Chapter 8: Quadrilaterals

1. What is a quadrilateral? Mention 6 types of quadrilaterals.

Solution:

A quadrilateral is a 4 sided polygon having a closed shape. It is a 2-dimensional shape.

The 6 types of quadrilaterals include:

- Rectangle
- Square
- Parallelogram
- Rhombus
- Trapezium
- Kite

2. The diagonals of which quadrilateral are equal and bisect each other at 90°?

Solution:

Square. The diagonals of a square are equal and bisect each other at 90°.

3. Identify the type of quadrilaterals:

(i) The quadrilateral formed by joining the midpoints of consecutive sides of a quadrilateral whose diagonals are perpendicular.

(ii) The quadrilateral formed by joining the midpoints of consecutive sides of a quadrilateral whose diagonals are congruent.

Solution:

(i) The quadrilateral formed by joining the midpoints of consecutive sides of a quadrilateral whose diagonals are perpendicular is a **rectangle**.

(ii) The quadrilateral formed by joining the midpoints of consecutive sides of a quadrilateral whose diagonals are congruent is a **rhombus**.

4. Find all the angles of a parallelogram if one angle is 80°.

Solution:

For a parallelogram ABCD, opposite angles are equal.

So, the angles opposite to the given 80° angle will also be 80°.

It is also known that the sum of angles of any quadrilateral = 360°.

So, if $\angle A = \angle C = 80°$ then,

$\angle A + \angle B + \angle C + \angle D = 360°$

Also, $\angle B = \angle D$

Thus,

$80° + \angle B + 80° + \angle D = 360°$

Or, $\angle B + \angle D = 200°$

Hence, $\angle B = \angle D = 100°$

Now, all angles of the quadrilateral are found which are:

$\angle A = 80°$

∠B = 100°

∠C = 80°

∠D = 100°

5. In a rectangle, one diagonal is inclined to one of its sides at 25°. Measure the acute angle between the two diagonals.

Solution:

Let ABCD be a rectangle where AC and BD are the two diagonals which are intersecting at point O.

Now, assume ∠BDC = 25° (given)

Now, ∠BDA = 90° – 25° = 65°

Also, ∠DAC = ∠BDA, (as diagonals of a rectangle divide the rectangle into two congruent right triangles)

So, ∠BOA = the acute angle between the two diagonals = 180° – 65° – 65° = 50°

6. Is it possible to draw a quadrilateral whose all angles are obtuse angles?

Solution:

It is known that the sum of angles of a quadrilateral is always 360°. To have all angles as obtuse, the angles of the quadrilateral will be greater than 360°. So, it is not possible to draw a quadrilateral whose all angles are obtuse angles.

7. Prove that the angle bisectors of a parallelogram form a rectangle.

Solution:

LMNO is a parallelogram in which bisectors of the angles L, M, N, and O intersect at P, Q, R and S to form the quadrilateral PQRS.
LM || NO (opposite sides of parallelogram LMNO)
L + M = 180 (sum of consecutive interior angles is 180o)
MLS + LMS = 90
In LMS, MLS + LMS + LSM = 180
90 + LSM = 180
LSM = 90
RSP = 90 (vertically opposite angles)
SRQ = 90, RQP = 90 and SPQ = 90
Therefore, PQRS is a rectangle.

8. In a trapezium ABCD, AB∥CD. Calculate ∠C and ∠D if ∠A = 55° and ∠B = 70°

Solution:

In a trapezium ABCD, ∠A + ∠D = 180° and ∠B + ∠C = 180°

So, 55° + ∠D = 180°

Or, ∠D = 125°

Similarly,

70° + ∠C = 180°

Or, ∠C = 110°

9. Calculate all the angles of a parallelogram if one of its angles is twice its adjacent angle.

Solution:

Let the angle of the parallelogram given in the question statement be “x”.

Now, its adjacent angle will be 2x.

It is known that the opposite angles of a parallelogram are equal.

So, all the angles of a parallelogram will be x, 2x, x, and 2x

As the sum of interior angles of a parallelogram = 360°,

$x + 2x + x + 2x = 360°$

Or, $x = 60°$

Thus, all the angles will be 60°, 120°, 60°, and 120°.

10. Calculate all the angles of a quadrilateral if they are in the ratio 2:5:4:1.

Solution:

As the angles are in the ratio 2:5:4:1, they can be written as-

2x, 5x, 4x, and x

Now, as the sum of the angles of a quadrilateral is 360°,

$2x + 5x + 4x + x = 360°$

Or, $x = 30°$

Now, all the angles will be,

$2x = 2 \times 30° = 60°$

$5x = 5 \times 30° = 150°$

$4x = 4 \times 30° = 120°$, and

$x = 30°$

11. ABCD is a quadrilateral in which P, Q, R, and S are the mid-points of sides AB, BC, CD, DA respectively. AC is a diagonal. Show that,

(I) SR $\parallel$ AC and SR = $\frac{1}{2}$AC

(II)PQ = SR

(III)PQRS is a parallelogram

Solution: In triangle ABC, P is the mid-point of AB and Q is the midpoint of BC. Then PQ $\parallel$ AC. and PQ = $\frac{1}{2}$AC

(I) In triangle ACD, R is the mid-point of CD and S is the mid-point of AD. Then SR $\parallel$ AC. SR = $\frac{1}{2}$AC

(II) Since, PQ = $\frac{1}{2}$AC and SR = $\frac{1}{2}$AC, then PQ = SR

(III) Since PQ $\parallel$ AC and SR $\parallel$ AC

Therefore, PQ $\parallel$ SR. Now PQ $\parallel$ SR and PQ = SR

therefore, PQRS is a parallelogram