y) is necessarily an irrational number.

Let us take, x = 2, and y = $\sqrt{2}$.
Then x + y = $2+\sqrt{2}$

Let , (x + y) = a be rational .
Then, $a^2 = \left(2+\sqrt{2}\right)^2$
or, $a^2 = 2^2 + \left(\sqrt{2}\right)^2 + 2(2)\left(\sqrt{2}\right)$
or, $a^2 = 4 + 2 + 4\sqrt{2}$
or, $a^2 = 6 + 4\sqrt{2}$
or, $\frac{a^2-6}{4} = \sqrt{2}$
Therefore, a is rational implies that $\frac{a^2-6}{4}$ is rational, which in turn implies that $\sqrt{2}$ is rational.
This is a contradiction.
Hence, (x + y) is irrational.

Question 2: Let x be rational and y be irrational. Is xy necessarily irrational? Justify your answer by an example.

Answer: No, (xy) is necessarily an irrational only when x ≠0.
Let x be a non-zero rational and y be an irrational.

If possible, let xy be a rational number.
Since, quotient of two non-zero rational number is a rational number.
Hence, $\left(\frac{xy}{x}\right)$ is a rational number implies that y is a rational number.
But, this contradicts the fact that y is an irrational number.
Hence our supposition is wrong.
Hence, xy is an irrational number.
But, when x = 0, then xy = 0, a rational number.

Question 3: State whether the following statements are true or false? Justify your answer.

- **(i)** $\frac{\sqrt{2}}{3}$ **is a rational number.**
- **(ii)** **There are infinitely many integers between any two integers**
- **(iii)** **Number of rational numbers between 15 and 18 is finite.**
- **(iv)** **There are numbers which cannot be written in the form** $\frac{p}{q}$ **,** $q \neq 0$, **p and q are both integers.**
- **(v)** **The square of an irrational number is always rational.**
- **(vi)** $\frac{\sqrt{12}}{\sqrt{3}}$ **is not a rational number as** $\sqrt{12}$ **and** $\sqrt{3}$ **are not integers.**
- **(vii)** $\frac{\sqrt{15}}{\sqrt{3}}$ **is written in the form** $\frac{p}{q}$ **,** $q \neq 0$ **and so it is a rational number.**

Answer: (i) False, here √2 is an irrational number and 3 is a rational number, we know that on dividing an irrational number by non-zero rational number we get an irrational number.
(ii) False, because between two consecutive integers (like 1 and 2), there does not exist any other integer.
(iii) False, because between any two rational numbers there exist infinitely many rational numbers.

(iv) True, because there are infinitely many numbers which cannot be written in the form p/q, q ≠0, p,q both are integers, and these numbers are termed irrational numbers.

(v) False, let an irrational number be $\sqrt{2}$ and $\sqrt[4]{2}$

a) $\left(\sqrt{2}\right)^2 = 2$, is a rational number.

b) $\left(\sqrt[4]{2}\right)^2 = \sqrt{2}$, is not a rational number.

Hence the square of an irrational number is not always a rational number.

(vi) False, $\frac{\sqrt{12}}{\sqrt{3}} = \frac{\sqrt{4\times3}}{\sqrt{3}} = \frac{\sqrt{4}\times\sqrt{3}}{\sqrt{3}} = 2 \times 1 = 2$, is a rational number.

(vii) False, $\frac{\sqrt{15}}{\sqrt{3}} = \frac{\sqrt{5\times3}}{\sqrt{3}} = \frac{\sqrt{5}\times\sqrt{3}}{\sqrt{3}} = \sqrt{5}$, is an irrational number.

Question 4: Classify the following numbers as rational or irrational with justification.

