Chapter 10- Circles

Exercise 10.1

Question 1: Fill in the blanks.
(i) The centre of a circle lies in ___ of the circle. (exterior/interior)
(ii) A point, whose distance from the centre of a circle is more significant than its radius lies in ____ of the circle, (exterior/interior)
(iii) The longest chord of a circle is a ____ of the process.
(iv) An arc is a ____ when its ends are the ends of a diameter.
(v) Segment of a circle is the region between an arc and ____ of the circle.
(vi) A circle divides the plane, on which it lies, in ____ parts.

Answer: (i) interior
(ii) exterior
(iii) diameter
(iv) semicircle
(v) the chord
(vi) three

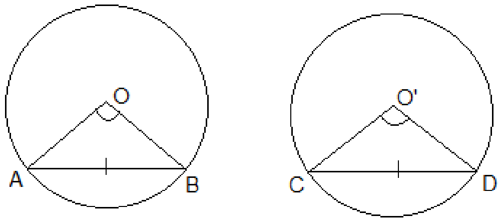
Question 2: Write True or False. Give a reason for your answers.
(i) Line segment joining the centre to any point on the circle is a circle radius.
(ii) A circle has only a finite number of equal chords.
(iii) If a circle is divided into three equal arcs, each is a major arc.
(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.
(v) Sector is the region between the chord and its corresponding arc.
(vi) A circle is a plane figure.

Answer: (i) True [All the points on the circle are always equidistant from the centre]
(ii) False [A circle can have an infinite number of equal chords]
(iii) False [Each part will be less than a semicircle]
(iv) True [Diameter = 2 x Radius, d=2r]
(v) False [The region between the chord and its corresponding arc is known as segment]
(vi) True [A circle is drawn on a plane]

Exercise 10.2

Question 1: Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Answer:

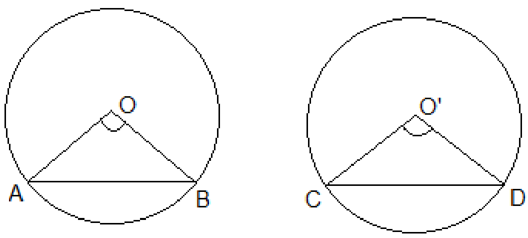


<u>Given</u>: Two congruent circles with centres O and O' and radii r, which have chords AB and CD respectively in such a way that AB = CD.
<u>To Prove</u>: ∠AOB = ∠CO'D

Proof: In ΔAOB and ΔCO'D, we have
AB = CD [Given]
OA = O'C [Each equal to r]
OB = O'D [Each equal to r]
therefore, ΔAOB ≅ ΔCO'D [By SSS congruence criteria]
or, ∠AOB = ∠CO'D [C.P.C.T.]

Question 2: Prove that, if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Answer:



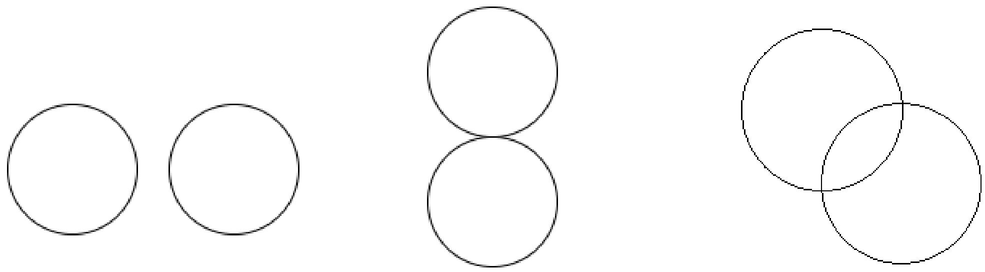
<u>Given</u>: Two congruent circles with centres O & O' and radii r which have chords AB and CD respectively such that ∠AOB = ∠CO'D.
<u>To Prove</u>: AB = CD

Proof: In ΔAOB and ΔCO'D, we have
OA = O'C [Each equal to r]
OB = O'D [Each equal to r]
∠AOB = ∠CO'D [Given]
Therefore, ΔAOB ≅ ΔCO'D [By SAS congruence]
Hence, AB = CD [C.P.C.T.]

<u>Exercise 10.3</u>

Question 1: Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common issues?

Answer: Let us draw different pairs of circles, as shown below:



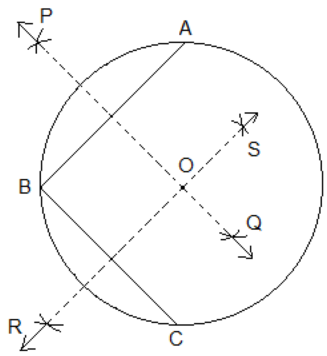
(i) (ii) (iii)

Figure	Max. no of common points
i	Nil
ii	One
iii	Two

Question 2: Suppose you are given a circle. Give the construction to find its centre.

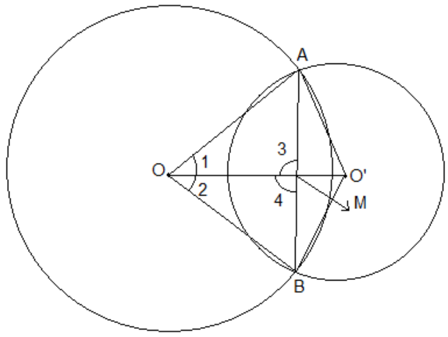
Answer: Step 1: Take any three points on the given circle. Let these points be A, B and C.
Step 2: Join AB and BC.
Step 3: Draw the perpendicular bisector, PQ of AB.
Step 4: Draw the perpendicular bisector, RS of BC such that it intersects PQ at O.

Thus, 'O' is the required centre of the given circle.



Question 3: If two circles intersect at two points, prove that their centres lie on the common chord's perpendicular bisector.

