Chapter 10 – Circles
Exercise – 10.1

Question 1: 1. How many tangents can a circle have?

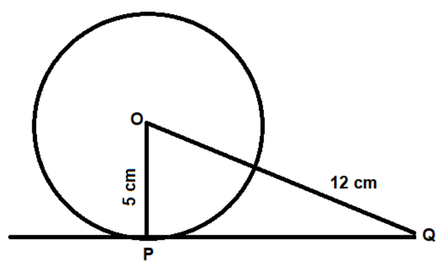
Answer: There can be infinite tangents to a circle.

Question 2: Fill in the blanks:
(i) A tangent to a circle intersects it in …………… point(s).
(ii) A line intersecting a circle in two points is called a ………….
(iii) A circle can have …………… parallel tangents at the most.
(iv) The common point of a tangent to a circle and the circle is called …………

Answer: (i) one
(ii) secant
(iii) two
(iv) point of contact

Question 3: A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is
(a) 12 cm
(b) 13 cm
(c) 8.5 cm
(d) $\sqrt{119}$ c

Answer:



From the figure, we can see that, OP ⊥ PQ

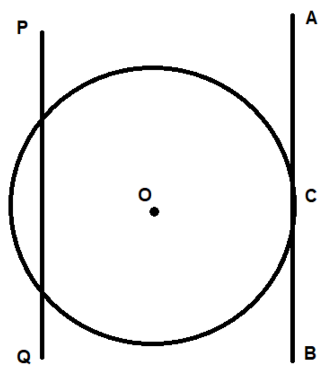
Using Pythagoras theorem in triangle ΔOPQ, we get,

$OQ^2 = OP^2+PQ^2$
or, $(12)^2 = 5^2+PQ^2$
or, $PQ^2 = 144-25$
or, $PQ^2 = 119$
or, $PQ = \sqrt{119}$ cm

So, option (D) √119 cm is the length of PQ.

Question 4: Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Answer:

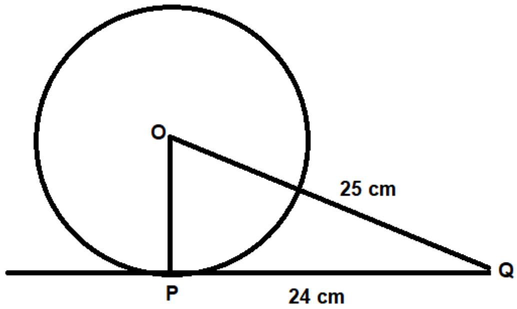


In the above figure, PQ and AB are two parallel lines. The line segment AB is the tangent at point C while the line segment PQ is the secant.

Exercise 10.2

Question 1: From a point Q, the length of the tangent to a circle is 24 cm, and the distance of Q from the centre is 25 cm. The radius of the circle is
(A) 7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm

Answer:

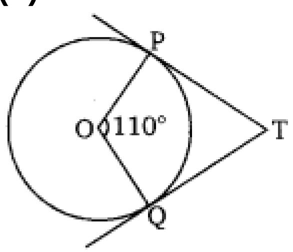


From the figure we can see that, OP ⊥ PQ
It is given that, OQ = 25 cm and PQ = 24 cm
Hence, by using Pythagoras theorem in △OPQ,

$OQ^2 = OP^2 + PQ^2$
or, $(25)^2 = OP^2 + (24)^2$
or, $OP^2 = 625 - 576$
or, $OP^2 = 49$
or, OP = 7 cm

So, option (A), i.e. 7 cm is the radius of the given circle.