(i) $\sqrt{196}$ **(ii)** $3\sqrt{18}$ **(iii)** $\sqrt{\frac{9}{27}}$

(iv) $\frac{\sqrt{28}}{\sqrt{343}}$ **(v)** $-\sqrt{0.4}$ **(vi)** $\frac{\sqrt{12}}{\sqrt{75}}$

(vii) 0.5918 **(viii)** $\left(1+\sqrt{5}\right) - (4+\sqrt{5})$

(ix) 10.124124… **(x) 1.010010001…**

Answer: (i) $\sqrt{196} = \sqrt{(14)^2} = 14$

Hence it is a rational number.

(ii) $3\sqrt{18} = 3\sqrt{(3)^2 \times 2} = 3 \times 3\sqrt{2} = 9\sqrt{2}$

Hence it is a irrational number.

(iii) $\sqrt{\frac{9}{27}} = \sqrt{\frac{9}{9\times3}} = \frac{1}{\sqrt{3}}$

Hence it is an irrational number.

(iv) $\frac{\sqrt{28}}{\sqrt{343}} = \frac{\sqrt{2\times2\times7}}{\sqrt{7\times7\times7}} = \frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$

Hence, it is a rational number

(v) - $\sqrt{0.4}$ = - $\sqrt{\frac{4}{10}}$ = - $\frac{2}{\sqrt{10}}$

Hence, quotient of rational and irrational numbers is an irrational number.

(vi) $\frac{\sqrt{12}}{\sqrt{75}} = \frac{\sqrt{4\times3}}{\sqrt{25\times3}} = \frac{2}{5}$

Hence, it is a rational number.

(vii) 0.5918 is a number with terminating decimal so it can be written in the form of $\frac{p}{q}$, where q ≠ 0, p and q are integers. Hence, it is a rational number.

(viii) $\left(1+\sqrt{5}\right) - (4+\sqrt{5})$ = 1 – 4 + $\sqrt{5}$ - $\sqrt{5}$ = -3

Hence it is a rational number.

(ix) 10.124124………., is a number with non-terminating recurring decimal expansion. Hence, it is a rational number.

(x) 1.010010001…………, is a number with non-terminating non-recurring decimal expansion. Hence, it is an irrational number.

Exercise 1.3 (Short answer type question)

Question 1: Find which of the variables x, y, z and u represent rational numbers and which irrational numbers.

(i) $\mathbf{x^2 = 5}$

(ii) $\mathbf{y^2 = 9}$

(iii) $\mathbf{z^2 = 0.04}$

(iv) $\mathbf{u^2 = \frac{17}{4}}$

Answer: (i) Given, $x^2 = 5$

On taking square root both sides we get, $x = \pm\sqrt{5}$………[irrational number]

(ii) Given, $y^2 = 9$

On taking square root both sides we get, $x = \pm 3$………[rational number]

(iii) Given, $z^2 = 0.04$

On taking square root both sides we get,

$z = \sqrt{0.04} = \sqrt{\frac{4}{100}} = \frac{2}{10}$ …………………..[rational number]

(iv) Given, $u^2 = \frac{17}{4}$

On taking square root both sides we get,

$u = \pm\sqrt{\frac{17}{4}} = \pm \frac{\sqrt{17}}{2}$ ………………………..[irrational number as numerator is irrational]

Question 2: Find three rational numbers between

(i) **-1 and -2** (ii) **0.1 and 0.11**

(iii) **5/7 and 6/7** (iv) **1/4 and 1/5**

Answer: (i) Let = -1 and x = -2

Here, x < y.

Since, $d = \frac{y-x}{n+1} = \frac{-1+2}{3+1} = \frac{1}{4}$

Three rational numbers between x and y are x + d, x + 2d and x + 3d.

Now, $x + d = -2 + \frac{1}{4} = \frac{-8+1}{4} = \frac{-7}{4}$

$X + 2d = -2 + \frac{2}{4} = \frac{-8+2}{4} = \frac{-3}{2}$

$X + 3d = -2 + \frac{3}{4} = \frac{-8+3}{4} = \frac{-5}{4}$

Hence, three rational numbers are $\frac{-7}{4}, \frac{-3}{2}, \frac{-5}{4}$

(ii) Let x = 0.1 and y = 0.11

Here x < y.