Answer: We have two circles with centres O and O’, intersecting at A and B. Therefore, AB is the common chord of two circles, and OO’ is the line segment joining their centres.
Let OO’ and AB intersect each other at M.



To prove that OO’ is the perpendicular bisector of AB, we need to join OA, OB, O’A and O’B. Now, in ∆QAO’ and ∆OBO’, we have
OA = OB [Radii of the same circle]
A = O’B [Radii of the same circle]
OO’ = OO’ [Common]
Therefore, ∆OAO’ ≅ ∆OBO’ [By SSS congruence criteria]
or, ∠1 = ∠2 , [C.P.C.T.]
Now, in ∆AOM and ∆BOM, we have
OA = OB [Radii of the same circle]
OM = OM [Common]
∠1 = ∠2 [Proved above]
Therefore, ∆AOM = ∆BOM [By SAS congruence criteria]
or, ∠3 = ∠4 [C.P.C.T.]
But ∠3 + ∠4 = 180° [Linear pair]
Thus, ∠3=∠4 = 90°
or AM ⊥ OO.’
Also, AM = BM [C.P.C.T.]
Hence, M is the mid-point of AB.
Thus, OO’ is the perpendicular bisector of AB.

Exercise 10.4

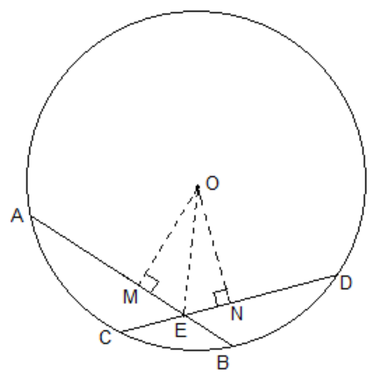
Question 1: Two circles of radii 5 cm and 3 cm intersect at two points, and the distance between their centres is 4 cm. Find the length of the common chord.

Answer: We have two intersecting circles with centres at O and O’(say) respectively. Let PQ be the common chord. Hence, in two intersecting circles, the line joining their centres is perpendicular to the common chord.

Therefore, ∠OLP = ∠OLQ = 90° and PL = LQ
Now, in the right ∆OLP, we have
$PL^2 + OL^2 = OP^2$
Or, $PL^2 + (4 – x)^2 = 5^2$
or, $PL^2 = 5^2 – (4 – x)^2$
or, $PL^2 = 25 - 16 – x^2 + 8x$
or, $PL^2 = 9 – x^2 + 8x$ ……………………………………………(1)
Again, in the right ∆O'LP,
$PL^2 = PO'^2 – LO'^2$
$= 3^2 – x^2 = 9 – x^2$ …………………………………………….…(2)
From (1) and (2), we have
$9 – x^2 + 8x = 9 – x^2$
or, $8x = 0$
or, $x = 0$
Or, L and O' coincide.
Therefore, PQ is a diameter of the smaller circle.
or, PL = 3 cm
But PL = LQ
Hence, LQ = 3 cm
Therefore, PQ = PL + LQ = 3cm + 3cm = 6cm
Thus, the required length of the familiar chord = 6 cm.

Question 2: If two equal chords of a circle intersect within the process, prove that one chord's segments are equal to corresponding parts of the other chord.

Answer:



Given: A circle with centre O and equal chords AB and CD intersect at E.
To Prove: AE = DE and CE = BE
Construction: Draw OM ⊥ AB and ON ⊥ CD. We need to join OE.

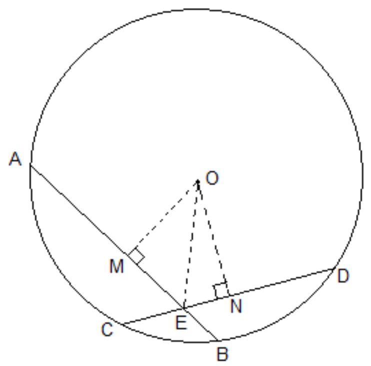
Proof: Since AB = CD [Given]
therefore, OM = ON [Equal chords are equidistant from the centre]

Now, in ∆OME and ∆ONE, we have
∠OME = ∠ONE [Each equal to 90°]
OM = ON [Proved above]
OE = OE [Common hypotenuse]
Therefore, ∆OME ≅ ∆ONE [By RHS congruence criteria]
or, ME = NE [C.P.C.T.]

Adding AM on both sides, we get
or, AM + ME = AM + NE
or, AE = DN + NE = DE
or, AB = CD ⇒ $\frac{1}{2}$AB = $\frac{1}{2}$DC
Therefore, AM = DN
And, AE = DE ……………………………………………………………(1)
Now, AB – AE = CD – DE
or, BE = CE ………………………………………………………..….(2)
From (1) and (2), we have
AE = DE and CE = BE

Question 3: If two equal chords of a circle intersect within the circle, prove that the line joining the intersection point to the centre makes equal angles with the chords.

Answer:

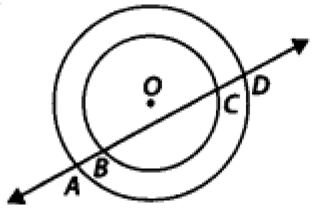


<u>Given</u>: A circle with centre O and equal chords AB and CD are intersecting at E.
<u>To Prove:</u> ∠OEA = ∠OED
<u>Construction:</u> Draw OM ⊥ AB and ON ⊥ CD. We need to join OE.

