CBSE class 9 maths solutions

CHAPTER 1: NUMBER SYSTEM

Exercise 1.2

Question 1: Is Zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and q $\neq 0$.

Answer: Consider the definition of a rational number.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Zero can be written as $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \dots$

Zero can be written as well $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \dots$

So, we arrive at the conclusion that 0 can be written in the form $\frac{p}{q}$, where p and q are integers. Therefore zero is a rational number.

Question 2: Find six rational numbers between 3 and 4.

Answer: We know there are infinite rational numbers between any two numbers. We have to find six rational numbers between 3 and 4.

So we multiply and divide the upper limit and lower limit by 7.

We get, $3 = 3 \times \frac{7}{7} = \frac{21}{7}$ And, $4 = 4 \times \frac{7}{7} = \frac{28}{7}$

Thus we can have 6 rational numbers between 3 and 4 as : $\frac{22}{7}$, $\frac{23}{7}$, $\frac{24}{7}$, $\frac{25}{7}$, $\frac{26}{7}$, $\frac{27}{7}$.

Question 3: Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Answer: We know there are infinite rational numbers between any two numbers. We have to find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

So, we multiply and divide the upper and lower limits by any number greater than 5 (say 6).

 $\frac{3}{5} = \frac{3}{5} \times \frac{6}{6} = \frac{18}{30}$ $\frac{4}{5} = \frac{4}{5} \times \frac{6}{6} = \frac{24}{30}$

Thus we can have five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ as: $\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}, \frac{23}{30},$

Question 4: State whether the following statements are true or false. Give reasons for your answers.

- i. Every natural number is a whole number
- ii. Every integer is a whole number.
- iii. Every rational number is a whole number.

Answer:

- We know, the natural numbers are: 1,2,3,4,5,....
 We also know that whole numbers are: 0,1,2,3,4,5,...
 So, we can see that natural numbers form a part of the whole number series.
 Hence every natural number is a whole number.
- We know that the whole number series: 0,1,2,3,4,5,...
 We also that the integer series : ..-5,-4,-3,-2,-1,0,1,2,3,4,5,...
 We get to see that the whole of whole number series lies within the integer series.
 Hence every whole number is a integer, but not every integer is a whole number, as negative integers are clearly not whole numbers.
- iii. We know that every rational numbers can be represented as $\frac{p}{q}$ where $q \neq 0$.

We also know that every whole number can be represented as $\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \dots$ Hence we can conclude that every rational number can be represented in the form of whole number, but every rational number is not a whole number.

Exercise 1.2

Question 1: State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form \sqrt{m} where *m* is a natural number.

(iii) Every real number is an irrational number.

Answer: (i) Here we need to consider the irrational numbers and the real numbers separately.

• As we know the irrational numbers are the numbers that cannot be converted in the form of $\frac{p}{q}$, where p and q are integers and q \neq 0. Example, $\sqrt{3}$, 1.01100110 • The real number is the collection of rational numbers and irrational numbers. Therefore, we conclude that, every irrational number is a real number.

(ii) We should consider a number line, on a number line, we can represent negative as well as positive numbers.

- Positive numbers are represented in the form of $\sqrt{1.1}$, $\sqrt{1.3}$, $\sqrt{1.5}$,
- We cannot get a negative number after taking square root of any number. Therefore, we can conclude that very number point on the number line is not of the form \sqrt{m} , where m is a natural number.

(iii) We can consider the irrational numbers and the real numbers separately.

- Irrational numbers are the numbers that cannot be converted in the form of $\frac{p}{q}$, where p and q are integers and q $\neq 0$.
- A real number is collection of rational numbers (eg, $\frac{1}{2}, \frac{1}{4}, \dots$) and irrational numbers (eg, $2\pi, 3\pi, \dots$)

So we can conclude that every irrational number is a real number. But every real number is not an irrational number.

Therefore, every real numbers is not an irrational number.

Question 2: Are the square roots of all positive integers' irrational? If not, give an example of the square root of a number that is a rational number.

Answer: Square root of every positive integer will not yield an integer (eg, $\sqrt{2}, \sqrt{3}, \sqrt{5}, ...$) which are called irrational numbers. But, $\sqrt{4}$ is 2, which is an integer. Thus, we can conclude that square root of every positive integer is not an irrational number.

Question 3: Show how $\sqrt{5}$ can be represented on the number line.

Answer: <u>STEP 1:</u> We need to draw a line segment AB of 2units on the number line. <u>STEP 2:</u> Then draw a perpendicular line segment BC at B of 1units. <u>STEP 3:</u> Then join the points C and A, to form a line segment AC. Now, aq10. According to Pythagoras theorem

 $AB^2 + BC^2 = AC^2$

Or, $2^2 + 1^2 = AC^2$

Or, $4 + 1 = 5 = AC^{2}$.