Since, d = $\frac{y-x}{n+1} = \frac{0.11-0.1}{3+1} = \frac{0.01}{4}$

Three rational numbers between x and y are x + d, x + 2d and x + 3d.

Therefore, x + d = $0.1 + \frac{0.01}{4} = \frac{0.4+0.01}{4} = \frac{0.41}{4} = 0.1025$

x + 2d = $0.1 + \frac{0.02}{4} = \frac{0.4+0.02}{4} = \frac{0.42}{4} = 0.105$

x + 3d = $0.1 + \frac{0.03}{4} = \frac{0.4+0.03}{4} = \frac{0.43}{4} = 0.1075$

Hence the three rational numbers : 0.1025 , 0.105 , 0.1075

(iii) Let x = $\frac{5}{7}$, and y = $\frac{6}{7}$

Here, x < y.

we have d = $\frac{y-x}{n+1} = \frac{\frac{6}{7}-\frac{5}{7}}{3+1} = \frac{\frac{1}{7}}{4} = \frac{1}{28}$

The three rational numbers between x and y are x + d, x + 2d and x + 3d.

Therefore, x + d = $\frac{5}{7} + \frac{1}{28} = \frac{20+1}{28} = \frac{21}{28}$

x + 2d = $\frac{5}{7} + \frac{2}{28} = \frac{20+2}{28} = \frac{22}{28}$

x + 3d = $\frac{5}{7} + \frac{3}{28} = \frac{20+3}{28} = \frac{23}{28}$

Hence the three rational numbers are $\frac{21}{28}, \frac{22}{28}, \frac{23}{28}$

(iv) Three rational numbers between $\frac{1}{4}$ and $\frac{1}{5}$ are $\frac{9}{40}, \frac{19}{80}$ and $\frac{17}{80}$

Question 3: Insert a rational number and an irrational number between the following

(i) 2 and 3

(ii) 0 and 0.1

(iii) 1/3 and ½

(iv) -2/5 and -1/2

(v) 0.15 and 0.16

(vi) √2 and √3

(vii) 2.357 and 3.121

(viii) .0001 and .001

(ix) 3.623623 and 0.484848

(x) 3.375289 and 6.375738

Answer: (i) A rational number between 2 and 3 is 2.1.

To find an irrational number between 2 and 3. Find a number which is non-terminating non-recurring lying between them.

Such number will be 2.040040004.............

(ii) A rational number between 0 and 0.1 is 0.03. An irrational number between 0 and 0.1 is 0.007000700007..........

(iii) A rational number between 1/3 and 1/2 is 5/12. An irrational number between 1/3 and 1/2 i.e., between 0-3 and 0.5 is 0.4141141114............

(iv) A rational number between -2/5 and 1/2 is 0. An irrational number between -2/5 and ½ i.e., between – 0.4 and 0.5 is 0.151151115...........

(v) A rational number between 0.15 and 0.16 is 0.151. An irrational number between 0.15 and 0.16 is 0.1515515551.......

(vi) A rational number between √2 and √3 i.e.,, between 1.4142...... and 1.7320...... is 1.5. An irrational number between √2 and √3 is 1.585585558..........

(vii) A rational number between 2.357 and 3.121 is 3. An irrational number between 2.357 and 3.121 is 3.101101110........

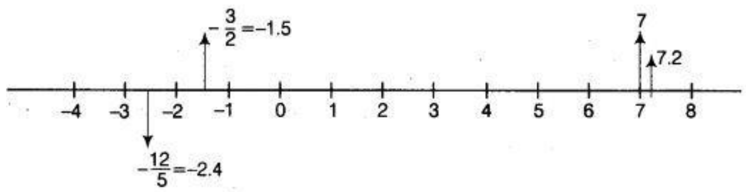
(viii) A rational number between 0.0001 and 0.001 is 0.00011. An irrational number between 0.0001 and 0.001 is 0.0001131331333...........