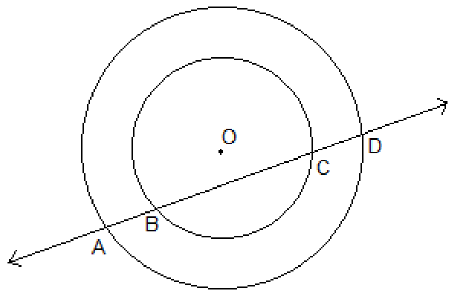
<u>Proof:</u> In ∆OME and ∆ONE,
OM = ON [Equal chords are equidistant from the centre]
OE = OE [Common hypotenuse]

∠OME = ∠ONE [Each equal to 90°]
Therefore, ∆OME ≅ ∆ONE [By RHS congruence criteria]
or, ∠OEM = ∠OEN [C.P.C.T.]
or, ∠OEA = ∠OED

Question 4: If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see figure).



Answer:

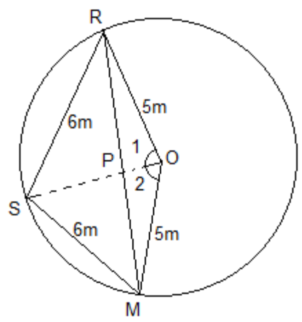


<u>Given</u>: Two circles with the common centre O. A line D intersects the outer ring at A and D and the inner circle at B and C.
<u>To Prove</u> : AB = CD.
<u>Construction</u>: We need to draw OM ⊥ l.
<u>Proof</u>: For the outer circle,
OM ⊥ l [By construction]
therefore, AM = MD …………………………..(1) [Perpendicular from the centre to the chord bisects the chord]

For the inner circle,
OM ⊥ l [By construction]
Therefore, BM = MC ……………………………………….(2) [Perpendicular from the centre to the chord bisects the chord]
Subtracting (2) from (1), we have
AM – BM = MD – MC
Hence, AB = CD

Question 5: Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Answer:



Let the three girls Reshma, Salma and Mandip be positioned at R, S and M respectively on the circle with centre O and radius 5 m such that RS = SM = 6 m, as given in the question.

Equal chords of a circle subtend equal angles at the centre.
therefore, ∠1 = ∠2
In ΔPOR and ΔPOM, we have
OP = OP [Common]
∠1 = ∠2 [Proved above]
OR = OM [Radii of the same circle]
Hence, ΔPOR ≅ ΔPOM [By SAS congruence criteria]
Therefore, PR = PM and
∠OPR = ∠OPM [C.P.C.T.]
As, ∠OPR + ∠OPM = 180° [Linear pair]
Therefore, ∠OPR = ∠OPM = 90°
or, OP ⊥ RM

Now, in ΔRSP and ΔMSP, we have
RS = MS [Each 6 cm]
PR = PM [As proved above]
SP = SP [Common]
Hence, ΔRSP ≅ ΔMSP [By SSS congruence criteria]
or, ∠RPS = ∠MPS [C.P.C.T.]
But ∠RPS + ∠MPS = 180° [Linear pair]
Therefore, ∠RPS = ∠MPS = 90°
SP passes through O.
Let OP = x m
Hence, SP = (5 – x)m
Now, in right ΔOPR, we have
$x^2 + RP^2 = 5^2$
$RP^2 = 5^2 - x^2$ ……………………………………………..(1)
In right ΔSPR, we have
$(5 - x)^2 + RP^2 = 6^2$
or, $RP^2 = 6^2 - (5 - x)^2$ ………………………………………..(2)
From (1) and (2), we get
or, $5^2 - x^2 = 6^2 - (5 - x)^2$
or, $25 - x^2 = 36 - [25 - 10x + x^2]$
or, $-10x + 14 = 0$

or, 10x = 14

or, $x = \frac{14}{10} = 1.4$

Now, $RP^2 = 5^2 - x^2$

or, $RP^2 = 25 - (1.4)^2$

or, $RP^2 = 25 - 1.96 = 23.04$

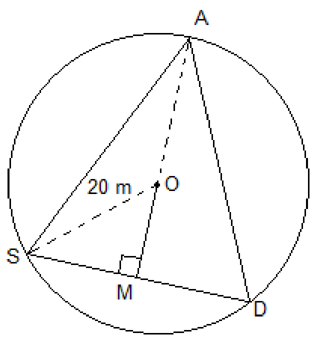
Hence, $RP = \sqrt{23.04} = 4.8$

Therefore, RM = 2RP = 2 x 4.8 = 9.6

Thus, the distance between Reshma and Mandip is 9.6 m.

Question 6: A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Answer:



Let Ankur, Syed and David are sitting at A, S and D respectively in the circular park with centre O such that AS = SD = DA

i. e., ∆ASD is an equilateral triangle.

Let the length of each side of the equilateral triangle be 2x. Need to draw AM ⊥ SD.

Since ∆ASD is an equilateral triangle.

Therefore, AM passes through O.

or, $SM = \frac{1}{2} SD = \frac{1}{2}(2x)$

or, SM = x

Now, in right ∆ASM, we have

$AM^2 + SM^2 = AS^2$ [Using Pythagoras theorem]

or, $AM^2 = AS^2 - SM^2 = (2x)^2 - x^2$

or, $4x^2 - x^2 = 3x^2$

or, $AM = \sqrt{3x}$ m

Now, $OM = AM - OA = (\sqrt{3x} - 20)$m

Again, in the right ∆OSM, we have

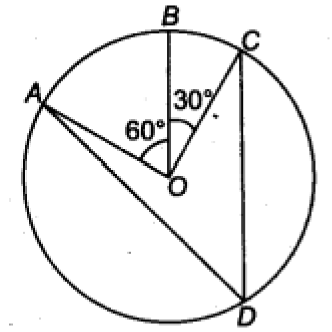
$OS^2 = SM^2 + OM^2$ [using Pythagoras theorem]

$20^2 = x^2 + (\sqrt{3x} - 20)^2$

or, 400 = x^2 + $3x^2$ – $40\sqrt{3x}$ + 400
or, $4x^2$ = 40 $\sqrt{3x}$
or, x = 10√3 m
Now, SD = 2x = 2 x 10√3 m = 20√3 m
Thus, the length of the string of each phone = 20√3 m

Exercise 10.5

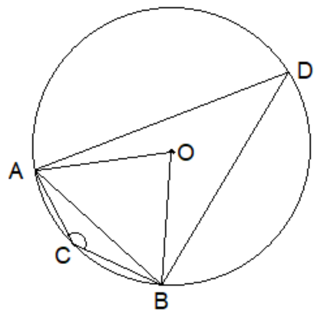
Question 1: In figure A, B and C are three points on a circle with centre 0 such that ∠BOC = 30° and ∠ AOB = 60°. If D is a point on the process other than the arc ABC, find ∠ ADC.