Therefore, AC = $\sqrt{5}$



Exercise 1.3

Question 1: Write the following in decimal form and say what kind of decimal expansion each has:

i. $\frac{36}{100}$	ii. <u>1</u>	iii. 4 1 /8
iv. <u>3</u>	v. $\frac{2}{11}$	vi. $\frac{329}{400}$

i) We have, $\frac{36}{100} = 0.36$ Thus, the decimal expansion of 36100 is terminating.

(ii) Dividing 1 by 11, we have

11)1.00000(0.090909	
-0	
10	
- 00	
100	
-99	
10	
- 00	
100	
- 99	
10	
-00	
-00	
100	
- 99	
1	
$\therefore \frac{1}{11} = 0.090909 = 0.009$	

Thus, the decimal expansion of $\frac{1}{11}$ is non-terminating repeating.

iii) We have, $4\frac{1}{8} = \frac{33}{8}$





Therefore, $4\frac{1}{8} = 4.125$, thus, the decimal expansion of $4\frac{1}{8}$ is terminating.

iv) Dividing 3 by 13, we get



Here, the repeating block of digits is 230769 therefore, $\frac{3}{13} = 0.23076923...$

Thus, the decimal expansion of $\frac{3}{13}$ is non-terminating repeating.

v) Dividing 2 by 11, we get



Here, the repeating block of digits is 18. Therefore, $\frac{2}{11} = 0.1818...$ Thus, the decimal expansion of $\frac{2}{11}$ is non-terminating repeating. vi) Dividing 329 by 400, we get



Therefore, $\frac{329}{400} = 0.8225$. Thus, the decimal expansion of $\frac{329}{400}$ is terminating.

Question 2: You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$ are, without actually doing the long division? If so, how?

Answer: We know that $\frac{1}{7} = \overline{0.142857}$ Or, $\frac{1}{7} = 0.142857$ Therefore, $\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times \overline{0.142857} = \overline{0.285714}$ $\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times \overline{0.142857} = \overline{0.428571}$ $\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times \overline{0.142857} = \overline{0.571428}$ $\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times \overline{0.142857} = \overline{0.714285}$ $\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times \overline{0.142857} = \overline{0.857142}$

Hence, without actually doing the long division we can predict the decimal expansions of the given rational numbers.

Question 3: Express the following in the form $\frac{p}{q}$ where *p* and *q* are integers and

q ≠ 0.

- i) 0.<u>6</u>
- ii) 0.47
- iii) 0.001

Answer:

i) Let $x = 0.\overline{6} = 0.6666...$ (I) As there is only one repeating digit, multiplying (I) by 10 on both sides, we get 10x = 6.6666... (II) Subtracting (I) from (II), we get 10x - x = 6.66666... - 0.6666...or, 9x = 6or, x = 69 = 23Thus, $0.\overline{6} = 23$.

ii) Let $x = 0.4\overline{7} = 0.4777...$ (I) As there is only one repeating digit, multiplying (I) by lo on both sides, we get 10x = 4.777Subtracting (I) from (II), we get 10x - x = 4.777... or, 9x = 4.3or, x = 4390Thus, $0.4\overline{7} = 4390$

iii) Let $x = 0.\overline{001} = 0.001001...$ (I) As there are three repeating digits, multiplying (I) by 1000 on both sides, we get 1000x = 1.001001... (I) Subtracting (I) from (II), we get 1000x - x = (1.001...) - (0.001...)or, 999x = 1 or, x = 1999 Thus, $0.\overline{001} = 1999$

Question 4: Express 0.99999 in the form $\frac{p}{q}$ Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Answer: Let x = 0.99999.... (I) As there is only one repeating digit, multiplying (I) by 10 on both sides, we get 10x = 9.9999... (II) Subtracting (I) from (II), we get 10x - x = (99999) - (0.9999)Or, 9x = 9Or, x = 1Thus, 0.9999 =1 As 0.9999... goes on forever, there is no such a big difference between 1 and 0.9999. Hence, both are equal.

Question 5: What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Answer: In $\frac{1}{17}$, In the divisor is 17.

Since, the number of entries in the repeating block of digits is less than the divisor, then the maximum number of digits in the repeating block is 16. Dividing 1 by 17, we have



The remainder 1 is the same digit from which we started the division. Therefore, $\frac{1}{17} = 0$. $\overline{0588235297}$

Thus, there are 16 digits in the repeating block in the decimal expansion of $\frac{1}{17}$. Hence, our answer is verified.

Question 6: Look at several examples of rational numbers in the form $(q^1 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Answer:

Let us take some examples of rational numbers with terminating decimal expansion.