Question 2: In the figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to
(a) 60°
(b) 70°
(c) 80°
(d) 90°



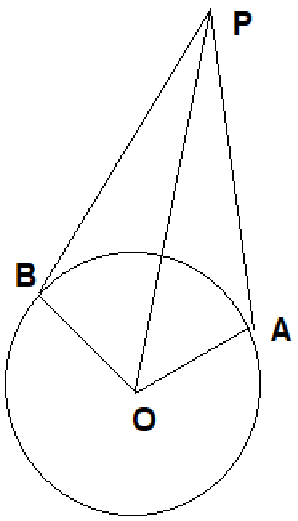
Answer: So, from the given figure, OP ⊥ PT and TQ ⊥ OQ
Therefore, ∠OPT = ∠OQT = 90°
Now, in the quadrilateral POQT, we know that the sum of the interior angles is 360°
So, ∠PTQ+∠POQ+∠OPT+∠OQT = 360°

Now, ∠PTQ + 90° + 110° + 90° = 360°
or, ∠PTQ = 70°

So, ∠PTQ is 70° option(B).

Question 3: If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then ∠ POA is equal to
(A) 50° (B) 60° (C) 70° (D) 80°

Answer:



After making the diagram according to the problem, in the diagram, OA is the radius to tangent PA and OB is the radius to tangents PB. So, OA ⊥ PA and OB ⊥ PB and ∠OBP = ∠OAP = 90°

Now, in the quadrilateral AOBP,
∠AOB+∠OAP+∠OBP+∠APB = 360°
or, ∠AOB + 260° = 360°
or, ∠AOB = 100°

Now, taking the triangles △OPB and △OPA,

AP = BP (Since the tangents from a point are always equal)
OA = OB (Which are the radii of the circle)
OP = OP (It is the common side)

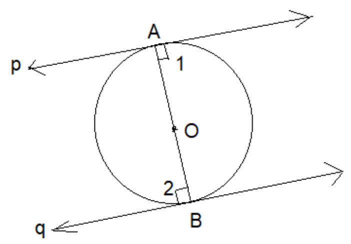
Therefore, △OPB ≅ △OPA [SSS congruency]
or, So, ∠POB = ∠POA
or, ∠AOB = ∠POA+∠POB
or, 2(∠POA) = ∠AOB

By putting the values, we get, $\angle POA = \frac{100°}{2} = 50°$

As angle ∠POA is 50° option(A) is the correct option.

Question 4: Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

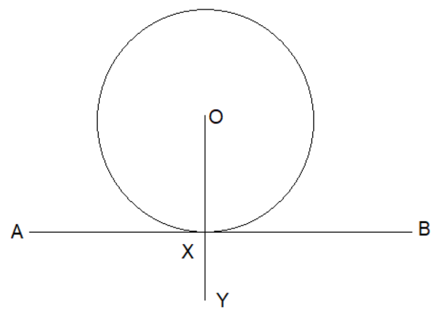
Answer:



AB is the diameter and p and q are the tangents.
OA ⊥ p and, OB ⊥ q
Therefore, ∠1 = ∠2 = 90°, hence, p ∥ q [As ∠1 and ∠2 are alternate angles]

Question 5: Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Answer:

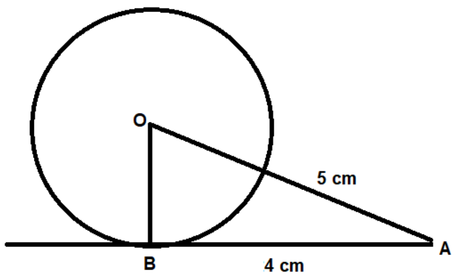


In the given figure, AXB is the tangent to the circle having centre O.
Hence, OX ⊥ AXB

Therefore, ∠OXB + ∠BXY = 90° + 90° = 180°, which implies that OXY is collinear, i.e., OX passes through the centre of the circle. (Proved)

Question 6: The length of a tangent from a point A at a distance 5 cm from the circle's centre is 4 cm. Find the radius of the circle

Answer:

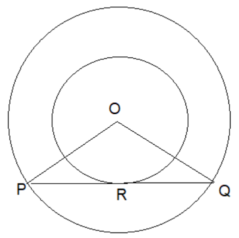


We know, OA = 5cm and AB = 4 cm
Now, In △ABO,
$OA^2 = AB^2 + BO^2$ [Using Pythagoras theorem]
or, $5^2 = 4^2 + BO^2$
or, $BO^2 = 25 - 16$
or $BO^2 = 9$
or, BO = 3

So, the radius of the given circle, i.e. BO is 3 cm.