Answer: We have a circle with centre O, such that
∠AOB = 60° and ∠BOC = 30°
Therefore, ∠AOB + ∠BOC = ∠AOC
Thus, ∠AOC = 60° + 30° = 90°
The angle subtended by an arc at the circle is half the angle subtended by it at the centre.
Therefore, ∠ ADC = $\frac{1}{2}$ (∠AOC) = $\frac{1}{2}$(90°) = 45°

Question 2: A chord of a circle is equal to the radius of the process, find the angle subtended by the chord at a point on the minor arc and a point on the major arc.

Answer:

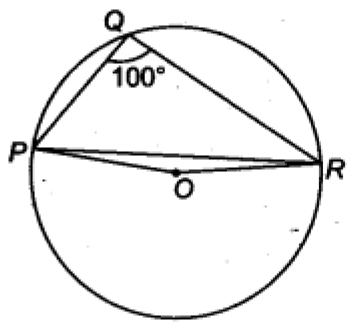


We have a circle having a chord AB equal to the radius of the process.
∴ AO = BO = AB
⇒ ∆AOB is an equilateral triangle.
Since each angle of an equilateral triangle is 60°.
⇒ ∠AOB = 60°
Since the arc ACB makes reflex ∠AOB = 360° – 60° = 300° at the centre of the circle and ∠ACB at a point on the minor arc of the circle.

Therefore, ∠ACB = $\frac{1}{2}$ [reflex ∠AOB] = $\frac{1}{2}[300°] = 150°$

Hence, the angle subtended by the chord on the minor arc = 150°.
Similarly, ∠ADB = $\frac{1}{2}$ [∠AOB] = $\frac{1}{2}$ x 60° = 30°
Hence, the angle subtended by the chord on the major arc = 30°

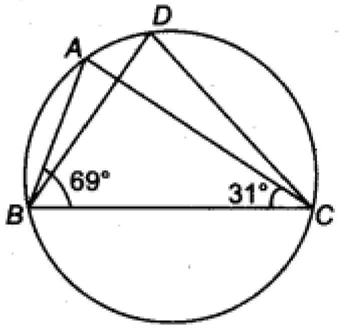
Question 3: In the figure, ∠PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠OPR.



Answer: The angle subtended by an arc of a circle at its centre is twice the angle subtended by the same arc at a point on the circumference.
Therefore, reflex ∠POR = 2∠PQR
But ∠PQR = 100°
Thus, reflex ∠POR = 2 x 100° = 200°
Since, ∠POR + reflex ∠POR = 360°
or, ∠POR = 360° – 200°
or, ∠POR = 160°
Since, OP = OR [Radii of the same circle]
or, In ∆POR, ∠OPR = ∠ORP [Angles opposite to equal sides of a triangle are equal]

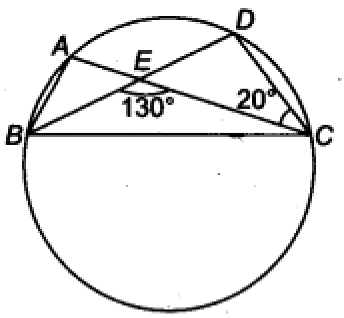
Also, ∠OPR + ∠ORP + ∠POR = 180° [Sum of the angles of a triangle is 180°]
or, ∠OPR + ∠ORP + 160° = 180°
or, 2∠OPR = 180° -160° = 20° [∠OPR = ∠ORP]
or, ∠OPR = $\frac{20°}{2}$ = 10°

Question 4: In figure, ∠ABC = 69°,∠ACB = 31°, find ∠BDC.



Answer: In ΔABC, ∠ABC + ∠ACB + ∠BAC = 180°
or, 69° + 31° + ∠BAC = 180°
or, ∠BAC = 180° – 100° = 80°
Since angles in the same segment are equal.
Therefore, ∠BDC = ∠BAC
or, ∠BDC = 80°

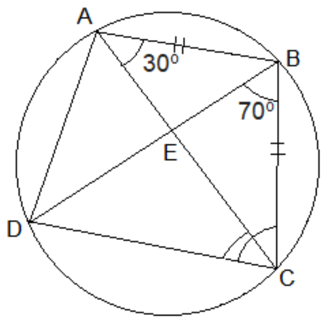
Question 5: In the figure, A, B and C are four points on a circle. AC and BD intersect at a point E such that ∠ BEC = 130° and ∠ ECD = 20°. Find ∠BAC.



Answer: ∠BEC = ∠EDC + ∠ECD [Sum of interior opposite angles is equal to exterior grade]
or, 130° = ∠EDC + 20°
or, ∠EDC = 130° – 20° = 110°
or, ∠BDC = 110°
Since angles in the same segment are equal.
Hence, ∠BAC = ∠BDC
or, ∠BAC = 110°

Question 6: ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠DBC = 70°, ∠BAC is 30°, find ∠BCD. Further, if AB = BC, find ∠ECD.