 $\frac{5}{2} = \frac{5 \times 5}{2 \times 5} = \frac{25}{10} = 2.5$ Denominator : 2¹ $\frac{2}{25} = \frac{2 \times 4}{25 \times 4} = \frac{8}{100} = 0.08$ Denominator : 5² $\frac{25}{40} = \frac{25 \times 25}{40 \times 25} = \frac{625}{1000} = 0.625$ Denominator : 2³x5¹

We see that all the rational numbers have 2 and 5 as the factors of the denominator. So for a rational number to have a terminating decimal form is that the prime factorisation of the denominator will have only powers of 2 or powers of 5 or both.

Question 7: Write three numbers whose decimal expansions are non-terminating non-recurring.

Answer: Any root of a natural number that is not a perfect square, is a non-terminating and non-recurring decimal.

Example:

 $\sqrt{2} = 1.414213 \dots$ $\sqrt{3} = 1.732050 \dots$ $\sqrt{5} = 2.236067 \dots$

Question 8: Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Answer: We have,



Three irrational numbers between 0.714285 and 0.81 are (i) 0.750750075000 (ii) 0.767076700767000 (iii) 0.78080078008000

Question 9: Classify the following numbers as rational or irrational

i) $\sqrt{23}$ ii) $\sqrt{225}$ iii) 0.3796 iv) 7.478478... v) 1.101001000100001...

Answer: i) 23 is not a perfect square. $\sqrt{23}$ is an irrational number.

(ii) $225 = 15 \times 15 = 15^2$ therefore, 225 is a perfect square. Thus, $\sqrt{225}$ is a rational number.

(iii) 0.3796 is a terminating decimal. therefore, it is a rational number.

(iv) $7.478478... = 7.\overline{478}$ Since, $7.\overline{478}$ is a non-terminating recurring (repeating) decimal. therefore, it is a rational number.

(v) Since, 1.101001000100001... is a non terminating, non-repeating decimal

number. hence, It is an irrational number.

Exercise 1.4

Question 1: Visualise 3.765 on the number line, using successive magnification.

Answer: 3.765 lies between 3 and 4.



i. 3.7 lies between 3 and 04

ii. 3.76 lies between 3.7 and 3.8

iii. 3.765 lies between 3.76 and 3.77

Question 2: Visualise $4.\overline{26}$ on the number line, up to 4 decimal places

Answer: $4.\overline{26}$ or 4.2626 lies between 4 and 5.



i. 4.2 lies between 4 and 5

ii. 4.26 lies between 4.2 and 4.3

iii. 4.262 lies between 4.26 and 4.27

iv. 4.2626 lies between 4.262 and 4.263

Exercise 1.5

Question 1: Classify the following numbers as rational or irrational:

i.
$$2 - \sqrt{5}$$

ii. $(3 + \sqrt{23}) - \sqrt{23}$
iii. $\frac{2\sqrt{7}}{7\sqrt{7}}$
iv. $\frac{1}{\sqrt{2}}$
v. 2π

Answer: (i) Since, it is a difference of a rational and an irrational number. $\therefore 2 - \sqrt{5}$ is an irrational number.

(ii) $(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23} = 3$, which is a rational number.

(iii) Since, $\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$, which is a rational number.

(iv) Since, the quotient of rational and irrational number is an irrational number. Therefore, $\frac{1}{\sqrt{2}}$ is an irrational number.

(v) Since, $2\pi = 2 \times \pi = Product$ of a rational and an irrational number is an irrational number.

 $\therefore 2\pi$ is an irrational number.

Question 2: Simplify each of the following expressions:

i.
$$(3 + \sqrt{3})(2 + \sqrt{2})$$

ii. $(3 + \sqrt{3})(3 - \sqrt{3})$
iii. $(\sqrt{5} + \sqrt{2})^2$
iv. $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

Answer: (i) $(3 + \sqrt{3})(2 + \sqrt{2})$ = $2(3 + \sqrt{3}) + \sqrt{2}(3 + \sqrt{3})$ = $6 + 2\sqrt{3} + 3\sqrt{2} + \sqrt{6}$ Thus, $(3 + \sqrt{3})(2 + \sqrt{2}) = 6 + 2\sqrt{3} + 3\sqrt{2} + \sqrt{6}$

(ii) $(3 + \sqrt{3})(3 - \sqrt{3}) = (3)^2 - (\sqrt{3})^2$ = 9 - 3 = 6 Thus, $(3 + \sqrt{3})(3 - \sqrt{3}) = 6$ (iii) $(\sqrt{5} + \sqrt{2})^2 = (\sqrt{5})^2 + (\sqrt{2})^2 + 2(\sqrt{5})(\sqrt{2})$ = 5 + 2 + 2 $\sqrt{10}$ = 7 + 2 $\sqrt{10}$ Thus, $(\sqrt{5} + \sqrt{2})^2$ = 7 + 2 $\sqrt{10}$

(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2 = 5 - 2 = 3$ Thus, $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = 3$

Question 3: Recall π is defined as the ratio of the circumference (say *c*) of a circle to its diameter (say *d*). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Answer: If we measure the length of a line with scale or with any other instrument, we only get an approximate irrational value, i.e, c and d both are irrational.