Question 7: Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches, the smaller circle

Answer:



We have, OP = OQ = 5cm [Radii of larger circle]
OR = 3 cm [Radii of smaller circle]

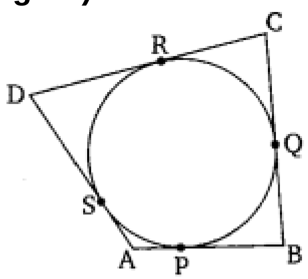
PQ is the tangent to the smaller circle.
Therefore, OR ⊥ PQ [by theorem]
In the tringles, ORP and ORQ

∠ORP = ∠ORQ [90^0]
OR = OR [Common]
OP = OQ [Radii of the same circle]
Therefore, $\Delta OPR \cong \Delta OQR$, HENCE, PR = RQ [CPCT]

Using Pythagoras theorem in triangle OPR,
$PR^2 = OP^2 - OR^2$
or, $PR^2 = 5^2 – 3^2$
or, $PR^2 = (25 – 9)$
or, PR = 4 cm

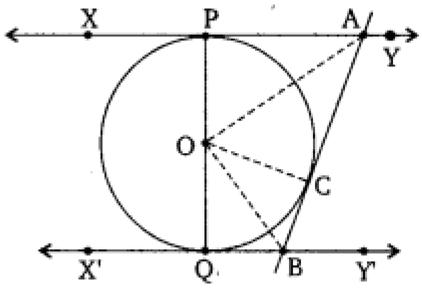
Therefore, PQ = 2PR = 2 × 4 = 8cm

Question 8: A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that AB + CD = AD + BC.



Answer: AP = AS ………..……..(1)[length of tangents from the same point are equal]
BP = BQ………………………………..(2)
CR = CQ……………………………………..(3)
DR = DS………………………………..(4)
Now, adding these four equations, we get,
AP + BP + CR + DR = AS + BQ + CQ + DS
or, (AP + BP)+(CR + DR) = (AS + DS)+(BQ + CQ)
or, AB + CD = AD + BC [proved]

Question 9: In the figure, XY and X'Y' are two parallel tangents to a circle, x with centre O and another tangent AB with the point of contact C intersecting XY at A and X'Y' at B. Prove that ∠AOB = 90°.



Answer: In △OPA and △OCA
OP = OC [radii of the same circle]
AO = AO [common side]
AP = AC [tangents from point A]

So, △OPA ≅ △OCA [SSS congruency]

Similarly, △OQB ≅ △OCB

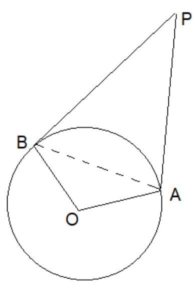
So, ∠POA = ∠COA ………………………………………….(1)
and, ∠QOB = ∠COB ……………………………………………..(2)

Since the line, POQ is the diameter of the circle.
So, ∠POA +∠COA +∠COB +∠QOB = 180°

Now, from equations (1) and equation (2) we get,
2∠COA+2∠COB = 180°
or, ∠COA+∠COB = 90°
or, ∠AOB = 90°

Question 10: Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Answer:



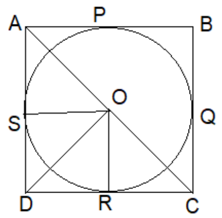
From the diagram, line segments OA and PA are perpendicular. So, ∠OAP = 90°
The line segments OB ⊥ PB and so, ∠OBP = 90°

Now, in the quadrilateral OAPB,
Therefore, ∠APB+∠OAP +∠PBO +∠BOA = 360° [As the sum of all interior angles will be 360°]

By putting the values,
∠APB + 180° + ∠BOA = 360°
So, ∠APB + ∠BOA = 180°

Question 11: Prove that the parallelogram circumscribing a circle is a rhombus.