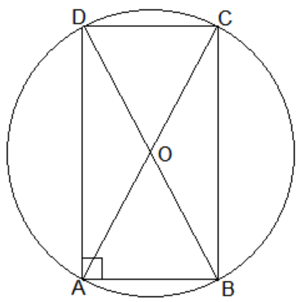
Answer:



Since angles in the same segment of a circle are equal. 70°
∴ ∠BAC = ∠BDC
⇒ ∠BDC = 30°
lso, ∠DBC = 70° [Given]
In ∆BCD, we have
∠BCD + ∠DBC + ∠CDB = 180° [Sum of angles of a triangle is 180°]
⇒ ∠BCD + 70° + 30° = 180°
⇒ ∠BCD = 180° -100° = 80°
Now, in ∆ABC,
AB = BC [Given]
∴ ∠BCA = ∠BAC [Angles opposite to equal sides of a triangle are equal]
⇒ ∠BCA = 30° [∵ ∠B AC = 30°]
Now, ∠BCA + ∠BCD = ∠BCD
⇒ 30° + ∠ECD = 80°
⇒ ∠BCD = 80° – 30° = 50°`

Question 7: If diagonals of a cyclic quadrilateral are diameters of the circle through the quadrilateral vertices, prove that it is a rectangle.

Answer:

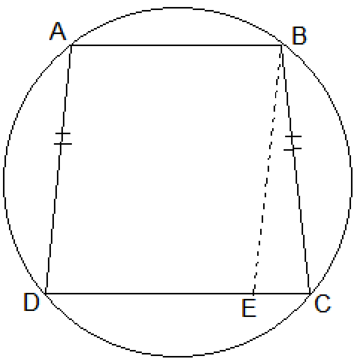


Given that, AC and BD are diameters.
or, AC = BD ……………………………………(1) [All diameters of a circle are equal]
Also, ∠BAD = 90° [Angle formed in a semicircle is 90°]
Similarly, ∠ABC = 90°, ∠BCD = 90°
and ∠CDA = 90°

Now, in ΔABC and ΔBAD, we have
AC = BD [From (1)]
∠ABC = ∠BAD [Each equal to 90°]
AB = BA [Common hypotenuse]
Therefore, ΔABC ≅ ΔBAD [By RHS congruence criteria]
or, BC = AD [C.P.C.T.]
Similarly, AB = DC
Thus, the cyclic quadrilateral ABCD is such that its opposite sides are equal and each of its angles is a right angle.
Hence, ABCD is a rectangle.

Question 8: If the non – parallel sides of a trapezium are equal, prove that it is cyclic.

Answer:

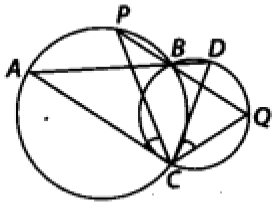


We have a trapezium ABCD such that AB ll CD and AD = BC.
Let us draw BE ll AD such that ABED is a parallelogram.
Since, the opposite angles and opposite sides of a parallelogram are equal.
therefore, ∠BAD = ∠BED ……………………………………………………………(1)
and AD = BE ………………………………………………………………….(2)
But AD = BC [Given] ……………………………………………………. (3)

Hence, From (2) and (3), we have BE = BC
or, ∠BCE = ∠BEC …………………………………….. (4) [Angles opposite to equal sides of a triangle are equal]
Now, ∠BED + ∠BEC = 180° [Linear pair]
or, ∠BAD + ∠BCE = 180° [Using (1) and (4)]
i.e., A pair of opposite angles of a quadrilateral ABCD is 180°.

Hence, ABCD is cyclic.
Therefore, the trapezium ABCD is cyclic.

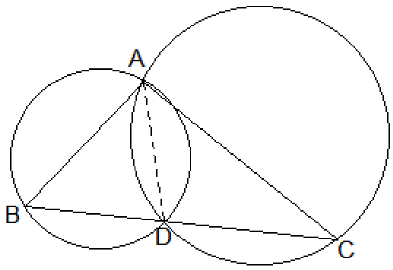
Question 9: Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to cross the processes at A, D and P, Q respectively (see figure). Prove that ∠ ACP = ∠QCD.



Answer: Since angles in the same segment of a circle are equal.
Therefore, ∠ACP = ∠ABP ……………………………………………………………..(1)
Similarly, ∠QCD = ∠QBD ………………………………………………………(2)
Since ∠ABP = ∠QBD ……………………………………………..(3) [Vertically opposite angles]
Therefore, from (1), (2) and (3), we have
∠ACP = ∠QCD

Question 10: If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Answer:



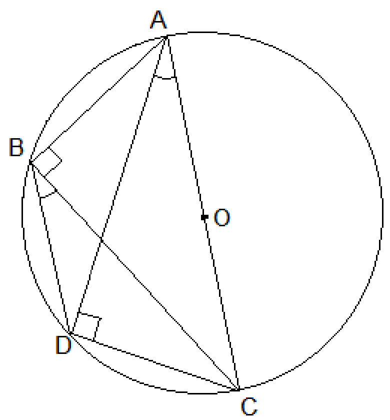
We have ∆ABC, and two circles described with a diameter as AB and AC respectively. They intersect at a point D, other than A. Now, let us join A and D.

Since AB is a diameter.
Therefore, ∠ADB is an angle formed in a semicircle.
or, ∠ADB = 90° ………………………………………………………………………(1)
Similarly, ∠ADC = 90° …………………………………………………………..(2)
Adding (1) and (2), we have
∠ADB + ∠ADC = 90° + 90° = 180°
i. e., B, D and C are collinear points.
Or, BC is a straight line.
Thus, D lies on BC.

Question 11: ABC and ADC are two right-angled triangles with standard hypotenuse AC. Prove that ∠CAD = ∠CBD.

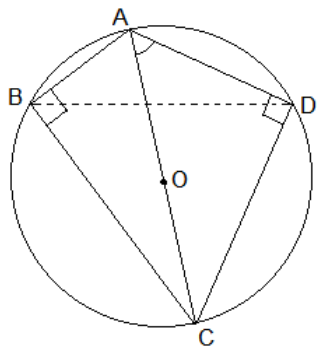
Answer: We have ΔABC and ΔADC such that they have AC as their common hypotenuse and ∠ADC = 90° = ∠ABC
Therefore, both the triangles are in a semi-circle.