(ix) A rational number between 3.623623 and 0.484848 is 1. An irrational number between 3.623623 and 0.484848 is 1.909009000..........

(x) A rational number between 6.375289 and 6.375738 is 6.3753. An irrational number between 6.375289 and 6.375738 is 6.375414114111.........

Question 4: Represent the following numbers on the number line 7, 7.2, -3/2 and -12/5

Answer: Firstly, we draw a number line whose mid-point is 0. Marks a positive numbers on right hand side of 0 and negative numbers on left hand side of 0.



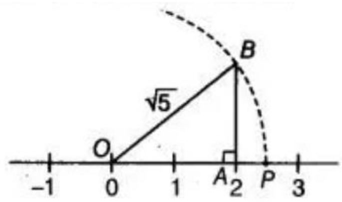
Step 1) Number 7 is a positive number. So we mark a number 7 on the right hand side of 0, which is a 7 units distance from zero.
Step 2) Number 7.2 is a positive number. So, we mark a number 7.2 on the right hand side of 0, which is a 7.2 units distance from zero.
Step 3) Number -3/2 or -1.5 is a negative number. So, we mark a number 1.5 on the left hand side of 0, which is a 1.5 units distance from zero.
Step 4) Number – 12/5 or -2.4 is a negative number. So, we mark a number 2.4 on the left hand side of 0, which is a 2.4 units distance from zero.

Question 5: Locate √5 on the number line.

Answer: Here, 5 = $2^2 + 1^2$
So, a right angled triangle ΔOAB in which OA = 2 units and AB = 1 unit, and $\angle OAB$ =90^0.
By Pythagoras theorem we get,

OB = $\sqrt{OA^2 + AB^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$.

Taking OB = $\sqrt{5}$ as radius and point O as centre draw an arc which meets the number line at point P on the positive side of it.
The point P represents $\sqrt{5}$ on the number line.



Question 7: Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) 0.2 **(ii) 0.888…** **(iii) $5.\overline{2}$**
(iv) $0.\overline{001}$ **(v) 0.2555…** **(vi) $0.1\overline{34}$**
(vii) 0.00323232… **(viii) 0.404040…**

Answer: (i) $0.2 = \frac{2}{10} = \frac{1}{5}$
(ii) Let x = 0.888……………………………………………..…..(1)
On multiplying both sides by 10 we get,
10x = 8.8888………………………………………………………(2)

Subtracting (1) from (2), we get -
10x – x = (8.88…) – (0.88…)
or, 9x = 8
or, $x = \frac{8}{9}$

(iii) $5.\overline{2} = \frac{47}{9}$
(iv) $0.\overline{001} = \frac{1}{999}$
(v) 0.2555… $= \frac{23}{90}$
(vi) $0.1\overline{34} = \frac{133}{990}$
(vii) 0.00323232…$= \frac{8}{2475}$
(viii) 0.404040… $= \frac{40}{99}$

Question 8: Show that 0.142857142857… = 1/7.

Answer: Let x = 0.142857142857 …………………..(i)
On multiplying both sides of eq. (i) by 1000000, we get
1000000 x = 142857.142857…………………………(ii)

On subtracting eq. (i) from eq. (ii), we get
1000000 x – x = (142857.142857…) – (0.142857..)
or, 999999 x = 142857
Therefore, $x = \frac{142857}{999999} = \frac{1}{7}$ (Hence proved)

Question 9: Simplify the following:
(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$
(ii) $\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$