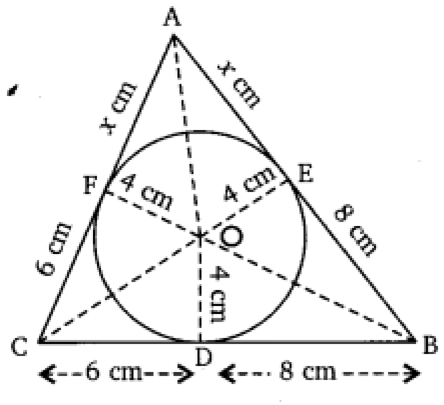
Answer:



Answer: In ΔORC and ΔOSA
∠ORC = ∠OSA [90^0]
OC = OA [O is the midpoint of AC]
OR = OS [Radii of same circle]
Therefore, ΔORC ≅ ΔOSA [By RHS congruency]
Hence, RC = AS……………………………..(1) [CPCT]
and, DR = DS …………………………………(2) [tangents from the points]

Adding eq. (1) and (2), we get,
RC + DR = AS + DS
or, DC = AD
or, AB = DC, AD = BC [ABCD is a parallelogram]
Hence, ABCD is a rhombus.

Question 12: A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.



Answer:

BD = 8cm and DC = 6cm
BE = BD = 8cm
CD = CF = 6cm
Let AE = AF = x cm
In ΔABC, a = 6+8 = 14cm
b = (x+6) cm
c = (x+8) cm

$s = \frac{a+b+c}{2} = \frac{14+x+6+x+8}{2} = \frac{2x+28}{2} = (x+14)\ cm$

ar(ΔABC) = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(x+14)\times x \times 8 \times 6} = \sqrt{48x \times (x+14)}\ cm^2$ … (i)

Again, ar$(\Delta ABC) =$ ar(ΔOBC) + ar(ΔOCA) + ar(ΔOAB)

$= \frac{1}{2}\times 4 \times a + \frac{1}{2}\times 4\times b + \frac{1}{2}\times 4 \times c$

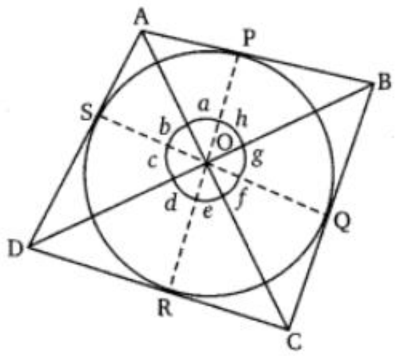
$= 2a + 2b + 2c = 2(a+b+c) = 2\times 2(x+14)$ … (ii)

From (i) and (ii) we get,

$\sqrt{48x(x+14)} = 4(x+14)$
or, $48x(x+14) = 4^2(x+14)^2$
or, $48x(x+14) = 16(x+14)^2$
or, $3x(x+14) = (x+14)^2$
or, $3x = (x+14)$
or, $2x = 14$
or, $x = 7$

Therefore, AB = x + 8 = 7 + 8 = 15cm
AC = x + 6 = 7 + 6 = 13cm

Question 13: Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.



Answer: From the figure, we observe that OA bisects ∠SOP.

So, ∠a =∠b …………………………(1)
∠c = ∠d……………………………………(2)
∠e =∠f………………………………..(3)
∠g =∠h………………………………(4)

Therefore, 2(∠a + ∠h + ∠e + ∠d) = 360^0
or, (∠a + ∠h) + (∠e + ∠d) = 180^0
or, ∠AOB + ∠DOC = 180^0

And, 2(∠b + ∠c + ∠g + ∠f) = 360^0
or, (∠b + ∠c) + (∠g + ∠f) = 180^0
or, ∠AOD + ∠BOC = 180^0