<u>CASE 1:</u>



Suppose both the triangles are in the same semi-circle. Hence, A, B, C and D are concyclic.
Let us join BD, and DC is a chord.
Therefore, ∠CAD and ∠CBD are formed in the same segment.
or, ∠CAD = ∠CBD

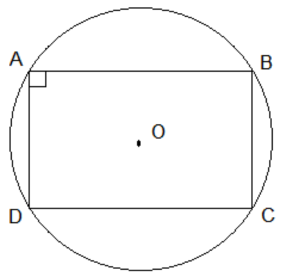
<u>CASE 2:</u>



Suppose both the triangles are not in the same semi-circle. Hence, A, B, C and D are concyclic. Let us join BD, and DC is a chord.
Therefore, ∠CAD and ∠CBD are formed in the same segment.
or, ∠CAD = ∠CBD

Question 12: Prove that a cyclic parallelogram is a rectangle.

Answer: We have a cyclic parallelogram ABCD. Since ABCD is a cyclic quadrilateral.

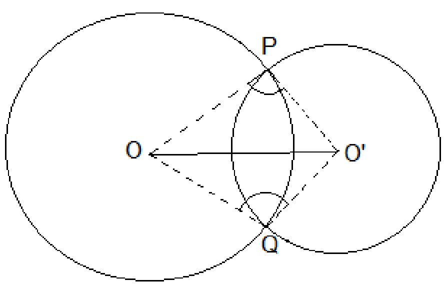


As we know, the sum of its opposite angles is 180°.
or, ∠A + ∠C = 180° ……………………………………………………………(1)
But ∠A = ∠C ……………………………………(2) [Opposite angles of a parallelogram are equal]
From (1) and (2), we have
∠A = ∠C = 90°
Similarly, ∠B = ∠D = 90° and each angle of the parallelogram ABCD is 90°.
Thus, ABCD is a rectangle.

Exercise 10.6

Question 1: Prove that the line of centres of two intersecting circles subtends equal angles at the two intersection points.

Answer: Given: Two circles with centres O and O' respectively such that they intersect each other at P and Q.
To Prove: ∠OPO' = ∠OQO'.
Construction: Join OP, O'P, OQ, O'Q and OO'.

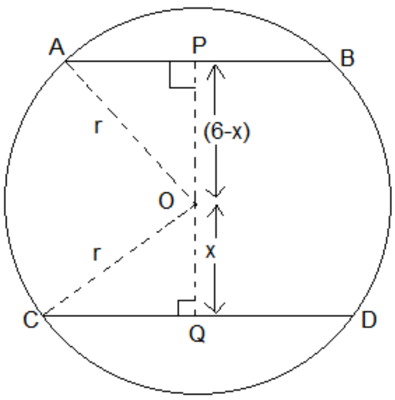


Proof: In ΔOPO' and ΔOQO', we have
OP = OQ [Radii of the same circle]
OO' = OO' [Common]
O'P = O'Q [Radii of the same circle]

Hence, Triangle OPO’ = Triangle OQO’ [By SSS congruence criteria]
or, ∠OPO’ = ∠OQO’ [C.P.C.T.]

Question 2: Two chords AB and CD of lengths 5 cm and 11 cm, respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Answer:



We have a circle with centre O. AB || CD and the perpendicular distance between AB and CD is 6 cm and AB = 5 cm, CD = 11 cm. [Given]

Let r cm be the radius of the circle.
Let us draw OP ⊥ AB and OQ ⊥ CD such that PQ = 6 cm and join OA and OC.
Let OQ = x cm
Hence, OP = (6 – x) cm
Therefore, The perpendicular drawn from the centre of a circle to a chord bisects the chord.

Therefore, $AP = \frac{1}{2}AB = \left(\frac{1}{2} \times 5\right) cm = \frac{5}{2} cm$

Similarly, $CQ = \frac{1}{2}CD = \left(\frac{1}{2} \times 11\right) cm = \frac{11}{2} cm$

In ΔCQO, we have $CO^2 = CQ^2 + OQ^2$

or, $r^2 = \left(\frac{11}{2}\right)^2 + x^2$

or, $r^2 = \frac{121}{4} + x^2$ ... (1)

In ΔAPO, we have $AO^2 = AP^2 + OP^2$

Or, $r^2 = \left(\frac{5}{2}\right)^2 + (6 - x)^2$

Or, $r^2 = \frac{25}{4} + 36 - 12x + x^2$ ... (2)

From (1) and (2) we have,

$\frac{25}{4} + 36 - 12x + x^2 = \frac{121}{4} + x^2$

Or, $-12x = \frac{121}{4} - \frac{25}{4} - 36$

Or, $12x = 12$ $\quad , \; or, x = 1$

Substituting the value of x in (1) we get

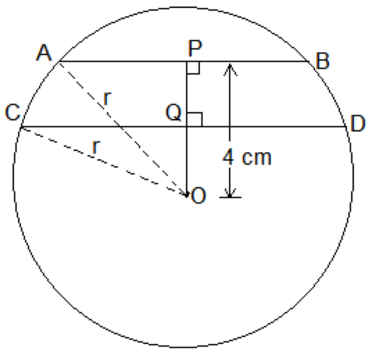
$r^2 = \frac{121}{4} + 1 = \frac{125}{4}$

Or, $r = \frac{5\sqrt{5}}{2}$

Thus the radius of the circle is $\frac{5\sqrt{5}}{2} cm$.