(iii) $\sqrt[4]{12} \times \sqrt[7]{6}$

(iv) $\mathbf{4\sqrt{28}+3\sqrt{7}+\sqrt[3]{7}}$

(v) $\mathbf{3\sqrt{3}+2\sqrt{27}+\frac{7}{\sqrt{3}}}$

(vi) $\mathbf{\left(\sqrt{3}-\sqrt{2}\right)^2}$

(vii) $\mathbf{\sqrt[4]{81}-8\sqrt[3]{216}+15\sqrt[5]{32}+\sqrt{225}}$

(viii) $\mathbf{\frac{3}{\sqrt{8}}+\frac{1}{\sqrt{2}}}$

(ix) $\mathbf{\frac{2\sqrt{3}}{3}-\frac{\sqrt{3}}{6}}$

Answer: (i) $\sqrt{45}$ - $3\sqrt{20}+4\sqrt{5}$
= $\sqrt{3\times3\times5}-\ 3\ \sqrt{5\times2\times2}\ +4\sqrt{5}$
= $3\sqrt{5}-\ 6\sqrt{5}$ + $4\sqrt{5}$
= $\sqrt{5}$

(ii) $\frac{\sqrt{24}}{8}+\ \frac{\sqrt{54}}{9}$
= $\frac{\sqrt{2\times2\times2\times3}}{8}+\frac{\sqrt{3\times3\times3\times2}}{9}$

= $\frac{2\sqrt{6}}{8}+\frac{3\sqrt{6}}{9}$
= $\frac{\sqrt{6}}{4}+\frac{\sqrt{6}}{3}$
= $\frac{3\sqrt{6}+4\sqrt{6}}{12}$
= $\frac{7\sqrt{6}}{12}$

(iii) $\sqrt[4]{12}\times\sqrt[7]{6}$
= $(12)^{\frac{1}{4}}\times\ (6)^{\frac{1}{7}}$
= $(2\times2\times3)^{\frac{1}{4}}\times(2\times3)^{\frac{1}{7}}$

= $2^{\frac{1}{4}}\times2^{\frac{1}{4}}\times3^{\frac{1}{4}}\ \times2^{\frac{1}{7}}\times3^{\frac{1}{7}}$
= $2^{\frac{1}{4}+\frac{1}{4}+\frac{1}{7}}\times3^{\frac{1}{4}+\frac{1}{7}}$
= $2^{\frac{18}{28}}\times3^{\frac{11}{28}}$
= $2^{\frac{9}{14}}\times3^{\frac{11}{28}}$

= $\sqrt[14]{2^9}\times\sqrt[28]{3^{11}}$
= $\sqrt[28]{2^{18}}\times\sqrt[28]{3^{11}}$ ……………………….[$\sqrt[n]{a}$ = $\sqrt[mn]{a^m}$]

= $\sqrt[28]{2^{18}\times3^{11}}$

(iv) $4\sqrt{28}+3\sqrt{7}+\sqrt[3]{7}$ $=\frac{8}{3\sqrt[3]{7}}$

(v) $3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}} = \frac{34\sqrt{3}}{3}$

(vi) $\left(\sqrt{3} - \sqrt{2}\right)^2 = 5 - 2\sqrt{6}$

(vii) $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225} = 0$

(viii) $\frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}} = \frac{5\sqrt{2}}{4}$

(ix) $\frac{2\sqrt{3}}{3} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$

Question 10: Rationalise the denominator:

(i) $\frac{2}{3\sqrt{3}}$

(ii) $\frac{\sqrt{40}}{\sqrt{3}}$

(iii) $\frac{3+\sqrt{2}}{4\sqrt{2}}$

(iv) $\frac{16}{\sqrt{41}-5}$

(v) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$

(vi) $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

(vii) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

(viii) $\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

(ix) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

Answer: (i) $\frac{2}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{9}$

(ii) $\frac{\sqrt{40}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{120}}{3} = \frac{\sqrt{2\times2\times2\times5\times3}}{3} = \frac{2}{3}\sqrt{30}$

(iii) $\frac{3+\sqrt{2}}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}+2}{8}$

(iv) $\frac{16}{\sqrt{41}-5} \times \frac{\sqrt{41}+5}{\sqrt{41}+5} = \frac{16(\sqrt{41}+5)}{\left(\sqrt{41}\right)^2-5^2} = \frac{16(\sqrt{41}+5)}{16} = (\sqrt{41} + 5)$