Therefore $\frac{c}{d}$ is irrational and hence π is irrational.

Thus, saying π is irrational is not a contradiction.

Question 4: Represent $\sqrt{9.3}$ on the number line.

Answer: <u>STEP 1:</u> First we have to draw a line segment AB = 9.3 units and extend it to C such that BC = 1 unit.

STEP 2: Then point out the mid-point of AC and mark it as O.

<u>STEP 3</u>: Then draw a semicircle taking O as centre and AO as radius. Draw BD \perp AC. <u>STEP 4</u>: Then draw an arc taking B as centre and BD as radius meeting AC produced at E such that BE = BD = $\sqrt{9.3}$ units.

DIAGRAM TOMOLAAAAAAAAAAAA.....

Question 5: Rationalise the denominators of the following:

i.
$$\frac{1}{\sqrt{7}}$$

ii. $\frac{1}{\sqrt{7}-\sqrt{6}}$
iii. $\frac{1}{\sqrt{5}+\sqrt{2}}$
iv. $\frac{1}{\sqrt{7}-2}$

Answer: (i) $\frac{1}{\sqrt{7}} = \frac{1 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{\sqrt{7}}{7}$

(ii)
$$\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$$

(iii)
$$\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$$

(iv)
$$\frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{\sqrt{7}+2}{7-4} = \frac{\sqrt{7}+2}{3}$$

Exercise 1.6

Question 1: Find:

i. $64^{\frac{1}{2}}$

iii.
$$125^{\frac{1}{3}}$$

Answer: (i) $64 = 8 \times 8 = 8^2$ therefore, $(64)^{1/2} = (82)^{1/2} = 8^{2 \times 1/2} = 8 [(a^m)^n = a^{m \times n}]$

ii. $32^{\frac{1}{5}}$

(ii) $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$ therefore, $(32)^{1/5} = (2^5)^{1/5} = 2^5 \times 1/5 = 2 [(a^m)^n = a^m \times n]$

(iii) $125 = 5 \times 5 \times 5 = 5^3$ therefore, $(125)^{1/3} = (5^3)^{1/3} = 5^3 \times {}^{1/3} = 5 [(a^m)^n = a^{m \times n}]$

 Question 2: Find:

 i. $9^{\frac{3}{2}}$ ii. $32^{\frac{2}{5}}$ iii. $16^{\frac{3}{4}}$ iv. $125^{\frac{-1}{3}}$

 Answer: (i) $9 = 3 \times 3 = 3^2$

therefore, $(9)^{3/2} = (3^2)^{3/2} = 3^2 \times 3^{1/2} = 3^3 = 27$ [(a^m)ⁿ = a^{mn}]

(ii) $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$ therefore, $(32)^{2/5} = (2^5)^{2/5} = 2^5 \times 2^{2/5} = 2^2 = 4$ [(a^m)ⁿ = a^{mn}]

(iii) $16 = 2 \times 2 \times 2 \times 2 = 2^4$ therefore, $(16)^{3/4} = (2^4)^{3/4} = 2^4 \times 3/4 = 2^3 = 8$ [(a^m)ⁿ = a^{mn}]

(iv) $125 = 5 \times 5 \times 5 = 53$ therefore, $(125)^{-1/3} = (5^3)^{-1/3} = 5^{3 \times (-1/3)} = 5^{-1} = \frac{1}{5} [a^{-n} \frac{1}{a^n}]$

Question 3: Simplify:

i. $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$ ii. $(\frac{1}{3^3})^7$ iii. $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ iv. $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

Answer: (i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = 2^{\frac{2}{3} + \frac{1}{5}} = 2^{\frac{13}{15}}$ $[a^{m} \cdot a^{n} = a^{(m+n)}]$

$$\begin{aligned} &\text{ii}\right) \left(\frac{1}{3^3}\right)^7 = (3^{-3})^7 = 3^{-3\times7} = 3^{-21} = \frac{1}{3^{21}} & \left[\frac{1}{a^n} = a^{-n}\right] \\ &\text{iii}\right) \frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}} \div 11^{\frac{1}{4}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{1}{4}} & \left[a^m \div a^n = a^{m-n}\right] \\ &\text{iv}\right) \frac{7^{\frac{1}{2}}}{7^{\frac{1}{2}}} \cdot 8^{\frac{1}{2}} = (7\times8)^{\frac{1}{2}} = (56)^{\frac{1}{2}} & \left[a^m \times b^m = (ab)^m\right] \end{aligned}$$