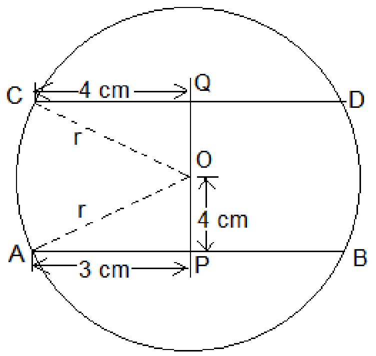
Question 3: The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance 4 cm from the centre, what is the distance of the other chord from the centre?

Answer:



We have a circle with centre O. Parallel chords AB and CD are such that the smaller chord is 4 cm away from the centre. Let r cm be the radius of the circle and draw OP ⊥ AB and join OA and OC. As OP ⊥ AB
Therefore, P is the mid-point of AB.
or, AP = $\frac{1}{2}$AB = $\frac{1}{2}$ (6cm) = 3 cm
Similarly, CQ = $\frac{1}{2}$CD = $\frac{1}{2}$ (8cm)= 4 cm
Now in ∆OPA, we have $OA^2 = OP^2 + AP^2$
or, $r^2 = 4^2 + 3^2$
or, $r^2 = 16 + 9 = 25$
or, r = $\sqrt{25}$ =5
Again, in ∆CQO, we have $OC^2 = OQ^2 + CQ^2$
or, $r^2 = OQ^2 + 4^2$

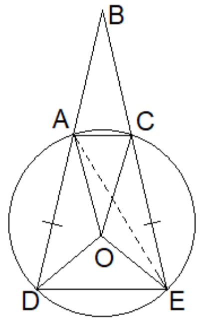
or, $OQ^2 = r^2 - 4^2$
or, $OQ^2 = 5^2 - 4^2 = 25 - 16 = 9$ [r = 5]
or, OQ
or, $\sqrt{9} = 3$
The distance of the other chord (CD) from the centre is 3 cm.



If we take the two parallel chords on either side of the centre, O, then,
In ΔPOA, $OA^2 = OP^2 + PA^2$
or, $r^2 = 4^2 + 3^2 = 5^2$
or, r = 5
In ΔQOC, $OC^2 = CQ^2 + OQ^2$
or, $OQ^2 = 4^2 + OQ^2$
or, $OQ^2 = 5^2 - 4^2 = 9$
or, OQ = 3

Question 4: Let the vertex of an angle ABC be located outside a circle and let the sides of the rise intersect equal chords AD and CE with the process. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Answer: <u>Given:</u> ∠ABC is such that when we produce arms BA and BC, they make two equal chords AD and CE.
<u>To prove</u>: $\angle ABC = \frac{1}{2} [\angle DOE - \angle AOC]$
<u>Construction</u>: We need to join AE.
<u>Proo</u>f: An exterior angle of a triangle is equal to the sum of interior opposite angles.
In ΔBAE, we have,
$\angle DAE = \angle ABC + \angle AEC$ ……………………………………………….(1)
The chord DE subtends ∠DOE at the centre and ∠DAE in the remaining part of the circle.



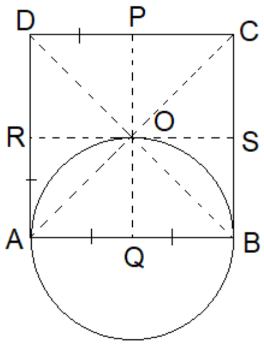
Therefore, ∠DAE = $\frac{1}{2}$ ∠DOE…………………………..(2)

Similarly, ∠AEC = $\frac{1}{2}$ ∠AOC …………………………………..(3)

So, from (1) (2) and (3)
$\frac{1}{2}$ ∠DOE = ∠ABC + $\frac{1}{2}$ ∠AOC
or, ∠ABC = $\frac{1}{2}$ ∠DOE – $\frac{1}{2}$ ∠AOC
or, ∠ABC = $\frac{1}{2}$ [(Angle subtended by the chord DE at the centre) – (Angle subtended by the chord AC at the centre)]
or, ∠ABC = $\frac{1}{2}$ [Difference of the angles subtended by the chords DE and AC at the centre]

Question 5: Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

Answer:

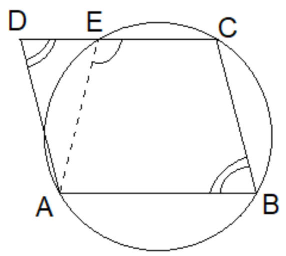


We have a rhombus ABCD and its diagonals AC and BD intersect at O. We need to take AB as diameter, a circle is drawn. Let us draw PQ || DA and RS || AB; both are passing through O. P, Q, R and S are the mid-points of DC, AB, AD and BC respectively,
As Q is the mid-point of AB.
or, AQ = QB ……………………………………………………(1)
Since AD = BC [ABCD is a rhombus]

or, $\frac{1}{2}$ AD = $\frac{1}{2}$ BC
or, RA = SB
or, RA = OQ …………………………………………………………(2) [PQ is drawn parallel to AD and AD = BC]
We have, AB = AD [Sides of a rhombus are equal]
or, $\frac{1}{2}$ AB = $\frac{1}{2}$ AD
or, AQ = AR ……………………………………………………………… (3)
From (1), (2) and (3), we have AQ = QB = OQ, i.e. A circle drawn with Q as the centre, will pass through A, B and O. Thus, the process passes through the of intersection 'O' of the diagonals of rhombus ABCD.