(v) $\frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{\left(2+\sqrt{3}\right)^2}{2^2-\left(\sqrt{3}\right)^2} = \frac{4+3+4\sqrt{3}}{4-3} = 7 + 4\sqrt{3}$

(vi) $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{\sqrt{2}^2-\sqrt{3}^2} = \sqrt{6}(\sqrt{2} - \sqrt{3})$

(vii) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{\left(\sqrt{3}+\sqrt{2}\right)^2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2} = \left(\sqrt{3} + \sqrt{2}\right)^2 = 3 + 2 + 2\sqrt{6} = 5 + 2\sqrt{6}$

(viii) $\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 9 + 2\sqrt{15}$

(ix) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} = \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}} = \frac{9+4\sqrt{6}}{15}$

Question 11: Find the values of a and b in each of the following:

(i) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a - 6\sqrt{3}$

(ii) $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5} - \frac{19}{11}$

(iii) $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = 2 - b\sqrt{6}$

(iv) $\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = a + \frac{7}{11}\sqrt{5}b$

Answer: (i) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = a - 6\sqrt{3}$

or, $\frac{5(7-4\sqrt{3})+2\sqrt{3}(7-4\sqrt{3})}{(7)^2-(4\sqrt{3})^2} = a - 6\sqrt{3}$

or, $\frac{35-20\sqrt{3}+14\sqrt{3}-24}{49-48} = a - 6\sqrt{3}$

or, $11 - 6\sqrt{3} = a - 6\sqrt{3}$

or, a = 11

(ii) $\frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}} = a\sqrt{5} - \frac{19}{11}$

Or, $\frac{3(3-2\sqrt{5})-\sqrt{5}(3-2\sqrt{5})}{3^2-2\sqrt{5}^2} = a\sqrt{5} - \frac{19}{11}$

Or, $\frac{9-6\sqrt{5}-3\sqrt{5}+10}{9-4\times5} = a\sqrt{5} - \frac{19}{11}$

Or, $\frac{19-9\sqrt{5}}{-11} = a\sqrt{5} - \frac{19}{11}$

Or, $\frac{9\sqrt{5}}{11} - \frac{19}{11} = a\sqrt{5} - \frac{19}{11}$

Or, a = $\frac{9}{11}$

(iii) b = $-\frac{5}{6}$

(iv) a = 0
b = 1

Question 12: If a = 2 + $\sqrt{3}$, then find the value of $\left(a - \frac{1}{a}\right)$

Answer: We have, a = 2 + $\sqrt{3}$

Then, $\frac{1}{a} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

= $\frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2}$

= 2 – $\sqrt{3}$

Hence, $a - \frac{1}{a} = (2+\ \sqrt{3}) - (2-\ \sqrt{3}) = 2+\sqrt{3}-2+\sqrt{3} = 2\sqrt{3}$

Question 13: Rationalise the denominator, taking $\sqrt{2} = 1.414, \sqrt{3} = 1.732, \sqrt{5} = 2.236$

(i) $\frac{4}{\sqrt{3}}$

(ii) $\frac{6}{\sqrt{6}}$

(iii) $\frac{\sqrt{10}-\sqrt{5}}{2}$

(iv) $\frac{\sqrt{2}}{2+\sqrt{2}}$

(v) $\frac{1}{\sqrt{3}+\sqrt{2}}$

Answer: (i) $\frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3} = \frac{4}{3} \times 1.732 = 2.309$

(ii) $\frac{6}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{6\sqrt{6}}{6} = \sqrt{2} \times \sqrt{3}$ = 1.414 × 1.732 = 2.449

(iii) $\frac{\sqrt{10}-\sqrt{5}}{2} = 0.46285\ \cong 0.463$

(iv) $\frac{\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{\sqrt{2}(2-\sqrt{2})}{2^2-\sqrt{2}^2} = \frac{2(\sqrt{2}-1)}{2} = \sqrt{2}-1 = 1.414-1 = 0.414$

(v) $\frac{1}{\sqrt{3}+\sqrt{2}} = 0.318$

Question 14: Simplify:

(i) $(1^3+2^3+3^3)^{\frac{1}{2}}$

(ii) $\left(\frac{3}{5}\right)^4 \left(\frac{8}{5}\right)^{-12} \left(\frac{32}{5}\right)^6$

(iii) $\left(\frac{1}{27}\right)^{-\frac{2}{3}}$

(iv) $\left[\left(625^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right]^2$

(v) $\frac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}}$

(vi) $\mathbf{64^{-\frac{1}{3}}\left[64^{\frac{1}{3}} - 64^{\frac{2}{3}}\right]}$

(vii) $\mathbf{\dfrac{8^{\frac{1}{3}} \times 16^{\frac{1}{3}}}{32^{-\frac{1}{3}}}}$

Answer: (i) $(1^3 + 2^3 + 3^3)^{\frac{1}{2}} = (1 + 8 + 27)^{\frac{1}{2}}$ = $(36)^{\frac{1}{2}}$ = 6

(ii) $\left(\frac{3}{5}\right)^4 \left(\frac{8}{5}\right)^{-12} \left(\frac{32}{5}\right)^6$ = $\left(\frac{3}{5}\right)^4 \left(\frac{5}{8}\right)^{12} \left(\frac{32}{5}\right)^6$ = $\frac{2025}{64}$

(iii) $\left(\frac{1}{27}\right)^{-\frac{2}{3}}$ = $\left(\frac{1}{3^3}\right)^{-\frac{2}{3}}$ = $(3^{-3})^{-\frac{2}{3}}$ = 9

(iv) $\left[\left(625^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right]^2$ = 5

(v) $\dfrac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}}$ = $3^{-\frac{1}{3}}$

(vi) $64^{-\frac{1}{3}}\left[64^{\frac{1}{3}} - 64^{\frac{2}{3}}\right]$ = -3

(vii) $\dfrac{8^{\frac{1}{3}} \times 16^{\frac{1}{3}}}{32^{-\frac{1}{3}}}$ = 16

<u>Exercise 1.4 – Long Answer type questions</u>

Question 1: Express 0.6 + $0.\overline{7} + 0.4\overline{7}$ in the form of $\frac{p}{q}$ where p and q are integers and q$\neq 0$.

Answer: Let, x = $0.\overline{7}$ = 0.7777….. ………………………..……….(1)
Multiplying 10 on both sides,
10x = 7.77…….. ……..……………………………………….(2)

On subtracting eq.(1) and eq.(2) we get,
10x – x = 7.777…. – 0.7777….
or, 9x = 7
or, x = $\frac{7}{9}$

Now, let y = $0.4\overline{7}$ = 0.4777…. ………………………………….(3)
Multiplying 10 on both sides we get,
10y = 4.7777….. ………………………………………………(4)
Multiplying 10 on both sides we get,
100y = 47.777…. ……………………………………………………..(5)

On subtracting eq.(4) from eq.(5), we get,
(100y – 10y) = (47.777…) – (4.777…)

or, 90y = 43

or, y = $\frac{43}{90}$

Thus, $0.6 + 0.\overline{7} + 0.4\overline{7} = \frac{6}{10} + \frac{7}{9} + \frac{43}{90} = \frac{54+70+43}{90} = \frac{167}{90}$

Question 2: Simplify:

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

Answer:

$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$ …………………………………….(1)

$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} \times \frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}} = \frac{7\sqrt{30}-21}{(\sqrt{10})^2-(\sqrt{3})^2} = \sqrt{30} - 3$

$\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \frac{2\sqrt{30}-10}{(\sqrt{6})^2-(\sqrt{5})^2} = 2\sqrt{30} - 10$

$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \times \frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}} = \frac{3\sqrt{30}-18}{(\sqrt{15})^2-(3\sqrt{2})^2} = (-\sqrt{30} + 6) = 6 - \sqrt{30}$