Question 6: ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

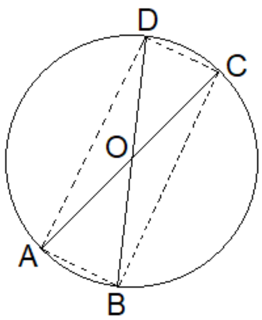
Answer:



We have a circle passing through A, B and C is drawn such that it intersects CD at E.
ABCE is a cyclic quadrilateral.
Therefore, ∠AEC + ∠B = 180° ……………………………….(1) [Opposite angles of a cyclic quadrilateral are supplementary]
But, it is given that ABCD is a parallelogram.
Hence, ∠D = ∠B ……………………………………………(2) [Opposite angles of a parallelogram are equal]
From (1) and (2), we have
∠AEC + ∠D = 180° …………………………………………………..(3)
But ∠AEC + ∠AED = 180° [Linear pair] …………………………………(4)
From (3) and (4), we have ∠D = ∠AED
i.e., The base angles of AADE are equal.
Hence, the opposite sides must be equal.
or, AE = AD

Question 7: AC and BD are chords of a circle which bisect each other. Prove that
(i) AC and BD are diameters,
(ii) ABCD is a rectangle.

Answer:



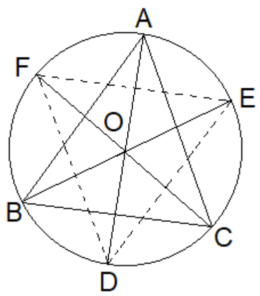
Given: A circle in which two chords AC and BD are such that they bisect each other. Let their point of intersection be O.
To Prove: (i) AC and BD are diameters.
(ii) ABCD is a rectangle.
Construction: Join AB, BC, CD and DA.

Proof: (i) In ∆AOB and ∆COD, we have
AO = CO [O is the mid-point of AC]
BO = DO [O is the mid-point of BD]
∠AOB = ∠COD [Vertically opposite angles]
Therefore, Using the SAS criterion of congruence,
∆AOB ≅ ∆COD
or, AB = CD [C.P.C.T.]
or, arc AB = arc CD ……………………………………………(1)
Similarly, arc AD = arc BC ……………………………………………(2)
Adding (1) and (2), we get
arc AB + arc AD = arc CD + arc BC
or, arc BAD = arc BCD
Or, BD divides the circle into two equal parts.
Hence, BD is a diameter.
Similarly, AC is a diameter.

(i) We know that $\Delta AOB \cong \Delta COD$
or, ∠OAB = ∠OCD [C.P.C.T]
or, ∠CAB = ∠ACD
AB || DC
Similarly, AD || BC
Therefore, ABCD is a parallelogram.
Since, opposite angles of a parallelogram are equal.
Hence, ∠DAB = ∠DCB
But ∠DAB + ∠DCB = 180° [Sum of the opposite angles of a cyclic quadrilateral is 180°]
or, ∠DAB = 90° = ∠DCB Thus, ABCD is a rectangle.

Question 8: Bisectors of angles A, B and C of an ∆ABC intersect its circumcircle at D, E and F, respectively. Prove that the angles of the ∆DEF are 90° – $\frac{1}{2}$ A, 90° – $\frac{1}{2}$2 B and 90° – $\frac{1}{2}$ C.

Answer:



Given: A triangle ABC inscribed in a circle, such that bisectors of ∠A, ∠B and ∠C intersect the circumcircle at D, E and F respectively.
Construction: Join DE, EF and FD.

Proof: Since angles in the same segment are equal.
Therefore, ∠EDA = ∠FCA ……………………………………………………………(1)
∠EDA = ∠EBA ……………………………………………………………(2)
Adding (1) and (2), we have
∠FDA + ∠EDA = ∠FCA + ∠EBA
or, ∠FDE = ∠FCA + ∠EBA

$= \frac{1}{2}\angle C + \frac{1}{2}\angle B$

$= \frac{1}{2}(\angle C + \angle B)$

$= \frac{1}{2}[180° - \angle A]$

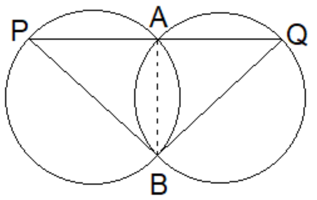
$= \left(90° - \frac{\angle A}{2}\right)$

Similarly, $\angle FED = \left(90° - \frac{\angle B}{2}\right)$ and $\angle EFD = \left(90° - \frac{\angle C}{2}\right)$

Thus, the angles of ΔDEF are, $\left(90° - \frac{\angle A}{2}\right)$, $\left(90° - \frac{\angle B}{2}\right)$, $\left(90° - \frac{\angle C}{2}\right)$

Question 9: Two congruent circles intersect each other at points A and B. Any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

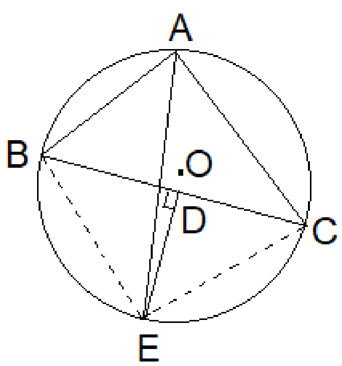
Answer: We have two congruent circles such that they intersect each other at A and B. A line segment passing through A meets the circles at P and Q. Let us draw the common chord AB for the convenience of the solution.



Since angles subtended by equal chords in the congruent circles are equal.
or, ∠APB = ∠AQB
Now, in ΔPBQ, we have ∠AQB = ∠APB
So, their opposite sides must be equal.
or BP = BQ.

Question 10: In any Δ ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the ΔABC.

Answer:



Here, we need to join BE and CE for the convenience of the solution.
Now, AE is the bisector of ∠BAC,
∠BAE = ∠CAE
also, arc BE = arc EC
hence, we can conclude that chord BE = chord EC

Now, we need to consider ΔBDE and ΔCDE,

DE = DE [common side]
BD = CD [Given]
BE = CE [Earlier proved]

So, by SSS congruency, ΔBDE ΔCDE.
Thus, ∠BDE = ∠CDE
Since, we know that, ∠BDE = ∠CDE = 180°
Or, ∠BDE = ∠CDE = 90°

Hence, DE ⊥ BC.