Now, from eq.(1),

$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

$= (\sqrt{30} - 3) - (2\sqrt{30} - 10) - (6 - \sqrt{30})$

$= \sqrt{30} - 3 - 2\sqrt{30} + 10 - 6 + \sqrt{30}$

= 1

Question 3: If $\sqrt{2} = 1.414$ and $\sqrt{3}$ = 1.732, the find the value of,

$$\frac{4}{3\sqrt{3}-2\sqrt{2}} + \frac{3}{3\sqrt{3}+2\sqrt{2}}$$

Answer: By doing L.C.M. we get, 2.06316 ≈ 2.063

Question 4: If a = $\frac{3+\sqrt{5}}{2}$, then find the value of $a^2 + \frac{1}{a^2}$

Answer: Given, a = $\frac{3+\sqrt{5}}{2}$ ……………………………………….(1)

Hence, $\frac{1}{a} = \frac{2}{3+\sqrt{5}}$

$= \frac{2}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$

$= \frac{6-2\sqrt{5}}{3^2-(\sqrt{5})^2}$

$= \frac{6-2\sqrt{5}}{4}$

$= \frac{3-\sqrt{5}}{2}$ ……………………………………………………..(2)

$a^2 + \frac{1}{a^2} = a^2 + \frac{1}{a^2} + 2 - 2 = \left(a + \frac{1}{a}\right)^2 - 2$

From eq(1) and eq(2),
$\left(\frac{3+\sqrt{5}}{2} + \frac{3-\sqrt{5}}{2}\right)^2 - 2 = \left(\frac{6}{2}\right)^2 - 2 = 3^2 - 2 = 9 - 2 = 7$

Question 5: If x = $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and y = $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find the value of $x^2 + y^2$ = ?

Answer: x = $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} = 3 + 2 + 2\sqrt{6}$ ……………………………….(1)

On squaring both sides, we get,
$x^2 = (5 + 2\sqrt{6})^2$
or, $x^2 = 49 + 20\sqrt{6}$ ……………………………………………..(2)

Therefore, y = $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{1}{x} = \frac{1}{5+2\sqrt{6}} = \frac{1}{5+2\sqrt{6}} \times \frac{5-2\sqrt{6}}{5-2\sqrt{6}} = \frac{5-2\sqrt{6}}{5^2-(2\sqrt{6})^2} = 5 - 2\sqrt{6}$

On squaring both sides, we get,
$y^2 = (5 - 2\sqrt{6})^2$
or, $y^2 = 49 - 20\sqrt{6}$ ……………………………………………………………(3)

On adding eq.(2) and eq.(3), we get,
$x^2 + y^2 = 49 + 20\sqrt{6} + 49 - 20\sqrt{6} = 98$

Question 6: Simplify $(256)^{-\left(4^{-\frac{3}{2}}\right)}$

Answer: $(256)^{-\left(4^{-\frac{3}{2}}\right)}$

$= (256)^{(-4)^{-\frac{3}{2}}}$

$= (256)^{-\left(2^{2\times-\frac{3}{2}}\right)}$

$= (256)^{-(2^{-3})}$

$= (2^8)^{-\left(\frac{1}{2^3}\right)}$

$= (2^8)^{-\frac{1}{8}}$

$= 2^{-1}$
$= \frac{1}{2}$

Question 7: Find the value of

$$\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$$

Answer: $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$

$= \frac{4}{(6^3)^{-\frac{2}{3}}} + \frac{1}{(16^2)^{-\frac{3}{4}}} + \frac{2}{(3^5)^{-\frac{1}{5}}}$

$= \frac{4}{(6)^{3\times(-\frac{2}{3})}} + \frac{1}{(16)^{2\times(-\frac{3}{4})}} + \frac{2}{(3)^{5\times(-\frac{1}{5})}}$

$= \frac{4}{6^{-2}} + \frac{1}{16^{-\frac{3}{2}}} + \frac{2}{3^{-1}}$

$= 4 \times 6^2 + 16^{\frac{3}{2}} + 2 \times 3^1$

= 144 + 64+ 6

= 214